Musonda’s ratio method of solving linear systems of equations in two and three variables

Joseph Musonda

Affiliation: none

Lusaka, Zambia

ORCID: 0009-0002-5366-4794

Josephmusonda1986@gmail.com

ABSTRACT

The purpose of this paper is simply to present to the mathematics community a novel approach of solving systems of linear equations in two and three variables. This method uses products of coefficients with determinants and constants with determinants. The method is derived through the cross multiplication of equations which results in finding the critical values (Dx, Dy and Dz). These critical values are later substituted in any initial given equation for the purpose of finding scale factor K. It is this scale factor that is multiplied with critical values to find real solutions. The analysis of this formula shows that it is certainly accurate for all problems it is invented for. Unlike the Crammer’s rule, substitution method, elimination method that requires three and four determinants to solve a two and three variable problems of linear systems of equations, for this method the two and three variable problems only two and three determinants respectively.

In two variables

$$x = \frac{c_1 Dx}{M}$$

$$y = \frac{c_1 Dy}{M}$$

In three variables

$$x = \frac{d_2 Dx}{M}$$

$$y = \frac{d_2 Dy}{M}$$

$$z = \frac{d_2 Dz}{M}$$

Coefficients a_n, b_n, c_n and constants is obtained from any of the single equation. The ratio method is formally derived and proven in the methodology section and Denominator (M) $=$ $a_1 Dx + b_1 Dy$ for a two variable equations and $a_1 Dx + b_1 Dy + c_1 Dz$ for 3 variable equations.

Keywords: Linear systems of equations, Ratios, Critical Values, Scale factor K, Crammer’s rule, elimination method, substitution method.

INTRODUCTION

Background: There are a number of ways in which the simultaneous equations can be used in our daily life. We can use this equations in dealing with cost and demand, in investment, deciding the best plan, deciding the best deal and so on. This paper has dealt with an alternative method that can be used to deal with such named problem above.

Literature review: There are so many types of methods that are been used to solve simultaneous equations. Among these methods are elimination method, substitution method, matrix method, graphical method, Crammer's rule, Gaussian elimination and so on. None of these methods is similar to the Musonda's ratio method.

Research gap: There is no method that has solved simultaneous equations using ratios hence this gap needs to be filled. Musonda's ratio method that uses products of coefficients and determinants has finally solved this problem.

Research Aim: Given the luck of research regarding the use of ratios to solve simultaneous equations, this research paper explains how this goal can be achieved.

Research objectives

1. To identify a way in which simultaneous equations can be solved using ratios.
2. To invent a formula based on ratios for solving simultaneous equations.
3. To evaluate the accuracy and effectiveness of this new formula through proving and solving problems.

Scope: This study will contribute to the body of knowledge in solving simultaneous equations. This formula invented solves all kinds of linear simultaneous equations.

METHODOLOGY

Two variables

Firstly, the two equations are cross multiplied and like terms are collected. The variables x and y are then made the subject of the formula to obtain the critical values known as determinants. These critical values are substituted in any into one of the initial equations to find the scale factor K. It is this scale factor that is multiplied with critical values also known as determinants to find the true solutions.

Let $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ and cross multiply them

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$c_2(a_1x + b_1y) = c_1(a_2x + b_2y)$$

$$a_1c_2x + b_1c_2y = c_1a_2x + b_2c_1y$$

$$a_1c_2x - c_1a_2x = b_2c_1y - b_1c_2y$$

$$(a_1c_2 - c_1a_2) = (b_2c_1 - b_1c_2)$$

$$\frac{x(a_1c_2 - c_1a_2)}{y(a_1c_2 - c_1a_2)} = \frac{y(b_2c_1 - b_1c_2)}{y(a_1c_2 - c_1a_2)}$$

$$\frac{x}{y} = \frac{(b_2c_1 - b_1c_2)}{(a_1c_2 - c_1a_2)} = \frac{Dx}{Dy}$$

Hence $Dx = b_2c_1 - b_1c_2$ and $Dy = a_1c_2 - c_1a_2.$ Dx and Dy are $called$ $critical$ $values$

