Nuclear reactions in gaseous stars: perspectives from kinetic theory and thermodynamics[1]

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Abstract: In the Standard Model of gaseous stars, temperature is primary both in the initiation of thermonuclear reactions to form heavier elements and the emission of radiation. These processes have been described using thermodynamic expressions. Yet, within any given thermodynamic relation, not only must units balance on each side, but so must thermodynamic character. Temperature, whether or not equilibrium conditions are established, must always be intensive in macroscopic thermodynamics and mass must be extensive. This ensures that the laws of thermodynamics are respected. The theory of temperatures and nuclear reactions within gaseous stars is constructed from the kinetic theory of an ideal gas, by which temperature is introduced, in combination with gravitational and Coulomb forces. The resulting thermodynamic relations impart a non-intensive character to temperature and a non-extensive character to mass. Consequently, the theory of nuclear reactions in gaseous stars is invalid. Deprived of the only theoretical means by which the Standard Model justifies stellar nuclear reactions, the theory of gaseous stars is not viable. The most reasonable alternative rests in lattice confinement fusion and the recognition that the stars are comprised of condensed matter, namely metallic hydrogen.

Key words: ideal gas; thermodynamics; stellar nuclear reactions; stellar temperature; liquid metallic hydrogen; lattice confinement fusion; intensive and extensive thermodynamic coordinates

I. INTRODUCTION

The balance of thermodynamic character in all thermodynamic equations or inequalities is essential [1]. Landsberg argued that this requirement should be regarded as the Fourth Law of Thermodynamics [2]. Similarly, Canagaratna [3] noted that "if one side of an equation is extensive (or intensive), then so must be the other side." Landsberg stated the 'Fourth Law' in this way: for a class of non-equilibrium states, and for equilibrium states, extensive and intensive properties exist. Furthermore, the laws of thermodynamics hold wherever local thermodynamic equilibrium is utilized [1, 4].

The kinetic theory of an ideal gas is defined in the presence of a container [5-13] without which pressure, temperature, volume, and density of the gas cannot be specified in accordance with the ideal gas law, $PV = nRT$, which is thermodynamically balanced. Moreover, there are no forces between particles in an ideal gas except when they collide elastically with one another or with the walls of their container [6-10]. The pressure of the gas is due to particle collisions with the walls of its container [6-15]. The introduction of gravitational and Coulomb forces between particles in the ideal gas to facilitate thermonuclear reactions within gaseous stars stands in stark violation of the kinetic theory itself. With the improper introduction of stellar temperature via the ideal gas law, the resulting thermodynamic relations produce non-intensive temperature and non-extensive mass, contrary to the laws of thermodynamics. Consequently, the theory of thermonuclear reactions within gaseous stars is invalid.

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II. STELLAR TEMPERATURES

The Standard Model maintains that the stars are hot balls of gaseous plasma. For two nuclei to fuse, the Coulomb potential energy barrier between the positively charged nuclei must be overcome. Gas temperature provides the mechanism through which the Standard Model initiates and sustains thermonuclear reactions. In the classical analysis, it is assumed that the thermal energy of the ideal gas provides the energy to overcome the Coulomb barrier. To apply the kinetic theory of the ideal gas, the reduced mass, μ_m, of the two nuclei is employed, along with the mean square of the relative speeds, $\overline{v^2}$, between them are utilized [14, 15]. In order to determine the temperature necessary for the gaseous plasma to sustain intra-stellar nuclear reactions, the ideal gas kinetic energy of the reduced mass is equated to the Coulomb potential energy [14, 15],

$$\frac{1}{2}\mu_m\overline{v^2} = \frac{3}{2}kT_{cl} = \frac{Z_1Z_2e^2}{r} \quad , \tag{1}$$

where Z_1 and Z_2 denote the number of protons in each nucleus respectively, r the distance separating the nuclei and e is the electron charge. The temperature of the gaseous plasma required to overcome the Coulomb barrier is then,

