A Modified Born-Infeld Model of Electrons Leads to a Modified Zitterbewegung of Electrons

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Abstract

Recently, a classical model of electrons has been proposed, which represents the peak of a rotating field solution of a modified Born-Infeld field theory by a point-like particle of half the mass of the whole field solution. While this model is consistent with many features of electrons, it does not include the theoretical frequency of an electron's Zitterbewegung, which is a central part of many other models of spinning point-like electrons. In an attempt to explain the absence of this frequency in the new model, this work hypothesizes that the Hamiltonian of a free electron in Dirac's theory of electrons could be modified by halving the electron's mass when describing its spin motion. This leads to a modified frequency (equal to the Compton frequency), which is consistent with not only the modified Born-Infeld model of electrons but also de Broglie's internal clock hypothesis.

1 Introduction

Zitterbewegung (literally "trembling motion") is a rapid oscillation at twice the Compton frequency in specific operators of Dirac's theory of electrons [Sak67]. Although it was identified already in 1928 by Gregory Breit [Bre28], there is no commonly accepted experimental evidence for it yet. While Zitterbewegung is often assumed to cause the Darwin term in atomic physics [Sak67], this term could also be caused by oscillations of different frequencies. Speculations about the reasons for this lack of experimental evidence include hypothesized relations to de Broglie's internal clock [dB25], the assumption that Zitterbewegung is too rapid to be observable (a curious assumption considering that de Broglie's internal clock is of a similar frequency and was proposed specifically to explain experimental observations), and speculations that Zitterbewegung is an artifact of assumptions and/or approximations of Dirac's theory of electrons. While Zitterbewegung at twice the Compton frequency is an undeniable part of Dirac's theory of electrons, there is no consensus on whether it is necessarily a part of all sufficiently elaborate models of electrons.

Thus, some models of spinning electrons oscillate at twice the Compton frequency as suggested by Zitterbewegung, for example the neo-classical model by Beck [Bec23], while other models of spinning electrons oscillate at the Compton frequency as suggested by de Broglie's internal clock hypothesis, for example the classical model of point-like electrons [Kra24a] that is based on a modified Born-Infeld field theory [Kra23]. Since the latter model is based in part on Beck's model, the reason for the different spin frequencies is particularly clear in a comparison of these two models. Specifically, Beck's model assumes that the whole mass of an electron is concentrated in one point, whereas the model based on modified Born-Infeld field theory assumes that the mass of an electron is spread out such that a spinning point-like particle of half the mass is a more accurate effective representation of the mass distribution of a spinning electron. It is this factor of $\frac{1}{2}$ that results in the ratio of 1:2 between the spin frequencies of these models.

Therefore, the main hypothesis of this work is that different assumptions about the mass distribution of an electron might also help to explain why the Zitterbewegung's frequency in Dirac's theory of electrons differs from the frequency of the modified Born-Infeld model of electrons and, more generally, de Broglie's internal clock hypothesis. Note that differing assumptions about this mass distribution might be perfectly valid as long as there are no convincing experimental results in support of one of these assumptions.

The next section summarizes related work. Section 3 presents a modified Zitterbewegung based on the assumption that an electron's mass should be halved when describing its spin motion as suggested by a modified Born-Infeld model of electrons [Kra23]. Section 4 discusses the results of Section 3, while Section 5 concludes this work.

2 Previous Work

2.1 Modified Born-Infeld Model of Electrons

The modified Born-Infeld model of electrons is based on a Lagrangian density $\mathscr{L}$:

$$\mathscr{L} \stackrel{\text{def}}{=} \frac{b^2}{\mu_0}\left(1 - \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}\right) \tag{1}$$

with the Born-Infeld parameter b specifying the maximum magnetic field strength, the vacuum permeability μ_0, and the electromagnetic four-potential $A = (V/c, \mathbf{A})$ [Kra24c].

The corresponding Euler-Lagrange equations were solved numerically in previous work [Kra23] resulting in a rotating field solution with a peak moving at the speed of light c on a circular orbit with radius $\hbar/(m\,c)$; i.e., the reduced Compton wavelength. Thus, the angular frequency of this circular motion is $m\,c^2/\hbar$; i.e., the angular Compton frequency as required by de Broglie's internal clock hypothesis [dB25]. At large distances, the solution appears to show the same Lorentz-type interaction with electromagnetic fields as relativistic electrons [Kra24b].

