Cosmic length $\mathscr{L}$ and constant ξ = *f*(G)

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Abstract : Following in the footsteps of Max Planck, through the path of dimensional analysis, this study reveals the factor $\xi = 1{,}5457109\times10^{11}$ which turns out to be omnipresent in cosmology, at all scales. This factor also appears in the form ξ^4, as the ratio between the Coulomb energy and the gravitational energy, acting on an electron-positron pair, separated by a unit distance. This same factor reveals, through several independent paths, a cosmic and constant length: $\mathscr{L}$ = 2483 Gyl. In addition to the valuable tool offered by dimensional analysis, this study uses the mass unit "electron" which allows to compare dimensionless ratios with masses expressed in the same unit. The study indicates that Newton's constant G, is determined with the parameters [M, L, T] of the electron and the factor ξ^4. Via the factor ξ^8 and the standard photon density, $\rho_{ph} = 4.1073\times10^8$ u / m³, the study predicts the mass density $\rho = 8.953389\times10^{-10}$ J/ m³. The application of the Friedmann-Lemaitre equation, reveals: H_o = 72.7411 km s-1 $Mpc^{-1} = 2.360126\times10^{-18}$ s^{-1}; the age of the Universe, 13.44 Gy; the critical density $\rho c = 8.953386\times10^{-10}$ J/ m³ < $\rho = 8.953389\times10^{-10}$ J/ m³. These results are close to the results obtained by the WMAP [1] space probe, namely: 13.77 Gy; Ho = 73±3 km s-1 Mpc^{-1} = $(2.35\pm0.10)\times10^{-18}$ s^{-1}. However, they are in disagreement with the results of the Planck 2018 mission: $H_o = 67.4\pm0.4$ km s^{-1} $Mpc^{-1} = (2.18\pm0.01)\times10^{-18}$ s^{-1}. This study also reveals what could be likened to a cosmic "string" of length $\mathscr{L}$ and Planck section, whose volume is equal to the Compton volume of the electron.

Introduction

The ratio between the Planck parameters [M, L T], and those of the electron, shows the factor $\xi^2 = 2.3892\times10^{22}$, according to:

$$\ell_p = \sqrt{\frac{G\hbar}{c^3}} = \frac{\lambda_e}{\xi^2}, \qquad (1)$$

$$m_p = \sqrt{\frac{\hbar c}{G}} = m_e \xi^2, \qquad (2)$$

$$t_p = \sqrt{\frac{G\hbar}{c^5}} = \frac{t_e}{\xi^2}, \qquad (3)$$

The reduced Planck constant ħ, is also determined with the [M, L, T] parameters of the electron, according to:

$$\hbar = \frac{m_e \lambda\!\!\!^-_e{}^2}{t_e}, \qquad (4)$$

With $t_e = \lambda\!\!\!^-_e / c = 1.2880886664 \times 10^{-21}$ s. Continuing along the path of Max Planck's dimensional analysis, the factor ξ^4 can be determined from the CODATA[2] value of G = 6.67430(15) $\times 10^{-11}$ $m^3\, kg^{-1}\, s^{-2}$, according to:

$$\xi^4 = \frac{\lambda\!\!\!^-_e{}^3}{G\ m_e\ t_e^2} = \frac{\lambda\!\!\!^-_e\, c^2}{G\ m_e} = 5{,}70838(15) \times 10^{44}, \qquad (5)$$

that to say : $\xi = 1{,}5457109(15) \times 10^{11}$. This confirms G, according to:

$$G = \frac{\lambda\!\!\!^-_e{}^3}{\xi^4\ m_e\ t_e^2} = \frac{\lambda\!\!\!^-_e\ c^2}{\xi^4\ m_e} = 6{,}67430(15) \times 10^{-11} kg^{-1} m^3 s^{-2}, \qquad (6)$$

The factor ξ^4 also appears as the ratio between the Coulomb energy and the gravitational energy, exerted between an electron and positron pair, separated by a unit length:

$$\xi^4 = \frac{\left(\dfrac{-e^2 \alpha}{4 \pi \epsilon_o}\right)}{-G\ m_e^2} = 5{,}70838(15) \times 10^{44}, \qquad (7)$$

with $\alpha = 137{,}035999177(21)$[3], the fine structure constant. (7) can be written by including the 4 forms of electron energy:

$$\frac{e^2}{4 \pi \epsilon_o\, r_0} = \frac{\xi^4 G\ m_e^2}{\lambda\!\!\!^-_e} = m_e c^2 = K_B\ T_e = 8{,}1871057879 \times 10^{-14} J, \qquad (8)$$

With $T_e = 5{,}9298974748 \times 10^9$ K. We note that the first term is constrained by the classical radius: $r_o = \lambda\!\!\!^-_e / \alpha$, not measurable today. Everything happens as if the initial interval r_o, had been enlarged to its measurable value today, i.e. $\lambda\!\!\!^-_e$. As $\xi^4 = f(1/G)$, we can write: ξ^4 G = Cte.