These critical values should now be substituted in one of the initial equations above in order to find the constant k.

$$a_1x + b_1y = c_1$$

$$[a_1(Dx) + b_1(Dy)]k = c_1$$

$$\text{K} = \frac{c1}{[a_1(Dx) + b_1(Dy)]}$$

$$x = x_c k$$

$$\text{x} = \frac{c_1 Dx}{a_1(Dx) + b_1(Dy)}$$

$$y = y_c k$$

$$\text{X} = \frac{\mathrm{C_1 D_x}}{\mathrm{a_1(D_x) + b_1(D_x)}} = \frac{\mathrm{C_1 D_x}}{\mathrm{M}}$$

$$\text{Y} = \frac{\mathrm{C_1 D_y}}{\mathrm{a_1(D_x) + b_1(D_x)}} = \frac{\mathrm{C_1 D_y}}{\mathrm{M}}$$

Proving the formula in two variables substitution method

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

Making x the subject of the formula from the first equation, $x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$

and by substituting x into second equation, y = $\frac{c_2a_1 - a_2c_1}{a_1b_2 - a_2b_1}$

for x

$$\frac{c_1Dx}{a_1(Dx)+b_1(Dy)} = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$\frac{c_1\left((b_2c_1 - b_1c_2\right)}{a_1(b_2c_1 - b_1c_2) + b_1(a_1c_2 - c_1a_2)} = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$\frac{c_1\left((b_2c_1 - b_1c_2\right)}{a_1b_2c_1 - a_1b_1c_2 + b_1a_1c_2 - b_1c_1a_2} = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$\frac{c_1\left((b_2c_1 - b_1c_2\right)}{a_1b_2c_1 - b_1c_1a_2} = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$\frac{c_1\left((b_2c_1 - b_1c_2\right)}{c_1(a_1b_2 - b_1a_2)} = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$\frac{b_2c_1 - b_1c_2}{a_1b_2 - b_1a_2} = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$ proven

Similarly, you can also prove for y by sing same steps taken to prove for x

Three variables

Two equations are picked from the three equations and cross multiplied in order to make an equation in two variables x and y where the constant will involve the z. The other two equations are picked and same steps are followed to make an equation in two variables. Now these two equations are further cross multiplied in order to find critical values of x and y. These critical values are later substituted in 4th or 5th equation to find another critical value of z. The critical values are finally substituted in any of the first three initial equations to find the scale factor K. These critical values are then multiplied by K to obtain the true solutions.

Let the following be simultaneous equations in three variables $a_1x + b_1x + c_1 = d1$, $a_2x + b_2x + c_2 = d2$ and $a_3x + b_3x + c_3 = d3$ and using the concept of ratios

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

First and second equations are cross multiplied

$$d_2(a_1x + b_1y + c_1z) = d_1(a_2x + b_2y + c_2z)$$

$$a_1d_2x + b_1d_2y + c_1d_2z = a_2d_1x + b_2d_1y + c_2d_1z$$

$$a_1d_2x - a_2d_1x + b_1d_2y - b_2d_1y = c_2d_1z - c_1d_2z$$

$$(a_1d_2 - a_2d_1)x + (b_1d_2 - b_2d_1) = (c_2d_1 - c_1d_2) \ldots \ldots \ldots \ldots (4)$$

Second and third equations are cross multiplied

$$d_3(a_1x + b_1y + c_1z) = d_1(a_3x + b_3y + c_3z)$$

$$a_1d_3x + b_1d_3y + c_1d_3z = a_3d_1x + b_3d_1y + c_3d_1z$$

$$a_1d_3x - a_3d_1x + b_1d_3y - b_3d_1y = c_3d_1z - c_1d_3z$$

$$(a_1d_3 - a_3d_1)x + (b_1d_3 - b_3d_1) = (c_3d_1 - c_1d_3) \ldots \ldots \ldots \ldots . (5)$$