$$T_{cl} = \frac{2Z_1Z_2e^2}{3kr} . \tag{2}$$

Note that by Eq. (1) this temperature is derived from supposed Coulomb potential energy between the particles of an ideal gas. Using two protons and $r \sim 1$ fm, results in a temperature, $T_{cl} \sim 10^{10}$ K. The astronomers conclude that the temperature of the Sun's gaseous interior, putatively at 1.58×10^7 K, is too low to overcome the Coulomb barrier, "to explain the solar luminosity" [14]. However, Eq. (2) is incorrect because temperature on the left side is intensive, a homogeneous function of degree zero, while the right side is not intensive. In fact, the right side is comprised of pure numbers and physical constants, which have no thermodynamic character, and of radius, which is a homogenous function of degree ⅓. This also reveals that the ideal gas cannot be endowed with Coulomb forces and associated potential energies without violating the kinetic theory and the laws of thermodynamics. In treating the charged nuclei as particles of the ideal gas, kinetic theory is violated because, as with the case of combining gravity with the ideal gas [1], there are no forces between the particles of the ideal gas except when they collide elastically with one another and with the walls of their container. Since there are no gravitational or Coulomb forces there are no related potential energies either between the particles of the ideal gas. To suppose that there are such forces between the particles is a violation of the kinetic theory of the ideal gas. Furthermore, potential energy cannot manifest a temperature yet Eq. (2) assigns temperature to a potential energy, which varies merely with the separation of two given charged particles, and for a given separation varies merely with the charges.

In an attempt to overcome the insufficient temperature for the solar interior to produce nuclear reactions, Eq. (2) is reformulated in quantum mechanical terms for a proton to tunnel through the Coulomb barrier. By this means, the temperature is lowered to match the inferred temperature of the Sun's gaseous interior such that thermonuclear reactions can be justified. The quantum mechanical relation between wavelength and momentum is $p = h/\lambda$. In terms of momentum, Eq. (1) becomes,

$$\frac{1}{2}\mu_m \overline{v^2} = \frac{3}{2}kT_{cl} = \frac{p^2}{2\mu_m} = \frac{Z_1 Z_2 e^2}{r}. \tag{3}$$

Substituting $p = h/\lambda$ and setting $r = \lambda$,

$$\frac{h^2}{2\lambda^2 \mu_m} = \frac{Z_1 Z_2 e^2}{\lambda}, \tag{4}$$

hence,

$$\lambda = \frac{h^2}{2\mu_m Z_1 Z_2 e^2}. \tag{5}$$

Since $r = \lambda$, substituting Eq. (5) into Eq. (2) gives the quantum mechanical temperature,

$$T_{qtm} = \frac{4\mu_m Z_1^2 Z_2^2 e^4}{3kh^2}. \tag{6}$$

Equation (6) is not thermodynamically balanced because the right side is composed of only pure numbers and physical constants, none of which have any thermodynamic character, whereas the left side has intensive thermodynamic character. Consequently Eq. (6) is certainly invalid. Yet setting $Z_1 = Z_2 = 1$ and $\mu_m = m_p/2$ for two protons, $T_{qtm} \sim 10^7$ K is obtained by the astronomers to substantiate their conclusion that nuclear reactions can occur inside a gaseous Sun at the temperature claimed therein: "In this case, if we assume the effects of quantum mechanics, the temperature required for nuclear reactions is consistent with the estimated central temperature of the Sun" [14].Such a conclusion is without scientific merit owing to the inadmissibility of Eqs. (1) to (6).

Treating the stars as gases and combining the ideal gas law with gravity, Eddington [16] advanced for the mean temperature T_m of polytropic gaseous stars, the equation,

$$T_m = \frac{\beta\mu GM}{(5-n)\Re R}, \tag{7}$$

where $0 \le n < 5$ and $\Re$ is the gas constant. According to Eddington Eq. (7) applies where "the material is a perfect gas and $\beta\mu$ is constant" and that "there is a minimum value of the mean temperature given by the form $n = 0$" [16].Chandrasekhar, emulating Eddington, writes Eq. (7) for the "mean temperature" of an ideal gas star as [17],

$$\overline{T} = \frac{\beta\mu HGM}{2kR}, \tag{8}$$

where H is the mass of the proton, k is Boltzmann's constant and β is a constant.[2] In similar fashion both authors adduce formulae for the central temperature of stars. Eddington says that "the minimum value of the central temperature in a star of mass M and radius R composed of a perfect gas of constant molecular weight μ, subject only to the condition that density and temperature do not decrease inwards" [16] is given by,