The length $\mathscr{L}$ obtained with the electron parameters and ξ^n

The following relation gives a length on the astronomical scale $\mathscr{L}$, which is necessarily a constant, in the form of a Schwarzschild black hole:

$$\mathscr{L} = \frac{2\ G \xi^8 m_e}{\alpha^2 c^2} = 2{,}34768 \times 10^{28}\ m, \qquad (9)$$

The length $\mathscr{L}$ (2483 Gyl) is confirmed by the relation (10), excluding the ξ^n factor:

$$\mathscr{L} = \frac{2\,\hbar^2}{\alpha^2 G\, m_e^3} = 2{,}34768 \times 10^{28}\ m, \qquad (10)$$

These two other relationships also confirm the length $\mathscr{L}$:

$$\mathscr{L} = \sqrt{\frac{4\,\xi^4\,\lambda\!\!\!^-_e{}^3\,c^2}{\alpha^4 G\, m_e}} = 2{,}3476 \times 10^{28}\ m, \qquad (11)$$

and:

$$\mathscr{L} = \frac{2\,\lambda\!\!\!^-_e\,\xi^4}{\alpha^2} = 2{,}34768 \times 10^{28}\ m \qquad (12)$$

(12) shows a direct connection between $\mathscr{L}$ and the electron Compton length $\lambda\!\!\!^-_e$, via the scale factor ξ^4. The length $\mathscr{L}$ represents a distance of 2483 light-years. Hereafter, (13) indicates that the volume of a "string" with a Planck section and length $\mathscr{L}$, is equal to the electron Compton volume, according to:

$$(\alpha\,\ell_p)^2\,\mathscr{L} = 2\,\lambda\!\!\!^-_e{}^3, \qquad (13)$$

However, equality requires that the Planck length be enlarged by the factor α. It should be noted that the prism of dimensional coherence alone does not necessarily give the coefficients such as, for example, α in (13). Finally, this relation confirms the standard value of the speed c.

$$c = \sqrt[4]{\frac{\lambda\!\!\!^-_e\,\xi^4\,m_e\,G}{t_e^2}} = 2{,}9979245800 \times 10^8\ m/s, \qquad (14)$$

or:

$$c = \sqrt[4]{\frac{\mathscr{L}\ m_e\,G\,\alpha^2}{2\,t_e^2}} = 2{,}9979245800 \times 10^8\ m/s, \qquad (15)$$

The mass $\mathscr{M} = f(\xi^8)$

Relation (9), homogeneous to a Schwarzschild black hole, can be written, up to a factor of 1/2:

$$\mathscr{L} = \frac{G\,\mathscr{M}}{c^2} = 2{,}34768 \times 10^{28}\ m, \qquad (16)$$

with :

$$\mathscr{M} = \frac{2\,\xi^8\,m_e}{\alpha^2} = 3{,}161374 \times 10^{55}\ kg, \qquad (17)$$

This mass $\mathscr{M}$ is greater by a factor of 11.4 than that generally estimated (all contributions combined), for the universe, i.e. M = 2.78 × 10^{54} kg. However, it verifies the following equality with the gravitational potential: $-G M^2/R$:

$$-\frac{2\,G\,\mathscr{M}^2}{\mathscr{L}} + \mathscr{M}\,c^2 + \mathscr{M}\,v^2 = 0\,, \qquad (18)$$

With v = c, in the framework of the non-relativistic kinetic energy of the expansion of the universe. The equilibrium of (18) is constrained by the magnitude of the length $\mathscr{L}$. The sum (mass energy + kinetic energy), is given by:

$$E = 2\,\mathscr{M}\,c^2 = 5{,}682603 \times 10^{72}\ \textit{joules}\,, \qquad (19)$$

Par le prisme de la simple analyse dimensionnelle, la longueur $\mathscr{L}$ apparaît comme le rayon d'un « univers trou noir ». L'existence de ce rayon fini $\mathscr{L}$, implique une densité de matière, supérieure à la densité critique qui elle, serait relative à un univers « plat ». L'égalité de la relation (18) indique qu'il n'existe pas de coefficient caché.

