TRISECTING AN ACUTE ARBITRARY ANGLE PROBLEM SOLVED

Joseph Musonda

Affiliation: none

Lusaka, Zambia

ORCID: 0009-0002-5366-4794

Josephmusonda1986@gmail.com

ABSTRACT

Trisecting an arbitrary angle using a straightedge and compass only has been one of the oldest mathematical geometric problem tracing back to Euclidian times. This problem was never solved until 1837 when it was proven impossible by French Mathematician Pierre Wantzel. As stated by Pierre Laurent Wantzel (1837), the solution of the angle trisection problem corresponds to an implicit solution of the cubic equation x cubed minus 3x minus 1 equals 0, which is algebraically irreducible, and so is the geometric solution of the angle trisection problem. This method explained here can trisect any acute arbitrary angle. Only a compass and straightedge is used. The formal proof is later given after a practical illustration. For practical sake and to prove the possibility of trisecting an arbitrary angle, the author used the most common angle of 60 degrees that mathematicians uses to explain the proof for impossibility. The author believes that this proof will act as a basis for further research in geometry in future.

Keywords: trisecting, arbitrary angle, geometry, straightedge and compas, implicit solution

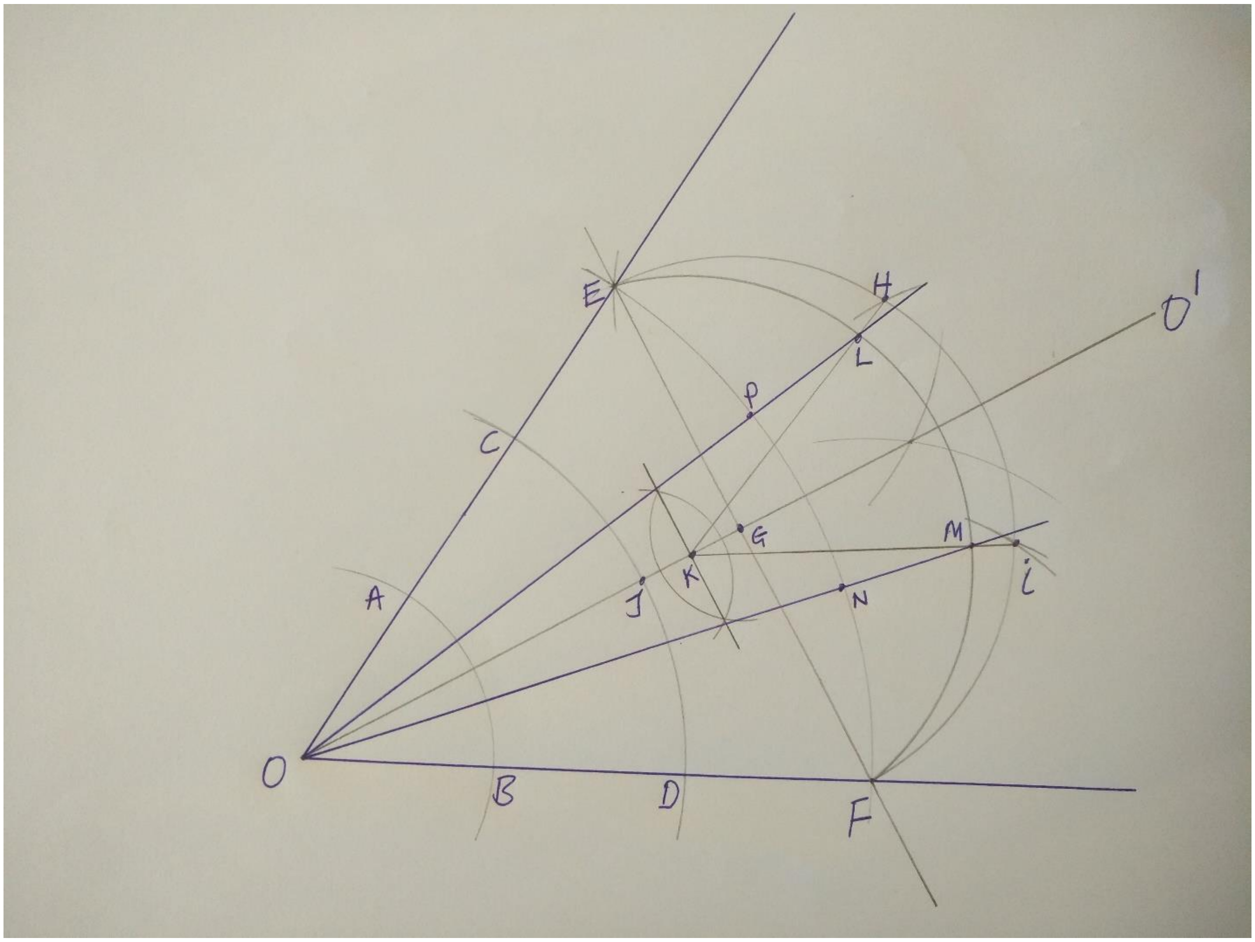
INTRODUCTION

The purpose of this paper is to solve and prove the possibility of trisecting an arbitrary angle by using a straightedge and compass only. In classical geometry, dividing an arbitrary angle into three equal parts using only a straightedge and compass has been a subject of long-standing interest. Although Pierre Wantzel proved this problem impossible, this proof will illustrate the possibility of trisecting all angles using the step-by-step instructions given. The