HOOKE-KEPLER LAW OF GRAVITATION: the rejected law of gravitation, which turned out to be more accurate and perfect than Newton's law.

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Abstract. *The role of Robert Hooke in the discovery of the law of universal gravitation is shown in a new light. New circumstances are revealed that relate to the priority dispute between Hooke and Newton. It is shown that Newton's law of gravitation is not the only law of gravitational interaction. There is another law of gravitation that was outside Newton's field of vision. The existence of this law was indicated by Robert Hooke in his correspondence with Newton. Robert Hooke pointed out that the law of gravitation should take into account the elliptical orbits of the planets and the inverse square law. In 1687, Newton presented the law of gravitation, which includes the inverse square law. But the parameters of the elliptical orbit were not included in Newton's law. Instead of the orbital parameters, Newton introduced mass into his law:* $\mathbf{F_N \propto mM/r^2}$. *As a result, a more perfect law of gravitation than Newton's law was not discovered. Here we present this law of gravitation. It has a beautiful and mathematically perfect form:* $\boldsymbol{F_{H\text{-}K} = mR^3/(T*r)^2}$. *I call this physical law the Hooke-Kepler law of gravitation. It includes the Kepler constant. It includes the inverse square law. It does not include the central mass. It does not contain the gravitational constant G. This is a more accurate and perfect law of gravitation than Newton's law, since distances and periods are known from observations with greater accuracy than mass. Thus, Hooke's path to the law of gravitation was more promising than Newton's. Robert Hooke was very close to discovering an alternative law of gravitation. Obviously, Hooke knew something about gravitation that Newton did not.*

Keywords: *Robert Hooke; Isaac Newton; astronomy; Newton's law of gravitation; Kepler's 3rd law.*

1. Introduction.

It is believed that with the advent of Newton's law of gravity, the desired law of gravitational interaction, to which Hooke and Newton were moving, was discovered. At the same time, it is believed that the law discovered by Newton is the only possible law of gravitational interaction. The possibility of the existence of an alternative law of gravity is not considered.

There is currently a major re-evaluation of Hooke's contribution to mechanics. Researchers of the work of Hooke and Newton did not pay attention to the fact that Hooke and Newton understood the future law of gravity differently. Researchers passed by the fact that Hooke and Newton were moving towards the future law of gravity in different ways. These different approaches and different paths must have led them to two different laws of gravity. Newton's approach ended with the discovery of only one of the two laws of gravity existing in Nature. The second law of gravity, to which Hooke's path objectively led, remained undiscovered.

Newton presented the law of universal gravitation in a verbal formulation in 1687. At the same time, as of 1687, there were not one, but two verbal formulas for the future law of gravitation. They differed significantly from each other. One verbal formulation was given by Newton in «*Philosophiæ Naturalis Principia Mathematica*». A second verbal formulation of the law of gravitation was given by

Hooke in correspondence with Newton before the publication of «*Philosophiæ Naturalis Principia Mathematica»*. In essence, these were verbal formulas for two different laws of gravitation. Newton's verbal formulation became the law of universal gravitation. Hooke failed to transform his knowledge into a law of gravitation. His verbal formulation was unreasonably forgotten after the appearance of Newton's law of universal gravitation. Newton and Hooke, without suspecting it, spoke *of two different laws of gravitation*.

Hooke and Newton took different paths to the law of gravitation. These different paths should have produced different laws of gravity. Newton discovered only one of the two laws of gravitation. Newton used masses as parameters in the law of gravitation. Hooke did not use masses, but persistently investigated the trajectories of the planets. This can be seen both in his works and in his experiments with a pendulum [1, 2].

Hooke hoped that Newton would help him explain the law of universal gravitation using the trajectories of the planets. Robert Hooke, in his correspondence with Newton, indicated that the law of gravity should take into account the orbits of the planets and the inverse square law [1]. Newton was more interested in masses as parameters of the law of gravity and the relationship of elliptical orbits with the inverse square law. Newton did not see the advantage of using the parameters of an elliptical orbit in the law of gravity and overestimated the role of mass as a parameter in the law of gravity. Newton, in his letter to Hooke dated December 13, 1679, stated regarding elliptical orbits: "*...it being of no great moment*..." [1].

As a result, the law of gravity that Hooke was working towards was not discovered. All that remains is the verbal formula for this law of gravity. Based on this verbal formula, it must be a completely different law, unlike Newton's. We show that the law of gravitation that Hooke was working towards is an exact and mathematically perfect law of gravitation, more perfect than Newton's law.

We present this law of gravitation and give it a complete explanation. We show that Hooke's path to the law of gravitation was more promising than Newton's. Hooke's path could in fact have led him to another law of gravitation, more perfect than Newton's law. Obviously, Hooke knew something about gravitation that Newton did not.

2. About Newton's original law of gravity $F_N \propto mM/r^2$.

Newton formulated his law of gravity in verbal form. In his formulation, the gravitational force is proportional to the masses and inversely proportional to the square of the distance. When trying to present Newton's law of gravity in symbolic form, a mistake is often made. It is mistakenly presented as: $\mathbf{F_N = mM/r^2}$. The formula for force uses the equal sign. This symbolic representation of Newton's law in the form of equality is a fairly common mistake. Newton actually said that the force of attraction is proportional to the masses. Newton only asserted the aspect of proportionality and did not assign any numerical value to the gravitational force. Therefore, the equal sign in the symbolic representation of the law proposed by Newton is unacceptable. The correct form of presentation of the original law proposed by Newton is as follows:

$$F_N \propto \frac{mM}{r^2} \quad (1)$$

Where: F_N is the force, m, M are the masses of bodies, r is the distance, $\propto$ is the proportionality sign.

As we can see, Newton presented the world with an unfinished law of gravity. Newton's verbal formula did not provide a quantitative description of the gravitational force. Formula (1) is not an exact formula for the law of gravitation. Using masses as parameters did not allow Newton to obtain an exact formula for the law of gravitation. In this unfinished form, Newton's law $\mathbf{F_N \propto mM/r^2}$ existed for almost 200 years. It turned into the exact equality $\mathbf{F = GmM/r^2}$ only after the gravitational constant G was introduced into it [3 - 6]. In fact, Newton's proportionality law $\mathbf{F_N \propto mM/r^2}$ with the appearance of the constant G was "*refined*" to the exact law of gravitation $\mathbf{F = GmM/r^2}$. The appearance of the constant G in Newton's law of gravitation was a revolutionary step. The constant G transformed Newton's unfinished law into an exact law of gravitational interaction between two bodies.

But even in its unfinished form, Newton's law became a real breakthrough in science. Gravitational interaction became one of the 4 fundamental interactions in Nature. At the same time, the illusion appeared and took root that Newton's law gives a complete description of gravitational interaction. It is still believed that this is the only formula for the force of gravitational interaction. Below we will show that there is another law of gravity that was outside Newton's field of vision and we will give its formula. The existence of this law of gravity was indicated by Robert Hooke in his correspondence with Newton long before the discovery of Newton's law. Robert Hooke gave a verbal formula for the future law of universal gravitation. Robert Hooke pointed out that the law of gravity should take into account the orbits of the planets and the inverse square law. Hooke did not mention mass as a parameter in the law of gravity.

3. Hooke's hint that Newton did not use.

It is known that in 1679-1680 Robert Hooke entered into correspondence with Newton. Hooke in his correspondence with Newton gave him two hints for the future law of gravity. The first hint: the law of gravity should include the inverse square law. The second hint: the law of universal gravitation must take into account the orbits of the planets. In practice, Hooke gave an approximate verbal description of the future law of gravity. This verbal description differs from the verbal description of the law of gravity (1) given by Newton. Newton introduced masses into his law of gravity. But Hooke wanted to see the parameters of the elliptical orbit in the law of universal gravitation itself.

Elliptical orbits were known at that time from Kepler's laws. The inverse square law was proposed by Bullialdus [3, 7]. But the possibility of their application in the future law of gravity was not obvious. Hooke was the first to guess that the law of gravity could be derived from Kepler's empirical laws. He persistently studied motion along elliptical trajectories and studied the inverse square law. To do this, he conducted experiments with a pendulum and used a graphical method to describe an elliptical orbit [1]. In his letters to Newton, he emphasized elliptical orbits. Newton confirmed that he had received Hooke's suggestion that motion along an ellipse should be taken into account: "*...as if he had found the motion in the Ellipsis, which inclined me to try it* ..." [8].