Now cross multiply equation (4) and (5) and z is eliminated by division

$$(c_3d_1 - c_1d_3)[(a_1d_2 - a_2d_1) + (d_2b_1 - b_2d_1)y]$$
$$= (c_2d_1 - c_1d_2)[(a_1d_3 - a_3d_1)x + (d_3b_1 - b_3d_1)y]$$

$$(c_3d_1 - c_1d_3)[(a_1d_2 - a_2d_1) + d_2b_1 - b_2d_1)y]$$
$$= (c_2d_1 - c_1d_2)[(a_1d_3 - a_3d_1)x + (d_3b_1 - b_3d_1)y]$$

$$[(c_3d_1 - c_1d_3)(a_1d_2 - a_2d_1)x + (c_3d_1 - c_1d_3)(d_2b_1 - b_2d_1)y]$$
$$= [(c_2d_1 - c_1d_2)(a_1d_3 - a_3d_1)x + (c_2d_1 - c_1d_2)(d_3b_1 - b_3d_1)y]$$

$$[(c_3d_1 - c_1d_3)(a_1d_2 - a_2d_1)]x - [(c_2d_1 - c_1d_2)(a_1d_3 - a_3d_1)]x = [(c_2d_1 - c_1d_2)(d_3b_1 - b_3d_1)]y - [(c_3d_1 - c_1d_3)(d_2b_1 - b_2d_1)]y$$

$$[(c_3d_1 - c_1d_3)(a_1d_2 - a_2d_1) - (c_2d_1 - c_1d_2)(a_1d_3 - a_3d_1)]x = [(c_2d_1 - c_1d_2)(d_3 - b_3d_1) - (c_3d_1 - c_1d_3)(d_2b_1 - b_2d_1)]y$$

$$(c_3d_1 - c_1d_3)(a_1d_2 - a_2d_1) - (c_2d_1 - c_1d_2)(a_1d_3 - a_3d_1)]x = [(c_2d_1 - c_1d_2)(d_3b_1 - b_3d_1) - (c_3d_1 - c_1d_3)(d_2b_1 - b_2d_1)]y$$

$$\frac{x}{y} = \frac{[(c_2d_1 - c_1d_2)(d_3b_1 - b_3d_1) - (c_3d_1 - c_1d_3)(d_2b_1 - b_2d_1)]}{(c_3d_1 - c_1d_3)(a_1d_2 - a_2d_1) - (c_2d_1 - c_1d_2)(a_1d_3 - a_3d_1)]}$$

$$= \frac{(c_2d_1d_3b_1 - c_2d_1b_3d_1 - c_1d_2d_3b_1 + c_1d_2b_3d_1 - (c_3d_1d_2b_1 - c_3d_1b_2d_1 - c_1d_3d_2b_1 + c_1d_3b_2d_1)}{(c_3d_1a_1d_2 - c_3d_1a_2d_1 - c_1d_3a_1d_2 + c_1d_3a_2d_1 - (c_2d_1a_1d_3 - c_2d_1a_3d_1 - c_1d_2a_1d_3 + c_1d_2a_3d_1}$$

$$= \frac{c_2d_1d_3b_1 - c_2d_1b_3d_1 - c_1d_2d_3b_1 + c_1d_2b_3d_1 - c_3d_1d_2b_1 + c_3d_1b_2d_1 + c_1d_3d_2b_1 - c_1d_3b_2d_1)}{c_3d_1a_1d_2 - c_3d_1a_2d_1 - c_1d_3a_1d_2 + c_1d_3a_2d_1 - c_2d_1a_1d_3 + c_2d_1a_3d_1 + c_1d_2a_1d_3 - c_1d_2a_3d_1}$$

$$\frac{x}{y} = \frac{c_2d_1d_3b_1 - c_2d_1b_3d_1 + c_1d_2b_3d_1 - c_3d_1d_2b_1 + c_3d_1b_2d_1 - c_1d_3b_2d_1)}{c_3d_1a_1d_2 - c_3d_1a_2d_1 + c_1d_3a_2d_1 - c_2d_1a_1d_3 + c_2d_1a_3d_1 - c_1d_2a_3d_1}$$

