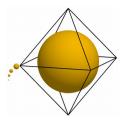
On the 14-Octahedron Backbone of the Bimetric 64-Tetrahedron Grid Universe

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Abstract: The 64-Tetrahedron Grid (64-TG) stands on a network of fourteen octahedral cavities, with tetrahedra filling up the interstices. Fibonacci numbers seem to drive the embedded Platonic and Archimedean solids. Tetrahedral packing densities are 0.53 in the 64-TG and 0.35 in the cuboctahedron. The inceptive construction protocol of the 64-TG universe involves a central coordinating unit. Jitterbug cycles require a non-rigid tetrahedron lattice. Puzzling equivalences amongst volumes & areas are found.

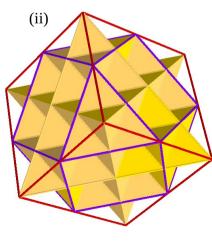
1. Introduction

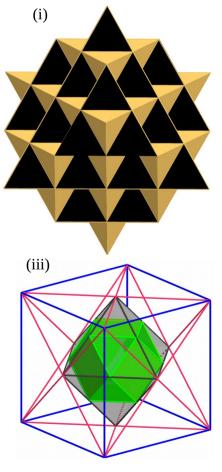
The highly symmetrical polyhedron commonly referred as 64-Tetrahedron Grid (64-TG) has become a mythical and mystical geometric structure of increasing interest. It is composed of 64 interlocking tetrahedra with edge =a/2 if taken as reference the edge of the embedded cuboctahedron equal to a^1 . Examination of the latticework reveals 240 segments and 63 nodes. Although without formal mathematical corroboration, the 64-TG is believed to be the fundamental blueprint of the cosmos, intricately weaving together the fabric of space and time [1]. The 64-TG is a remarkable latticework manifesting bipolar, fractal, and seemingly holographic characteristics (Fig.1) [2].

In addition to the stellated octahedron, the grid integrates several Platonic solids, in particular tetrahedron, octahedron, and cube, as well as an Archimedean solid known as cuboctahedron. The cube_{av2} perfectly encompasses the entire 64-TG with the 6 square faces aligned with the 6 faces of the cuboctahedron_a but rotated 45° (Fig.1ii). A regular octahedron_a and a smaller cuboctahedron_{a/2} are nested at the center of the latticework, one inside the other (Fig.1 iii)

Figure 1: (i) 3-D illustration of the 64-TG with its

inherent dual polarity reflected by the two opposed colors; (ii) The embedded cuboctahedron (purple segments), and the perfect inscription into a cube (red segments); (iii) The central nesting of a smaller cuboctahedron_{a/2} (green color) within an octahedron_a (gray color)

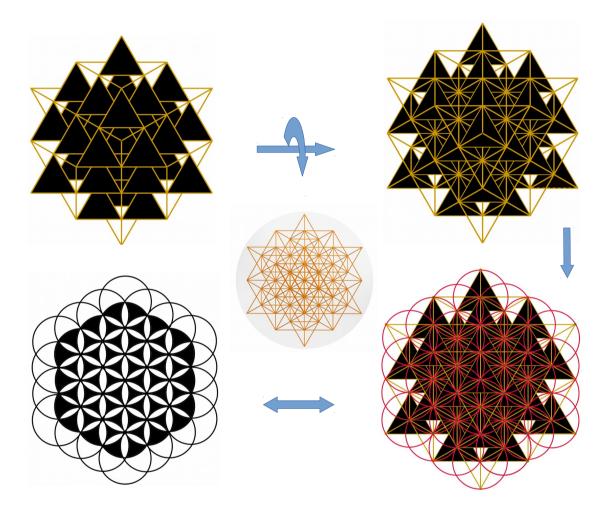




¹ The cuboctahedron (edge=a) will be the reference scale for the rest of the article. In order to distinguish the different polyhedron scales within the 64-TG, an index with edge value will be used

The radial projection of each tetrahedron onto its own circumsphere (known as spherical tiling) produces an interconnected pattern of overlapping spheres. Fig.2 is a 2-D representation of such a pattern, obtained after slight rotation of the 64-TG composed of alternate black and transparent tetrahedra_{a/2}. This front view exposes 49 nodes and 37 intertwined spheres (the other ones being aligned abaft). These figures are common representations of the 64-TG and its associated symbols known as "Flower-of-Life" in the sacred geometry realm.