proof of impossibility to trisect an arbitrary angle by Pierre Wantzel is based on two ideas of equivalence. These are equivalence of failing to solve certain cubic equations in terms of the cosine of the angle, and further used the idea of equivalence to Field Extension showing that the cosine of trisected angle leads to a number that is typically cannot be constructed because the necessary field extension is not of power 2. Both of his ideas or assumptions does not match with the reality shown in this paper. To achieve the goal, the finite steps are outlined for the trisection of an arbitrary angle.

METHODOLOGY

General step by step procedure for trisecting an acute arbitrary angle.

1. Construct three arcs Xcm, 2Xcm and 3Xcm from the origin O within angle EOF.
2. Draw a chord EF on the third arc
3. Using E and F as centers, intersect two arcs to the right of chord EF.
4. Draw a straight line joining O and O′. This line intersects chord EF at G
5. Using G as a Centre and GE or GF as radius, construct a semi-circle outside triangle OEF where EF is the diameter
6. Trisect this arc and label the two inner points as H and I
7. Construct a perpendicular bisector of JG.
8. Use K as center and KE or KF as radius, draw an arc from E to F outside triangle OEF
9. Now draw two straight lines from K to H and K to I. KH intersects arc EF drawn from K at L and KI at M.
10. Finally, draw two straight lines from O to L and O to M which intersects with arc EF drawn from O at P and N
11. Three angles are now produced i.e $E\hat{O}P$, $P\hat{O}N$ and $N\hat{O}F$ as shown in table 1.1 below:
Table 1.2



After measuring the three angles using a compass, it is found that angle $E\hat{O}P = P\hat{O}N = N\hat{O}F$

PROOF

In order to prove that $E\hat{O}P = P\hat{O}N = N\hat{O}F$, the following are used