$$\frac{x}{y} = \frac{c_3d_1b_2d_1 - c_2d_1b_3d_1 - c_3d_1d_2b_1 + c_2d_1d_3b_1 + c_1d_2b_3d_1 - c_1d_3b_2d_1}{c_3d_1a_1d_2 - c_2d_1a_1d_3 - c_3d_1a_2d_1 + c_2d_1a_3d_1 + c_1d_3a_2d_1 - c_1d_2a_3d_1}$$

$$\frac{x}{y} = \frac{d_1(c_3b_2d_1 - c_2b_3d_1 - c_3d_2b_1 + c_2d_3b_1 + c_1d_2b_3 - c_1d_3b_2)}{d_1(c_3a_1d_2 - c_2a_1d_3 - c_3a_2d_1 + c_2a_3d_1 + c_1d_3a_2 - c_1d_2a_3)}$$

$$\frac{x}{y} = \frac{c_3b_2d_1 - c_2b_3d_1 - c_3d_2b_1 + c_2d_3b_1 + c_1d_2b_3 - c_1d_3b_2}{c_3a_1d_2 - c_2a_1d_3 - c_3a_2d_1 + c_2a_3d_1 + c_1d_3a_2 - c_1d_2a_3}$$

$$\frac{x}{y} = \frac{d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)}{a_1(c_3d_2 - c_2d_3) - d_1(c_3a_2 - c_2a_3) + c_1(d_3a_2 - d_2a_3)}$$

$$\frac{x}{y} = \frac{Dx}{Dy} = \frac{d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)}{a_1(c_3d_2 - c_2d_3) - d_1(c_3a_2 - c_2a_3) + c_1(d_3a_2 - d_2a_3)}$$

Where Dx is determinant for x and Dy is determinant for y. Now substitute Dx and Dy in either eqn (4) or (5) in order to find Dz

$$(\mathrm{a_1d_2 - a_2d_1})\mathrm{x} + (\mathrm{b_1d_2 - b_2d_1})\mathrm{y} = (\mathrm{c_2d_1 - c_1d_2})\mathrm{z} \ldots\ldots\ldots\ldots(4)$$

$$(\mathrm{a_1d_2 - a_2d_1})[\mathrm{d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)}] + (\mathrm{b_1d_2 - b_2d_1})[\mathrm{a_1(c_3d_2 - c_2d_3) - d_1(c_3a_2 - c_2a_3) + c_1(d_3a_2 - d_2a_3)}] = (\mathrm{c_2d_1 - c_1d_2})\mathrm{z}$$

$$[\mathrm{d_1(c_3b_2 - c_2b_3)(a_1d_2 - a_2d_1) - b_1(c_3d_2 - c_2d_3)(a_1d_2 - a_2d_1) + c_1(d_2b_3 - d_3b_2)(a_1d_2 - a_2d_1)}] + [\mathrm{a_1(c_3d_2 - c_2d_3)(b_1d_2 - b_2d_1) - d_1(c_3a_2 - c_2a_3)(b_1d_2 - b_2d_1) + c_1(d_3a_2 - d_2a_3)(b_1d_2 - b_2d_1)}] = (\mathrm{c_2d_1 - c_1d_2})\mathrm{z}$$

$$
\begin{aligned}
&(d_1a_1d_2c_3b_2 - d_1a_2d_1c_3b_2 - d_1c_2b_3a_1d_2 + d_1c_2b_3a_2d_1 - b_1a_1d_2c_3d_2 + b_1a_2d_1c_3d_2 + b_1a_1d_2c_2d_3 \\
&\quad - b_1c_2d_3a_2d_1 + c_1a_1d_2d_2b_3 - c_1a_2d_1d_2b_3 - c_1a_1d_2d_3b_2 + c_1d_3b_2a_2d_1 + a_1b_1d_2c_3d_2 \\
&\quad - a_1b_2d_1c_3d_2 - a_1b_1d_2c_2d_3 + a_1b_2d_1c_2d_3) - d_1b_1d_2c_3a_2 + d_1b_2d_1c_3a_2 \\
&\quad + d_1b_1d_2c_2a_3 - d_1b_2d_1c_2a_3 + c_1b_1d_2d_3a_2 - c_1b_2d_1d_3a_2 - c_1b_1d_2d_2a_3 \\
&\quad + c_1b_2d_1d_2a_3)] = (c_2d_1 - c_1d_2)
\end{aligned}
$$