[2] Eddington and Chandrasekhar call the ideal gas a perfect gas.

$$T_1 = 0.32 \frac{\mu G M}{\Re R}, \tag{9}$$

and that “for the polytrope $n = 3$” [16],

$$T_1 = 0.856 \frac{\mu G M}{\Re R}. \tag{10}$$

Compare Eq. (10) with Chandrasekhar’s for the central temperature [17],

$$T_c = 0.8543 \frac{\beta \mu H G M}{k R}. \tag{11}$$

In any event it is evident that equations (7) through to (11) are also inadmissible because the right side in each is not intensive, contrary to the laws of thermodynamics. In these equations mass M is a homogenous function of degree 1, whereas radius R is a homogeneous function of degree ⅓. All other terms on the right are either pure numbers or physical constants, which have no thermodynamic character. So on the left one has an intensive property, temperature, a homogenous function of degree 0, and on the right, one ends up with a homogenous function of degree ⅔. Note that these equations also assign temperature to gravitational potential energy even though potential energy cannot in reality manifest a temperature. In deriving equations (7) through to (11) Eddington and Chandrasekhar invalidly combined the ideal gas with gravity between its particles. In doing so they also produced violations of the laws of thermodynamics. Astronomy and astrophysics to this day continue to combine the ideal gas with gravity and Coulomb forces in violation of the kinetic theory and the laws of thermodynamics.

III. THERMONUCLEAR REACTIONS INSIDE GASEOUS STARS

Based on invalid Eq. (6) nuclear reaction rates are proposed. The cross-section σ of a nucleus is argued to be a function of the kinetic energy $K = ½\, \mu_m \overline{v^2}$ of the nuclei treated as ideal gas: $\sigma(K)$. The proposed relation first has the cross section inversely proportional to K [14, 15],

$$\sigma(K) \propto 1/K\ . \tag{12}$$

The cross-section is also proportional to the tunnelling probability,

$$\sigma(K) \propto \exp\left(-2\pi^2 U_c / K\right). \tag{13}$$

Here U_c is the Coulomb potential energy barrier,

$$U_c = \frac{Z_1 Z_2 e^2}{r}. \tag{14}$$

Setting $r \sim \lambda = h/p$ with $p = \mu_m v$, $\left(v = \sqrt{\overline{v^2}}\right)$,

$$\frac{U_c}{K} = \frac{2 Z_1 Z_2 e^2}{h v}. \tag{15}$$

Now

$$v = \sqrt{2K/\mu_m}\ , \tag{16}$$

so Eq. (15) becomes,

$$\frac{U_c}{K} = \frac{2Z_1Z_2e^2}{h\sqrt{2K/\mu_m}}\ . \tag{17}$$

Substituting Eq. (17) into Eq. (13) gives,

$$\begin{aligned} &\sigma(K) \propto \exp\left(-\beta/\sqrt{K}\right) \\ &\beta = \frac{2^{3/2}\pi^2\mu_m^{1/2}Z_1Z_2e^2}{h} \end{aligned}\ . \tag{18}$$

Since β is composed only of pure numbers and physical constants, it has no thermodynamic character, and the entire exponential has no units thereby having no thermodynamic character. Combining expressions (12) and (13), and supposing $S(K)$ to be some "slowly varying function of energy" [14, 15], the cross section becomes,

$$\sigma(K) = \frac{S(K)}{K}\exp\left(-\beta/\sqrt{K}\right). \tag{19}$$

Now Eq. (19) is invalid because Eqs. (15) and (17) are false due to combining the kinetic energy of the ideal gas with Coulomb potential energy. Although charged particles experience a Coulomb force and give rise to a Coulomb potential energy, no such forces or potentials exist between the particles of the ideal gas, which collide only elastically. Similarly, the 'tunnelling probability' itself is invalid because the exponent is produced by combining the ideal gas law with Coulomb forces between the ideal gas particles.