In more recent work [Kra24c], the linear momentum of the rotating (but globally resting) field solution was computed by numerically evaluating elements of the field's canonical stress-energy tensor. The resulting momentum points in the direction of the velocity of the peak of the rotating field solution and has an absolute value close to $m\,c/2$. The intrinsic angular momentum of the solution was evaluated similarly and appears to match the spin of electrons $\hbar/2$.

2.2 Classical Model of Point-Like Electrons Based on the Modified Born-Infeld Model of Electrons

The mentioned linear momentum and angular momentum of the field solution of the modified Born-Infeld model of electrons are consistent with a hypothetical particle of mass $m/2$ moving at the speed of light c on the same orbit as the peak of the field solution. Thus, the motion of such a particle could represent the linear momentum and angular momentum of a much more complex rotating field solution. This observation motivated a classical model of point-like electrons [Kra24a] that describes the motion of the modified Born-Infeld model of electrons as the sum of a "local spin motion" of a spinning point-like particle and a "global motion" of its spin center as suggested by Beck's neo-classical model [Bec23].

The key difference to Beck's model is that the electron's mass is reduced to $m/2$ in all equations related to the local spin motion while it is kept at m in all equations related to the global motion of the whole electron. As mentioned, these differences are motivated by and consistent with the computed linear momentum and angular momentum of the rotating field solution of the modified Born-Infeld model of electrons as well as the motion of its peak and the global motion of the whole field solution in external fields. Furthermore, the new model is consistent with de Broglie's internal clock hypothesis. However, the model lacks any occurrence of the frequency of Zitterbewegung, which is discussed next.

2.3 Zitterbewegung in Dirac's Theory of Electrons

As mentioned, Zitterbewegung is a rapid oscillation in specific operators of Dirac's theory of electrons [Sak67]. For the purpose of this work, it is sufficient to consider the case of a free electron. In the notation of Sakurai [Sak67, Section 3.6], the Hamiltonian H for a free electron in Dirac's theory is

$$H = c\,\alpha_j p_j + \beta\,m\,c^2 \tag{2}$$

with the speed of light c, the mass m of the particle, its momentum $\mathbf{p} = (p_1, p_2, p_3)$, an implicit summation over j, and the 4×4 matrices β and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ [Sak67, Section 3.2]:

$$\beta = \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \qquad \alpha_k = i\gamma_4\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix} \tag{3}$$

where $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4}$$

and I is the 2×2 identity matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{5}$$

$\boldsymbol{\alpha}$ is of particular interest because it "is the velocity in units of c" [Sak67, page 115] since

$$\dot{x}_k = \frac{i}{\hbar}[H, x_k] = \frac{i\,c}{\hbar}[\alpha_j p_j, x_k] = c\,\alpha_k. \tag{6}$$

The time derivative $\dot{\alpha}_k$ is computed by Sakurai [Sak67, page 116] as follows:

$$\begin{aligned} \dot{\alpha}_k &= \frac{i}{\hbar}[H, \alpha_k] & (7)\\ &= \frac{i}{\hbar}\left(-2\alpha_k H + \{H, \alpha_k\}\right) & (8)\\ &= \frac{i}{\hbar}\left(-2\alpha_k H + 2c\,p_k\right). & (9) \end{aligned}$$

Interpreting this equation as a differential equation for $\alpha_k(t)$ and solving it for the initial value $\alpha_k(0)$ results in:

$$\alpha_k(t) = c\,p_k H^{-1} + \left(\alpha_k(0) - c\,p_k H^{-1}\right) e^{-2i\,H\,t/\hbar}. \tag{10}$$

Multiplication by c and another integration step yields the coordinate $x_k(t)$:

$$x_k(t) = x_k(0) + c^2 p_k H^{-1} t + \frac{i\,c\,\hbar}{2}\left(\alpha_k(0) - c\,p_k H^{-1}\right) H^{-1} e^{-2i\,H\,t/\hbar}. \tag{11}$$

For an eigenstate of the Hamiltonian H to the eigenvalue E, the exponential $e^{-2i\,H\,t/\hbar}$ in these operators represents an oscillation at the angular frequency $2|E|/\hbar = 2\,m\,c^2/\hbar$, which is called "Zitterbewegung" because it "appears to imply that the free electron executes very rapid oscillations in addition to the uniform rectilinear motion" [Sak67, page 117]. (For some specific states, these oscillations are absent as discussed by Sakurai [Sak67, Section 3.7].)