The law of gravitation, published in 1687, shows that Newton rejected one of Hooke's hints. Of Hooke's two hints, only the inverse square law is included in Newton's law of gravitation. As a result, the law of gravitation that included the parameters of the elliptical orbit hinted at by Hooke remained undiscovered. Perhaps the situation can be clarified by Newton's letter to Hooke dated December 13, 1679. In it, Newton writes [1]: *«Your acute Letter having put me upon considering thus far ye species of this curve, I might add something about its description by points quam proximè. But the thing being of no great moment I rather beg your pardon for having troubled you thus far with this second scribble.…».*

What was of great importance to Hooke meaning, for Newton "... *being of no great moment* ...". Newton did not see the advantage of using the parameters of an elliptical orbit and overestimated the role of mass as a parameter in the law of gravity. Newton used only the first hint - the inverse square law. Obviously, Hooke's first hint confirmed his own opinion. Newton's manuscripts show that Newton came to the understanding of the need to use the inverse square law in the law of gravity before 1669 [9, 10]. It was this hint from Hooke that caused the dispute about priority. Newton rejected the second hint.

He did not use Hooke's second hint about elliptical orbits. Instead of using orbital parameters in the law of gravity, he used masses. It is known that Kepler's laws do not include masses. The use of masses in the law of gravity led to the fact that instead of an exact law of gravity, Newton proposed only a proportional dependence of force on masses (formula (1)).

At the same time, Newton considered motion along an ellipse. There is a known entry in his notebook from 15 years before his correspondence with Hooke: "*if a body moves in an ellipse, then the force at each point... can be found*" [11]. Later (November 28, 1679) in correspondence with Hooke, Newton claimed that a body under the influence of gravity would move in a spiral. Hooke discovered that Newton was mistaken. Reporting Newton's letter at a meeting of the Society, Hooke criticized it, claiming that the body would fall not in a spiral, but in an "Eccentric Elliptoid" [12].

Newton concluded that the force of gravity is proportional to the masses, but he failed to find a more perfect formula for this force because he did not use the parameters of an elliptical orbit.
And this is against the background of the fact that Kepler's laws had been known since 1619. And this is against the background of the fact that Hooke persistently pointed out the need to take into account elliptical motion. Why was Newton satisfied with an inaccurate law of gravity $\mathbf{F_N \propto mM/r^2}$, using mass as parameters? Why did he reject Hooke's hint and not use the parameters of an ellipse? This remains a mystery.

Perhaps the answer should be sought in the text [13]. from Newton's 1684 manuscript, known as the " Copernican scholium": "*...the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

This entry indicates that Newton did not believe in the possibility of obtaining an exact law of gravity, since he did not consider orbits to be elliptical. Perhaps for this reason, he rejected Hooke's hint and did not introduce the parameters of an ellipse into the law of gravity.

4. Two different approaches should have led to two different laws of gravity.

Newton introduced masses as parameters in his law of gravity. In his letter, Hooke did not mention masses in the context of the law of universal gravitation. Hooke believed that the law of universal gravitation should be represented by the inverse-square law and Kepler's orbits of the planets. It is known that Kepler's laws do not include masses. As we see, Hooke and Newton had different ideas about the future law of universal gravitation.

Robert Hooke's 1685 manuscript shows that he worked hard to develop a quantitative theory of motion using the geometric method of constructing an ellipse [1, 14]. He studied elliptical orbits diligently. The paper [1] shows Hooke's geometric construction of an elliptical orbit in September 1685 (Fig. 1):

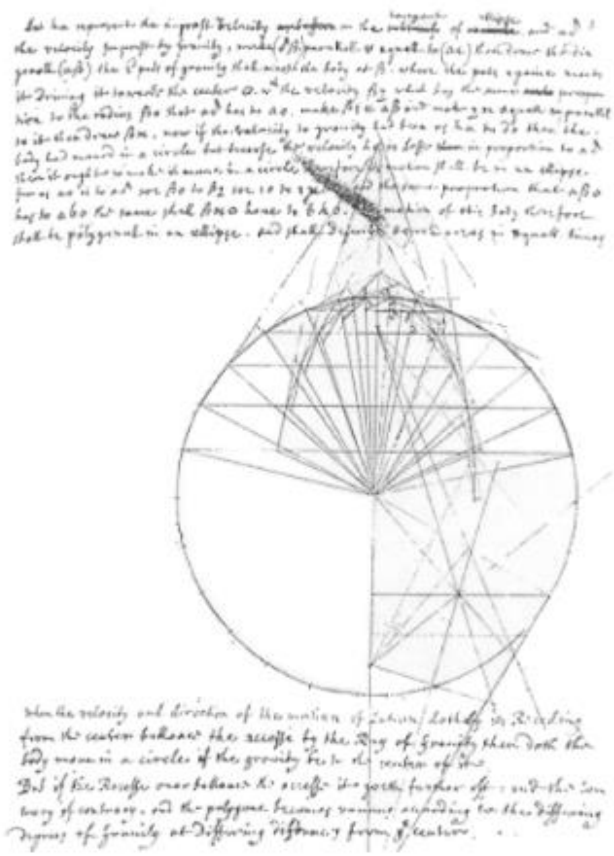

Fig. 4. Hooke's September 1685 geometrical construction and graphical evaluation of the orbital motion of a body in a central field of force that varies linearly with distance. The original is in Hooke's manuscripts in Trinity College Library, Cambridge, Ms O.11a.1/16. Reproduced by permission of the Master and Fellows of Trinity College.

Fig. 1. Geometrical construction of an elliptical orbit, made by Hooke in September 1685.

Could such close attention of Robert Hooke to Keplerian orbits have led to the discovery of a law of gravity different from Newton's? We show that Keplerian dynamics could indeed have led Hooke to discover a law of gravity better than Newton's.

5. Hooke-Kepler law of gravitation.

Let us consider in more detail the second hint that Hooke so persistently suggested to be taken into account in the law of gravitation. Maybe Hooke was wrong? After all, Newton rejected his hint and used masses instead of elliptical orbit parameters? Is it possible to propose a formula for the law of gravitation in which the orbital parameters would be used instead of mass, as Hooke wanted?
To answer these questions, let us consider what the law of gravitation would look like in symbolic form, adequate to Hooke's verbal formulations.

According to Hooke's first hint, the law of gravitation should include the inverse square law. According to Hooke's second hint, the law of gravitation should include the parameters of an orbit. We will consider a special case, applying it to an elliptical Keplerian orbit. The central mass should not be included in the law of gravitation, since masses are not included in Kepler's laws. Direct use of Kepler's laws does not lead to the formula for gravitational force. The problem is solved by Kepler's constant [15], which directly follows from Kepler's 3rd law.

Using Hooke's verbal description of the future law of gravity, it is not difficult to obtain a symbolic formula for the alternative law of gravity. Based on Hooke's verbal formulations, the alternative law of gravity can be represented by the following symbolic equivalent:

$$F_{H-K} = \frac{mR^3}{T^2 r^2}$$

Fig. 2. Hooke-Kepler law of gravity. Where: m is the mass of the body, R and T are orbit parameters, r is the distance.

As we can see, Hooke was not mistaken with the second hint. The fundamental law of gravity that Hooke hinted at does exist. The parameters of the elliptical orbit together with the inverse square law do lead to a new law of gravity. Robert Hooke was very close to discovering an alternative law of gravity. The Hooke-Kepler law of gravitation (Fig. 2) includes Kepler's constant R^3/T^2 . It incorporates the inverse-square law. It does not take into account the central mass. It does not include the gravitational constant G. This precise and beautiful law of gravity was not discovered by Newton, despite Hooke's hint. The reason is that Newton used masses as parameters and did not use Hooke's second hint about elliptical orbits.

Paying tribute to the genius of Robert Hooke, I call this mathematically perfect law of gravity the *Hooke-Kepler law*. This is a more precise and perfect law of gravity than Newton's law, since distances and periods are known from observations with greater accuracy than mass. If Newton had heeded both of Hooke's hints, he could have offered the world two laws of gravity. One approximate law of gravity $\mathbf{F_N \propto mM/r^2}$, and the second exact law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$.