Utilising the invalid Eq. (19) a consequently invalid nuclear relation rate integral is advanced by the astronomers [14, 15],

$$r_{ix} = \left(\frac{2}{kT}\right)^{3/2}\frac{n_i n_x}{\sqrt{\mu_m \pi}}\int_0^\infty S(K)\exp\left(-\beta/\sqrt{K}\right)\exp\left(-K/kT\right)dK\ . \tag{20}$$

Here n_i and n_x are the number densities of the beam particles and the target particles respectively. The $\exp\left(-K/kT\right)$ term appears in the Maxwell-Boltzmann molecular speed distribution [6-11, 18-20]. The β term "depends upon the composition of the gas" [14], which is treated throughout as ideal in order to invoke the relevant equations for the ideal gas in the first place. Since β contains both Coulomb potential energy terms and ideal gas terms it has no place in physics. Consequently, Eq. (20) has no valid physical or theoretical foundation.

A strongly peaked curve, called the Gamow peak, is generated by the $\exp\left(-\beta/\sqrt{K} - K/kT\right)$ component of Eq. (20). The actual peak of the curve occurs at a kinetic energy of [14, 15],

$$K_o = \left(\frac{\beta kT}{2}\right)^{2/3}. \tag{21}$$

Equation (21) is not the kinetic energy of an ideal gas particle because it results from combining the kinetic energy of the ideal gas with the Coulomb potential energy. Multiplying top and bottom of the right side of Eq. (21) by $3^{2/3}$ gives,

$$K_o = \left(\frac{\beta}{3}\right)^{2/3}\left(\frac{3kT}{2}\right)^{2/3}. \tag{22}$$

The term inside the second set of parentheses is obviously the kinetic energy of an ideal gas particle. Yet Eq. (21) is identified as the kinetic energy of a particle of the ideal gas, in the Gamow peak, in order to overcome the Coulomb barrier. Owing to this, by virtue of its very derivation, Eq. (21) violates the kinetic theory of the ideal gas, so it is invalid.

Where $S(K)$ is not a slowly varying function of kinetic energy across the Gamow peak, local maxima at specific energies are said to occur, mimicking the traits of orbital energy levels of electrons in corresponding to energy levels within the nucleus [14]. These various strong peaks are claimed to be due to resonance between the energy of incident nuclei and differences in energy levels within the nucleus. Since the Gamow peak is itself invalid these conclusions are without any scientific basis.

The "central temperature for an ideal gas in hydrostatic equilibrium" [15] is given by,

$$T \approx \frac{GM\mu m_H}{5kR}. \tag{23}$$

It is immediately evident that Eq. (23) stands in violation of the laws of thermodynamics because, once again, the right side is not intensive. This has also been noted by Robitaille [21]. Furthermore, in similar fashion to Eq. (2), potential energy cannot manifest a temperature yet Eq. (23) assigns temperature to a gravitational potential energy. Nevertheless, astronomers proceed on Eq. (23) and assert that at high density matter is degenerate, claiming that for such matter "the thermal energy of an electron must be less than its Fermi energy" [15], so that,

$$kT < \varepsilon_{F,e} = \frac{p_{F,e}^2}{2m_e} = \frac{\hbar^2\left(3\pi^2 n_e\right)^{3/2}}{2m_e}. \tag{24}$$

Here $p_{F,e}$ is the Fermi momentum of the electron, n_e the electron number density and m_e the electron mass. Expression (24) was obtained from the ideal gas law by improperly replacing neutral mass therein with the electron mass, thereby introducing Coulomb forces between the particles of the gas contrary to the ideal gas law. Owing to total charge neutrality, $n_e = n_p = \rho/m_H$, where n_p is the proton number density. Substituting n_p into Eq. (24) gives,

$$\left(\frac{m_H}{\rho}\right)^{1/3} > \frac{\hbar}{\left(2m_e kT\right)^{1/2}}. \tag{25}$$