$$
\begin{aligned}
&-d_1c_2b_3a_1d_2 + d_1c_2b_3a_2d_1 - b_1c_2d_3a_2d_1 + c_1a_1d_2d_2b_3 - c_1a_2d_1d_2b_3 - c_1a_1d_2d_3b_2 + a_1b_2d_1c_2d_3 \\
&\quad + d_1b_1d_2c_2a_3 - d_1b_2d_1c_2a_3 + c_1b_1d_2d_3a_2 - c_1b_1d_2d_2a_3 + c_1b_2d_1d_2a_3 \\
&\quad = (c_2d_1 - c_1d_2)z
\end{aligned}
$$

$$
\begin{aligned}
&a_1b_2d_1c_2d_3 - d_1c_2b_3a_1d_2 + d_1c_2b_3a_2d_1 - b_1c_2d_3a_2d_1 - d_1b_2d_1c_2a_3 + d_1b_1d_2c_2a_3 + c_1a_1d_2d_2b_3 \\
&\quad - c_1a_2d_1d_2b_3 - c_1a_1d_2d_3b_2 + c_1b_1d_2d_3a_2 - c_1b_1d_2d_2a_3 + c_1b_2d_1d_2a_3 \\
&\quad = (c_2d_1 - c_1d_2)z
\end{aligned}
$$

$$
\begin{aligned}
&c_2d_1(a_1b_2d_3 - b_3a_1d_2 + b_3a_2d_1 - b_1d_3a_2 - d_1b_2a_3 + b_1d_2a_3) - c_1d_2(a_1d_3b_2 - a_1d_2b_3 + a_2d_1b_3 \\
&\quad - b_1d_3a_2 + b_1d_2a_3 - b_2d_1a_3) = (c_2d_1 - c_1d_2)z
\end{aligned}
$$

$$(c_2d_1 - c_1d_2)(a_1b_2d_3 - b_3a_1d_2 - b_1d_3a_2 + b_1d_2a_3 + b_3a_2d_1 - d_1b_2a_3) = (c_2d_1 - c_1d_2)z$$

$$(a_1b_2d_3 - b_3a_1d_2 - b_1d_3a_2 + b_1d_2a_3 + b_3a_2d_1 - d_1b_2a_3) = z$$

$$a_1(b_2d_3 - b_3d_2) - b_1(d_3a_2 - d_2a_3) + d_1(b_3a_2 - b_2a_3) = z$$

Therefore $Dx = d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)$

$$Dy = a_1(c_3d_2 - c_2d_3) - d_1(c_3a_2 - c_2a_3) + c_1(d_3a_2 - d_2a_3)$$

$$Dz = a_1(b_2d_3 - b_3d_2) - b_1(d_3a_2 - d_2a_3) + d_1(b_3a_2 - b_2a_3)$$

These critical values needs to be substituted in one of the initial equations on x, y and z in order to find the critical the constant k.

$$[a_1x + b_1y + c_1z]k = d_1$$

$$[a_1Dx + b_1Dy + c_1Dz]k = d_1$$

$$\frac{[a_1Dx + b_1Dy + c_1Dz]k}{[a_1Dx + b_1Dy + c_1Dz]} = \frac{d_1}{[a_1Dx + b_1Dy + c_1Dz]}$$

$$k = \frac{d_1}{[a_1Dx + b_1Dy + c_1Dz]}$$

$$x = DxK$$

$$\boldsymbol{x = \frac{d_1Dx}{a_1Dx + \; b_1Dy + \; c_1Dz}}$$

$$\boldsymbol{y = DyK}$$

$$\boldsymbol{y = \frac{d_1Dy}{a_1Dx + \; b_1Dy + \; c_1Dz}}$$

$$\boldsymbol{y = DyK}$$

$$\boldsymbol{z = \frac{d_1Dz}{a_1Dx + \; b_1Dy + \; c_1Dz}}$$

Take note that you can use any equation to find solutions as shown below by keeping Dx, Dy and Dz constant.

