On superposition and entanglement of polarized photons without hidden variables

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Abstract

Polarized photons have properties that can be explained, among other things, by their indistinguishability. These include superposition and entanglement. The locality of entangled photons can thus be explained without hidden parameters. We propose that the superposition state can be understood as the entirety of all mixtures of indistinguishable perpendicularly polarized photon beams. Superposition of indistinguishable photon beams can be demonstrated experimentally using a Mach-Zehnder interferometer. This explains how the polarization of the input state reappears at the output of a Mach-Zehnder interferometer.

Keywords: Entanglement; quantum nonlocality superposition; EPR

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1. Introduction

In order to explain entanglement, it is crucial that the states of an entangled system are defined before a measurement and are not random. Otherwise, there are no correlations between the measurement results on both sides of an entangled system. Various approaches, including Bell's, assume that a hidden parameter must be introduced for a realistic explanation of entanglement, which is assigned to both partner particles at the source of an entangled photon pair [1], [2] . This was also performed in [3- 4] and allowed the development of models that correctly explained quantum correlations. That is sufficient to disprove Bell's theorem, which states that a realistic local model for the prediction of quantum correlations is impossible. Although there are a number of approaches that consider nonlocality to be a physical phenomenon of entangled particles, none has so far been completely convincing [5]. There are also doubts that quantum correlations necessarily result in nonlocal behavior [6].

In this paper, a new approach is pursued that does not require hidden variables and still predicts defined states before a measurement. This is achieved by the model describing the entirety of all possible states, from which the measured state is extracted by selection before the measurement. This model is only valid for particles whose quantum states are indistinguishable and which can therefore assume common states. The model is described with four model assumptions (in italics). Both entangled photons and non-entangled photons in superposition were considered. This model also explains the measurement results obtained using a Mach-Zehnder interferometer.

2. Method - Model description

Model assumption MA1 *(describing superposition)*
A photon beam with polarization α can be regarded as a mixture of two indistinguishable photon beams, one with polarization β and the share $\cos^2(\beta-\alpha)$ and the other with polarization β-$\pi/2$ and the share $\sin^2(\beta-\alpha)$ for arbitrary α and β. A polarizer set to β/β-$\pi/2$ selects the photon beams with polarization β and β-$\pi/2$ from the original beam. All pairs of beams are equivalent.

MA1 reproduces Malu's law. From the quantum perspective, a system with the polarization α can be regarded as a superposition of two systems with polarizations β and β-$\pi/2$. MA1 means that the projective measurement selects states that already exist. All individual photons that pass through a polarizer have a polarization before the measurement. However, due to the indistinguishability of the photons in a superposition, it is not possible to predict which photon will go into which exit of a polarizer.

Figure 1 shows geometrically how a photon beam of polarization α can be seen as a mixture of two indistinguishable photon beams with polarization β and β-π/2 for arbitrary α and β. Mixtures of indistinguishable states in R3 are equivalent to superposition in Hilbert space. This is because the proportions of the different components are the same in both the representations. Born's rule is already included in the model, whereas it must be added to the quantum state in the Hilbert space to predict the probabilities of the measurement results.

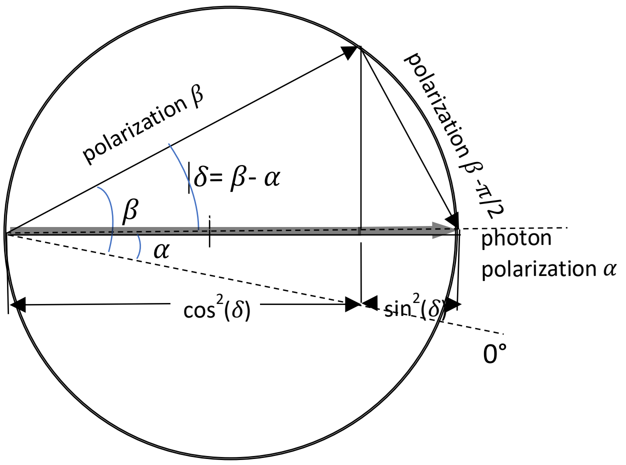

Figure 1: Geometric illustration of how a photon beam of polarization α can be represented as a mixture of photon beams of polarization β with the share $\cos^2(\beta-\alpha)$ and the other with polarization β-π/2 and the share $\sin^2(\beta-\alpha)$ for arbitrary α and β. The arrows in the figure show the vector addition of the superimposed states in Hilbert space which also holds for arbitrary α and β..

Complementary to MA1 we define for arbitrary α and β

Model assumption MA2: *(describing the absolute value of the common polarization)*
Indistinguishable photon beams with fractions $\cos^2(\beta-\alpha)$ of polarization β and $\sin^2(\beta-\alpha)$ of polarization β -π/2 assume the common polarization α or -α.

The sign of the common polarization on both sides is given for Bell states by model assumption MA3.

