Hopf Fibrations, Non-Homotopy and Mass

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Abstract

The Standard Model treats particle rest mass as a free parameter, offering no fundamental explanation. An alternative approach is proposed that derives rest mass from geometric and topological constraints. A 3-sphere intersecting three-dimensional space is considered the foundation of mass, where the intersection forms a Hopf fibration. Resistance to force arises from non-homotopy. Rest mass is an emergent property of topology rather than an intrinsic quantity. Despite this exotic framework, rest mass remains a form of Newtonian inertial mass. This approach eliminates the need for the Higgs field and quark confinement, replacing them with a unified mathematical structure for baryon rest mass. The intersection's topological signature is evident in mass-splitting relationships, where lighter hyperons are functions of the proton, neutron, and electron masses. Derived mass values for Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, Ω^-, Λ^0 are within experimental uncertainty. The mathematical structure of Σ and Ξ baryons naturally resolves the neutron-proton mass difference problem. Further refinements reduce the number of free parameters from ten to just two — the neutron and electron masses. Finally, we examine the mechanism of energy dissipation. In the case of the proton, the dissipation factor is found to equal the electron mass, suggesting a deeper connection between charge, mass, and energy release. These results indicate baryon rest mass follows from fundamental geometric constraints.

Keywords— rest mass, inertial mass, Hopf fibration, non-homotopy.

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1 Introduction.

The problem of baryon rest mass is typically addressed within the Standard Model, where mass generation involves spontaneous symmetry breaking and confinement mechanisms. However, baryon rest mass remains an intractable issue. In the Standard Model, mass arises from Lie group symmetry breaking via the Higgs mechanism, with quark masses determined by empirical Yukawa couplings.[1, 2, 3] However, the Higgs field interacts only weakly with the strong force, meaning over 99% of baryon mass originates from quark confinement energy. Since Yukawa couplings are experimentally determined rather than derived from first principles, and since quark confinement remains analytically intractable, the Standard Model is generally unable to predict why particles have the specific masses they do.

This paper takes a different approach to the baryon rest mass problem. Instead of treating mass as a free parameter linked to Higgs interactions, this framework interprets mass as an emergent property of topological constraints. The approach also differs from conventional quantum chromodynamics (QCD) where baryon mass is attributed to gluonic energy and quark interactions. The central thesis is that baryon rest mass is a form of inertial mass that arises from topological differences between field and particle. The topological framework replaces the Higgs field and quark confinement.

In Section 2, we examine the intersection of a 3-sphere and three-dimensional space, a situation described by a Hopf fibration. When a force interacts with a Hopf fibration, non-homotopy prevents smooth deformations, generating resistance to acceleration. Rest mass is therefore a consequence of interactions between force and 3-sphere constrained by the Hopf fibration.

In Section 3, a mathematical treatment of this idea shows intersection mass includes an additional component, which we call "hypermass." The hypermass signature provides a means to test the theory.

In Section 4 a mathematical framework is further developed with lighter hyperon rest mass expressed as a function of the electron, proton, and neutron masses. Incorporating the hypermass signature, these functions yield rest mass values for the Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, Ω^-, and Λ^0 baryons (Eqs. 1, 2, 3, 4, 7, 13, 15), all within experimental uncertainty. Eqs. (6) and (8) demonstrate the framework is non-arbitrary whilst establishing invariant relationships between mass components. Additionally, Eq. (8) provides an elegant resolution to the neutron-proton mass difference problem.

The mathematical frameworks leads to a scaling operator S_M introduced in Section 5. S_M rescales rest masses into dimensionless magnitudes that resemble MeV values. The resemblance requires a new approach to inertial mass dimensionality, which is developed through a synthesis of Gaussian and SI unit systems.

Finally, Section 6 examines how energy exits an intersection, finding that the dissipation factor for a proton coincides with the electron mass. While this hints at deeper connections between mass, charge, and topology, the present analysis focuses strictly on addressing the problem of mass.

2 Hopf Fibration and Mass

Instead of invoking the Higgs scalar field, this theory assumes an $\mathbb{R}^3$ vector space (3-space) with a continuous, smooth, and contractible topology. We also consider the intersection of a 3-sphere and 3-space. A situation described by a the Hopf fibration.