The wavelength of an electron is,

$$\lambda_e = \frac{\hbar}{p_e}. \tag{26}$$

Note that Eq. (25) has the dimension of length, being the internuclear separation, which must therefore exceed "the thermal wavelength of the electron $\hbar/p_e \approx \hbar/\sqrt{2m_e kT}$" [14]. The density is,

$$\rho = \frac{3M}{4\pi R^3}. \tag{27}$$

Using Eqs. (23) and (27) to eliminate ρ and R, inequality (25) yields the minimum mass, M_{min}, of a gaseous star, composed of a fuel characterised by mean molecular weight μ in terms of mass fractions, for a thermonuclear ignition temperature T, thus,

$$M_{\min} \approx \frac{1}{\mu^{3/2}}\left[\frac{375\hbar^3 k^{3/2}}{4\pi m_H^4 m_e^{3/2} 2^{3/2} G^3}\right]^{1/2} T^{3/4}. \tag{28}$$

All terms on the right side except T are either pure numbers or physical constants. Hence, the right side is intensive because $T^{3/4}$ is intensive; but the left side is not intensive since mass is extensive. Therefore relation (28) is false.

Putting into expression (28) the values of the physical constants gives,

$$M_{\min} \approx 5.36\times 10^{23}\frac{T^{3/4}}{\mu^{3/2}}. \tag{29}$$

In number of solar masses N this becomes,

$$N = \frac{M_{\min}}{M_*} \approx 2.69\times 10^{-7}\frac{T^{3/4}}{\mu^{3/2}}, \tag{30}$$

where M_* is the Sun's mass. In the case of hydrogen, the astronomers use μ = ½ and $T = 10^7$ in Eq. (30) to obtain [15],

$$N = \frac{2.69\times 10^{-7}\left(10^7\right)^{0.75}}{0.5^{1.5}} = 0.14 \text{ solar masses.}$$

For Population I stars in hydrogen, astronomers take $\mu = 0.67$ and $T = 10^7$ in Eq. (30) and obtain [15],

$$N = \frac{2.69\times 10^{-7}\left(10^7\right)^{0.75}}{0.67^{1.5}} = 0.09 \text{ solar masses,}$$

and so on for helium, carbon, oxygen and silicon as nuclear fuels at various ignition temperatures. All the resulting minimum masses are without scientific credibility.

The penetration factor is, from expressions (18),

$$\exp\left(-\beta/\sqrt{K}\right)$$

$$\beta = \frac{2^{3/2}\pi^2\mu_m^{1/2}Z_1Z_2e^2}{h} \quad . \tag{31}$$

According to the astronomers, nuclear reactions mostly occur on the lower energy tail of the penetration factor "because the Maxwellian velocity distribution gives relatively few particles with high energies" [15]. Approximate temperatures at which nuclear reactions occur are then asserted to be given by [15],

$$T \approx \xi\frac{\mu_m c^2 Z_1^2 Z_2^2}{k}, \tag{32}$$

where the numerical coefficient ξ is of the order of 10^{-6}. From Eq. (32) a temperature for proton-proton reactions is obtained; T ~ 10^7 K [15]. However, it is again clear that Eq. (32) is inadmissible because the left side is intensive while the right side is not intensive, as it has no thermodynamic character at all being made up solely of constants. Consequently, determination of temperatures for nuclear reactions by species other than the pp chain from Eq. (32) is also invalid.

IV. THE CHAIN REACTIONS

Its invalidity notwithstanding, Eq. (20) serves astronomy as the basis for determination of energy generation rates for a variety of nuclear reactions such as PP I, PP II, PP III, the supposed CNO cycle and the triple alpha process [14, 15]. The resulting mathematical relations are utilized to propose energy production dependence on temperature and density. For instance, from the energy generation rates obtained it is concluded that the CNO cycle is significantly more dependent on temperature than the pp chain. From this it is said that low-mass stars are dominated by the pp chains owing to their lower temperatures and more massive stars are dominated by the CNO cycle owing to their higher temperatures. Astronomers regard the Sun and lower-main-sequence stars as being dominated by the pp chains and stars on the upper-main sequence as being dominated by the CNO cycle [14, 15]. These conclusions have no scientific merit owing to the inadmissibility of Eq. (20) and associated stellar temperature relations.