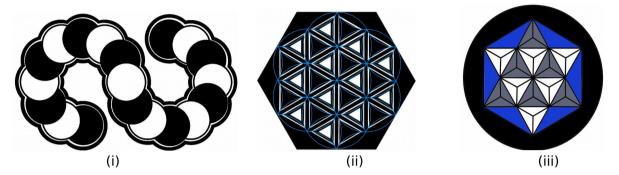
Figure 2: Common representations of the 64-TG and associated "Flower-of-Life" symbols



2. The inherent "bipolar" nature of the 64-TG

The 64-TG exhibits tetrahedron units with opposite attributes commonly depicted by two different colors. The nature of these opposing characteristics is currently unclear, but could involve physico-chemical properties such as electromagnetism, mass/energy, density, electrical charge, gravity, etc. The simultaneous creation and omnipresence of opposites in the universe, both at the cosmic or quantum scale, is inherent to the very nature of the origin of the universe itself. The creation of energy from a preexisting cosmic Substance at maximum entropy requires asymmetric divisions, hence opposite "polarities" [3-5]. Therefore every manifestation of mass/energy in the universe can only arise along with its opposite. For the sake of simplicity and consistency, the noun "polarity" used in the text will define this duality, although polarity normally applies more specifically to electrical charges. Fig.3 illustrate this duality.

Figure 3: (i) Depiction using opposing colors of the dual process leading the simultaneous creation of opposite constituents and to the emergence of mass/energy in the universe. These opposites have generative and destructive properties, acting in successive and alternate preponderance. Action prompts reaction and disintegration follows creation; (ii) 2-D depiction of tetrahedral units within the 64-TG. Each tetrahedron is totally surrounded by tetrahedrons of opposite polarity; (iii) The stellated octahedron is a fundamental geometric figure nested within the 64-TG at different scales. Opposites are also painted in B&W



The Big Bang theory and its associated mathematical singularity have been incompatible with the first law of thermodynamics, despite numerous attempts such as the classical bounce theory [6] to circumvent this issue. In the Big Split model [3-5], the singularity vanishes due to the existence of a preexisting Substance at zero energy and maximum entropy pervading the whole space, should we say making up the entire spacetime. This original Substance, as the precursor of both the observable cosmos and invisible universe(s), generates via initial asymmetric self-division two opposite daughter substances, which simultaneously provide the potential energy to the early universe, satisfying concurrently the first law of thermodynamics. Further subdivisions of the two daughter Substances will provide opposites pairs at various scales, and the vacuum components will emerge from the division and recombination pathways [5].

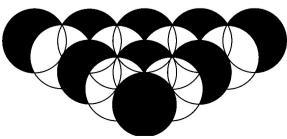
3. The bimetric nature of the 64-TG universe

Although more difficult to detect in the universe from the earth, one specific property of the vacuum relates to negative mass/energy. Since the 1950s a number of physicists including J-P Petit [7], P. Marquet [8], J.S. Farnes [9], H. Bondi [10], W.B. Bonnor [11], S. Hossenfelder [12], A. Sakharov [13] have supported the existence of negative mass/energy in the universe. In addition to the original Einstein field equation, a set of two field equations displaying negative sign has been derived via different methods [7-8]. This set of two coupled field equations could be written:

(1)
$$\begin{cases} \mathsf{R}_{\mu\nu}^{(\star)} - \frac{1}{2} \mathsf{R}^{(\star)} \mathsf{g}_{\mu\nu}^{(\star)} = +\chi [\mathsf{T}_{\mu\nu}^{(\star)} + \mathsf{T}_{\mu\nu}^{(-)}] \\ \mathsf{R}_{\mu\nu}^{(-)} - \frac{1}{2} \mathsf{R}^{(-)} \mathsf{g}_{\mu\nu}^{(-)} = -\chi [\mathsf{T}_{\mu\nu}^{(\star)} + \mathsf{T}_{\mu\nu}^{(-)}] \end{cases}$$

The 64-TG universe is composed of 64 adjacent tetrahedral cells with alternate polarities/properties, which spherical analogs overlap (Fig.4)

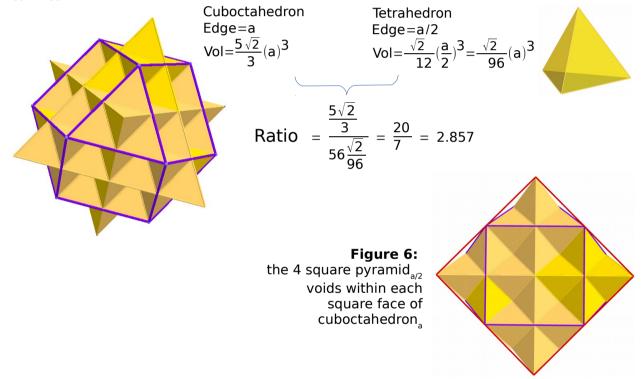
Figure 4: Single layer arrangement of spherical analogs of tetrahedron units within the 64-TG