The Hopf fibration is a fundamental topological structure mapping the 3-sphere (S^3) onto the 2-sphere (S^2) , with circle fibres (S^1) constraining the transformation. The Hopf fibration is a mapping:

$$\pi : S^3 \to S^2$$

where each point on S^3 is mapped to a point on S^2, and an entire circle fiber (S^1) of points in S^3 collapses onto that same point. In explicit terms, we describe S^3 using two complex coordinates:

$$S^3 = \{(z_1, z_2) \in \mathbb{C}^2 \mid |z_1|^2 + |z_2|^2 = 1\}$$

where z_1 and z_2 satisfy the unit norm condition. The Hopf map projects these onto S^2 using:

$$\pi : (z_1, z_2) \in S^3 \mapsto (2z_1\bar{z}_2, |z_1|^2 - |z_2|^2) \in \mathbb{C} \times \mathbb{R}.$$

Since $\mathbb{C} \times \mathbb{R}$ is equivalent to a 2-sphere, this provides a projection from S^3 to S^2.

A key feature of the Hopf fibration is that transformations in S^2 cannot always be extended smoothly to S^3. This topological obstruction, known as non-homotopy, prevents continuous deformations between fibre bundles. This obstruction is the mechanism by which the intersection resists acceleration due to an external force.

Baryon rest mass arises when an external force, unable to continuously deform the system, undergoes a discrete transition to act on S^3 through the S^2 projection. Baryon rest mass is therefore a form of inertial mass. The explanation blends a Newtonian inertial frame with geometry. As a force reaches the 3-space boundary to act on the 3-sphere, it is seen to dissipate in the ratio $F \cdot \frac{2\text{-}sphere}{3\text{-}sphere}$ over the totality of the 2-sphere. Where the 2-sphere is exposed to relativistic effects this ratio is not constant. However, the 3-sphere remains a constant denominator. For each point on the 2-sphere the dissipation is in the ratio $F \cdot \frac{2\text{-}sphere}{3\text{-}sphere}$ divided by the $2\text{-}sphere$, i.e. $\frac{F}{3\text{-}sphere}$. Despite dissipation, the force still results in an acceleration, given by $a = \frac{F}{3\text{-}sphere}$. Applying Newton's second law, $\frac{F}{a} = M$, we identify the 3-sphere as the measure of the intersection's rest mass.

3 The Origin of Hypermass

To establish a structured approach to baryon rest mass, we first introduce five fundamental equations describing mass-related quantities at an intersection. The first fundamental equation defines mass as a function of geometric constraints:

$$M = 2\pi^2 r^3$$

where r represents the characteristic radius of the mass distribution. The spatial volume occupied by the intersection follows the standard formula for a three-dimensional sphere:

$$V = \frac{4\pi}{3} r^3.$$

The density of the system, defined as the ratio of mass to volume, takes the form:

$$\rho = \frac{3\pi}{2}.$$

This provides an intrinsic numerical factor that will play a role in later mass relations. The third equation introduces hypermass (H), a quantity that captures the deviation of mass from volume:

$$H = M - V.$$

This difference suggests that mass is not solely determined by spatial volume but involves an additional contribution. A crucial relationship, which we refer to as the H-signature, follows from these definitions. It expresses mass in terms of hypermass and density:

$$M = H\left(\frac{\rho}{\rho - 1}\right).$$

This equation reveals that hypermass plays a direct role in determining mass values through a well-defined scaling factor. The H-signature serves as a constraint on mass values, ensuring that hypermass follows a structured relationship with mass and density. This framework provides a basis for deriving mass relationships that will be tested in the next section.