The factor $\frac{i\,c\,\hbar}{2}H^{-1}$ may be used to estimate the maximum amplitude of this Zitterbewegung by setting $H^{-1} = \frac{1}{E}$ and taking the absolute value. This results in an estimated amplitude of $\frac{c\,\hbar}{2\,|E|} = \frac{\hbar}{2\,m\,c}$; i.e., half the reduced Compton wavelength.

3 Modified Zitterbewegung

While Zitterbewegung is a well established part of Dirac's theory of electrons, it lacks generally accepted experimental confirmation. One potential reason for this lack of experimental evidence might be that calculations like the one presented in Section 2.3 take Dirac's theory of electrons beyond the limits of its applicability. One of these limits (nowadays known as "Schwinger limit") is caused by too strong electromagnetic fields as noted already in 1931 by Sauter [Sau31]. Sauter estimated that the field strength of the classical electric field of a point-like elementary charge reaches a critical limit at a distance of about 8×10^{-14} m, which corresponds to about one-third of the amplitude of an electron's Zitterbewegung calculated in Section 2.3. Therefore, an electron's own electromagnetic field might already be strong enough to affect predictions of Dirac's theory at this length scale. In particular, the calculated angular frequency $2\,m\,c^2/\hbar$ might be correct only for ideal "Dirac electrons," but it might be incorrect for observable electrons in the real world.

As mentioned in Section 1, the present work aims to improve the calculation of this angular frequency for real-world electrons by replacing the assumption that the mass of an electron is concentrated in one point. To this end, the same approach is employed as in the model of point-like electrons [Kra24a] that was summarized in Section 2.2; i.e., the "local spin motion" is modeled by a point-like particle of half the mass of the whole electron, while the "global motion" is represented by the motion of a point-like particle of the full mass. As mentioned in Section 2.2, this approach is motivated by the motion of the peak of the field solution of the modified Born-Infeld model of electrons, which is discussed in Section 2.1.

As shown in Section 2.3, the angular frequency of an electron's Zitterbewegung may be calculated with the Hamiltonian H of a free electron of mass m and linear momentum $\mathbf{p} = (p_1, p_2, p_3)$:

$$H = c\,\alpha_j p_j + \beta\, m\, c^2. \tag{12}$$

Since $\mathbf{p}$ is constant for a free electron, the "global motion" is considered a straight trajectory. Thus, the following calculation describes this free motion plus a "local spin motion" of a particle of mass $\tilde{m}$, which is set to half the mass m of an electron; i.e., $\tilde{m} = m/2$. Therefore, the linear momentum $\tilde{\mathbf{p}}$ of this spinning particle is half the linear momentum $\mathbf{p}$ of the whole electron; i.e., $\tilde{\mathbf{p}} = \mathbf{p}/2$. Thus, the Hamiltonian $\tilde{H}$ of the spinning particle is just half the Hamiltonian H of the whole electron:

$$\tilde{H} = c\,\alpha_j \tilde{p}_j + \beta\, \tilde{m}\, c^2 = c\,\alpha_j \frac{p_j}{2} + \beta\, \frac{m}{2}\, c^2 = \frac{1}{2} H. \tag{13}$$

The time derivative of position $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ of the spinning particle is related to the $\boldsymbol{\alpha}$ matrices in analogy to Eq. (6):

$$\dot{\tilde{x}}_k = \frac{i}{\hbar}[\tilde{H}, \tilde{x}_k] = \frac{i\,c}{\hbar}[\alpha_j \tilde{p}_j, \tilde{x}_k] = c\,\alpha_k, \tag{14}$$

which is why the Zitterbewegung of the spinning particle may be analyzed analogously to the calculation in Section 2.3.

Specifically, the time derivative $\dot{\tilde{\alpha}}_k(t)$ is given by:

$$\begin{aligned} \dot{\tilde{\alpha}}_k(t) &= \frac{i}{\hbar}[\tilde{H}, \tilde{\alpha}_k(t)] && (15)\\ &= \frac{i}{\hbar}\left(-2\tilde{\alpha}_k(t)\tilde{H} + 2c\,\tilde{p}_k\right) && (16)\\ &= \frac{i}{\hbar}\left(-\tilde{\alpha}_k(t) H + c\, p_k\right). && (17) \end{aligned}$$