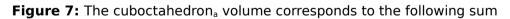


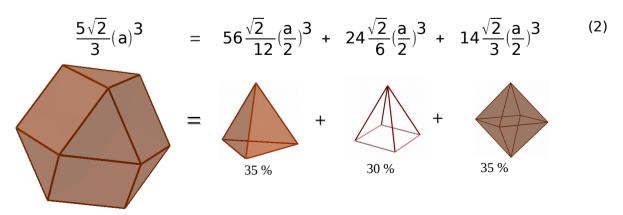
4. The puzzling network of 14 octahedral cavities in the 64-TG

One peculiar characteristic of the 64-TG is the stunning presence of 14 octahedral_{a/2} "cavities" or "voids" underlying the tetrahedral packing. Examination of the cuboctahedron_a volume embedded in the 64-TG reveals an extra volume relative to the total volume occupied by 56 tetrahedra_{a/2} (8/64 tetrahedra_{a/2} are located outside the cuboctahedron_a). As a matter of fact the cuboctahedron_a volume is 2.857 times the volume occupied by 56 tetrahedra_{a/2} (Fig.5). The extra volume corresponds precisely to 14 octahedra_{a/2} + 24 square pyramids_{a/2} (Eq.2). From the 14 octahedra_{a/2} 8 are nested within the 8 stellated octahedra_a (Fig.7-8) and 6 are embedded in the octahedron_a at the center of the 64-TG (Fig.8-9). On the other hand, the 24 square pyramids_{a/2} correspond to the 4 pyramida_{a/2} cavities facing each cuboctahedron_a square face (Fig.6).

Figure 5: The cuboctahedron_a volume is 2.857 times the volume occupied by 56 tetrahedra_{a/2}. The extra volume is precisely 14 octahedra_{a/2} plus 24 square pyramids_{a/2} cavities







We can determine from Fig.7 that the volume occupied by the 24 square pyramid_{a/2} cavities correspond to 30% (vol= $\sqrt{2}/2$ with a=1) of the cuboctahedron_a volume. On the other hand, the 56 tetrahedra_{a/2} and the 14 octahedra_{a/2} occupy both 35% (vol= $7\sqrt{2}/12$) of the cuboctahedron_a. The fact that the 56 tetrahedra_{a/2} and the 14 octahedra_{a/2} occupy similar volumes within the 64-TG is also puzzling.

The following are graphical representations of the octahedral cavities within the 64-TG. Eight of those octahedra_{a/2} are nested within the 8 stellated octahedra_a making up the the full 64-TG shape (Fig.8). These octahedra_{a/2} are all interconnected via their vertices (Fig.9) and each of their triangular face correspond to one tetrahedron_{a/2} face.

Figure 8:

The full 64-TG is erected upon the 8 stellated octahedra_a. Colors are used only to differentiate individual location.

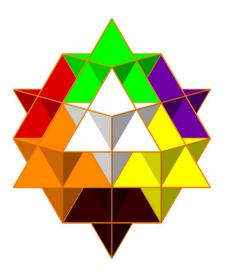
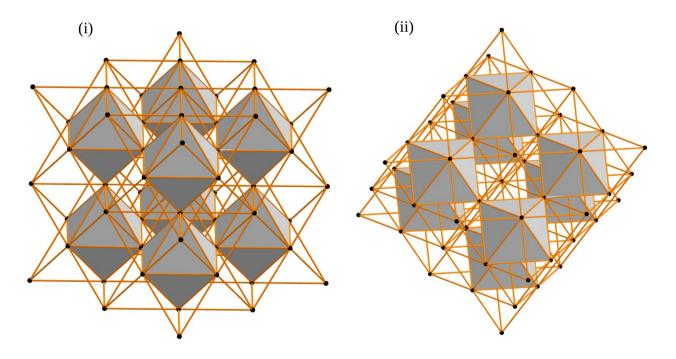


Figure 9: The 8 interconnected octahedra_{a/2} network viewed from different angles (i) and (ii)