4 H-signatures found in the rest mass data.

In this section, we test whether the theoretical H-signatures appear in experimental baryon rest mass data. Their presence would support the hypothesis that this framework is non-arbitrary and that hypermass plays a fundamental role in structuring baryon masses. Eqs. (6, 7, 8) reveal distinct H-signatures in the Σ baryon mass values using the 2022 CODATA values for proton and neutron mass: $M_p = 938.272\,089\,43$ MeV, $M_n = 939.565\,421\,94$ MeV.[4]

$$M_{\Sigma^+} = (2M_p - M_n)\left(\frac{\rho}{\rho - 1}\right) \approx 1189.3712. \tag{1}$$

$$M_{\Sigma^0} = M_n\left(\frac{\rho}{\rho - 1}\right) \approx 1192.6546. \tag{2}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)\left(\frac{\rho}{\rho - 1}\right) \approx 1197.5797. \tag{3}$$

Eq. (1) precisely matches the Particle Data Group (PDG) fit for M_{Σ^+} (1189.37 $\pm$0.07).[5] The PDG fit for M_{Σ^0} is 1192.642 $\pm$0.024. Eq. (2) is particularly close to Wang 1192.65 $\pm$0.020.[6] However, Eq. (3) deviates by more than four standard deviations from the current PDG fit (1197.449 $\pm$ 0.030). While this suggests that refinements may be necessary, it is important to note that the PDG fit is influenced by three independent results, two of which systematically lower the reported value. Schmidt (1197.43) and Gurev (1197.417) yield values that are significantly lower than the one derived here.[7, 8] The Schmidt result dates back to 1965, and the Gurev value serves primarily as a proof of method. Notably, M_{Σ^-} lies within one standard deviation of Gall (1197.532 $\pm$ 0.057),[9] suggesting suggesting the value derived here is within the range of experimental variability.

Next, we examine whether Eqs. (4, 5) correctly predict Ξ rest masses. The fact that volume subtraction appears reinforces the idea that these are hypermasses.

$$M_{\Xi^0} = M_{\Sigma^0}\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1314.8104. \tag{4}$$

$$M_{\Xi^-} = M_{\Sigma^-}\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1321.0622. \tag{5}$$

M_{Σ^0} is within one standard deviation of the PDG fit (1314.86 ± 0.20) and close to Fanti (1314.82 $\pm$0.06)[10]. Eq. (6) follows naturally from Eqs. (2, 3, 4, 5) establishing they are integral to the non-homotopic thesis.

$$\frac{M_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} = \frac{\rho}{\rho - 1}. \tag{6}$$

However, Eq. (6) leads to an undefined solution, requiring an additional correction. This issue provides a deeper insight into the structure of mass adjustments.

The present PDG fit for M_{Ξ^-} (1321.71 ± 0.07) draws on a 2006 study of 4.8k events.[11] Given an improbable nine-standard-deviation discrepancy, Eq. (7) introduces the electron mass energy equivalent ($M_e = 0.510\ 998\ 950\ 69$ MeV, 2022

CODATA). Whilst this may initially appear to be an arbitrary fudge factor, the correction has a deeper physical significance. To keep track an asterisk * identifies formulae that include the electron mass adjustment.

$$M^*_{\Xi^-} = (M_{\Sigma^-} + M_e)\left(\frac{\rho}{\rho - 1}\right) - V_p \approx 1321.7109. \tag{7}$$

Eqs. (2, 3, 4, 7) trivially resolve (8) providing a solution to the neutron - proton mass difference problem. $M^*_{\Sigma^-}$ averts the threat of infinity due to Eq. (6).

$$M_p + \frac{M_e}{3\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{\rho}{\rho - 1}\right)} = M_n. \tag{8}$$

Eq. (8) suggests a direct connection between Σ and Ξ mass energies in the proton-neutron transformation, implying a deeper relationship with weak force interactions. Eqs. (9, 10, 11) shuffle Ξ adjustments affirming all three Σ rest masses and with electron correction are integral to the same mathematical framework. This equivalence also hints that weak force interactions may play a role in setting the Σ baryon mass. (A $\frac{3}{4}M_e$ adjustment is indicated by the diamond superscript$^\diamond$).

$$M^*_{\Sigma^+} = \left(M_p + \frac{M_e}{3\left(\frac{M_{\Xi^-} - M^*_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{\rho}{\rho - 1}\right)}\right)\left(\frac{\rho}{\rho - 1}\right) = Eq.\,(6). \tag{9}$$

$$M^*_{\Sigma^0} = \left(M_p + \frac{M_e}{3\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{\rho}{\rho - 1}\right)}\right)\left(\frac{\rho}{\rho - 1}\right) = Eq.\,(7). \tag{10}$$

$$M^*_{\Sigma^-} = \left(M_p + \frac{M_e}{3\left(\frac{M^*_{\Xi^-} - M^\diamond_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}} - \frac{\rho}{\rho - 1}\right)}\right)\left(\frac{\rho}{\rho - 1}\right) = Eq.\,(8). \tag{11}$$