And the solution $\tilde{\alpha}_k(t)$ for the initial value $\tilde{\alpha}_k(0)$ is:

$$\tilde{\alpha}_k(t) = c\, p_k H^{-1} + \left(\tilde{\alpha}_k(0) - c\, p_k H^{-1}\right) e^{-i\,H\,t/\hbar}. \tag{18}$$

Multiplication by c and another integration step yields the coordinate $\tilde{x}_k(t)$:

$$\tilde{x}_k(t) = \tilde{x}_k(0) + c^2 p_k H^{-1} t + i\, c\,\hbar\left(\tilde{\alpha}_k(0) - c\, p_k H^{-1}\right) H^{-1} e^{-i\,H\,t/\hbar}. \tag{19}$$

Thus, the modified Zitterbewegung of $\tilde{\alpha}_k(t)$ and $\tilde{x}_k(t)$ are oscillations at the angular frequency $|E|/\hbar = m\,c^2/\hbar$; i.e., the electron's angular Compton frequency. The maximum amplitude of the oscillation of $\tilde{x}_k(t)$ is $\frac{c\,\hbar}{|E|} = \frac{\hbar}{m\,c}$; i.e., the reduced Compton wavelength.

4 Discussion

Compared to the standard Zitterbewegung of "ideal Dirac electrons" (as described in Section 2.3), the modified Zitterbewegung presented in Section 3 predicts oscillations at half the frequency and twice the amplitude. However, it is unclear whether these changes lead to currently observable deviations from the predictions of Dirac's theory for various reasons:

1. There is no consensus on whether it is possible to directly observe the Zitterbewegung of electrons (in particular its frequency).

2. Indirect effects that are attributed to the Zitterbewegung of electrons (e.g., the Darwin term in atomic physics) might also be caused by the modified Zitterbewegung.
3. In particular, most electromagnetic interactions of the whole electron with external electromagnetic fields affect only the "global motion" of the electron, which is not changed by the proposed modifications.

This is not to say that experimental tests of the proposed modified Zitterbewegung are impossible. In fact, any direct observation of the frequency of an electron's Zitterbewegung is likely to be sufficient to decide whether the proposed modification is, in fact, an improvement.

Since the frequency of the modified Zitterbewegung is equal to the Compton frequency and, therefore, equal to the frequency of de Broglie's hypothetical internal clock, another possibility is that the modified Zitterbewegung and/or its effects have already been experimentally observed but were not correctly identified.

5 Conclusion

This work proposes a modification of the calculation of the frequency of electrons' Zitterbewegung in Dirac's theory of electrons. This modification is based on a classical model of point-like electrons [Kra24a], which assumes that the mass of an electron is spread out in a specific way instead of being concentrated in one point. The resulting frequency of the modified Zitterbewegung is half the frequency of the standard Zitterbewegung and, therefore, equal to the Compton frequency, which is also the frequency of de Broglie's internal clock hypothesis. If true, this modified frequency has implications for observations of Zitterbewegung; including the possibility that its effects have already been observed.

References

[Bec23] James L. Beck. Neo-classical relativistic mechanics theory for electrons that exhibits spin, zitterbewegung, dipole moments, wavefunctions and Dirac's wave equation. *Foundations of Physics*, 53(57), 2023.

[Bre28] Gregory Breit. An interpretation of Dirac's theory of the electron. *Proceedings of the National Academy of Sciences*, 14(7):553–559, 1928.

[dB25] Louis de Broglie. Recherches sur la théorie des Quanta. *Annales de Physique*, 10(3):22–128, January 1925. English translation by A. F. Kracklauer.

[Kra23] Martin Kraus. A modified Born-Infeld model of electrons and a numerical solution procedure. viXra:2312.0149, 2023.

[Kra24a] Martin Kraus. A modified Born-Infeld model of electrons as foundation of a classical model of point-like electrons. viXra:2411.0048, 2024.

[Kra24b] Martin Kraus. A modified Born-Infeld model of electrons featuring a Lorentz-type force. viXra:2401.0101, 2024.

[Kra24c] Martin Kraus. A modified Born-Infeld model of electrons with intrinsic angular momentum. viXra:2403.0005, 2024.

[Sak67] Jun John Sakurai. *Advanced Quantum Mechanics*. Addison-Wesley Publishing Company, Reading, Massachusetts, 1967.

[Sau31] Fritz Sauter. Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs. *Zeitschrift für Physik*, 69:742–764, 1931.

A Revisions

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