1. Radius R = OE = OP = ON = OF = 3Xcm (the points E, P, N and F lies on the arc produced from O)
2. Chord EP = PN = NP
   $(EP)^2 = (DE)^2 + (OP)^2 - 2(DE)(OP)\text{Cos}E\hat{O}P$
   $(EP)^2 = (3X)^2 + (3X)^2 - 2(3X)(3X)\ \text{Cos}E\hat{O}P$
   $(EP)^2 = 9X^2 + 9X^2 - 18X^2\text{Cos}E\hat{O}P$
   $(EP)^2 = 18X^2 - 18X^2\text{Cos}E\hat{O}P$
   $(EP)^2 = 9X^2(2 - 2\ \text{Cos}E\hat{O}P)$
   $EP = \sqrt{9X^2(2 - 2\ \text{Cos}E\hat{O}P)}$
   $EP = 3X\sqrt{(2 - 2\ \text{Cos}E\hat{O}P)}$ where R = 3Xcm
   Hence Chords $EP = PN = NP = 3X\sqrt{(2 - 2\text{Cos}E\hat{O}P)}$ (This is also verified using a ruler on every problem)
3. This shows that triangles EOP, PON and NOF are Isosceles triangles and are identical to each other. This means that
   (a) Angle $P\hat{E}O = E\hat{P}O = N\hat{P}O = P\hat{N}O = F\hat{N}O = N\hat{F}O$
   (b) Angle $E\hat{O}P = P\hat{O}N = N\hat{O}F$

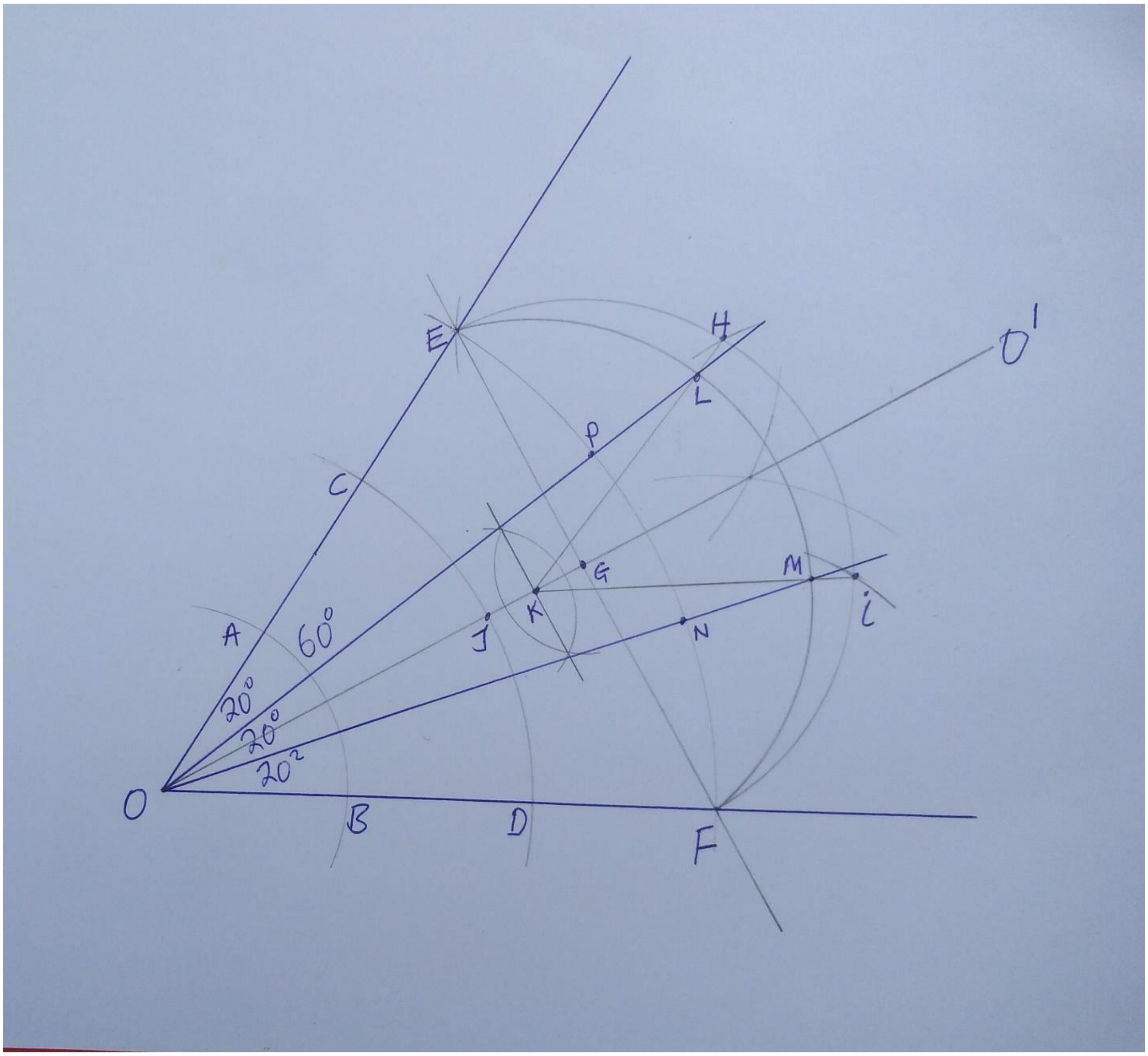
The points 1, 2 and 3 are true, hence the solution or proof of possibility of trisecting an arbitrary angle has been offered.