As we see, Newton had the opportunity to give the world not a proportional dependence F N ∝ mM/r 2 , but an exact law of universal gravitation back in 1687. To do this, it was enough to use Kepler's law. Robert Hooke clearly indicated this in his letter. Historians have paid more attention to the first hint: the inverse square law. Newton took this hint from Hooke into account. The inverse square law is equally compatible with both masses and orbital parameters. The second hint about the need to take into account motion along an elliptical orbit in the law of gravity was ignored. Newton rejected this hint. And in vain! As a result of Newton rejecting Hooke's second hint, the more perfect law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ was not discovered. Hooke knew something more profound about gravity, but he failed to transform his knowledge into the law of gravity. One can only guess what a great impetus to the development of science would have been received in the second half of the 17th century if Newton had accepted both of Hooke's hints.

6. Verbal formulas of Hooke and Newton

The peculiarity of Hooke's discovery, presented in his verbal formulations, is that he combined two ideas into a single system:

1. The force of attraction decreases proportionally to the square of the distance $1/r^2$: *«...my supposition is that the Attraction always is in a duplicate proportion to the Distance from the Center Reciprocal...»*. (Letter from Hooke to Isaac Newton, January 6, 1680).

2. Under the influence of gravity, planets move in an orbit close to an ellipse: «*... and consequently that the Direction will be a Curve Line very near resembling an Ellipsis»*. (Letter from Hooke to Isaac Newton, January 6, 1680).

Hooke's verbal formulations were not the law of gravitation. However, these verbal formulations pointed the way to both the physical law of two-body gravitation and the law of universal gravitation. In the hypothetical case of an idealized Keplerian orbit, this was the path to the physical law of two-body gravitation. For a real orbit, "very near resembling an Ellipsis," this was the path to the law of universal gravitation. Hooke hoped that Newton would help him express these verbal formulations in mathematical form. But instead of "mathematizing" Hooke's verbal formulas, Newton gave a modified verbal formula for the law of gravitation based on masses.

Following Hooke's letters, Newton also made a unification, but he did not use the orbital parameters along with the inverse-square law, but used the mass of the central body. The radical difference between Newton's verbal formula and Hooke's verbal formula is Newton's emphasis on masses rather than orbits:

: 1. The force of gravity decreases proportionally to the square of the distance $1/r^2$

2. The force of attraction is proportional to the masses.

Newton solved the two-body gravitational problem. His verbal formulations, like Hooke's, were not a complete law of gravitation. They pointed the way only to a law of two-body gravitation. It was not even a complete law of two-body gravitation. It was a proportional relationship $F_N \propto mM/r^2$. Not Newton, but popularizers, too far removed from an understanding of gravity, were too hasty and called the proportional relationship $F_N \propto mM/r^2$ the law of universal gravitation. Only many years later (in 1885) did the proportional relationship $F_N \propto mM/r^2$ become a complete law of two-body gravitation: $F = GmM/r^2$.

A comparison of Hooke's and Newton's verbal formulations reveals the superiority of Hooke's formula over Newton's. Hooke's verbal formula yields both a law of two-body gravitation and a physical law accounting for the gravitation of all bodies, while Newton's formula yields only a law of local gravitation for two bodies. Newton, solving the two-body problem, proved that if the force is inversely proportional to the square of the distance ($1/r^2$), then the body moves along an ellipse, parabola, or hyperbola. Judging by his response to Hooke's letter, Newton considered the orbital parameters to be secondary. For Hooke, the orbit was a primary, observable fact (thanks to Kepler). Hooke did not consider the parameter ($1/r^2$) to be a substitute for the orbital parameters. Hooke did not believe that the inverse-square law "absorbs" or replaces the geometric parameters of the ellipse itself. Hooke considered a parameter of the form ($1/r^2$) as an additional parameter to the orbit. In his letters to Newton, Hooke literally pushed him toward this synthetic model: *to combine a parameter of the form ($1/r^2$) with the geometry of the curve (ellipse).*

7. The inverse-square law is only part of Hooke's verbal formula.

When scholars of Robert Hooke's work discuss his contribution to the discovery of the law of universal gravitation, they traditionally mention the inverse-square law, which Hooke emphasized in his letter to Newton. However, they often overlook Hooke's second emphasis on the connection between gravity and the orbital motion of the planets. The inverse-square law is only part of Hooke's verbal formula. Another, no less important, part of Hooke's verbal formula concerned the connection between gravity and the trajectory of motion: "*...and consequently that the direction will be a curved line very near resembling an ellipsis...*"

Hooke made a revolutionary discovery that led him to conclude that the law of gravity must take into account the elliptical orbit: «...compounding the celestiall motions of the planetts of a direct motion by the tangent & an attractive motion towards a central body». (Hooke's letter to Isaac Newton, November 24, 1679). This discovery changed Newton's thinking and led him to discover the law of gravity.

Hooke's insight is admirable. Hooke used the formulation "*very near resembling an ellipse*", knowing that Kepler's laws describe perfect ellipses. He did not emphasize perfect ellipses, as Newton did. Hooke's insight here proved closer to reality: *perfect elliptical orbits do not exist in the universe. The actual orbits of the planets are not elliptical.* His formulation, "*very near resembling an Ellipsis,*" is not the vague formulation of an insecure person, but an acknowledgement of the complexity of the real world. He had a profound understanding of gravity. He understood that nature is more complex than geometrically perfect ellipses on paper. He understood that complexity is introduced by the gravity of multiple bodies.

As for the connection between the force and the trajectory of motion, this part of Hooke's verbal formula was either not understood by Newton, or he deliberately distanced himself from Hooke's formulation. Newton abandoned the use of orbital parameters in favor of masses.

Hooke's intuition is astounding! Indeed, acceleration can be represented not only as a combination of the parameter $1/r^2$ with mass ($a = GM/r^2$), but also as a combination of the parameter $1/r^2$ with the orbital parameters (R^3/T^2) from Kepler's law ($a = R^3/T^2r^2$).

After the discovery of Newton's law of gravitation, Hooke's verbal formula, in its full formulation emphasizing the connection between the force of attraction and the orbital motion of the planets, was forgotten for more than 300 years, even though it held the key to the real and precise law of universal gravitation.

Underestimating and misunderstanding Hooke's emphasis that the force of universal gravitation should be determined using the inverse square law in conjunction with orbital parameters, rather than in conjunction with mass, cost science dearly. This led to the fact that instead of the law of universal gravitation, Newton discovered the law of two-body gravitation, and popularizers unreasonably elevated the law of two-body gravitation to the rank of the law of universal gravitation. The exaggeration of the status of the local law of gravity became more important than the truth and overshadowed the need to discover a true law of universal gravitation that takes into account the gravity of all bodies in the universe.

8. Comparison of Hooke's and Newton's verbal formulas.

There are similarities and differences between Hooke's and Newton's verbal formulas. This is shown in Fig. 3.