$$\boldsymbol{x = \frac{\mathrm{d_1Dx}}{\mathrm{a_1Dx + b_1Dy + c_1Dz}} = \frac{\mathrm{d_1Dx}}{M}}$$

$$\boldsymbol{y = \frac{\mathrm{d_1Dy}}{\mathrm{a_1Dx + b_1Dy + c_1Dz}} = \frac{\mathrm{d_1Dy}}{M}}$$

$$\boldsymbol{x = \frac{\mathrm{d_1Dz}}{\mathrm{a_1Dx + b_1Dy + c_1Dz}} = \frac{\mathrm{d_1Dz}}{M}}$$

Proving the formula in three variables using Crammer's rule

For x

$$x = \frac{Dx}{D} = \frac{d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)}{a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)}$$

$$therefore, \frac{d_1Dx}{a_1Dx + \; b_1Dy + \; c_1Dz} = \frac{d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)}{a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)}$$

Using LHS

$$\frac{d_1Dx}{a_1Dx + \; b_1Dy + \; c_1Dz} = \frac{d_1[d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)]}{\begin{aligned} &a_1[d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)] \\ &+ b_1[a_1(c_3d_2 - c_2d_3) - d_1(c_3a_2 - c_2a_3) + c_1(d_3a_2 - d_2a_3)] \\ &+ c_1[(a_1(b_2d_3 - b_3d_2) - b_1(d_3a_2 - d_2a_3) + d_1(b_3a_2 - b_2a_3)] \end{aligned}}$$

$$= \frac{d_1[d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)]}{\begin{aligned} &a_1[(d_1c_3b_2 - d_1c_2b_3) - (b_1c_3d_2 - b_1c_2d_3) + (c_1d_2b_3 - c_1d_3b_2)] \\ &+ b_1[(a_1c_3d_2 - a_1c_2d_3) - (d_1c_3a_2 - d_1c_2a_3) + (c_1d_3a_2 - c_1d_2a_3)] \\ &+ c_1[(a_1b_2d_3 - a_1b_3d_2) - (b_1d_3a_2 - b_1d_2a_3) + (d_1b_3a_2 - d_1b_2a_3)] \end{aligned}}$$

$$= \frac{d_1[d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)]}{\begin{gathered} a_1d_1c_3b_2 - a_1d_1c_2b_3 - a_1b_1c_3d_2 + a_1b_1c_2d_3 + a_1c_1d_2b_3 - a_1c_1d_3b_2 \\ +b_1a_1c_3d_2 - b_1a_1c_2d_3 - b_1d_1c_3a_2 + b_1d_1c_2a_3 + b_1c_1d_3a_2 - b_1c_1d_2a_3 \\ + c_1a_1b_2d_3 - c_1a_1b_3d_2 - c_1b_1d_3a_2 + c_1b_1d_2a_3 + c_1d_1b_3a_2 - c_1d_1b_2a_3 \end{gathered}}$$

$$= \frac{d_1[d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)]}{a_1d_1c_3b_2 - a_1d_1c_2b_3 - b_1d_1c_3a_2 + b_1d_1c_2a_3 + c_1d_1b_3a_2 - c_1d_1b_2a_3}$$

$$= \frac{d_1[d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)]}{d_1[a_1c_3b_2 - a_1c_2b_3 - b_1c_3a_2 + b_1c_2a_3 + c_1b_3a_2 - c_1b_2a_3]}$$

$$= \frac{\boldsymbol{d_1(c_3b_2 - c_2b_3) - b_1(c_3d_2 - c_2d_3) + c_1(d_2b_3 - d_3b_2)}}{\boldsymbol{a_1(c_3b_2 - c_2b_3) - b_1(c_3a_2 - c_2a_3) + c_1(b_3a_2 - b_2a_3)}}$$

The formula has been proven. Using the same steps, we can also prove for both y and z.

