Cellular Generation Number as an Invariant Measure of Biological Time in Relativistic Inertial Systems: Biological Systems Do Not Experience Time Dilation

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A B S T R A C T

This work presents a thought experiment where the number of cellular duplications or generations (G) is used as a biological clock to investigate the effects of relativistic environments on biological time. We demonstrate that, although physical clocks in different reference systems measure varying times due to relativistic time dilation, biological time remains invariant and corresponds to the "proper" time. This invariance holds not only across inertial reference frames but also extends to non-inertial, accelerated, and gravitational systems. The invariance arises because G is defined as the ratio of the growth time to the duplication time, ensuring that any relativistic effects influencing these intervals mathematically cancel out.

These findings challenge the classic interpretation of Einstein's twin paradox, which suggests differential aging due to relativistic velocities. In reality, while physical clocks indicate differing times, biological time, and thus the biological age of living organisms, remains unaffected, aligning consistently with proper time. Although bacterial cultures were used as a model in this study, the results are generalizable to all cellular systems, provided identical growth conditions are maintained. This study provides new insights into the interplay between biological processes and relativistic effects, establishing G as a reliable and invariant measure of biological time across all reference frames.

Keywords

Biological time

Biological clock

Time dilation

Relativistic effects

Proper time

Invariant aging

Twin paradox

Abbreviations and Glossary

c: Speed of light in vacuum, approximately 300,000 km/s.

Equivalence Principle: A principle enunciated by Einstein stating that all effects of a uniform gravitational field are identical to the effects of a uniform acceleration of the coordinate system. This principle generalizes a result of Newtonian gravitation theory, where a uniform acceleration of the coordinate system gives rise to a gravitational field.

g: Acceleration due to gravity at the surface of the Earth, approximately 9.81 m/s².

G: Number of bacterial generations or duplications.

K (Gravitational Constant): The constant of proportionality in Newton's law of universal gravitation, with a value of $6.67430 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$

Proper Time (τ): Time measured by a clock <u>in the same reference frame</u> as the measured event, whether inertial or non-inertial.

Relative time (t): Time measured by a clock <u>in a reference frame different</u> from that of the measured event, whether inertial or non-inertial. It therefore depends on the relative motion between the clock and the event, or on the acceleration or gravitational field to which the clock is subjected.

Relativistic Time Dilation: The phenomenon where time appears to pass more slowly for an object in motion or in a gravitational field, compared to a stationary observer or one far from gravitational fields.

1. Introduction

In the context of special relativity, "proper" time refers to the time interval measured by a clock moving alongside an object, effectively at rest relative to the events being timed. This measurement remains invariant across all inertial reference frames. According to relativity, time appears to slow down for objects moving at relativistic speeds, a phenomenon experimentally confirmed for subatomic particles such as muons. When muons are produced in the upper atmosphere by cosmic ray interactions, they travel toward the Earth's surface at speeds approaching that of light. Due to their short half-life, one would expect that very few of them would reach the Earth's surface. However, observations indicate a significant number of muons reaching sea level, a result explained by the relativistic time dilation affecting muons traveling at high velocities.

While the proper time of the muons remains constant, the time observed in the Earth's rest frame appears dilated due to their high speed. In atmospheric muons, this effect allows them to travel greater distances before they decay (Rossi & Hall, 1941). In a controlled synchrotron environment, time dilation extends their apparent lifetime (Bailey et al., 1977). More recently, relativistic time dilation has also been demonstrated in lithium ion beams (Botermann et al., 2014).

Whereas relativity has had a significant impact on many areas of physics and technology, there remains a notable gap in the scientific literature regarding both experimental and theoretical studies on biological clocks in relation to relativistic effects. Recent studies, such as Maestrini et al. (2018) and Ajdžanović, V.Z. et al. (2023), have examined aging through concepts loosely inspired by relativity, such as notions of time dilation and contraction. However, these models do not involve systems at relativistic speeds or accelerations in the physical sense (m/s^2). Instead, they focus on biological aging rates affected by pathological conditions like cancer and chronic inflammation. In this context, "acceleration" is used metaphorically, referring to biological factors that speed up cellular deterioration. Such processes compress what can be seen as "biological time", leading to faster aging but without any connection to actual relativistic velocities or physical accelerations.