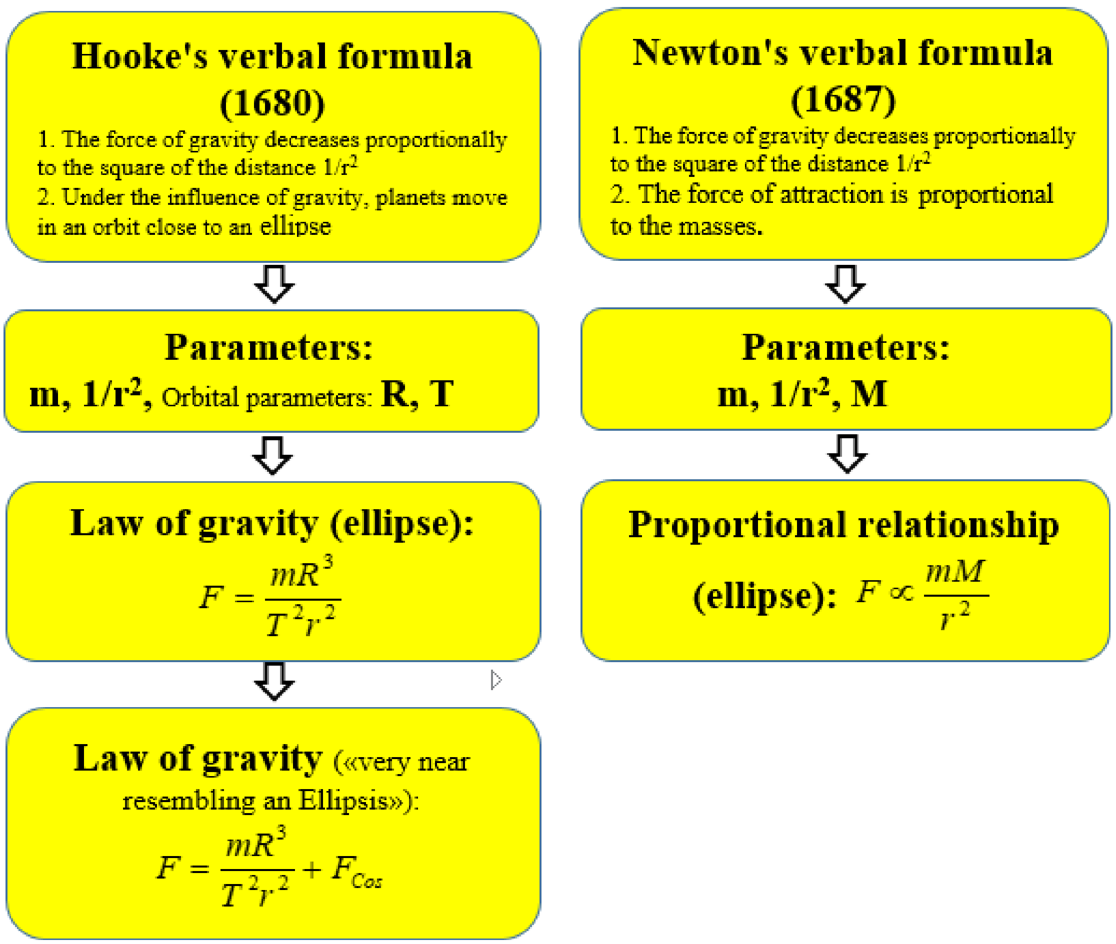

Fig. 3. Comparison of Hooke's and Newton's verbal formulas. Where: m, M is the mass of the bodies, R and T are orbit parameters, F_{Cos} is the cosmological force, r is the distance.

The common feature is the inverse square law. The difference is that Newton emphasized the masses in his verbal formula, while Hooke emphasized the orbits in his verbal formula. The following parameters follow from the verbal formulas: Newton's - (m, 1/r^2, M), Hooke's - (m, 1/r^2, Orbital parameters: R, T). Using the parameters (m, $1/r^2$, M), Newton's verbal formula led to the proportional dependence $F_N \propto mM/r^2$. This was not the law of universal gravitation. It was not even a complete law of two-body gravitation. The maximum expected result from Newton's verbal formula is the law of two-body gravitation, represented by masses ($F = GmM/r^2$). Proportional dependence $F_N \propto mM/r^2$ turned into the complete law of two-body gravitation $F = GmM/r^2$ only in 1875. In 1875, the Potential of Newton's verbal formula was fully realized.

Hooke's approach is more promising. Hooke's verbal formula allows one to derive both the complete law of two-body gravitation and the law of universal gravitation. The maximum expected result of Hooke's verbal formula is the actual law of universal gravitation. Hooke's verbal formula states: "very close to an ellipse." This points not to the gravitation of two bodies, but to the gravitation of all bodies in the universe. As an intermediate result, for an idealized elliptical orbit, Hooke's verbal formula leads to a new law of gravitation of two bodies, represented by the orbital parameters: $F = mR^3/T^2r^2$. For a real orbit close to an ellipse, Hooke's verbal formula leads to the law of gravitation of all bodies in the universe: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$. The deviation of the orbit from an ideal ellipse is caused by the additional gravitation of all other bodies in the universe F_{Cos}. In formula, this additional force is represented by the second component: $mc^2\sqrt{\Lambda}$.

The main conclusions that follow from a comparison of Hooke's and Newton's verbal formulas are the following:

1. Hooke's verbal formula can be transformed into a law of universal gravitation. Hooke's model is a model of gravity for all bodies in the universe.
2. Newton's verbal formula does not lead to a law of universal gravitation; it leads to a law of two-body gravitation. The two-body model is a simplified model of gravity.

9. Comparison of two laws of gravity.

Hooke and Newton had different approaches to the law of gravity. Newton and Hooke, without even realizing it, were talking about two different laws of gravity. Hooke relied on Kepler's laws. Newton used masses as parameters. Newton managed to discover the law of gravity $\mathbf{F_N \propto mM/r^2}$. But it was not the exact law of gravity. This was an approximate proportional relationship between the force and the two masses. Hooke was very close to discovering the exact law of gravity. But he failed to do so. Hooke turned to Newton for help. But Newton also failed to discover the exact law of gravity, the signs of which Hooke suggested to him.

From a comparison of the two laws it follows that the law of gravity $\mathbf{F_N \propto mM/r^2}$ proposed by Newton is clearly inferior to the Hooke-Kepler law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ (Fig. 4).

$$F_{H-K} = \frac{mR^3}{T^2 r^2} \qquad F_N \propto \frac{mM}{r^2}$$

Fig. 4. Hooke-Kepler law of gravitation and Newton's law of gravitation.

The law of gravitation (1), proposed by Newton, was not a quantitative physical law. It was presented as an approximate verbal formula. This is only a proportional relationship, not a complete equation. Law (1), proposed by Newton, is an incomplete law of gravitational interaction of two bodies. Newton's law became an exact equality only after the introduction of the gravitational constant G.

Hooke-Kepler law of gravitation (Fig. 2) is a quantitative physical law. Hooke-Kepler law is an exact and complete law of gravitational interaction of two bodies. For this reason, this law of gravity does not require a gravitational constant G.

Newton's law includes masses and the inverse square law as parameters. Hooke-Kepler's law of gravity does not include central mass. Hooke-Kepler's law includes elliptical orbit parameters and the inverse square law as parameters. It was these features that Robert Hooke pointed out in his letter to Newton.

Hooke-Kepler law of gravitation (Fig. 2) and Newton's law of gravitation (1) are not equivalent physical laws. These are different laws of gravitation. There is no equal sign between them:

$$\frac{mM}{r^2} \neq \frac{mR^3}{T^2 r^2} \qquad (2)$$

The common feature of the two laws of gravity is the inverse square law.

10. The two laws of two-body gravitation, derived from Hooke's and Newton's verbal formulas, complement each other.

In the two-body gravitation model, Hooke's and Newton's verbal formulas do not contradict each other. In this simplified model of gravitation, Hooke's verbal formula and Newton's formula complement each other. The two-body gravitation laws, $F = GmM/r^2$ and $F = mR^3/T^2r^2$, derived from these verbal formulas, also complement each other rather than replace each other. In the presence of an orbit, the gravitational interaction is more accurately described by the formula $F = mR^3/T^2r^2$. This is a convenient way to calculate the force when the mass of the central object is unknown. With a known mass of the central object, the formula $F = GmM/r^2$ describes the gravitational interaction with and without orbital motion. Here, the limiting factor is the low accuracy of the constant G.

In the all-bodies gravitation model of the universe, the picture is different. Newton's verbal formula, formulated for the two-body problem, is inapplicable to the gravity of all bodies, as confirmed by the unsolvable N-body problem. Hooke's verbal formula is applicable to the gravity of all bodies. It clearly indicates the deviation of the actual trajectory from a perfect ellipse. This is a direct indication of the presence of an additional force, in addition to the attractive force between the two bodies.

11. Hooke's formulation of the revolutionary law of gravitation.

Hooke's verbal formulation of the future law of gravitation differed from Newton's verbal formulation. Hooke spoke of orbits. Newton spoke of masses. The only coincidence was in the use of the inverse square law. Hooke knew something about gravitation that Newton did not. This "*something*" was related both to the role and place of Kepler's laws in the future law of gravitation and to the need to account for orbits close to an ellipse. Essentially, Hooke viewed the future law of two-body gravitation as a combination of Kepler's law and the inverse-square law. And the law of all-body gravitation, according to Hooke, must take into account orbits close to an ellipse and the inverse-square law. In essence, Hooke, speaking of elliptical orbits, proposed the use of two laws in the future law of gravitation: Kepler's law and the inverse square law.

Newton had a different opinion. Newton rejected Hooke's main hint about elliptical orbits. Newton replied to Hooke in a letter dated December 13, 1679, that "*... being of no great moment...*". Newton saw masses instead of elliptical orbit parameters in the future law of gravity. It is known that there are no masses in Kepler's laws. The result of using masses instead of elliptical orbit parameters was the appearance in 1687 of an unfinished law of gravity $\mathbf{F \propto mM/r^2}$. The opportunity to discover the exact and perfect law of gravity that Hooke was working towards was missed. Hooke's verbal formulation actually led to the revolutionary physical law $F = mR^3/T^2r^2$, which included Kepler's law and the inverse square law.