The other 6 octahedra_{a/2} are located within the central octahedron_a. The remaining volume of the octahedron_a is occupied by 8 tetrahedra_{a/2}. Each of those 6 octahedra_{a/2} meet via one vertex to the central point of the cuboctahedron_a and via the opposite vertex the center of the cuboctahedron_a square face. Therefore the central octahedron_a harbors 6 octahedra_{a/2} and 8 tetrahedra_{a/2} as depicted in Fig.10-11

Figure 10: Depiction of the central octahedron_a harboring 6 octahedra_{a/2} and 8 tetrahedra_{a/2} from the 64-TG perspective (i), and from the central cuboctahedron_a perspective (ii)

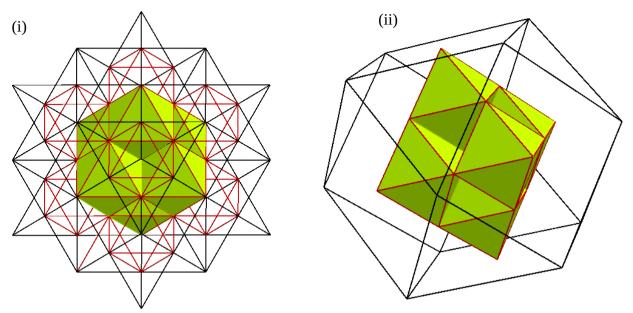
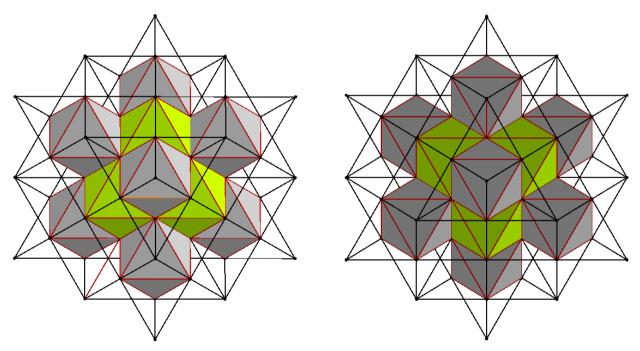


Figure 11: Views of 14 the octahedral cavities network embedded in the 64-TG (4 of them are aligned behind the structure)



It is worth noticing that in the 64-TG, the 6 square faces of the nested cuboctahedron_a have no rigid structure, as opposed to the triangular faces. In fact, each square face embeds 4 square pyramid_{a/2} cavities and is therefore "free". This important feature will allow the dynamic Jitterbug helical contraction/expansion to fold along the square face diagonals. As a matter of fact, for the Jitterbug transition to occur, the cuboctahedron_a square faces need rigid edges and flexible joints, but no rigid face. On the contrary, the rigidity of triangular faces does not impact the motion since the latter only rotate during the process.

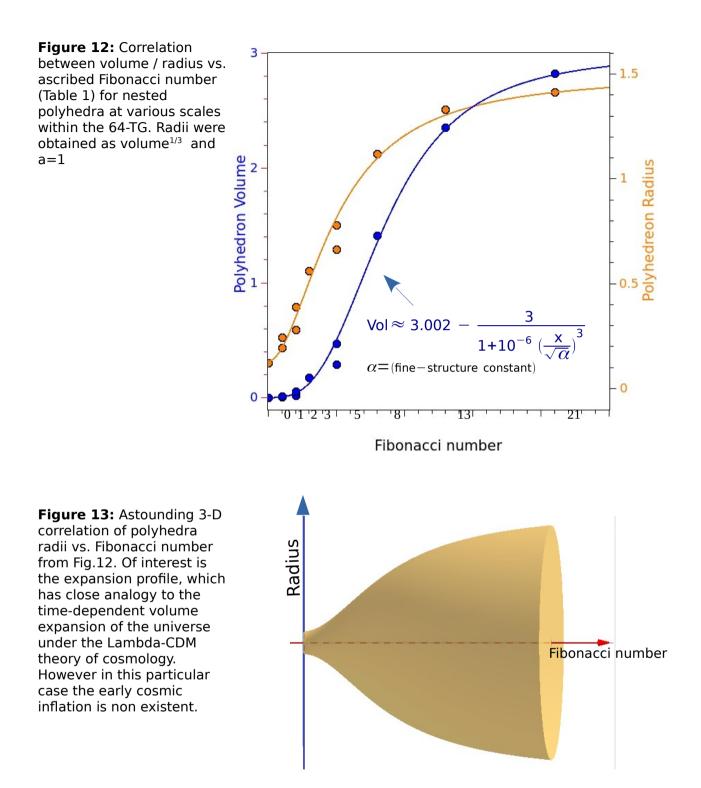
5. The 64-TG and embedded Platonic/Archimedean solids: Volumes, Radii, & Fibonacci numbers

As described above, several polyhedra at various scales are nested within the 64-TG. Their volumes can be expressed using early Fibonacci numbers or combination thereof, using a=1 (Table 1, column 3).