Eq. (12) is another alternative formulation for M_{Σ^0}.

$$M_{\Sigma^0} = \left(\frac{3H_{\Sigma^+} + 2H_{\Sigma^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right) = Eq.\,(7). \tag{12}$$

Eq. (12) serves as a template for Eq. (13), which predicts an Ω^- mass that aligns with the PDG fit 1672.45 ± 0.29 MeV.

$$M^*_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M^*_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right) \approx 1672.4824. \tag{13}$$

Eqs. (12, 13) are the second clue suggesting Ξ particles are hypermasses for two heavier particles with masses 1668.9787 MeV and 1677.7380 MeV. The uncertainties of a number of potential candidates, viz., N(1675), N(1680), Λ(1670), Σ(1660), Σ(1670),[5] are presently too wide to be definitive. The Ξ(1318) resonance is also a plausible candidate for $H_{\Omega^-} \approx 1317.5706$ with Ω^- the middle mass of a triple {1668.9787, 1672.4824, 1677.7380} and their hypermasses the Ξ triple {1314.8104, 1317.5706, 1321.7109}.

The electron mass correction introduced in $M^*_{\Xi^-}$, $M^*_{\Omega^-}$ in Eq. (13), leads to a necessary modification in the volume term. The volume V_p in Eqs. (4, 15) is now replaced with $V_{\Omega^-} + \frac{\rho}{\rho-1}$ in Eq. 14. However, the zero approximation holds only when measured in MeV.

$$\{M_e, M_p, M_n\}\ \text{MeV} \ \Rightarrow\ M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V^*_{\Omega^-} - \frac{\rho}{\rho - 1} \approx 0. \tag{14}$$

Given 2022 CODATA adjustments in MeV, Eq. (19) = -0.000209. The suspicion Eq. (20) = 0 is reinforced by a formula that gets close to the observed Λ mass. The PDG Λ^0 fit based on a study of 38k events is 1115.683 MeV ± 0.006.[12] Eq. (20) is a match. [It is assumed subtracting $H^*_{\Xi^-}$ from $M^*_{\Omega^-}$ entails cancelling negative charges to leave the mass for a neutral particle].

$$M^*_{\Lambda^0} = M_n + \frac{M^*_{\Omega^-} - H^*_{\Xi^-} - V_{\Xi^0}}{2} \approx 1115.683\,338. \tag{15}$$

There is good reason to think Eqs. (14, 15) belong to the same mathematical framework. Eq. (16) derived from (8, 13, 15) presents a problem that makes the point.

$$M_e - \frac{2M_n + M_{\Omega^-} - H_{\Xi^-} - V_{\Xi^0} - 2M^*_{\Lambda^0}}{1 - \frac{2}{5}\left(\frac{\rho}{\rho-1}\right)^2} \approx 0 \tag{16}$$

The term M_{Ω^-} is without an asterisk and is derived when Eq. (5) replaces (7) for the template used at (13), and $H_{\Xi^-} = H^*_{\Xi^-} - M_e$. Eq. (16) approximates to zero, but unlike (14) it works for all units, not just MeV. If Eq. (16) is used to diall-in the marginally less certain neutron mass (M'_n), using the 2022 CODATA electron and proton adjustments, the same number close to 939.565 421 761 resolves Eqs. (19, 21). It is reasonable to conclude Eqs. (14, 15, 16) belonging to a unified mathematical framework that favours a neutron slightly less massive than the 2022 CODATA adjustment 939.565 421 94 $\pm$ 0.000 000 48.

This completes the survey of the lighter hyperons with Eqs. (1, 2, 3, 4, 7, 13, 15) providing the derived rest mass values for the respective hyperons.