As hydrogen is converted into helium by the pp chain or the CNO cycle, the mean molecular weight μ of the stellar gas increases. According to the astronomers, if the temperature and density do not change, the star falls out of hydrostatic equilibrium since, according to the ideal gas law, the central pressure decreases, so the star must gravitationally collapse to restore equilibrium. This purported gravitational collapse supposedly raises the temperature and density of the star. By means of Eq. (6) astronomers claim [14] that helium burning begins when the temperature is ~ 64 times that which they say is necessary to burn hydrogen.[3] With continuing increase of helium and hence temperature increase, the triple alpha process commences to convert helium into carbon, and so on to carbon burning and oxygen burning. Since Eq. (6) is false, all the arguments of the astronomers are unsound. Stars do not gravitationally collapse.

V. CONCLUSION

The expressions for temperature and mass advanced by the astronomers violate the laws of thermodynamics as they render temperature non-intensive. They also assign temperatures to potential energies despite potential energy having no relation to temperature. Consequently, the

[3]Although 'burn' is used by the astronomers, nothing is burning here – the process is thermonuclear, not combustion.

theory of nuclear reactions in gaseous stars is incorrect. It also renders the theory of stellar evolution invalid by proxy so that the Standard Model account of stellar evolution on the HR diagram cannot be correct. This is clear when bearing in mind that the Standard Model invokes supposed changes in nuclear reactions within purported gaseous stars as one nuclear fuel is replaced by another with the aging of stars. Furthermore, the Standard Model theory of production of all the elements from primordial matter and first-generation stars is also erroneous owing to violations of thermodynamics and kinetic theory in advancing a gaseous constitution of the stars. This subverts the primordial matter contention and thereby the foundations of cosmology itself. The stars produce all the elements. Consequently, primordial matter and related first-generation stars are without scientific validity. Without primordial matter cosmology is otiose.

Deprived of any theoretical means to produce thermonuclear reactions in stellar interiors, the Standard Model of the stars has no valid scientific basis. Astronomy and astrophysics must concede that the stars are not gaseous.

The demise of the gaseous model brings the condensed matter model of the stars forefront [22]. Condensed matter stars are essentially incompressible, as they are built from degenerate one-component plasmas. Consequently, stars cannot 'gravitationally collapse' to form ultra-dense compact objects [23]. White dwarfs and what the Standard Model calls neutron stars have been incorrectly classified since their internal constitutions have been theorised using the invalid gaseous stellar model. Similarly, stars cannot self-compress by 'gravitational collapse' to form black holes [23, 14]. Black holes in their turn are alleged to also have a class in primordial matter, the so-called primordial black holes [25-27].

The only material that offers explanation of stellar observations is liquid metallic hydrogen [22, 28]. Liquid metallic hydrogen possesses a hexagonal planar structure similar to graphite. Within metallic hydrogen protons are fixed at lattice points about which they vibrate linearly; facilitating quantum mechanical tunnelling and lattice confinement fusion [22, 28, 29]. Lattice confinement fusion is the only means by which the stars can sustain thermonuclear reactions, and such processes have been confirmed in the laboratory [22, 28-33].

REFERENCES

[1] P.-M. Robitaille and S. J. Crothers, Phys. Essays, **32**, 158 (2019).

[2] P. T. Landsberg, *Monographs in Statistical Physics and Thermodynamics, Volume 2: Thermodynamics, With Quantum Statistical Illustrations*, (Interscience Publishers, Inc., New York, 1961).

[3] S. G. Canagaratna, J. Chem. Educ., **69**, No.12, 957, (1992).

[4] P. T. Landsberg, Nature, **238**, 229, (1972).