Further, Fibonacci numbers (0-21) were tentatively assigned to polyhedra radii nested within the 64-TG (Table 1, column 6). Correlations at Fig.12 were obtained for volumes and radii (as volume^{1/3}) vs. ascribed Fibonacci numbers. Curve functions were tentatively calculated and appear to integrate the fine-structure constant α . Interestingly, the radius curve shape closely resembles the time-based evolution of the universe in the Lambda-CDM model of cosmology (Fig.13) without inflationary epoch. It is known that the cosmic inflation theory has been long criticized, even by some of the original creators [14]

Table 1: Volumes of polyhedra nested within the 64-TG before and after Jitterbug helical contraction. In column 6, Fibonacci numbers were assigned to polyhedra volumes/radii and found to fit correlations in Fig.12

1	2	3	4	5	6
Polyhedron	Edge	Volume (a=1)	Volume Calcd	Radius =Vol ^{1/3} (a=1)	Assigned Fibonacci Number
Cube _{v2a}	√2a	2 \sqrt{2}	2.828	1.41	21
Cuboctahedron _a	а	$5\frac{\sqrt{2}}{3}$	2.357	1.33	13
Stellated Octahedron _{2a}	2a	√2	1.414	1.12	8
Octahedron _a	а	$\frac{\sqrt{2}}{3}$	0.471	0.78	5
Cuboctahedron _{a/2}	a/2	$5\frac{\sqrt{2}}{3*8}$	0.295	0.66	5
Stellated Octahedron _a	а	$\frac{\sqrt{2}}{8}$	0.177	0.56	3
Octahedron _{a/2}	a/2	$\frac{\sqrt{2}}{3*8}$	0.059	0.39	2
Stellated Octahedron _{a/2}	a/2	$\frac{\sqrt{2}}{8*8}$	0.022	0.28	2
Tetrahedron _{a/2}	a/2	$\frac{\sqrt{2}}{3 * 2^5}$	0.0147	0.24)
Octahedron _{a/4}	a/4	$\frac{\sqrt{2}}{3*8^2}$	0.0074	0.19)
Tetrahedron _{a/4}	a/4	$\frac{\sqrt{2}}{3*8*2^5}$	0.0018	0.12	0



Of particular interest is the determination of the precise volume occupied by the 64-TG, which can be deducted from different ways. One of them starts from the cuboctahedron_a while another one uses the 8 stellated octahedra_a. They are summarized in Table 2. The 64-TG volume calculated is $5\sqrt{2a^3}/4$ or $1.768(a)^3$ with "a" defined as cuboctahedron_a edge. It is worth noticing here again the similar volumes occupied by the large stellated octahedron_{2a} and the sum of 8 stellated octahedra_a.

Polyhedron	Edge	Volume	Volume 64-TG
Cuboctahedron _a	а	$\frac{5\sqrt{2}}{3}a^{3}$ (+)	
8 Tetrahedron _{a/2}	a/2	$\frac{\sqrt{2}}{12}a^3$ (+)	
24 Square Pyramid _{a/2} voids	a/2	$\frac{\sqrt{2}}{2}a^3 \qquad (-)$	$\frac{5\sqrt{2}}{4}a^3$
			4
8 Stellated Octahedron _a	а	$\sqrt{2}a^{3}$ (+)	
6 Octahedron _{a/2}	a/2	$\frac{\sqrt{2}}{4}a^3 \qquad (+)$	

Table 2: Determination of the 64-TG volume

6. Tetrahedral packing

As calculated in Table 3, the tetrahedral packing density in the 64-TG is particularly low, due to the presence of a large network of 14 octahedral cavities. This tetrahedral packing density is 0.53 in the 64-TG, and falls even lower with respect to the cuboctahedron_a (0.35), octahedron_a (0.25), and large stellated octahedron_{2a} (0.166).