5 Dimensionless mass looks like MeV.

When Eq. (14) equals zero, a precise scaling operator can be introduced, as formulated in Eqs. (17) and (18).

$$S_M = \left(\frac{\{M_x\}}{M_{\Sigma^0}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M_{\Xi^0} - V^*_{\Omega^-}} \right)\left(\frac{\rho}{\rho - 1}\right). \tag{17}$$

$$S_M = \left(\frac{\{M_x\}}{M_{\Sigma^-}\left(\frac{M^*_{\Xi^-} - M_{\Xi^0}}{M_{\Sigma^-} - M_{\Sigma^0}}\right) - M^*_{\Xi^-} - V^*_{\Omega^-}} \right)\left(\frac{\rho}{\rho - 1}\right). \tag{18}$$

The scaling operator S_M transforms rest mass values into a dimensionless number that is proportional to MeV. For instance, Eq. (19) yields ≈ 0.511.

$$\left(\frac{M_e}{M_{\Sigma^0}\left(\frac{M^*_{\Xi^-}-M_{\Xi^0}}{M_{\Sigma^-}-M_{\Sigma^0}}\right)-M_{\Xi^0}-V^*_{\Omega^-}}\right)\left(\frac{\rho}{\rho-1}\right)\approx 0.511. \tag{19}$$

When input values are in MeV, S_M naturally produces precise results under the condition that Eq. (14) equals zero. However, the output value is dimensionless and S_M is not restricted to MeV inputs. For instance, when M_e, M_p are 2022 CODATA adjustments in u, and the neutron mass is dialled-in to the more certain electron and proton vales, Eq. (19) $= 0.510\,998\,950\,69$, when $M'_n = 1.008\,664\,915\,876\,394\,072$. The ultra precision is overdone, but M'_n is within a standard deviation of 2022 CODATA and a match for the 2014 CODATA adjustment, viz., $1.008\,664\,915\,88\ \pm 0.000\,000\,000\,49$. Continuing to use S_M with $M'_n\ u$ to calibrate the proton mass gives 938.272 089 427 and the same number found before close to 939.565 421 761 for the neutron mass. In kg, and again allowing M'_n to be dialled-in to the necessary degree of precision, Eq. (19) $= 0.510\,998\,950\,69$ when $M'_n \approx 1.674\,927\,500\,254 \times 10^{-27}$. This compares to the 2022 CODATA adjustment $1.674\,927\,500\,56 \times 10^{-27} \pm 0.000\,000\,000\,85 \times 10^{-27}\ kg$.

A natural question arises: why does the dimensionless number generated by S_M align with the MeV scale? While a full exploration of this connection is beyond the scope of this paper, the argument presented here provides an initial framework. This formulation offers a path toward a deeper understanding of non-relativistic inertial mass.

$$\frac{\substack{number\ of\\ pushes} \times power\ of\ push \times \substack{duration\\ of\ push}}{mass\ energy\ determined\ by\ the\ type\ of\ energy\ doing\ the\ pushing} \tag{20}$$

For the model to be fully consistent, all four terms must be dimensionally distinct, ensuring that no two terms share identical subsets of physical units. This constraint inherently rules out the Gaussian mechanical cgs system, where certain physical quantities are not treated as independent dimensional entities. Additionally, a more simplified model should not exist—meaning that dimensional cancellation must occur only when all four terms are combined, not at intermediate steps. The resulting dimensionless number is a sign the model is fully resolved. Having just dismissed the Gaussian mechanical framework, the philosophy of balanced dimension is now desirable if systems that attempt to follow the model are to be scaled correctly. In the SI system, electric and magnetic field dimensions are imbalanced. The electric field $\mathbf{E}$ has dimensions $(M \cdot L \cdot T^{-3} \cdot A^{-1})$, while the magnetic field $\mathbf{B}$ carries an additional factor of $(M \cdot T^{-2} \cdot A^{-1})$. This discrepancy arises because $\mathbf{E}$ carries an additional factor of $L \cdot T^{-1}$ leading to the familiar relation $\mathbf{E}/\mathbf{B} = c$. This unresolved dimensional mismatch ultimately ties into the scale at which the ampere is defined. This dimensional imbalance is directly linked to how the ampere is defined in SI units, influencing the scaling of electromagnetic interactions.Before 2019, the ampere was defined experimentally as the current needed to produce a magnetic force of $2 \times 10^{-7}\ N \cdot m$ between two parallel conductors. The strength of the electric force that drives the current is proportional to the size of magnetic force, where $\mathbf{E}/\mathbf{B} = c$. More recently the ampere is a constant inversely proportional to the defined elementary charge. Nonetheless, this number continues to be in the SI scale and corresponds to the imbalanced $\mathbf{E}/\mathbf{B}$ quotient. Untangling the imbalance is not as simple as $\mathbf{E}/\mathbf{B} \times 1/c$, but almost. The ampere is rescaled with a Gaussian/SI synthesis that converts statvolts to volts.[13]