Results

Example

Solve the following simultaneous equations

(a) $x + 3y = 11$

$4x - 7y = 6$

$\mathrm{Dx} = \begin{vmatrix} 11 & 3 \\ 6 & -7 \end{vmatrix}$ = 11(-7) – 6(3) = -77 – 18 = -95

$\mathrm{Dy} = \begin{vmatrix} 1 & 11 \\ 4 & 6 \end{vmatrix}$ = 1(6) – 4(11) = 6 – 44 = - 38

M = 1(-95)+3(-38) = -209

$$\mathrm{x} = \frac{\mathbf{C}_1\mathrm{Dx}}{\mathrm{a_1(Dx) + b_1(Dy)}} = \frac{11(-95)}{(-209)} = \mathbf{5}$$

$$\mathrm{y} = \frac{\mathbf{C}_1\mathrm{Dy}}{\mathrm{b_1(Dy) + a_1(Dx)}} = \frac{11(-38)}{(-209)} = \mathbf{2}$$

(a) $3\mathrm{x} + 2\mathrm{y} - \mathrm{z} = 11$

$2\mathrm{x} - 3\mathrm{y} + \mathrm{z} = 7$

$5\mathrm{x} + \mathrm{y} - 2\mathrm{z} = 12$

$$\mathrm{Dx} = \begin{vmatrix} 11 & 2 & -1 \\ 7 & -3 & 1 \\ 12 & 1 & -2 \end{vmatrix}$$

$$11[(-3)(-2)-1(1)]-2[7(-2)-12(1)]-1[7(1)-12(-3)]=11(5)-2(-26)-1(43)=\ 64$$

$$\text{Dy} = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 7 & 1 \\ 5 & 12 & -2 \end{vmatrix}$$

$$3[7(-2)-12(1)]-11[2(-2)-5(1)]-1[2(12)-5(7)]=3(-26)-11(-9)-1(-11)=32$$

$$\text{Dy} = \begin{vmatrix} 3 & 2 & 11 \\ 2 & -3 & 7 \\ 5 & 1 & 12 \end{vmatrix}$$

$$3[(-3)(12)-1(7)]-2[2(12)-5(7)]+11[2(1)-5(-3)]=3(-43)-2(-11)+11(17)]=80$$

$$\text{M} = 3(64)+2(32)-1(80) = 176$$

$$x=\frac{d_1\text{Dx}}{\text{M}}=\ \frac{11(64)}{176}=\ \frac{704}{176}=4$$

$$y=\frac{d_1\text{Dy}}{\text{M}}=\frac{11(32)}{176}=\ \frac{352}{176}=2$$

$$z=\frac{d_1\text{Dz}}{\text{M}}=\frac{11(80)}{176}=\ \frac{880}{176}=5$$

DISCUSSION

The aim of this paper was successfully achieved as can be observed through solving problems and proving the formula. The solutions above shows no absolute error as compared to other solutions that can be obtained using other researcher's methods. This is reason enough to accept that Musonda's ratio method is real or authentic. If Crammers method is compared with Musonda's method, Musonda's ratio method uses one determinant less than Crammer's rule. Musonda's ratio method uses coefficients from a particular equation to multiply with determinants while Crammer's method do not.

CONCLUSION

Like the objective Aim states, this conceptual paper aimed at inventing a formula of solving simultaneous equations which involves ratios. The Aim is fully achieved based on solutions obtained in examples and by proving the Musonda's ratio method. This formula can now be used in our Mathematics literature to solve linear simultaneous equations. The steps involved in inventing the Musonda’s ratio method can also be an eye opener in inventing other mathematical formulas in future.

REFERRENCE

1. Ashby, Steven F. The generalized SRT iteration for linear systems of equations. Urbana, IL(1304 W. Springfield Ave, Urbana 61801-2987):Dept. of computer science, University of Illinois at Urbana-Champaign, 1986.
2. Codenotti, Bruno. Parallel complexity linear system solution. Singapore: World Scientific, 1991.
3. Greenbaum, Anne. Iterative methods for solving linear systems. Philadelphia, PA: society for industrial and applied mathematics, 1997.
4. Lapan, Glenda. It’s in the system: Systems of linear equations and inequalities. Boston, Massachusetts: Pearson, 2014.
5. Skorniakov, L.A. Systems of linear equations. Mir Publishers, 1988.