Polyhedron	64-TG	Cuboctahedron _a	Stellated Octahedron _{2a}	Octahedron _a
Volume	$\frac{5\sqrt{2}}{4}a^3$	$\frac{5\sqrt{2}}{3}a^3$	$\sqrt{2}a^3$	$\frac{\sqrt{2}}{3}a^3$
# Tetrahedron _{$a/2 within the polyhedron$}	64	56	16	8
Tetrahedron _{a/2} total volume	$\frac{2\sqrt{2}}{3}a^3$	$\frac{7\sqrt{2}}{12}a^3$	$\frac{\sqrt{2}}{6}a^3$	$\frac{\sqrt{2}}{12}a^3$
Tetrahedral Packing Density	$\frac{8}{15} = 0.53$	$\frac{7}{20} = 0.35$	$\frac{1}{6} = 0.166$	$\frac{1}{4} = 0.25$

Table 3:	Tetrahedron _{a/2}	packing densitie	s within the 64-TG and	d its nested polyhedra
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7. Surface areas and holographic imprints

It turns out that the area of the cube_{_{2a}} side (=2a^2) is equivalent to the sum of 16 internal tetrahedron_{a/2} faces within each cuboctahedron_a square face (Fig.14). Further, these 16 triangular faces point inward, whereas the 3 tetrahedron_{a/2} faces on each cuboctahedron_a triangular side point outward. From this observation we can infer that the flow of energy and/or information throughout the 64-TG would follow this in/out path.

Figure 14: Coincidentally, the sum of 16 tetrahedron_{a/2} faces inside the purple square face of the cuboctahedron_a is identical to the opposing surface area of the red cube_{v2a} side (= $2a^2$)

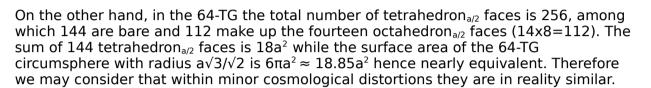


Figure 15: The surface area of the 64-TG circumsphere is almost equivalent to the sum of the 144 tetrahedron_{a/2} bare faces

Then if we consider the 144 bare triangles holographically imprinted on the surface of the circumsphere, then the surface area of each individual spherical triangle is

$$\Delta = \frac{6\pi a^2}{144} = \frac{\pi a^2}{24}$$
 (3)

The surface area Δ can also be expressed with the following [15] :

$$\Delta = R^{2}[(A+B+C)-\pi] = R^{2}E$$
 (4)

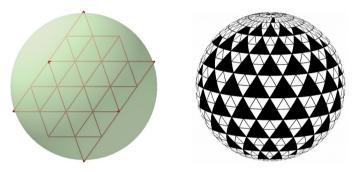
with A,B,C being the angles in radian at the 3 vertices along the surface of the sphere, and E is known as the spherical excess.

Then (3)+(4) gives

$$E = \frac{\Lambda}{R^2} = \frac{\frac{\pi a^2}{24}}{\frac{3a^2}{2}} = \frac{\pi}{36}$$
 (5)

if we consider then A=B=C for equilateral triangles, then from (4) and (5) we obtain the angle of spherical triangles

$$(A+B+C) = E + \pi = \frac{\pi}{36} + \pi = \frac{37\pi}{36}$$
then $A=B=C = \frac{1}{3}(\frac{37\pi}{36}) \approx 1.076 \text{ radian} \approx 61.666^{\circ}$
(6)



8. Jitterbug transformation applied to the 64-TG

Buckminster Fuller was likely the first to illustrate and publish the dynamic transformation of the cuboctahedron through rigid-edge helical contraction into the icosahedron and further down into the octahedron, and vice-versa, which he named Jitterbug. It was not only a remarkable intuition in itself, but he also had this second sight that Jitterbug could apply to the dynamics of the "Universe", as the following quote from 1975 about the Vector Equilibrium suggests [16]: "The vector equilibrium is the zero starting point for happenings or nonhappenings: it is the empty theater and empty circus and empty Universe ready to accommodate any act and any audience."

As previously cited, the square faces of the nested cuboctahedron_a in the 64-TG are nonrigid as they are exempt of embedded geometrical structure. This characteristics permits free folding along the square diagonals during the Jitterbug dynamics. However, the internal tetrahedral packing of the 64-TG must also allow the helical contraction and expansion motions. The details of this process are still unclear but ought to depend on the 14 octahedron_{a/2} moieties, as the tetrahedron_{a/2} latticework contracts by a factor of 2. Nested in the 64-TG, the octahedron_a and the cuboctahedron_{a/2} appear as the end geometries of the helical contraction of the 64-TG, as illustrated in Fig.16 below.

Figure 16: Illustration of the Jitterbug helical contraction applied to the 64-TG, from cuboctahedron_a to icosahedron_a and octahedron_a. In this process, the cuboctahedron_a is contracted into the already existing octahedron_a while the tetrahedral network appear to rearrange into the existing cuboctahedron_{a/2}, contracting by a factor of 2.