Model assumption MA3: *(controlling the sign of the common polarization)*
Each Bell state is a mixture of indistinguishable constituent photon pairs in equal shares whose components have the same polarization 0° or 90° for Φ+ and Φ- and an offset of π/2 for Ψ+ and Ψ-. The constituent photon pairs make up the initial state. From the conservation of the spin angular momentum we obtain for Ψ- and Φ+ the same sign of the polarization of the beams, and for Ψ+ and Φ- the opposite sign in the original coordinate system.

This has been described in detail in [4]. It has been shown that rotational invariance and conservation of spin angular momentum are equivalent and denote the same physical situation. From these results entanglement swapping and teleportation have been derived.

3. Results – Calculating probabilities of matching events with entangled photons

Figure 2 shows the coordinate systems and nomenclature of experiments with polarization-entangled photons [4].

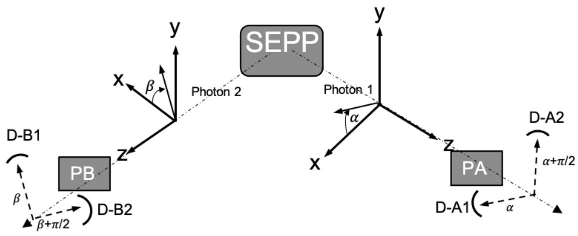

Figure 2: The SEPP (source of entangled photon pairs) emits entangled photons propagating towards the adjustable polarizers PA and PB and detectors DA-1 and DA-2 on wing A and DB-1 and DB-2 on wing B. A coincidence measuring device (not seen in the picture) encounters matching events. The polarization angles are defined in the x–y-plane, which is perpendicular to the propagation direction of the photons. The coordinate systems are left-handed with the z-axis in propagation direction for each wing, with the x-axis in horizontal and the y-axis in vertical direction.

Entangled photons on either side can be understood as a mixture of indistinguishable horizontally polarized and vertically polarized photons in equal parts. We set PA to the value α. From MA1 we obtain that the photon beam with horizontal polarization can be understood as a mixture of indistinguishable beams of polarization α and polarization α–π/2. The fraction of horizontally polarized photons which hit the polarizer set to α is $\cos^2(\alpha-0)$. Vertically polarized photons can be understood as a mixture of indistinguishable beams of polarization α and polarization α–π/2. The fraction of vertically polarized photons which hit the polarizer set to α is $\cos^2(\alpha-\pi/2)= \sin^2(\alpha)$.

For the singlet state, we obtain the corresponding beam of the partner photons on side B from the initial conditions. The fraction of horizontally polarized photons on side B matches that of vertically polarized photons on side A, which is $\sin^2(\alpha)$. The fraction of vertically polarized photons on side B is

$\cos^2(\alpha)$, matching the fraction of horizontally polarized photons on side A. From MA2 and MA3, we obtain that the common polarization of the partner photons on side B is $\alpha+\pi/2$.

Now we set PB to β. From MA1, we obtain that the fraction of photons with polarization β contributing to the photon flux with polarization $\alpha+\pi/2$ is $\cos^2(\beta-\alpha-\pi/2) = \sin^2(\alpha-\beta)$. This is the probability of matching events at polarizer PA and polarizer PB. The expectation value for a joint measurement with photon 1 detected behind detector PA at α and partner photon 2 detected behind detector PB at β is as obtained from ([3], equation (13))

$$E(\alpha,\beta) = -\cos(2(\alpha-\beta)) \tag{1}$$

This matches the predictions of quantum mechanics.

4. Results – Explaining the Mach-Zehnder-Interferometer

The model also explains the Mach-Zehnder interferometer (MZI) with polarizing beam splitters (PBS) without interference, as shown in Figure 3. In a Mach–Zehnder interferometer, a photon beam with polarization α is split into a beam of horizontally polarized photons with a fraction of $\cos^2(\alpha)$ and a beam of vertically polarized photons with a fraction of $\sin^2(\alpha)$ [7]. At the output, the two separate photon beams are recombined in the PBS, and the original polarization from the input is restored.

Model assumption MA4: *(controlling the sign of the output of a MZI without interference)*
The vertically polarized photons carry the sign of α. When generating the common polarization α, the sign of α is retained. This is achieved by maintaining the sign of the phase difference between left and right polarized components of a linearly polarized photon beam.

The relationship between the state of linearly polarized photons and the state of circularly polarized photons is

$$\cos(\alpha)*|H\rangle + \sin(\alpha)*|V\rangle = (\exp(-i*\alpha)*|R\rangle + \exp(i*\alpha)*|L\rangle)/\sqrt{2} \text{ where} \tag{2}$$

$$|H\rangle = 1/\sqrt{2} *(|R\rangle + |L\rangle) \text{ and} \tag{3}$$

$$+|V\rangle = -i/\sqrt{2} *(|R\rangle - |L\rangle) = (\exp(-i*\pi/2)*|R\rangle + \exp(i*\pi/2)*|L\rangle)/\sqrt{2} \quad \text{and} \tag{4}$$

$$-|V\rangle = -i/\sqrt{2} *(|L\rangle - |R\rangle) = (\exp(-i*\pi/2)*|L\rangle + \exp(i*\pi/2)*|R\rangle)/\sqrt{2}. \tag{5}$$

From equation (2), we obtain the phase difference between the left and right polarized components as 2α. Thus, α and the phase difference exhibited the same sign. This sign is retained by the vertically polarized photon eqs. (4) and (5), respectively. For $\alpha > 0$ eq. (4) applies and for $\alpha < 0$ eq. (5) applies.

