Two New Derivations of Ramanujan's Formula $\pi^4 \approx 97.5 - 1/11$

Janko Kokošar[1]

[1] *SI-4281 Mojstrana, Slovenia, janko.kokosar@gmail.com*

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Abstract

The paper presents two procedures, or derivations with educated guessing, where Ramanujan's formula for π^4 is the final result. The first derivation uses the integer approximation for π^3, and 22/7 as an approximation for π, followed by a simple modification to match Ramanujan's formula. The second derivation uses an integer approximation for π^5 alongside with 22/7. Once again, the modification to Ramanujan's formula for π^4 is quite simple. The aim is also to explore whether further approximations of this formula exist. Some hints are provided on this subject.

1 Introduction

In some previous articles, I explored Ramanujan's formula for π^4 [1–4]. My first article [1] describes how I discovered this formula by chance, without prior knowledge of its existence. Since then, I have explored it further because it is both very accurate and simple. This raises the intriguing question of whether additional approximations for this formula exist [2–4]. This problem is also interesting because maybe it could help estimate the probability that guessed physical formulas have a physical basis [5]. In this article I propose two methods to derive Ramanujan's formula. These derivations incorporate also a guessing to arrive at the results.

In the appendices, I have also added still some formulas which try to built a system for identifying higher approximations for Ramanujan's Eq. (1).

2 Calculation

Ramanujan's formula [1–4] can also be expressed as:

$$\pi^4 = (97.5 - 1/11) \times (1 + 1.28233957 \cdots \times 10^{-9}). \tag{1}$$

Alternatively, it can be written as

$$\pi^4 = (97 + \frac{9}{22}) \times (1 + 1.28233957 \cdots \times 10^{-9}), \tag{2}$$

or as

$$\pi^4 = \frac{2143}{22} \times (1 + 1.28233957 \cdots \times 10^{-9}). \tag{3}$$

This formula is very simple and very accurate., maybe it is even the simplest and the most accurate among the approximations for π. There we exclude formulas that are not parts of the completely exact formulas for π. Its precision and simplicity even suggest it might be part of an exact formula for π.

Let us examine the values of some consecutive integer powers of π.

$$\pi^2 = 9.86960\cdots = 10 \times (1 - 0.01303\cdots). \tag{4}$$

$$\pi^3 = 31.00627\cdots = 31 \times (1.00020247\cdots). \tag{5}$$

$$\pi^5 = 306.01968\cdots = 306 \times (1 + 6.43294 \times 10^{-5}\cdots). \tag{6}$$

$$\pi^6 = 961.38919\cdots . \tag{7}$$

$$\pi^8 = 9488.53101\cdots . \tag{8}$$

2.1 Derivation 1

First, calculate the value of π^4 using the following two approximations for π^3 and π

$$\pi^4 \approx 31 \times \frac{22}{7} = 97 + \frac{9}{21}. \tag{9}$$

To obtain Eq. (2), we modify the result by just changing 21 to 22, i.e. subtracting 9/21 and adding 9/22, which together equal:

$$\frac{-9}{21 \times 22}. \tag{10}$$

Thus, this is the only modification to obtain Ramanujan's formula for π^4.

2.2 Derivation 2

Now, calculate the value of π^4 using the approximation for π^5 and again using the approximation 22/7 for π

$$\pi^4 \approx 306 \times \frac{7}{22} = \frac{2142}{22}. \tag{11}$$

According to Eq. (3), only 2142 needs to be modified to 2143, so 1/22 needs to be added. Thus, Eqs. (9) and (11) show that only minor modifications are needed to obtain Eq. (1).

2.3 Derivation 3

Now let us first calculate the value of π^5 using the product of integer approximations for π^2 and π^3, and then divide by the same approximation for π, i.e. with 22/7

$$\pi^4 \approx 10 \times 31 \times \frac{7}{22} = 310 \times \frac{7}{22} = \frac{2170}{22}. \tag{12}$$

According to Eq. (3), 2170 must be modified to 2143, so 27/22 must be subtracted. However, this modification is no longer as small as in Eq. (11), and is at least significantly larger than in Eq. (9), so this modification is no longer something special, but this can be said for Eqs. (9) and (11). Thus, according to Eqs. (9) and (11), very little needs to be modified to obtain Eq. (1).