In this study, we use bacterial growth as a "biological clock" to demonstrate that biological time, represented by the number of bacterial generations (G), is invariant across all inertial frames. Thus, G serves as a biological timekeeper, independent of relativistic time dilation. This study proposes a thought experiment with two identical bacterial cultures: one situated on Earth (System T) and the other onboard a spaceship moving at relativistic speed v relative to Earth (System A). All culture conditions, such as temperature, nutrient medium composition, and oxygenation, are kept exactly the same in both systems. This setup allows us to isolate the effects of proper time on biological growth, helping to validate the invariance of G as a measure of biological time across inertial systems.

The twin paradox describes a scenario where one twin remains on Earth while the other travels at relativistic speed and later returns. According to special relativity, each twin observes the other's clock as running slower due to time dilation. This symmetry creates the apparent paradox: if each twin perceives the other as aging more slowly, how can the traveling twin ultimately age less?

The resolution lies in the fact that the traveling twin undergoes phases of acceleration and deceleration during their journey, which break the symmetry of the situation. These phases involve transitions between inertial reference frames, distinguishing the traveling twin from the one who remains on Earth. As a result, the traveling twin experiences less elapsed time over the course of the journey, consistent with the predictions of relativity.

In this study, we intentionally exclude the acceleration and deceleration phases of the traveling twin’s journey to focus exclusively on inertial systems. This approach allows us to analyse the effects of relativistic time dilation on biological processes in a simplified context, isolating its impact within purely inertial frames.

The anticipated result of this thought experiment is that biological time, as measured by the number of bacterial generations (G), remains invariant in both reference frames, despite the relativistic effects observed in physical clocks. Moreover, this invariance is not limited to inertial reference frames; it also extends to non-inertial, both accelerated and gravitational systems. This occurs because biological time is represented by G, which is defined as the ratio of the growth time to the duplication time. As a result, any relativistic effects influencing these two time intervals cancel each other out mathematically, ensuring that *G* remains invariant across all reference frames (see chapter 3.2).

Therefore, in the famous twin paradox (Einstein, 1905), while the times measured by Earth and the spaceship differ, the biological ages of the two twins remain the same.

2. Results

2.1. Invariance of Bacterial Duplication Number Across Inertial Reference Frames

The number of bacterial duplications or generations *G* is invariant across all inertial reference frames, as it is measured relative to the proper time of the respective system. Thus, the number of duplications *G* serves as a consistent measure of biological time progression, independent of the reference frame. In other words, if we determine the number of duplications under specific and same culture conditions on Earth or on a spaceship traveling at a relativistic velocity *v*, we will use a clock synchronized with the system in which the study is conducted (either on Earth or aboard the spaceship), which will measure the proper time. Consequently, the number of duplications will be identical in both systems, provided that the growth conditions are the same. This means that the bacterial duplication number is "invariant" with respect to any inertial reference frame. Such invariance highlights the consistency of biological processes across different contexts, and from this, it follows that biological time, represented by the number of generations *G*, does not depend on the relativistic effects experienced by physical measuring instruments, as we will demonstrate later.

The following section details the experimental setup and calculations designed to test the invariance of G across inertial frames.

2.2. Description of the Thought Experiment and Calculations

In special relativity, proper time τ (measured within reference system) and relative time t (measured by an observer external to the moving system) are related by the formula:

$$t = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (1)$$

Where t represents the relative time measured by an observer moving at velocity v relative to the object, τ is the proper time experienced by the object, and c is the speed of light in vacuum.

The thought experiment is designed as follows:

- **System T (on Earth):** A bacterial culture grows with a doubling or generation time equal to τ hours of proper time.
- **System A (on a spaceship moving at speed *v* relative to Earth):** An identical Earth bacterial culture grows at the same doubling rate τ, measured with respect to the spaceship's proper time. To ensure identical conditions in both experiments, the spaceship laboratory simulates an acceleration equivalent to Earth's gravity ($1g$), eliminating potential variations in bacterial growth due to differing gravitational forces. This setup controls for both relativistic and biochemical effects of gravity on bacterial growth.

Furthermore, to avoid growth differences due to acceleration as the spaceship reaches velocity *v*:

- The experiment starts once the spaceship reaches a constant velocity *v*.
- It is conducted entirely at this constant relativistic speed (*v*).

The experiment duration, denoted as the growth period τ_{growth}, is identical in both systems and measured as proper time within each system. At the end of this growth period τ_{growth}, the cultures are frozen to preserve the bacterial count and prevent further division, allowing for a direct comparison based solely on each system's proper time.

2.2.1. System on Earth as Observed from the Spaceship's Reference Frame (System A)

For an observer on the spaceship, the proper time τ_{growth} on Earth becomes:

$$t'_{growth} = \frac{\tau_{growth}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2)$$

Where t'_{growth} is the relative growth time measured with respect to the spaceship reference frame, and τ_{growth}represents the proper time duration of the experiment, which is the same for both systems.