The revolutionary Hooke-Kepler law of universal gravitation may have emerged in 1687 alongside and complemented Newton's law $F \propto mM/r^2$. At the time, both Kepler's laws and the inverse-square law were known. But it wasn't so obvious that they were part of the law of universal gravitation. Only Robert Hooke pointed this out in his letters.

12. Bertrand and Koenigs' Gravitational Problems

Hooke's verbal formulation with an emphasis on elliptical orbits only provided a statement of the problem. Hooke expected to receive a solution from Newton. But Newton was solving the two-body problem at the time. Hooke's verbal formulation implied finding the law of gravitation based on a known trajectory of motion. This is the inverse two-body problem. The problem of finding the law of gravitation based on a known trajectory of a body's motion had not yet been formulated at that time. Such a problem appeared in the late 1870s. It was formulated by Bertrand J. [16]. The first and second gravitational problems of Bertrand are known [16]. Bertrand's first problem was formulated for trajectories that are conic sections. Bertrand's second problem was formulated for trajectories that are closed curves. In general, for trajectories represented by algebraic curves, this problem is known as the Koenigs problem [16]. These are inverse problems to the two-body problem.

13. The third solution to the Bertrand and Koenigs gravitation problems.

It is known that the solution to the Bertrand and Koenigs problems gives a force law, which can be either Hooke's law or Newton's law of gravitation [16]. The two solutions found to the Bertrand and Koenigs problems only confirmed the force laws known at that time, but did not give new predictions. In fact, the Bertrand and Koenigs problems have not two, but three solutions. The third solution exists for closed elliptical trajectories of motion. This third solution to the Bertrand and Koenigs gravitation problems is presented above as: F = mR^3/T^2r^2. The third solution directly includes the parameters of the elliptical trajectory of motion.

14. The gravitational constant G includes the Kepler constant R^3/T^2.

The constant G is called the Newtonian constant of gravitation [17]. But Newton did not even mention it. The constant G appeared in the formula for the law of gravitation much later than the discovery of the law of universal gravitation. Henry Cavendish in 1798 also did not know about the existence of the constant G and did not measure it. He measured the density of the Earth. The constant

G appeared in the formula $F = GmM/r^2$ almost 200 years after the discovery of the law of universal gravitation and 75 years after the Cavendish experiment.

J. A. M. Pereira in [18] showed the history of the appearance of the constant G. In 1803, S. D. Poisson presented the formula for the law of universal gravitation $F = f\,mM/r^2$ with the coefficient *f*, which later became the constant G [18, 19].

In 1873, A. Cornu and J. B. Baille also used the symbol (*f*) for the coupling constant in Newton's law of gravitation (4 - 6, 18).

The law of universal gravitation in the form familiar to us $F = GmM/r^2$ was presented by A. K ̈onig, F. Richarz, [20, 21], J. H. Poynting [18, 22].

Fig. 5 shows the evolution of the law of universal gravitation from its discovery by Newton in 1687 to the present day.

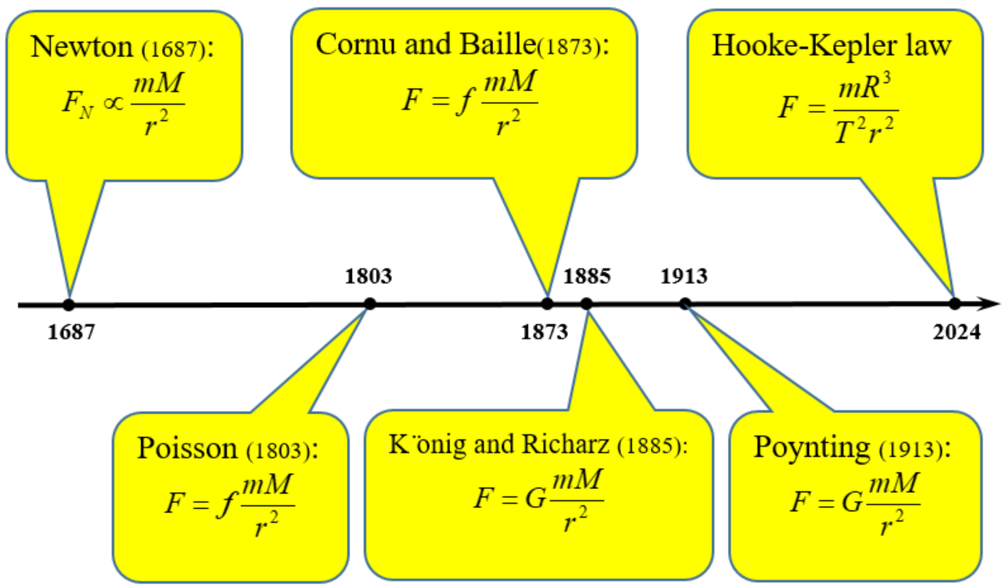

Fig. 5. Evolution of the law of universal gravitation from Newton's law to the Hooke-Kepler law.

The constant G is now considered a fundamental constant [17]. Limits on the change of G have been confirmed. Analysis of observations shows that the gravitational constant G has changed by less than one part in ten billion per year over the past nine billion years [23].

There is an ongoing debate about the fundamental status of the constant G [24 - 26]. In [27], it was shown that the constant G can be represented using the fundamental physical constants of the electron by the following formula:

$$\mathbf{G = r_e^3/t_e^2 m_e D_o} \qquad (3)$$

Where: r_e is the radius of the electron, $t_e = r_e/c$, me is the mass of the electron, Do is the large Weyl number (D_0 = 4.16561... x 10^42).

The constant G is presented in formula (3) by electromagnetic constants. The formula for calculating G includes the Kepler constant for the electron r_e^3/t_e^2.

In [15] it is shown that the constant G can be presented using the parameters of the Universe by the following formula:

$$\mathbf{G = R_u^3/T_u^2 M_u} \qquad (4)$$

Where: R_u is the radius of the Universe, T_u is the time of the Universe, M_u is the mass of the Universe.

Formula (4) includes the Kepler constant for the Universe R_u^3/T_u^2. Formulas (3) and (4) show that G is a composite constant. A similar equation is valid for the solar system:

$$\mathbf{G = R^3/T^2M_\odot} \qquad (5)$$

Where: R, T are the parameters of the planet's orbit, $M_\odot$ is the mass of the Sun.

Equations (3) - (5) show that the gravitational constant G is directly related to the Kepler constant [15]. The universality of the constant G leads to an interesting connection between the parameters of the electron, the parameters of the solar system and the parameters of the Universe:

$$\mathbf{r_e^3/t_e^2m_eD_o = R^3/T^2M_\odot = R_u^3/T_u^2M_u} \quad (6)$$

Equation (6) lacks a formula that includes the parameters of the galaxy. The universality of the constant G indicates that this equation must include a formula that includes the parameters of the galaxy, represented by the Kepler constant. This equation is valid for all structural objects (X) in the Universe:

$$\mathbf{r_e^3/t_e^2m_eD_o = R^3/T^2M_\odot = \ldots = R^3_x/T^2_xM_x = \ldots = R_u^3/T_u^2M_u} \qquad (7)$$

What is surprising is the presence in the equation of the electron parameters in the same row with the parameters of objects in the Universe.

15. From Hooke's and Newton's verbal formulas to two different laws of gravitation.

Hooke's verbal formula was ignored by scientists both in Newton's time and later. Newton did not accept Hooke's hint and missed the opportunity to give the world an accurate and perfect law of two-body gravitation. Only in 1853 H. Garcet presented Newton's gravitational force as: $\boldsymbol{F} = \boldsymbol{4\pi^2}(R^3/\boldsymbol{T^2})(\boldsymbol{m/r^2})$ [18, 28]. H. Garcet obtained this formula for the special case of a circular orbit. This is indicated by the coefficient $\boldsymbol{4\pi^2}$. Scientists of that time did not pay attention to the fact that this formula is very close to Hooke's verbal formula. Once again, an opportunity was missed to give the world an accurate and perfect law of two-body gravitation, more perfect than Newton's law.

H. Garcet was a professor of mathematics in Paris who published "Leçons nouvelles de cosmographie", a textbook on astronomy. H. Garcet was a cousin of Jules Verne. Jules Verne used his cousin H. Garcet's knowledge of astronomy when writing his books [29].

The path from Hooke's verbal formula to the new law of gravitation of two bodies $F_{HK} = mR^3/T^2r^2$ turned out to be very long (Fig. 6).