An important consideration is that due its high symmetry, the 64-TG is not chiral and therefore cannot have another enantiomer. However, the litterbug motion is symmetry breaking, with the spiral contraction operating either to the left or to the right of the 64-TG, creating a chiral transformation on either side. But chirality only operates during the litterbug transformation. At the end of the transformation, the litterbug can pursue further the helical expansion on the same side or back in the reverse direction, both directions creating the original 64-TG. One direction creates a continuous chiral 64-TG universe always spinning in the same direction alternating contraction/expansion cycles, while the reverse direction produces the other chiral structure and then oscillate between one chiral structure and the other. The period of this process is currently unknown.

Is the 64-TG universe permanently spinning in the same direction with successive helical contraction/expansion motions, or is the spinning alternating between one contraction direction and reversed direction expansion? The former would create a permanent rotational energy and angular momentum, while the latter would create and alternating rotational energy. Nevertheless, this successive dynamics of helical contraction/expansion could correspond to the famous "breathing" of the universe mentioned in various eastern philosophies. With respect to the Λ -CDM model of cosmology, this process could correspond to the so called Big Bounce theory.

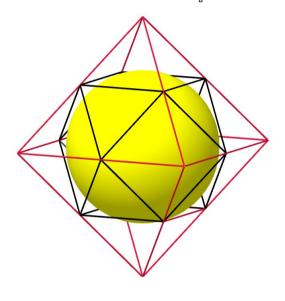
Of further interest is the octahedron, insphere volume. The inradius of the octahedron_a is given by $r_i=a\sqrt{6}$ (or $1/\sqrt{6}$ if a=1) and the insphere volume is $(2\pi/9\sqrt{6}=0.285)$, which is very close to the inscribed cuboctahedron_{a/2} volume $(5\sqrt{2}/24=0.295)$. But most importantly. the ratio of the insphere volume to the stellated octahedron_a volume ($\sqrt{2}/8$) is very close to the golden mean Φ (Eg.7). This result is meaningful as it relates to the initial buildup of the 64-TG (para.9) where the central spherical cosmic energy enables the formation of the eight stellated octahedron_a scaffolding making up the complete 64-TG lattice. Each stellated octahedron_a is a building block for the 64-TG superstructure. In order for the ratio to be exactly Φ the inradius r_i must be defined by Eq.8, which is 0.40873.. instead of 0.40824.. The universe boundaries might in reality be a little distorted from exact geometrical structures due to its curvature.

$$\frac{\frac{2\pi}{9\sqrt{6}}}{\frac{\sqrt{2}}{8}} = \frac{8\pi}{9\sqrt{3}} = 1.612 \approx \Phi$$
 (7)

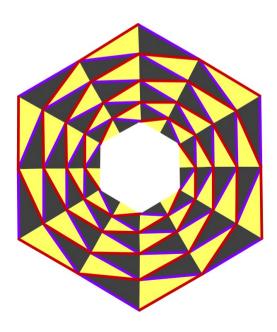
It's also legitimate to consider that during the Jitterbug process, tetrahedra of same polarity would spin on one side, while those of opposite polarity would spin toward the opposite direction, as illustrated by Fig.18

Figure 18:

Illustration of the Jitterbug transformation of the dual polarity tetrahedron lattice within the 64-TG, with tetrahedra of opposite polarity spiraling in opposite directions. **Figure 17:** The octahedron_a insphere seems to relate to the cuboctahedron_{a/2} and the stellated octahedron_a



$$r_i = \left(\frac{3\sqrt{2}}{64\pi} (1+\sqrt{5})\right)^{1/3}$$
 (8)



9. Initial build-up of the 64-TG universe

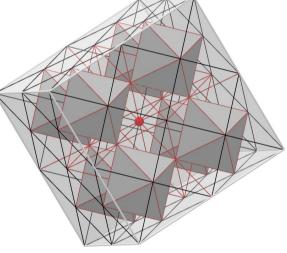
"There is no logical way to the discovery of elemental laws. There is only the way of intuition, which is helped by the feeling for the order lying behind appearances"-Einstein 1933 [17].