3 Conclusion

We now wonder ourselves whether either of these two derivations holds the key to explaining the very small disagreement in Eq. (1). Disagreement for 22/7 equals:

$$\frac{22}{7} = \pi \times 1.000402499\cdots . \tag{13}$$

It seems that we cannot explain the very small disagreement in Eq. (1) in this way, since the disagreements $\sim 10^{-4}$ in Eqs. (5), (6), (13) are much larger than $\sim 10^{-9}$ in Eq. (1).

To imagine the accuracy of the approximations 31 and 306, let us see how the values in Eqs. (7) and (8) differ from $31 \times 31 = 961$ and $31 \times 306 = 9486$. The disagreement is not large, but the accuracy of these two approximations probably does not justify the accuracy of Ramanujan's formula. This gives us another feeling for the approximation.

However, considering the added number of computational elements, each of the two corrections is minor and changes very little, only for one neighbouring digit. Thus, we essentially obtained two new derivations for Eq. (1). However, the question remains whether Eq. (1) has the higher approximations.

A future step could involve examining all approximate formulas for π, Ref. [6], to estimate their number of simpler elements, and to use disagreements to determine whether Eq. (1) is particularly accurate and simple, or not.

The second option is to find next approximations instead of 31 and 306, and this is unclear to me. However the next approximation insted of 22/7 is clear, this is 333/106 as can also be found in Ref. [6]. Besides, we need a fairly simple correction, for instance, according to Eq. (11) it was an addition of 1/22.

References

[1] Janko Kokosar, "An Unintentional Repetition of the Ramanujan Formula for π, and Some Independent Mathematical Mnemonic Tools for Calculating with Exponentiation and with Dates," *viXra*, 1–11 (2020); *Preprint* https://vixra.org/abs/2005.0208.

[2] Janko Kokosar, "A Somewhat Intuitive Visual Representation of the Formulae for π^3 and Ramanujan's π^4," *viXra*, 1–9 (2022); *Preprint* https://vixra.org/abs/2212.0017.

[3] Janko Kokosar, "A New Attempt to Check Whether Ramanujan's Formula Pi^4 Approx 97.5-1/11 is a Part of Some Completely Accurate Formula," *viXra*, 1–7 (2024); *Preprint* https://vixra.org/abs/2402.0081.

[4] Janko Kokosar, "Similarity of a Ramanujan Formula for π with Plouffe's Formulae, and Use of This for Searching of Physical Background for Some Guessed Formula for the Elementary Physical Constants," *viXra*, 1–12 (2022); *Preprint* https://vixra.org/abs/2206.0147.

[5] Janko Kokosar, "Guessed formulae for the elementary particle masses, interpretation and arguments of them and a new view on quantum gravity," *viXra*, 1–19 (2011); *Preprint* https://vixra.org/abs/1103.0025.

[6] Wikipedia, Approximations of π (2024) https://en.wikipedia.org/wiki/Approximations_of_%CF%80

[7] *Web Page* https://www.wolframalpha.com/examples/mathematics/numbers/arbitrary-precision.

A Checking for the existence of higher approximations of Equation (1)

Let us use numerators and denominators of the first 12 fractions of the series of the best rational approximations of π. These fractions are, [6]:

$$\frac{3}{1}, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{103993}{33102}, \frac{104348}{33215}, \frac{208341}{66317}, \frac{312689}{99532}, \frac{833719}{265381}, \frac{1146408}{364913}, \frac{4272943}{1360120}, \frac{5419351}{1725033}. \tag{14}$$

Using Eq. (9) multiplied by 22, and inserting each fraction a/b from the above series, we have:

$$22\left(\frac{2143}{22} - 31\frac{a}{b}\right) = \tag{15}$$

The results are:

$$\frac{97}{1}, -\frac{3}{7}, \frac{26}{53}, \frac{49}{113}, \frac{7180}{16551}, \frac{14409}{33215}, \frac{28769}{66317}, \frac{21589}{49766}, \frac{115125}{265381}, \frac{158303}{364913}, \frac{295017}{680060}, \frac{748337}{1725033}. \tag{16}$$

Similarly, using Eq. (11), multipled by 22, and inserting each fraction a/b from the above series, we have:

$$22\left(\frac{2143}{22} - 306\frac{b}{a}\right) = \tag{17}$$

The results are:

$$-\frac{101}{1}, \frac{1}{1}, \frac{3}{37}, \frac{49}{355}, \frac{14335}{103993}, \frac{3596}{26087}, \frac{3191}{23149}, \frac{43103}{312689}, \frac{114925}{833719}, \frac{13169}{95534}, \frac{589009}{4272943}, \frac{747037}{5419351}. \tag{18}$$

These resulting fractions might reveal patterns or rules that could lead to better approximations. Using these fractions, it might be possible to determine some rules and apply them to improve approximations $\pi^3 = 31$ and $\pi^5 = 306$.