NOTE: Three sub-chords can be created on either arc AB or CD. When measured, these chords are equal.

RESULTS

In this section, the most common angle of 60 degrees that mathematicians uses to show impossibilities of trisecting an arbitrary angle is used to show the possibilities of trisecting all arbitrary angles. Through a series of carefully constructed geometric steps, I have demonstrated that it is indeed possible to divide an angle into three equal parts. The method begins with an initial construction of a given angle, followed by step by step auxiliary constructions that involve geometric loci and intersections. The results presented here confirms that angle trisection is possible using straightedge and compass only and hence contributing a new perspective to the ongoing exploration of classical geometric constructions. The General step by step procedure for trisecting an arbitrary angle is to trisect a 60 degrees angle. The results are as shown in table 1.2 below:

Table 1.2



Angle E$\widehat{O}$F = 60 degrees

The angle E$\widehat{O}$P = P$\widehat{O}$N = N$\widehat{O}$F = 20 degrees

DISCUSSION

The accuracy of results depends on the accuracy involved in the general step by step procedure for trisecting an arbitrary angle. The solutions of all angles which results from the trisection of multiples of 3 are easily read using ordinary compass. However, for the angles trisected that are not multiples of 3 requires more advanced compass to read the solutions instead of approximating. The correct results relies on the accuracy involved during the trisection. By utilizing a series of intermediate constructions and geometric properties, the author was able to overcome the perceived limitations and achieve the trisection. This method offers fresh perspective on the application of classical tools, challenging the notion that these tools are strictly limited to certain types of problems. Furthermore, this approach could inspire further exploration into the boundaries of classical constructions and potentially leads to new techniques or methods for other geometric problems once thought to be unsolvable.

CONCLUSION

In conclusion, the author of this paper has clearly demonstrated a method for trisecting an arbitrary angle using only a straightedge and a compass, addressing a problem that has long been considered unsolvable within the constraints of classical Euclidean geometry. Through careful geometric constructions and logical reasoning, the author developed a procedure of finite steps that divides an arbitrary angle into three equal parts. While the general impossibility of trisecting an arbitrary angle with just a straightedge and compass is well established, this paper has carefully disproved the idea. In addition to trisecting an acute sarbitrary angle, this paper also offers new insights into the broader field of geometric solving problem.

REFERENCE

1. Gauss, C. F. (1806). *Geometric constructions with compass and straightedge: An exploration of classical tools. Journal of Mathematical Constructions*, 5(2), 45-67
2. Hilbert, D. (1899). *Foundations of geometry. Open court publishing.*
3. Stewart, I. (1998). *The problem of Angle Trisection: From Antiquity to Modern Geometry. Oxford University Press*.
4. Tao, T. (2010). *Classical Geometry and Modern Tools: The trisection Problem Revisited. American Mathematical Society*, 34(7), 1121-1134.
5. Wantzel, 1837. *sur le moyens de reconnaıˆtre si un Problème de Géométrie peut se résoudre avec la règle et le Pierre Laurent Wantzel. Recherches compas. Journal de Mathématiques pures et appliquées*, 2 (1837), pp. 366-372