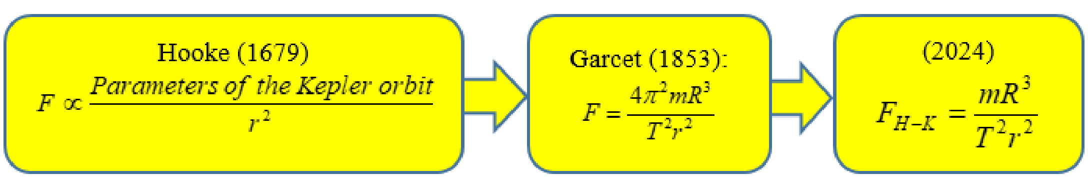

Fig. 6. The path from the verbal formulas of Hooke and Newton to two different laws of gravitation of two bodies.

Thus, Hooke's 1679 - 1680 formula was historically the first verbal formula for the law of universal gravitation. Hooke's 1679 - 1680 verbal formula preceded Newton's 1687 formula. Hooke's verbal formula pointed the way to an accurate law of universal gravitation. The same cannot be said of Newton's verbal formula, which was transformed into an accurate law of universal gravitation only many years later. An accurate law of universal gravitation could have emerged as early as 1679 - 1680 if Newton had taken into account both of Hooke's indications.

Newton gave his verbal formula of the law of gravity 7 years later, in 1687. Newton's verbal formula was not the exact law of gravity. The path from Newton's verbal formula to the law of two-body gravity also turned out to be very long. Newton's verbally formulated law of proportionality $F_N \propto mM/r^2$ turned into the exact law of gravitational interaction of two bodies many years later (Fig. 7). This happened with the appearance of the gravitational constant in the formula ($F = fMm/r^2$ (1803), $F = fMm/r^2$ (1873), $F = GMm/r^2$ (1885)).

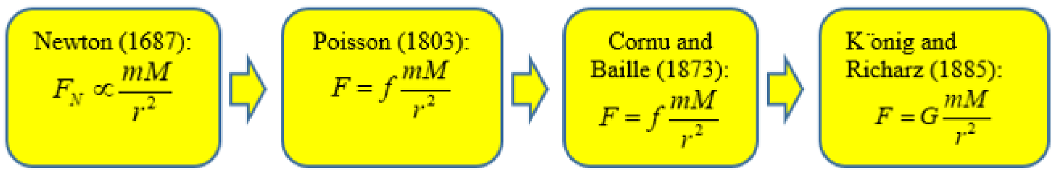

Fig. 7. The path from Newton's verbal formula to the law of two-body gravity.

Even today, the Hooke-Kepler law $F_{H-K} = mR^3/T^2r^2$ is a more perfect law of gravitation than Newton's law $F = GMm/r^2$. This is due to the fact that distances and periods are known from observations with greater accuracy than masses. This is also due to the unsolved problem of obtaining a more accurate value for the gravitational constant G ($G = 6.674\ 30(15) \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$).

The verbal formulas of Hooke and Newton eventually turned into two symbolic formulas for the law of two-body gravitation (Fig. 8). These are two equivalent formulas for the two-body law of gravity in the sense that they both apply to an idealized elliptical orbit.

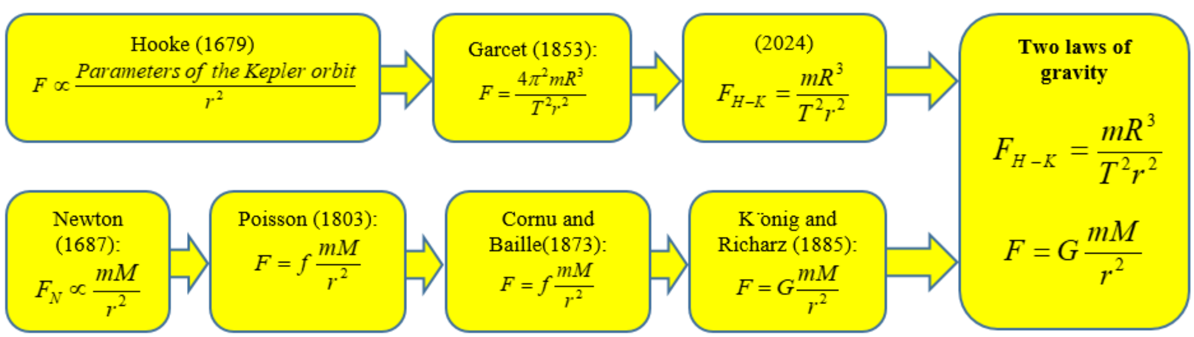

Fig.8. From the verbal formulas of Hooke and Newton to two laws of gravitation.

16. The fundamental secret of gravitation, known to Robert Hooke.

What did Hooke know about gravitation that Newton didn't? Hooke spoke of "universal attraction." He understood gravity as the general attraction of everything to everything else. He distinguished the local gravity of two bodies from general attraction. He knew that the actual orbits of the planets are shaped by the gravitational force of all bodies, not just one gravitating body.

Hooke knew that the law of universal gravitation should be sought in the orbits of the planets. Hooke's verbal formula shows that he knew the path to the law of universal gravitation lay through the use of the parameters of orbital motion in the law of universal gravitation. It was this fundamental secret of gravitation, this knowledge, that motivated him to tirelessly study orbital motion. This was a true scientific breakthrough in the understanding of gravity. Hooke shared this knowledge with Newton. But Newton rejected Hooke's proposal. Newton was seeking a solution to the two-body problem. He used mass, not orbital parameters. However, Hooke sought to derive a law of gravitation for all bodies, not a law of local gravitation for individual bodies. In this respect, his understanding of gravity differed radically from Newton's.

Newton's path objectively could not lead to a law of universal gravitation. In the two-body problem, the gravitational orbit is closed, which does not correspond to the actual orbits of celestial bodies. Even the refined formula $F = GmM/r^2$ is not a law of universal gravitation in the sense of Hooke's understanding of gravity. The formula $F = GmM/r^2$ describes the gravity of only one local source of attraction and does not take into account that bodies are simultaneously attracted to all other bodies in the universe.

We now see that Hooke's verbal formula, when using closed Keplerian orbits, leads to a new law of two-body gravitation: $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$. The Hooke-Kepler law of gravity, $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$, is the complement of the formula $F = GmM/r^2$. This same verbal Hooke's formula, when using real open orbits of bodies, can lead to a new law of universal gravitation.

17. The Hooke-Kepler law of gravity leads to a new law of universal gravitation without the constant G.

In [35], we showed that the Hooke-Kepler law leads to a new law of universal gravitation without the constant G. The insufficient accuracy of the constant G ceases to be a limiting factor in gravitation. The new law of universal gravitation has a two-component structure (Fig. 9).

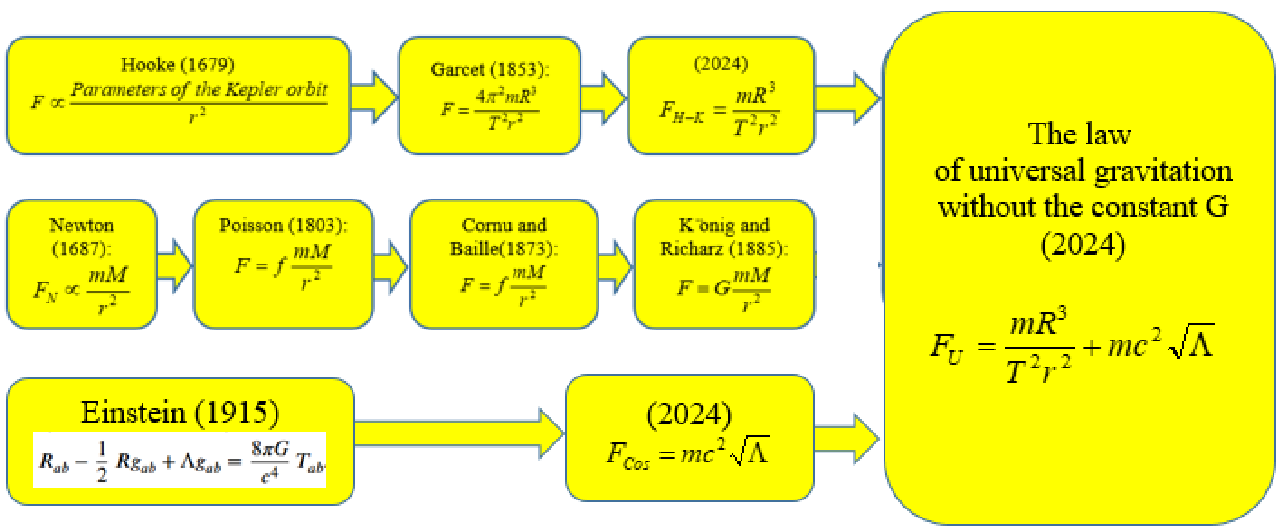

Fig. 9. Two-component law of universal gravitation without constant G. Where: m is the mass of the body, R and T are orbit parameters, r is the distance, c is the speed of light, and Λ is the cosmological constant.