From the spaceship perspective, the relativistic doubling time of the bacterial culture on Earth appears as:

$$t' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

Where, t' is the relative duplication time measured with respect to the spaceship reference frame, and τ denotes the proper time in which the bacteria double in number, which is also equal for both systems.

Over a period observed on Earth, the number of bacterial generations (G) is given by:

$$G' = \frac{t'_{growth}}{t'} = \frac{\tau_{growth} \cdot \sqrt{1 - \frac{v^2}{c^2}}}{\tau \cdot \sqrt{1 - \frac{v^2}{c^2}}} = \frac{\tau_{growth}}{\tau} \tag{4}$$

Consequently, an observer on the spaceship will see that the bacteria on Earth double exactly τ_{growth}/τ times.

2.2.2. System on the spaceship as seen from Earth (System T):

For the observer on Earth, the proper time τ_{growth} in spaceship corresponds to:

$$t_{growth} = \frac{\tau_{growth}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

From the terrestrial perspective, the relative doubling time of the bacterial culture in spaceship becomes:

$$t = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6}$$

During this relative time period, the number of bacterial generations (G) is given by:

$$G = \frac{t_{growth}}{t} = \frac{\tau_{growth}}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\tau} = \frac{\tau_{growth}}{\tau} \tag{7}$$

Thus, the observer on Earth finds that the bacterial generation number is $G = \tau_{growth}/\tau$, identical to that of the culture on the spaceship (see Eq. 4). In fact:

$$G = G' = \frac{\tau_{growth}}{\tau} \tag{8}$$

This result is highly significant, as it demonstrates that the bacterial generation number *G*, under identical culture conditions, remains invariant across all inertial reference frames. Consequently, *G* serves as a reliable measure of biological time, consistently corresponding to the proper time of the reference system, regardless of the time dilation observed between different systems.

2.2.3. Freezing and Final Result

At the end of the growth period τ_{growth} in each system, the cultures are frozen to prevent further bacterial divisions and to maintain the final count. The final number of bacteria is thus given by:

$$N_{final} = N_0 \cdot 2^{\frac{\tau_{growth}}{\tau}} \tag{9}$$

where N_0 is the initial number of bacteria.

This count remains identical in both systems, as the observed doublings τ_{growth}/τ are invariant with respect to proper time. Freezing preserves this count, allowing for an accurate comparison between the two systems without influences due to the deceleration of the spaceship. Thus, the final number of bacteria serves as a "biological clock," representing a measure of the time elapsed in each system.

2.3. Numerical Example

Consider an example with 1,000 initial bacteria (N_0) for both cultures, a proper growth time τ_{growth} of 2 hours, a doubling time of τ equal to 0.5 hours, and a spaceship traveling at a velocity of 0.9*c*.

Using the formula for relativistic time dilation, the proper biological time of each system appears dilated in the other reference frame:

2.3.1. Numerical Example: System on the Spaceship as Seen from Earth (System T):

For an observer on Earth, the proper growth time τ_{growth} = *2 hours* on the spaceship appears dilated by an amount calculable using Eq.5:

$$t_{growth} = \frac{\tau_{growth}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2}{\sqrt{1-0.9^2}} \cong 4.588\ hours \quad (10)$$

Using Eq.6, we can determine the doubling time of the culture on the spaceship as observed from Earth:

$$t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0.5}{\sqrt{1-0.9^2}} \cong 1.147\ hours \quad (11)$$

During this growth period (4.588 hours, Eq.10), and based on the generation number calculated using Eq.11 (1.147 hours), we can determine how many times the bacteria on the spaceship double:

$$G = \frac{t_{growth}}{t} = \frac{4.588}{1.147} = 4\ times \quad (12)$$

Thus, the observer on Earth sees that the bacteria on the spaceship double 4 times.

2.3.2. Numerical Example: System on Earth as Seen from the Spaceship (System A)

Below, we repeat the same calculations as before, but from the perspective of the spaceship.

For an observer on the spaceship, the proper growth time $\tau_{growth} = 2\ hours$ on Earth can be calculated using Equation 2:

$$t'_{growth} = \frac{\tau_{growth}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2}{\sqrt{1-0.9^2}} \cong 4.588\ hours \quad (13)$$

Using Eq.3, we can determine the doubling time of the culture on Earth as observed from spaceship:

$$t' = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0.5}{\sqrt{1-0.9^2}} \cong 1.147\ hours \quad (14)$$

Thus, the observer in spaceship sees that the bacteria on Earth double exactly in:

$$G' = \frac{t'_{growth}}{t'} = \frac{4.588}{1.147} = 4\ times \quad (15)$$

Thus, the bacteria on the spaceship double exactly 4 times, identical to the doubling observed in the Earth culture.