$$1\ statvolt = \frac{299792458}{10^6}\ Volts. \tag{21}$$

Equation (21) introduces the crucial 10^6 scaling factor, which adjusts the volt and, consequently, the ampere. This correction brings the system into alignment with a balanced dimensional framework. The result of the rescale aligns with a system of balanced dimensions. Clearly electron-volts rescaled by the 10^6 factor are going to look like S_M values. Whether this relationship extends beyond the hyperon mass spectrum remains an open question. But for a generic magnitude that applies to baron mass the procedure followed at (22) cancels dimensions along with any reference to SI values. On derived lines the generic dimensions are shown.

$$
\begin{array}{llr}
\frac{2999792458}{c}\ V & & 1 \\
= 1\ \frac{V\cdot s}{m} & L^2 \cdot M \cdot T^{-3} \cdot A][T][L^{-1}] & 2 \\
\times\ 10^{-6}\ \frac{1}{N\cdot s} & & 3 \\
= 10^{-6}\ \frac{V}{N\cdot m} & [L^2 \cdot M \cdot T^{-3} \cdot A][L^{-1} \cdot M^{-1} \cdot T^2][L^{-1}] & 4 \\
\times\ 1.60219 \cdot 10^{-19}\ A \cdot s & & 5 \\
= 10^{-6}\ \frac{eV}{N\cdot m} & [[A \cdot T] \cdot [L^2 \cdot M \cdot T^{-3} \cdot A]][L^{-1} \cdot M^{-1} \cdot T^2][L^{-1}] & 6 \\
\times\ \ 510\,998.950\,69\ eV & & 7 \\
=\ \ 0.510\,998\,950\,69\ \frac{eV}{N\cdot m} & & 8 \\
=\ \ 0.510\,998\,950\,69\ \frac{A\cdot W\cdot s}{A\cdot J} & [A][L^2 \cdot M \cdot T^{-3}][T][A^{-1}][L^{-2} \cdot M^{-1} \cdot T^2] & 9 \\
=\ \ 0.510\,998\,950\,69 & & 10
\end{array}
\tag{22}
$$

The conversion factor 299792458 in line 1 is inherently dimensionless but explicitly tied to the SI definition of the speed of light. To remove this dependence, division by$c$ ensures that the final expression remains universally applicable. By line 2, the expression transforms into volt-seconds per meter, which corresponds to the magnetic vector potential. At line 3 the statvolts to volts conversion factor 10^{-6} is introduced as an inverse impulse. By line 4, the expression correctly relates electric potential to force per unit distance, establishing consistency with physical expectations. Line 5 introduces elementary charge which converts volts to electron-volts at line 6. At this line all the basics dimensions already cancel and the next few line add no more dimensions. Line 7 introduces the electron mass energy but this could be any mass energy in electron-volts. Line 8 gives the scaled rest energy number. The basic dimensions are reframed at line 9 to agree with the non-homotopic inertial mass model, as (23).

$$
\frac{A \cdot W \cdot s}{A \cdot J} = \frac{\substack{number\ of \\ charges} \cdot wattage \cdot duration}{electric\ mass\ energy} \tag{23}
$$

In words: a particle's rest mass (non-homotopic inertial mass) is measurable by its resistance to being accelerated by an electric current. This is measured as an amount of electric mass energy. The amount is determined by the number of elementary charges for the current applied, multiplied by the power of each charge, multiplied by the duration each charge is applied. This formulation provides a self-consistent resolution, with all dimensions canceling to yield a fully dimensionless result. From the perspective of non-homotopic inertial mass, conventional rest mass units do not fully resolve the deeper structure of mass-energy interactions. However, further analysis is required to determine whether S_M offers a unique or universal scaling principle.