18. The origin of the term: "Law of Universal Gravitation."

Although the law of universal gravitation is today associated with Newton, Hooke arrived at the concept of the "*universality*" of gravity many years before the publication of Newton's Principia.

Hooke was the first to formulate that gravity is a universal force acting on all celestial bodies, holding planets in orbit and determining the shape of their orbits by this universal force.

Hooke first discussed gravity as a universal force that gives celestial bodies their spherical shape in 1665 in his book "Micrographia". In "Micrographia", he does not yet use the precise formulation "*attraction which is universal*," but he lays the foundation for this idea by discussing gravity as a universal property of matter to aggregate into spheres. In Observation VI, Hooke makes a bold assumption for that time: "*...Attraction is inherent not only to the Earth, the Sun, the Moon and the planets, but also to every smallest body in the Universe...*"

In 1674, in his work "An Attempt to Prove the Motion of the Earth from Observations," Hooke already wrote about "*an attraction which is universal*." He asserted that all celestial bodies exert a force of attraction toward their centers, and that they also attract all other celestial bodies. Instead of "*universal gravitation*," he used the term "*universal attraction*."

Above, we showed that Hooke and Newton gave verbal descriptions of different laws of gravitation. Newton solved the two-body problem. Hooke did not speak of a law of two-body gravitation. He understood that in the universe, there is no isolated interaction between two bodies. There is a universal attraction between all bodies in the universe. Hooke understood the key point: the orbits of celestial bodies carry information about the gravitational force of all bodies in the universe. In his letter to Newton, he expected Newton to formulate a universal law of gravitation that takes into account the orbital motion of bodies. But instead of the law of universal gravitation, based on the orbital motion of bodies, Newton posited a two-body law of gravitation based on the mass of the gravitating body.

We now know that the law of two-body gravitation, $\boldsymbol{F = GmM/r^2}$, yields a gravitational force characteristic of a closed elliptical orbit of a celestial body. But this does not correspond to the actual motion of celestial bodies. Real orbits of celestial bodies are not closed. Newton found a solution to the two-body gravitational problem that only approximately relates to the actual gravity of the universe. In the universe, there is no isolated gravitational interaction between two bodies. All bodies in the universe participate in gravitational interaction. This discovery is due to Robert Hooke. The gravitational force of two bodies is always "added" to the gravitational forces of other bodies in the universe. But the law of two-body gravitation does not recognize these additions. The law $\boldsymbol{F = GmM/r^2}$ is applicable only on small scales, where the contribution of the force F to universal gravitation is greatest.

To Newton's credit, he didn't call his proportional relationship $\mathbf{F_N \propto mM/r^2}$ the law of universal gravitation. How then did the laws of two-body gravitation ($\mathbf{F_N \propto mM/r^2}$, $\boldsymbol{F = GmM/r^2}$), which ignore the gravitational pull of all bodies in the universe, come to be called the laws of universal gravitation? How did this confusion come about?

Newton didn't do this. Others did. Roger Cotes, the editor of the second edition of Philosophiae Naturalis Principia Mathematica (1713), did. In the preface, he emphasizes the universality of Newton's law of universal gravitation. Voltaire, in his Philosophical Letters (1734), explained Newton's ideas using Hooke's concept of "universal attraction." Voltaire failed to mention that "universal attraction" is Robert Hooke's term, and it refers not to the law of universal gravitation between two bodies, but to the attraction between all bodies in the universe. Thus, Voltaire also authored the historical injustice that kept Hooke's contribution in the shadows for over three centuries.

Thus, due to Roger Cotes's words, the law of two-body gravitation, $\mathbf{F_N \propto mM/r^2}$, and later $F = GmM/r^2$, was unjustifiably called the law of universal gravitation. However, the true law of universal gravitation, which takes into account the fact that every body in the universe attracts every other body, as Robert Hooke claimed, was never discovered.

Thus, by unjustifiably calling the law of two-body gravitation, $F = GmM/r^2$, the law of universal gravitation, we are not repeating Newton's words, but rather the words of Roger Cotes, who was a great admirer of Newton.

19. Conclusion.

Hooke in his letter offered Newton cooperation. Newton rejected cooperation. He chose competition. In his correspondence with Newton, Hooke gave two hints about the future law of gravitation: the inverse square law and elliptical orbits like Kepler. Such hints of Hooke were supposed to lead to the exact law of gravitation. Newton included the inverse square law in his law of gravitation, but did not include the parameters of the elliptical orbit. Instead of the parameters of the elliptical orbit, he used mass. Newton did not see the decisive role of the parameters of the elliptical orbit and overestimated the role of mass as a parameter in the law of gravitation. As a result, the world received an incomplete and inaccurate law of gravitation in the form of a proportional relationship: $\mathbf{F_N \propto mM/r^2}$. This Newtonian law of gravity existed for a long time in an incomplete and imprecise form until the constant G appeared in it. In those years, nothing prevented Newton from giving the world an exact law of gravitation. At that time, both Kepler's laws and the inverse square law were well known. And most importantly, there was Hooke's hint. Kepler's laws and the inverse square law opened the way to a more exact and mathematically perfect law of gravitation in the form: $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$. But for this, Newton would have to accept both hints that Hooke gave him. In this case, the law of gravitation in the form: $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ would have very little Newtonian content. It would have been Hooke's law. Perhaps Newton's rivalry played its fatal role and prevented him from giving the world the exact and mathematically perfect law of gravitation that Hooke pointed out to him.

The Hooke-Kepler law $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ combines both Hooke's contribution to understanding gravitational forces and Kepler's laws of planetary motion. For this reason, I call it the **"*Hooke-Kepler law*".** This law combines the parameters of the elliptical orbit (**R** and **T**) with the inverse-square law, which states that the force of gravity decreases as the square of the distance between objects.

Hooke and Newton had a dispute over the priority of the inverse-square law in Newton's imprecise and incomplete law $\mathbf{F_N \propto mM/r^2}$. And the exact and mathematically perfect law of gravitation $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$, which also contains the inverse-square law, remained undiscovered. Hooke did not know how close he was to the exact and more valuable law of gravitation, which includes the parameters of the elliptical orbit and the inverse-square law. Newton did not see it either. He included mass in his law instead of the parameters of the elliptical orbit.

In 1853, H. Garcet presented Newton's gravitational force as: $F = 4\pi^2(a^3/T^2)(m/r^2)$ [18, 28, 34]. This formula for gravitational force is completely different from Newton's verbal formula. At the same time, this formula for gravitational force is very close to Robert Hooke's verbal formula. In fact, this was an alternative law of gravitation hinted at by Hooke. Unfortunately, H. Garcet did not see this and did not emphasize it. For the second time, the opportunity to give the world an accurate and mathematically perfect law of gravitation hinted at by Hooke was missed.

After Newton's law was refined and presented as $\mathbf{F = GMm/r^2}$, it became clear how closely the law of gravitation and Kepler's laws were connected. Now a student or even a schoolboy can show the connection between Newton's law and Kepler's laws. It is easy to derive both Newton's law from Kepler's law and Kepler's law from Newton's law. In the derivation, it is easy to see Kepler's constant $\mathbf{R^3/T^2}$. In the derivation, this constant $\mathbf{R^3/T^2}$ appears in the formulas every time and reminds us that its rightful place is in the new law of gravitation $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$.

Hooke saw and predicted the close connection between Kepler's laws and the future law of gravitation even before Newton's law of gravitation was discovered. Even before Newton's law of gravitation was discovered, Hooke told Newton that the law of gravitation must take into account the elliptical orbits of the planets and the inverse square law. This was a hint at a completely different law of gravitation than the one Newton later discovered. It was a hint at the precise and mathematically perfect law of gravitation $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$, not at the approximate law of proportionality $\mathbf{F_N \propto mM/r^2}$ that Newton discovered.