The final bacterial count in each system is then calculated using Eq.9, based on the identical result from Eq.12 and Eq.15, confirming four doublings.

$$N_{final} = N_0 \cdot 2^{\frac{\tau_{growth}}{\tau}} = 1{,}000 \cdot 2^4 = 16{,}000 \qquad (16)$$

Consequently, both the total bacterial count and the bacterial generation number *G* remain exactly the same across the two inertial reference frames considered in this example. This indicates that, unlike relative time, the elapsed biological time measured by *G* remains unchanged across both systems. In other words, the two cultures have "aged" by the same amount.

Based on the numerical provided example, here is a table showing the data for proper and relative times in the two systems (Earth and spaceship) for bacterial growth rates.

Table 1: Example of the Invariance of Bacterial Generation Number (*G*) Across Inertial Reference Frames.

System	Proper Time (τ)	Relative Time (t)	Doubling Time	Generations number (*G*)	Final Number of Bacteria
Spaceship (v = 0.9c)	2 hours	4.588 hours	1.147 hours	4	16,000
Earth	2 hours	4.588 hours	1.147 hours	4	16,000

3. DISCUSSION

This thought experiment shows that, although physical clocks in different inertial systems measure different times due to relativistic time dilation, biological time, indicated by the bacterial generation number *G*, remains invariant and corresponds to the "proper" time. The bacterial generation number *G* is not only identical in the two inertial frames considered here but also remains unchanged across any other inertial frame, as it consistently corresponds to the proper time in each system. In this study, we used bacteria, but the same results would be obtained using any other microorganism or eukaryotic cells in culture, as long as the growth conditions are the same in both inertial systems.

This conclusion revisits the classic interpretation of Einstein's twin paradox, which posits that the twin traveling at relativistic speeds ages more slowly compared to the twin who remains on Earth, as indicated by physical clocks. In reality, both twins would have the same biological age at the end of their journeys, despite the differing times displayed by the physical clocks in the two systems. Thus, biological time aligns with the proper time of each system and remains unaffected by the relativistic time dilation impacting physical measuring devices.

3.1. *Deriving the Time Dilation Equation in a Centrifuge with Radius r and Acceleration a*

In this chapter, we derive the equation for the time dilation experienced by an object in a rotating system, specifically within an ultracentrifuge. The premise for this derivation is that, according to the equivalence principle of general relativity, an acceleration causes time dilation that is exactly the same as that produced by an equivalent gravitational field (Misner, Thorne, & Wheeler, 1973, Chapter 16). Therefore, a time dilation effect could manifest as a result of the strong centripetal accelerations generated during the operation of these ultracentrifuges. For the following derivation, we will use Newtonian gravitational formulas, as they are more than sufficient for the purposes of our analysis.

We start from the equation for time dilation in a gravitational field:

$$t = \frac{\tau}{\sqrt{1 - \frac{2KM}{rc^2}}} \tag{17}$$

where t is the time measured by an observer far from the mass, τ is the proper time measured in proximity to the mass, K is the universal gravitational constant, M is the mass of the body generating the gravitational field, r is the distance from the center of the mass, and c is the speed of light in a vacuum.

According to the equivalence principle, a gravitational field and an accelerated system are equivalent; this means that the effects of a gravitational field can be replicated by a uniformly accelerating system. Therefore, we can transform the term KM/r^2 into terms of acceleration. The gravitational force acting on an object of mass *m* is given by the law of universal gravitation:

$$F = \frac{KMm}{r^2} \quad (18)$$

According to Newton's second law, the force can also be expressed as:

$$F = ma \quad (19)$$

where *a* is acceleration.

Equating the two previous expressions (Eq. 18 and 19) for force *F*, we obtain:

$$ma = \frac{KMm}{r^2} \quad (20)$$

By eliminating *m* from both sides, we find the acceleration *a*:

$$a = \frac{KM}{r^2} \quad (21)$$

Substituting *a* into the equation 17 allows us to rewrite the gravitational time dilation *t* in terms of acceleration:

$$t = \frac{\tau}{\sqrt{1 - \frac{2\mathrm{ar}}{c^2}}} \quad (22)$$

From Eq.22, we determine the time dilation factor (t/τ) due to centripetal acceleration over a distance *r*.

$$\frac{t}{\tau} = \frac{1}{\sqrt{1 - \frac{2ar}{c^2}}} \quad (23)$$

If $t/\tau = 1$, there is no time dilation, and time flows the same way in both systems. If $t/\tau > 1$, it means that for an external observer, time appears "dilated," or flows more slowly in the accelerated system.