6 How Energy Exits an Intersection

To conclude, we consider how energy exits the intersection in the case of the proton. While this digression could have been avoided to keep the paper tightly focused on rest mass, energy egress inevitably raises questions about the nature of charge. This discussion, therefore, serves as an initial exploration into how charge might emerge from the same topological framework that governs rest mass.

Since an intersection is a physical system, we assume a principle of symmetry preservation, though this principle is secondary to physical necessity. When energy enters the 3-sphere, it does so via a discrete event—a topological puncturing—that allows a force to adapt to the higher-dimensional topology. At the moment of egress, before exiting, the energy retains the $\mathbb{R}^4$ topology of a Hopf circle. For energy to leave, this circle must be broken in a second discrete event, where the topology transitions from genus one (a loop) to genus zero (no hole). This transition poses a potential threat to fibration symmetry, as fibres in a Hopf fibration are equidistant and locally parallel.[14] Thus, a missing fibre could disrupt fibre spacing.

Given the physical system, symmetry preservation is not necessarily perfect. However, it is reasonable to assume energy follows a principle of least disruption when exiting. This likely requires more than one break in a single Hopf circle, as well as multiple broken circles occurring simultaneously. Imperfections in the system may introduce hysteresis, reinforcing the tendency to reuse established pathways. While the exact pattern of release is not pursued here, a general structural feature of the exit mechanism needs to be established.

The 3-space ball volume V^3 bounded by the 2-sphere S^2 now plays a critical role. The set of $\mathbb{R}^3$ points (x, y, z) that form this volume is embedded in a higher-dimensional system. Specifically, the 3-sphere S^3 is composed of $\mathbb{R}^4$ points (w, x, y, z), and there exists a subset where the $\{x, y, z\}$ triple duplicates the V^3

set in $\mathbb{R}^3$. This duplication represents a projection of V^3 within S^3. This projection establishes a preferred energy exit pathway, which we label *the way out*. It consists of a proper subset of S^3 that contains only the V^3 projection. Meanwhile, the rest of S^3 forms a disjoint set, which we label *no way out*, where the triple $\{x, y, z\}$ does not duplicate a point in V^3. This distinction imposes a preference system on the Hopf fibration, dictating that energy egress occurs through specific coordinates along the V^3 projection.

Energy introduced into the 3-sphere does not immediately exit. Instead, the influx of energy creates pressure, pushing internalised energy toward the available exit pathways. Since only energy already at the shell can exit, the shell behaves like a capacitor, temporarily storing energy before release. The release trigger occurs when the capacitor reaches maximal density—defined in the simplest sense as all available space being occupied. At this threshold, energy is expelled in discrete bursts, suggesting that charge-related energy dissipation is inherently quantised. However, this introduces a paradox: if incoming energy is met by outbound energy at the same boundary, energy cannot enter and therefore cannot leave. This issue is resolved by considering the role of the w-dimension in facilitating energy egress. Unlike incoming forces, which are constrained to 3-space and lack a w-coordinate, outbound energy possesses a w-coordinate. This provides a unique egress mechanism: energy exits the 3-sphere along the w-dimension while remaining "attached" to 3-space via its initial $\{x, y, z\}$ coordinates now zeroed. From a 3-space perspective, the escaping energy packets appear to have no spatial location (since their $\{x, y, z\}$ values are zero), but they retain a well-defined exit point. Assuming that the limitation of the speed of light applies to motion along w, this means that the energy packets originate from a precise location but become increasingly uncertain over time. At this point, we make the following hypothesis: *the increasing uncertainty of the energy packet's location corresponds to the decreasing strength of the force associated with the proton's charge, following an inverse-square law.* This provides a topological basis for Coulomb's law.