20. Conclusions.

1. With the discovery of Newton's law of gravitation, the illusion arose that the theory of gravity had received the desired and only possible physical law. Such a breakthrough in science turned the heads of scientists for a long time. With the advent of Newton's law, interest in Hooke's verbal formulations faded. There were many attempts to modify Newton's law [7, 30 - 33]. Even the law of inverse squares was questioned. The possibility of the existence of a law of gravity other than Newton's law was not even considered.

2. Researchers of the work of Hooke and Newton did not pay attention to the fact that Hooke and Newton understood the future law of gravity differently. Researchers passed by the fact that Hooke and Newton were moving towards the future law of gravity in different ways. These different approaches and different paths must have led them to two different laws of gravity.

3. There were two different verbal formulations of the law of gravity. One formulation belonged to Newton: the force is proportional to the masses and inversely proportional to the square of the distance. The other verbal formulation belonged to Hooke. Robert Hooke pointed out that the law of gravity should take into account the orbits of the planets and the inverse square law.

4. Different verbal formulas could not refer to the same law of gravitation. They were formulations for two different laws of gravitation. Researchers of the work of Newton and Hooke did not pay attention to the fact that Newton and Hooke spoke about two different laws of gravitation.

5. Using masses as parameters led Newton to a law of proportionality of the form: F N $\propto$ mM/r^2. But the world accepted this as the law of universal gravitation. To turn this proportional formula into an exact equation, the gravitational constant **G** was needed. Newton's law existed in an unfinished form for about 200 years until the constant **G** appeared in it.

6. Hooke's approach should have led to an exact law of gravity of the form: $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$. Robert Hooke was very close to discovering this exact alternative law of gravity. He turned to Newton for help. But Newton did not take advantage of Hooke's second hint.

7. The law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ surpasses in its perfection the law proposed by Newton: $\mathbf{F_N \propto mM/r^2}$. The Hooke-Kepler law surpasses in its perfection the law of gravity containing the

constant **G** ($F_N = GMm/r^2$), since distances and periods are known from observations with greater accuracy than mass and the constant **G**.

8. If Newton had accepted both of Hooke's hints, the world would have received the exact and perfect law of gravity $\mathbf{F_{H\text{-}K} = mR^3/T^2r^2}$ back **in 1687**. Nothing stood in the way. At that time, both Kepler's laws and the inverse square law of Bullialdus were well known. Hooke's verbal formulations gave the necessary hint. But Newton used mass in the law of gravity instead of the parameters of an elliptical orbit. In 1687, a real opportunity to discover the revolutionary law of gravity $\mathbf{F_{H\text{-}K}=mR^3/T^2r^2}$ was missed.

21. Final Remarks.

1. Scholars of Hooke and Newton's work have not given due consideration to the fact that in the history of science, there were two different verbal formulas for the law of universal gravitation. One was Hooke's, the other Newton's. Chronologically, Hooke's verbal formula was first.

2. The peculiarity of Hooke's discovery, presented in his verbal formulations, is that he combined two ideas into a single system:

2.1. The force of attraction decreases proportionally to the square of the distance $1/r^2$: *«...my supposition is that the Attraction always is in a duplicate proportion to the Distance from the Center Reciprocal...»*. (Letter from Hooke to Isaac Newton, January 6, 1680).

2.2. Under the influence of gravity, planets move in an orbit close to an ellipse: «*... and consequently that the Direction will be a Curve Line very near resembling an Ellipsis»*. (Letter from Hooke to Isaac Newton, January 6, 1680).

3. Researchers of Robert Hooke's works have traditionally paid attention to the inverse square law. Much less attention was paid to Hooke's emphasis on the connection between gravity and the orbital motion of planets [36]. A dependence of the form ($1/r^2$) is characteristic of an elliptical orbit, as Newton demonstrated. However, in the law of gravitational force, a parameter of the form ($1/r^2$) does not replace the parameters of an elliptical orbit, but is an additional parameter, as Robert Hooke pointed out. In his letters to Newton, Hooke gave a verbal formula for the real law of universal gravitation. To implement Hooke's verbal formulation, Newton only needed to: *a) introduce the inverse-square law into the law of gravitation; b) introduce orbital parameters instead of mass into the law of gravitation.* Surprisingly, despite Kepler's laws being known at the time, Newton did not do this. Instead of the law of universal gravitation, Newton gave a verbal formula for the proportional relationship for the gravitation of two bodies. As a result, both the real law of universal gravitation and the second law of two-body gravitation: $F = mR^3/T^2r^2$, which is more perfect than Newton's law, remained undiscovered.

4. Hooke's emphasis on the connection between the force of universal gravitation and orbital motion was a true scientific breakthrough for its time. Hooke's verbal formulation combined both the special case of a closed elliptical orbit (gravity of two bodies) and the general case of non-closed orbits (gravity of all bodies). In the particular case of an idealized closed elliptical orbit, Hooke's verbal formula leads to a new law of gravitation: $F = mR^3/T^2r^2$. This is the second law of gravity, more accurate than Newton's law. The same verbal formulation by Hooke for open orbits led to the real law of universal gravitation: $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$. The law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ takes into account the gravity of all bodies in the Universe and the accelerated

expansion of the Universe. The law of universal gravitation $F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}$ does not contain the gravitational constant G.

5. Newton used part of Hooke's verbal formula (the inverse-square law) and went to considerable lengths to obscure Hooke's contribution. Hooke's verbal formula, in its full formulation, was forgotten for more than 300 years, although it contained the key to the real and exact law of universal gravitation.

6. Newton, in his verbal formula of the law of gravitation in 1687, indicated that the force is proportional to the masses of the bodies and inversely proportional to the square of the distance: $F_N \propto mM/r^2$. This was not the law of universal gravitation. It was not even a complete law of two-body gravitation. Newton's verbal formula was refined and only many years later transformed into the complete law of gravity $F = GmM/r^2$. This was done by Poisson (1803), Cornu and Baille (1873), K¨onig and Richarz (1885). The path that Newton chose did not lead to the law of universal gravitation, it was the path to the law of gravitation of two bodies.

7. In Proposition XV of Book I «Philosophiae Naturalis Principia Mathematica», Newton proved that if the force is inversely proportional to the square of the distance ($1/r^2$), then the squares of the revolution times in elliptical orbits are proportional to the cubes of their major axes. Mathematically, this means that the ratio R^3/T^2 is a constant for all bodies orbiting the same center of force. It remains a mystery why, with such a proof, Newton did not use the orbital parameters (R^3/T^2) in the law of gravitation along with the inverse square law ($1/r^2$). The law of gravitation $F = mR^3/T^2r^2$ could have been discovered as early as 1687! Essentially, the force had to be expressed using Kepler's law and the inverse square law. This is precisely what Hooke pointed out in his letter. We note that the law of gravitation $F = GmM/r^2$, due to the low accuracy of the constant G, is still inferior to the law of gravitation $F = mR^3/T^2r^2$. Perhaps Newton was hampered by his competition with Hooke, and in the law of gravity $F = mR^3/T^2r^2$ everything is Hookean, and there is no Newtonian mass.. Perhaps Newton deliberately distanced himself from Hooke's formulations out of personal animosity and wanted the law of gravitation to be presented as his own achievement, rather than as an "improvement" on Hooke's hypotheses or Kepler's observations.

8. Newton's use of masses as parameters in the law of gravitation leads to ambiguity. The actual trajectories of planetary motion are formed by the gravitational forces of all bodies in the universe. The same trajectory can be explained by different combinations of masses and density distributions. There is an infinite number of force fields that can generate the same orbit. Hooke knew that the force of universal gravitation was "*encoded*" in the actual trajectories of planetary motion. By emphasizing masses instead of orbits, Newton was doomed to produce a law of local gravitation, not a law of universal gravitation.

9. Hooke's verbal formula and Newton's verbal formula are not contradictory. They represent two different paths to the same goal. The laws of gravitation that follow from Hooke's and Newton's verbal formulas, although different, complement each other in practical applications. In the presence of an orbit, gravitational interaction is more accurately described by the law $F = mR^3/T^2r^2$. This is a convenient way to calculate the force when the mass of the central object is unknown. With a known mass of the central object, the law $F = GmM/r^2$ describes gravitational interaction both with and without orbital motion. The limiting factor here is the low accuracy of the constant G.

10. Ignoring and forgetting Hooke's verbal formulation and the substitution of terms led to the fact that two fundamental laws of gravity remained undiscovered in the theory of gravity: $F = mR^3/T^2r^2$, $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$. Among them is the real law of universal gravitation $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$, the path to which was paved by Robert Hooke 7 years before the publication of the "Principia".

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