[5] T. L. Brown, , H. Eugene LeMay, B. E. Bursten, S. Langford, D. Sagatys, and N. Duffy, *Chemistry: the central science: a broad perspective* (Pearson Education Australia, Sydney, NSW, 2007).

[6] J. H. Jeans, *The Dynamical Theory of Gases*, (Cambridge University Press, London, 1904).

[7] J. H. Jeans, *The Kinetic Theory of Gases*, (Cambridge University Press, London, 1940).

[8] L. Boltzmann, *Lectures on Gas Theory*, (Dover Publications, Inc., New York, 1995).

[9] F. W. Sears, *An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics*, (Addison-Wesley Publishing Company, Inc., Cambridge, MA, 1955).

[10] M. W. Zemansky and R. H. Dittman, *Heat and Thermodynamics*, (6th ed, McGraw-Hill Book Company, 1981).

[11] S. H. Raden and R. T. Folk, *Physics for Scientists and Engineers*, (Prentice-Hall, Inc., Englewood Cliffs, NJ, 1982).

[12] R. Smith, *Conquering Chemistry*, (McGraw-Hill Book Company Pty Limited, Sydney, NSW, 1987).

[13] L. Pauling, *General Chemistry*, (W. H. Freeman and Company, San Francisco, CA, 1953).

[14] B. W. Carroll and D. A. Ostlie, *An Introduction to Modern Astrophysics*, (1st ed, Addison-Wesley Publishing Company, Inc., 1996).

[15] R. Bowers and T. Deeming, *Astrophysics I: Stars*, (Jones and Bartlett Publishers, Inc., Boston, MA, 1984).

[16] A.S. Eddington, *The Internal Constitution of the Stars*, (Dover Publications Inc., New York, 1959).

[17] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure*, (Dover Publications, Inc., New York, 1957).

[18] R. H. Fowler, *Statistical Mechanics* (Cambridge University Press, London, 1929).

[19] L. D. Landau and E. M. Lifshitz, Course *of Theoretical Physics, Volume 5, Statistical Physics Part 1* (3rd ed, Butterworth Heineman, Oxford, 1980).

[20] G. Joos, *Theoretical Physics*, (3rd ed, Dover Publications, Inc., New York, 1986).

[21] P.-M. Robitaille, APS April Meeting 2018, **63**, 4, Columbus, Ohio, X13.00005.

[22] P.-M. Robitaille, Progress in Physics, **4**, 90, (2013).

[23] P.-M. Robitaille, see https://www.youtube.com/watch?v¼DtMee3rrHDY for “Sky Scholar” (2018).

[24] S. J. Crothers and P.-M. Robitaille, Phys. Essays, **33**, 2, 143 (2020). DOI: http://dx.doi.org/10.4006/0836-1398-33.2.143

[25] S. W. Hawking, *A Brief History of Time from the Big Bang to Black Holes,* (Bantam Press, London, 1988).

[26] R. M. Wald, *General Relativity*, (The University of Chicago Press, Chicago, IL,1984).

[27] R. d’Inverno, *Introducing Einstein’s Relativity*, (Clarendon Press, Oxford, 1992).

[28] P.-M. Robitaille, Progress in Physics, **3**, 60, (2011).

[29] P.-M. Robitaille, see https://www.youtube.com/watch?v¼080YgC45EJ8 for “Sky Scholar” (2021).

[30] V. B. Belyaev, M. B. Miller, J. Otto, and S. A. Rakityansky, Phys. Rev. C, **93**, 034622, (2016).

[31] V. Pines, M. Pines, A. Chait, B. Steinetz, R. C. Hendricks, G. C. Fralick, T. Benyo, B. Baramsai, P. B. Ugorowski, M. D. Becks, R. E. Martin, N. Penney, and C. E. Sandifer, Phys. Rev. C, **101**, 044609 (2020).

[32] B.M. Steinetz, T. Benyo, A. Chait, and R. C. Hendricks, Phys. Rev. C, **101**, 044610, (2020).

[33] NASA, https://www1.grc.nasa.gov/space/science/lattice-confinement-fusion/