It has already been established that the force entering the 3-sphere dissipates in the ratio

$$\frac{S^2}{S^3}.$$

It is this process that introduces energy to the system. Given the preamble regarding the exit pathway, external energy dissipation, once internalised, is determined by S^2 and V^3 in the ratio:

$$\frac{V^3 - S^2}{S^2}.$$

Here, $V^3 - S^2$ represents the stored energy awaiting release, while S^2 defines all possible egress points. This quotient corresponds to the proton's dissipation factor, much like the volume of a water barrel relative to the size of a leak determines the rate of depletion. Particularly, this ratio takes on a meaningful value when energy is expressed in terms of electron mass energy. In the case of the proton

$$\frac{M_p}{M_e} = 2\pi^2 r^3, \quad r \approx 4.531,$$

we obtain the simplified dissipation factor:

$$\frac{V - S}{S} = \frac{r-3}{3} \approx 0.5103.$$

The system's sensitivity to dissipation is characterised by the reciprocal of the dissipation factor:

$$\frac{S}{V - S}.$$

Subtracting $\frac{1}{V}$ modifies this depletion time, introducing a resilience factor:

$$x_0 = \frac{1}{\frac{3}{r-3} - \frac{3}{4\pi r^3}} \approx 0.510\,998\,97.$$

This factor represents the system's natural resistance to dissipation. The term $1/V$ describes the *rate at which energy is depleted per unit stored*, meaning that the larger V, the faster the dissipation. In simple terms, $1/V$ determines how sensitive the system is to energy loss. Subtracting $1/V$ from the depletion rate modifies the characteristic time of dissipation, effectively acting as a stabilising factor that prevents rapid energy loss. The proximity to the electron mass makes a connection between the damping effect and the electron mass energy and suggests the proton and electron are related through the mechanism that maintains equilibrium against dissipation.

If $\frac{1}{V}$ is further modified, introducing an additional delay to dissipation in the form

$$\frac{1}{V+z}, \quad \text{where} \quad z = 4\pi^2\left(\frac{\rho}{\rho-1}\right),$$

then $\frac{\rho}{\rho-1}$ provides an adjustment indicating the system appears to store more energy than its physical volume suggests. Since the fibres S^1 are distributed over S^2, the total fibre-weighted surface measure is $4\pi^2$, this adjustment accounts for the effect of the total circle-fibre contribution to energy dissipation capturing the complete contribution of the Hopf fibration to energy release. The introduction of this adjustment leads to a recursive damping equation, in Eq. (27):

$$x_{n+1} = \frac{1}{\left(\frac{3}{r-3} - \cfrac{1}{\cfrac{4\pi r^3}{3} + \cfrac{x_n}{4\pi^2\left(\frac{\rho}{\rho-1}\right)}}\right)}. \tag{24}$$

For M_p/M_e and letting $r = 4.530\,989\,086\,15$, the sequence rapidly stabilises. By x_1 the series has already sufficiently stabilised to match the CODATA 2022 electron mass energy value $(0.510\,998\,950\,69)$ well within its experimental uncertainty. This level of precision suggests the recursive damping mechanism that stabilises the proton naturally reproduces the observed electron mass energy value, giving this particular value a role in energy dissipation regulation.

If this model applies in reverse—where energy enters an intersection via w and exits as an ordinary 3-space force—then the rest mass of the reverse system is determined by the same damping factor. Conservation of energy implies that for every proton, there is a limit of 1836 electrons. This naturally suggests a charge-to-mass balance mechanism that sheds light on a topological origin of the proton-electron mass ratio.

7 Conclusion

This paper presents an alternative framework for understanding rest mass, departing from the Higgs mechanism and the quark confinement model. Whilst a substantial departure, it is compensated for by the ability to derive mass values that closely match experimental observations, an explanation for the long-standing neutron-proton mass difference problem, and potential insight into the proton-electron mass ratio. Additionally, the possibility of reducing ten free parameters to just two suggests a significant simplification in the fundamental structure of mass determination.

A key outcome of this approach is the introduction of the S_M scaling operator, which emerges as a natural, dimensionless framework for particle rest mass. The final calculation for the electron mass, derived from recursive damping effects within the topological framework, points toward a deeper underlying structure that requires further exploration. While this theory is still in its early stages, its predictive success suggests a promising direction for future research.

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