B Checking of correlations with the number 363894

Another approach is to consider the following formula used in Ref. [3]:

$$22\pi^4 = 2143 + 1/363893.918594827538226091114823084696230459280615070 0\cdots . \tag{19}$$

The constant in the denominator is quite close to 363894. I suspect that this is an important one for further study of the next approximations of Eq. (1). I divided 363894 by all the numerators and denominators in Eq. (14). Especially, use of the 5th fraction give the best proximity to integers or to half-integers:

$$\frac{363894}{103993} = 3.5 \times (1 - 0.000223916\cdots). \tag{20}$$

$$\frac{363894}{33102} = 11 \times (1 - 0.000626164\cdots). \tag{21}$$

A good result is also found for the 10th fraction:

$$\pi\frac{363894}{1146408} = 1 - 0.00279244\cdots . \tag{22}$$

$$\frac{363894}{364913} = 1 - 0.00279244\cdots . \tag{23}$$

Perhaps the probability of these coincidences using only 12 pairs is very small, but this requires further proof.

Using the 4th, 5th, and 10th fractions as approximations for π, inserting them into equations like Eq. (19), and calculating disagreements from zero yields:

$$22\left(\pi^4-\left(\frac{355}{113}\right)^4\right)=-1/1373.852555\cdots. \tag{24}$$

$$22\left(\pi^4-\left(\frac{103993}{33102}\right)^4\right)=1/634193.890260\cdots. \tag{25}$$

Equation (25) indicates that the 5th fraction provides a better result than Eq. (19), other 4 fractions do not. Using $97.5-1/11$ instead of π^4 yields:

$$22\left(97.5-\frac{1}{11}-\left(\frac{103993}{33102}\right)^4\right)=-1/853789.582194\cdots. \tag{26}$$

It is compared to the 4th power of 5th fraction. These two values are close to each other. This is promising for some kind of formula behind it. The 10th fraction provides a much smaller disagreement than at the fractions before:

$$22\left(\pi^4-\left(\frac{1146408}{364913}\right)^4\right)=-1/227531882.824186\cdots. \tag{27}$$

However, it remains unclear how to utilize information from Eqs. (22) and (23).

C A random equation that arose in this research

During further search for higher correction, the following equation also emerged

$$\pi^4-\left(\frac{333}{106}\right)^4-\left(\frac{333}{106}\right)^{-4}-\frac{1}{2}\left(\frac{333}{106}\right)^{-8}-\left(\frac{333}{106}\right)^{-12}=\frac{1}{18402067.74904217\cdots}. \tag{28}$$

Replacing π^4 with $2143/22$, we obtain

$$\frac{2143}{22}-\left(\frac{333}{106}\right)^4-\left(\frac{333}{106}\right)^{-4}-\frac{1}{2}\left(\frac{333}{106}\right)^{-8}-\left(\frac{333}{106}\right)^{-12}=-\frac{1}{14170367.60245289\cdots}. \tag{29}$$

The difference between Eqs. (28) and (29) is minimal. However, the significant aspect is that a relatively simple and symmetric formula was derived in this way. This raises questions about its deeper meaning, potential bias, or even programming errors in [7].

In this context, only the approximation $333/106$ was used, but it is possible to combine more approximations of π.

Expanding one more term in Eq. (28), we obtain:

$$\pi^4-\left(\frac{333}{106}\right)^4-\left(\frac{333}{106}\right)^{-4}-\frac{1}{2}\left(\frac{333}{106}\right)^{-8}-\left(\frac{333}{106}\right)^{-12}-5\left(\frac{333}{106}\right)^{-16}=-\frac{1}{821363471.96334491\cdots}. \tag{30}$$

If we take the previous and the next approximations for π, $22/7$ and $355/113$, we no longer obtain such a series for a multiple of the fourth power:

$$\pi^4-\left(\frac{355}{113}\right)^4=-\frac{1}{30224.75621508\cdots}. \tag{31}$$

However, it seems that this method could work for arbitrary integer powers of these approximations of π. Further analysis is needed to explore this, but we should be aware that polynomials can fit too much to data.