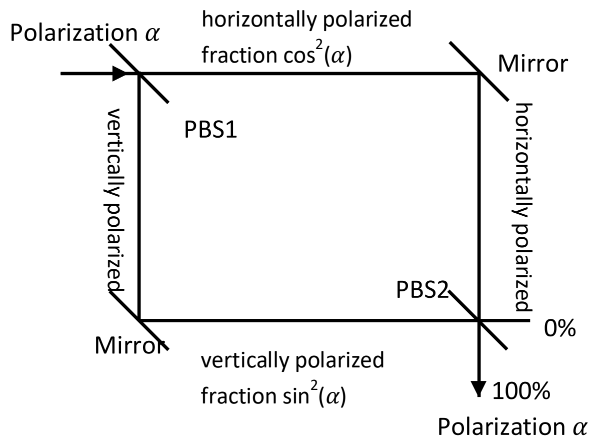

Figure 3: Beam paths at a Mach-Zehnder Interferometer (MZI) with polarizing beam splitters without interference

The input beam on the first PBS (PBS1) may have a polarization direction α. According to MA1, this beam of photons with polarization α is a mixture of indistinguishable beams of photons of horizontal polarization with fraction $\cos^2(\alpha)$ and vertical polarization with fraction $\sin^2(\alpha)$. Owing to MA4, the sign of α is retained by the vertically polarized photons. Horizontally polarized photons are transmitted at PBS1, whereas vertically polarized photons are reflected by PBS1. The mirrors do not change the polarization. The input of the second PBS (PBS2) are horizontally polarized photons with fraction $\cos^2(\alpha)$ and vertically polarized photons with fraction $\sin^2(\alpha)$. Horizontally polarized photons are transmitted, whereas vertically polarized photons are reflected by PBS2. Therefore, these photons both reach the same output of PBS2 and are indistinguishable. Thus, they have a uniform polarization direction, α according to MA2 and MA4.

5. Discussion and Conclusions

It could be shown that superposition can be understood as the entirety of all mixtures of indistinguishable perpendicularly polarized photon beams with arbitrary polarization. The photon beams, whose polarization correspond to the position of the polarizer, are filtered out by selection with a polarizer.

Each photon of a selection has a specific polarization state before the measurement, namely the one that corresponds to the polarizer position. This reflects Malus' law. From the quantum perspective, the projective measurement selects states that already exist.

Superposition as a mixture of indistinguishable photon beams can be demonstrated experimentally using a Mach-Zehnder interferometer. That can explain how the polarization of the input state reappears at the output in a Mach-Zehnder interferometer. It also uses the fact that a mixture of indistinguishable, perpendicularly polarized photon beams assume the common polarization, which results from the mixing ratio. The quantum world differs from the classical world in particular in that there are indistinguishable particles in the quantum world, which then assume common properties, which is not possible in the classical world.

With the polarizer setting defined there is no simultaneous existence of incompatible polarization states; therefore, there is no collapse of the wave function and the many-worlds theory is obsolete. The behavior of photons is predetermined, but not predictable. This is due to the indistinguishability of the photons.

From the model point of view there are also no nonlocal effects with the entangled photons. This is because the physical states of the photons are defined before each measurement. The connection between the two branches of a Bell experiment arises from the conservation of spin angular momentum and is mediated by the mixing ratio of horizontally and vertically polarized photons on each side. This does not require additional hidden variables.

Einstein, Podolsky, and Rosen (EPR) [8] first raised the question of whether the state function in quantum physics is complete. From the perspective of the model (MA1), it can be concluded that a photon beam with a certain polarization can be understood as a mixture of orthogonally polarized, indistinguishable photon beams. All the pairs of orthogonally polarized photons are equivalent. This means that the state function of a photon beam with a certain polarization describes all possible mixtures of orthogonally polarized photon beams (MA1). The measurement selects one of those by setting a polarizer. This fact is not modelled in quantum theory. In this respect, the model is a supplement to quantum theory as EPR envisioned it. No hidden variables are required for this. The model does not describe individual systems, but rather ensembles of photon beams. This view corresponds with Einstein's opinion [9].

The presented model does not replace QM, but confirms it and provides statements about the predetermination and locality of quantum states that QM does not provide. The model is based exclusively on physical principles and does not require any hidden variables. The paper presented is thus a conclusively substantiated contribution to the interpretation of quantum mechanics.

Ethical compliance
No human participants are involved
Data Access statement
No Data were produced

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