Ratio between photon and mass densities

For this ratio to make sense, it is appropriate to translate these two types of density into the same unit. This is made possible by expressing the mass no longer with the arbitrary unit "kg", but with the universal unit "electron mass". For example, the mass of the proton is commonly expressed in electron units, i.e.: 1836.152673421, dimensionless. Thus the mass density can be expressed in volume units, like the photon density. Let us assume that the mass of (17), $\mathscr{M} = f(2\xi 8/\alpha^2)$, is a survivor of an initial mass $\mathscr{M}o = f(4\,\xi^8)$ having undergone an annihilation rate α^2. Thus the number of initial photons would be equal to:

$$N_{ph} = 4\,\xi^8 - 2\,\frac{\xi^8}{\alpha^2} \approx 4\,\xi^8 = 1{,}303425 \times 10^{90}\,, \qquad (20)$$

The photon density [4] is of the order of: ρ_{ph} = 4.1073 ×10^8 units / m³. By taking the number of photons from (20), we obtain a radius R(T) noted R_T according to:

$$R_T = \sqrt[3]{\frac{N_{ph}}{\rho_{ph}}} = 1{,}469524 \times 10^{27}\,m\,, \qquad (21)$$

This radius of 155 Gyl is 12 times larger than the Hubble radius. Below, (22) indicates the mass density, according to:

$$\mu_T = \frac{\mathscr{M}}{R_T^3} = 9{,}961989 \times 10^{-27}\,kg/m^3\,, \qquad (22)$$

(23) expresses (22) in the unit "electron", replacing the unit "kg":

$$\rho_T = \frac{\mu_T}{m_e} = 1{,}093596 \times 10^4\,u/m^3\,, \qquad (23)$$

Hereinafter (24) expresses (23) in joules/m³:

$$\rho = \rho_T\, c^2 = 8{,}953389 \times 10^{-10}\, J/m^3, \qquad (24)$$

This density is consistent with WMAP measurements of the critical density given at: ρc = (9.0±0.7)×10^{-10} J/m³.

Verification with the Friedmann-Lemaitre equation:

The Friedmann Lemaitre equation has 3 unknowns, H², K and ρ:

$$\left(H^2 + \frac{K c^2}{\alpha^2}\right) = \frac{8\pi G}{3c^2}\rho, \qquad (25)$$

However, taking ρ from (24) and K = R_T^{-2}, as positive spatial curvature, we obtain H² according to:

$$H^2 = = \frac{8\pi G}{3c^2}\rho - \frac{R_T^{-2}\, c^2}{\alpha^2} = 5{,}57019518 \times 10^{-36}\, s^{-2}, \qquad (26)$$

or H = 2.360125×10^{-18} s^{-2} or 72.741 km s^{-1} Mpc^{-1} and 13.44 Gy. These results are close to the WMAP measurements, i.e.: 13.77 Gy; H_o = 73±3 km s^{-1} Mpc^{-1} or: (2.35±0.10)×10^{-18} s^{-1}. Thus the critical density can be calculated according to:

$$\rho_c = \frac{3H^2 c^2}{8\pi G} = 8{,}953386 \times 10^{-10}\, J/m^3, \qquad (27)$$

Given the positive curvature, the critical density is lower than the mass density of (24) i.e.: ρ = 8.953389×10^{-10} J/m³.

$$\frac{\rho}{\rho_c} = \Omega_m = 1 + 3{,}97876 \times 10^{-7}, \qquad (28)$$

(28) indicates that the curvature is positive.

Conclusion

The logical-deductive progression starts from relations (1 to 5) which reveal the factor ξ in the forms ξ^2 and ξ^4 in relation to the parameters of the electron. Relation (8) clearly indicates that the source of the factor α is explained by a primordial phase where the classical radius replaced the Compton length of the electron. This is similar to an increase in the elementary interval by annihilation of the ratio α in one dimension. Relations (9, 10, 11), are consistent with the factor α² in two dimensions. Relation (17) assumes that a number of masses, equivalent to 4 ξ^8 "electron" units, were annihilated into 4 ξ^8 initial photons to leave only the equivalent of 4 ξ^8/α^2 surviving elementary masses. Knowing the photon density, relation (24) deduces the mass density (all contributions combined). Thus, with the help of the Friedman-

Lemaitre equation, we deduce H^2. One of the keys that allow the announced results consists of using the unit "electron" to replace the unit "kg", arbitrary of its state. This allows in particular to compare the photon density with the mass density. The results of this study depend on the value of the photon density expressed in number per unit of volume. Even by varying this data, the curvature always remains positive. Relation (13) curiously gives the equality of volume between the Compton volume of the electron and that of the "string" of length $\mathscr{L}$ and Planck section.

Bibliographie

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