Away from the lame Big Bang theory of cosmology, it is of utmost interest to understand the come about of the 64-TG universe, whose beautiful fractal appearance can be misleading. As a matter of fact, it turns out that the most effective way to build up the 64-TG universe is through initial construction of the 8 stellated octahedron_a backbone and assemble them together into a magnificent fractal and holographic design. This process involves a stepwise construction of a skeleton made of eight interlinked octahedra_{a/2} organized into a perfect cube (Fig.19), and further development of stellations at the octahedron_{a/2} triangular faces. This process obviously requires a central coordinating unit that will provide building blocks with "energy" packets of alternating polarity to exact locations (Fig. 20-21)

Figure 19: (right) A

specific view of the interlinked octahedra_{a/2} at the heart of the eight star tetrahedra_a making up the 64-TG, with a central energy coordinating unit (not to scale) in red color

Figure 20: (below left) illustration of the construction of the 8 octahedron_{a/2} backbone of the 64-TG via central coordinating spherical units and energy packets exchange.



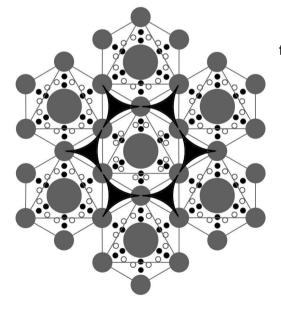
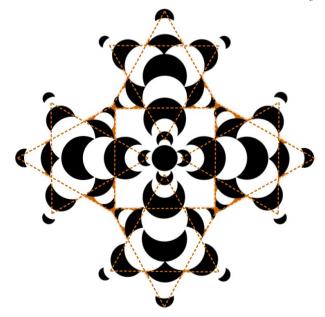


Figure 21: (below right) Other illustration of the formation of the stellated octahedra_a through provision of alternate polarity "energy" packets from a central reservoir and coordinating unit. (only the lower half part is depicted i.e. 4/8 stellated octahedra_a).

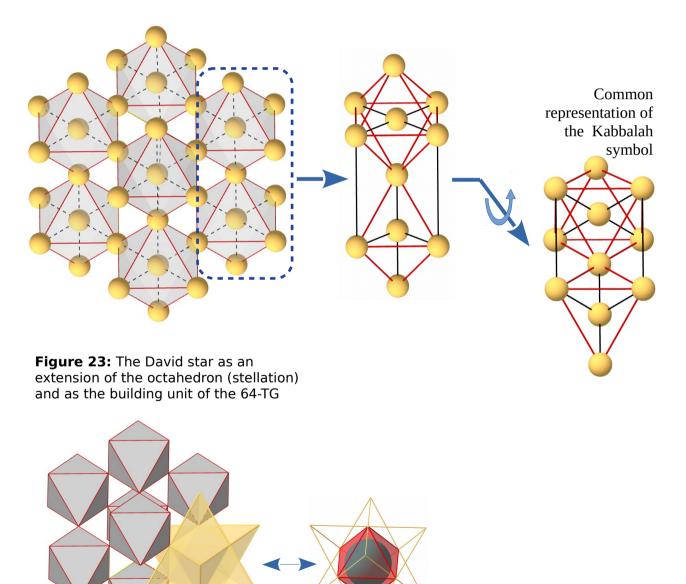


10. Sacred geometry symbols: the Star of David and the Kabbalah find their source in the octahedron backbone from the 64-TG

The octahedron network, backbone of the 64-TG, is also the basis of religious symbols which find their source in the construction of our universe, and the Jewish Kabbalah and Star of David are no exception. Therefore, those symbols are universal and cannot be appropriated since they are intuitive illustrations of the structure of the vacuum.

It has already published at [18] the construction of the 2-D Kabbalah from the double octahedron moiety through orthogonal projection of the 3-D structure and 35° bottom up rotation. This double octahedron moiety is found in the 3-D octahedron_{a/2} lattice within the 64-T as depicted in Fig.22. The David Star is also a 2-D representation of the star tetrahedron, also known as merkaba, which is illustrated in Fig.23

Figure 22: The Kabbalah symbol takes its origin in the octahedral lattice within the 64-TG



11. Conclusion

The octahedron is a fundamental structure in the 64-TG and in the universe. The collection of 64 tetrahedra making up the 64-TG sits on the underlying 14 octahedron network. The Jitterbug motion of the cuboctahedron also contracts into an octahedron. What is the exact purpose/role of these 14 octahedra? Do they facilitate the Jitterbug motion of the universe? Are they reservoir of antimatter or black holes? Are they different dimensions of a multiverse? Some eastern philosophies mention the existence of "14 worlds", is there a link and what does it mean?

Figure 24: The octahedron appears as a most fundamental structure of the universe at various scales, from which and to which all other cosmic structures might evolve.

"What I see in nature is a magnificent structure that we can comprehend only very imperfectly, and that must fill a thinking person with a feeling of humility..." Einstein [19].



12. References

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