While this thought experiment demonstrates, at least theoretically, the invariance of biological time in inertial systems, it is also essential to explore how biological processes might behave in relativistic accelerated or gravitational systems.

Studies indicate that microorganisms exposed to accelerations of thousands of g experience significant biological alterations in growth compared to cultures maintained under non-accelerated conditions. Notably, a study has examined the resilience of prokaryotic life under extreme gravitational forces, revealing surprising findings regarding their ability to grow and survive even under conditions vastly exceeding Earth's gravity. Deguchi et al. (2011) investigated the growth patterns of various microorganisms, such as *Escherichia coli*, *Paracoccus denitrificans*, and *Shewanella amazonensis*, under hyperaccelerative conditions in centrifuges, reaching accelerations as high as 403,627 × g.

At 403,627 × g, *E. coli* shows highly suppressed growth after 60 hours of incubation, while *Paracoccus denitrificans* exhibits significantly slowed growth, although cell proliferation continues under these extreme conditions.

To reach 403,627 × *g*, the authors reported using a Beckman XL-80 ultracentrifuge (Deguchi et al., 2011). One of the rotors capable of achieving these accelerations is the *Type 90 Ti*, which has a maximum radius of 7.6 cm.

Using Eq.23, we calculate the extent of the time dilation observed relative to the bacteria under these accelerations.

$$\frac{t}{\tau} = \frac{1}{\sqrt{1 - \frac{2a \cdot r}{c^2}}} = 1.0000000000033436 \qquad (24)$$

This suggests that the effects of time dilation at these high accelerations are negligible for biological processes; therefore, the significant differences in growth observed by Deguchi et al. (2011) cannot be attributed to relativistic effects. Moreover, there is no theoretical impediment preventing biological time from aligning with the proper time of the system, even in accelerated or gravitational environments. However, due to the profound biological effects caused by the strong accelerations required to observe relativistic effects, the type of experiment proposed in this study may not be feasible for investigating potential relativistic effects on biological time in non-inertial systems.

3.2. *Invariance of Bacterial Duplication Number in Non-Inertial Reference Frames*

As previously noted, it is challenging to experimentally study the potential relativistic effects on biological time in non-inertial systems. However, in this chapter, we mathematically demonstrate that even in non-inertial reference frames, biological time measured through *G* remains unchanged.

Although relativistic time dilation can theoretically affect both the growth period and the duplication time in accelerated systems, *G* remains invariant because it is defined as a ratio. Since *G* is the ratio between the growth period and the duplication time, any relativistic effects on these two quantities mathematically cancel out. This ensures that *G* (biological time) remains unchanged even in non-inertial reference frames.

In summary, in an accelerated reference frame, *G* is given by:

$$G = \frac{t_{growth}}{t} = \frac{\tau_{growth}}{\sqrt{1-\frac{2ar}{c^2}}} \cdot \frac{\sqrt{1-\frac{2ar}{c^2}}}{\tau} = \frac{\tau_{growth}}{\tau} \tag{25}$$

and, in a gravitational reference frame:

$$G = \frac{t_{growth}}{t} = \frac{\tau_{growth}}{\sqrt{1-\frac{2\mathrm{K}M}{rc^2}}} \cdot \frac{\sqrt{1-\frac{2\mathrm{K}M}{rc^2}}}{\tau} = \frac{\tau_{growth}}{\tau} \tag{26}$$

Thus, in a generic non-inertial reference frame, *G* remains identical to its value in an inertial reference frame (see Eq. 8).

In conclusion, this study highlights the invariance of biological time, as represented by the cellular generation number *G*, across both inertial and non-inertial reference frames. While relativistic time dilation influences physical clocks, *G* remains unaffected due to its mathematical definition as the ratio between the growth time and the duplication time, which ensures the cancellation of any relativistic effects on these intervals.

This invariance underscores the fundamental connection between biological processes and proper time, offering a universal metric for measuring biological time regardless of relativistic conditions. By establishing G as a reliable and invariant measure, this work provides a theoretical framework that could be extended to diverse biological systems, including more complex multicellular structures or tissues.

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Declaration of competing interest

The author declares that there are no competing interests.

Data availability

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