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TWO NEW LAWS OF GRAVITATION GIVE A COMPLETE DESCRIPTION OF GRAVITATIONAL INTERACTION IN THE UNIVERSE

Abstract. *For more than 300 years, the formula for the force of gravitational interaction is represented by Newton's Uniform Law of Universal Gravitation. During this period the facts that Newton's law of gravitation gives predictions that do not agree with observations have been revealed. Here we show that Newton's law of gravitation is not the only law of gravitational interaction in the universe. In addition to Newton's law of gravitation, a new law of gravitation is obtained:* F_{Cos}*=(mc^2)√Λ. The two laws of gravitation (*F_N*=GmM/r^2,* F_{Cos}*=(mc^2)√Λ) revitalize classical gravity and develop Newtonian dynamics towards a complete model of gravity. Newton's law of gravitation together with the new law of gravitation provide a complete and consistent description of the gravitational interaction in the universe. The real law of universal gravitation is presented in a new form. The law of universal gravitation is represented by two equivalent formulas:* F_U*=GmM/r^2+(mc^2)√Λ;* F_U*=mR^3/T^2*r^2+(mc^2)√Λ. The law of universal gravitation turned out to be much more complicated than Newton claimed.*

Keywords: *The law of universal gravitation; The law of cosmological force; Kepler's 3rd law; Cosmological equations; cosmological constant; Parameters of the observable universe; dark matter; galaxy rotation curve; Pioneer anomaly.*

1. Introduction

The dominant force in the Universe is gravity. Newton's law of gravitation was a real breakthrough in science.

$$F_N = G\frac{mM}{r^2} \qquad (1)$$

where: **G** is the Newtonian constant of gravitation; **m, M** is the mass; **r** is the distance.

It should be noted that the constant G was not in the law of universal gravitation proposed by Newton. Although the constant G is called the Newtonian gravitational constant, it appeared in formula (1) much later. Newton himself did not use symbolic equations at all [1]. Newton used verbal formulations. Cavendish also did not know about the existence of the constant G and did not measure it. He measured the density of the Earth. The product ***GM*** was available for measurement, so for Cavendish the question of the constant G did not arise.

Only in 1873 A. Cornu and J. B. Baille first introduced the symbol (f) for the coupling constant in Newton's law of gravitation [2 - 4]. The modern notation G for the gravitational constant appeared in the notation of Newton's law only 200 years after the appearance of Newton's laws [5 - 7]. Accordingly, only 200 years after Newton, the formula for the law of gravitation (1) acquired its modern form. The need for the constant G arose to reconcile dimensions. This was facilitated by the introduction of the CGS system in 1874. Now the constant G is considered a fundamental constant.

Studies have been conducted on the possible variation of its value. Strict restrictions on the variation of G have been confirmed. Analysis of observations shows that the gravitational constant G has changed by less than one ten-billionth part per year over the past nine billion years [8].

Newton's law of gravitation captivates with its simplicity and mathematical perfection. Gravitational interaction has become the fourth fundamental interaction. Newton's law of gravitation allows us to explain and predict with great accuracy the movements of celestial bodies within the solar system. The attractiveness of Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

2. What is wrong with Newton's law of universal gravitation?

For a long time, it was believed that Newtonian dynamics was a complete description of all types of motion occurring in the Universe. Experimental observations show that this is not the case. Newtonian dynamics were found to make predictions that were inconsistent with observations [9 - 14].

When extending Newton's law to the Universe, an insoluble gravitational paradox arose, which cast doubt on the applicability of the Newtonian model on cosmological scales. The gravitational paradox turned out to be the most serious difficulty with Newton's gravity model [15, 16].

In 1859, U. Le Verrier first reported an anomalous shift in the perihelion of Mercury. Observations showed a discrepancy with the calculations of Newton's law. The anomalous shift in the perihelion of Mercury did not obey Newton's law of universal gravitation [11]. Newcomb discovered a perihelion shift not only for Mercury, but also for other planets [17]. This means that some additional force acts on the planets. Newton's formula (1) "does not see" this additional force. An unexpected conclusion followed from the observations. The real trajectories of the planets are not elliptical. The real trajectories of the planets are not closed. An unknown additional force deviates the planets from the ideal elliptical trajectory. Kepler did not know this. Newton did not know this. Kepler believed that the orbits of the planets are elliptical. Newton believed that the orbits of the planets are elliptical. Both Newton's law and Kepler's third law are applicable to ideal elliptical trajectories. For this reason, they are approximate models.

The limit of applicability of Newton's law of gravity was especially acute in the study of spiral galaxies. The rotation curve of a spiral galaxy has a significant discrepancy with Newton's law [18, 19]. Serious difficulties of the Newtonian gravity model indicate that Newtonian dynamics is unsuitable for solving cosmological problems. Simple and perfect in mathematical representation, Newton's law of gravity has limitations and applicability limits. Newton's law of gravity describes the interaction of two point masses. Formula (1) does not take into account the influence of other bodies in the universe. But they exist! And there are a lot of them! The applicability limits of Newton's law of gravity end when moving to distributed masses. The reason for the inapplicability of Newton's law in cosmology are the following limitations:

- masses are considered point masses;
- this is the law of gravitational interaction of two bodies, while in reality many bodies and even the entire Universe participate in gravitational interaction.
- the formula includes the inverse square law.

The inverse square law was formulated in 1645 by Ismail Bullialdus [22]. The inverse square law turned out to be very productive for solving the problem of gravitational interaction of two bodies.

The same inverse-square law became an insurmountable obstacle when trying to extend the effect of Newton's law of gravitation to the Universe. Repeated attempts were made to modify Newton's law and make it applicable in cosmology. In 1745, Alexis Clairaut [20] proposed a modification of Newton's law, in which the inverse-square law was changed. In 1894, Hall A. [21] proposed replacing the square of the distance with a slightly higher power. Hugo von Seeliger and Carl Gottfried Neumann proposed a modification of the law with a faster decrease in gravity with distance than Newton's [22]. Milgrom developed the MOND theory, which provides for a modification of Newtonian dynamics at low accelerations [9, 10].

No clarifications or cosmetic edits to Newton's law of gravitation made it applicable in cosmology. The simple and mathematically perfect formula of Newton's law turned out to be inapplicable in cosmology. The inverse square law, the law of gravitational interaction of two bodies, point masses are the main limiting factors that prevent the extension of Newton's law of gravity to the Universe. Obviously, as applied to the Universe, a different law of gravity is in effect than Newton's law of gravity.

3. Newton's law of gravity "sees" only a part of the force of universal gravitation.

Despite the serious difficulties of Newton's gravity model, there is no reason to doubt the correctness of Newton's law of gravity. Newton's law of gravity is the correct and mathematically perfect law of gravity as applied to the gravitational interaction of two bodies. The boundaries of its applicability do not extend to the universe. The boundaries of its applicability are limited to the interaction of two bodies. At the same time, in reality, more than two bodies participate in gravitational interaction. On a universal scale, any body is affected by the gravitational force of many (very many!) bodies, located both nearby and at a distance. Moreover, the entire Universe participates in gravitational interaction! This is not taken into account in Newton's formula (1). This cannot be taken into account by formula (1), which includes the parameters of two bodies, but does not include the parameters of the Universe.

Therefore, Newton's law of gravity gives only a part of the force of universal gravitation. This is the force of gravitational interaction of two bodies. Newton's law "does not see" the full force of universal gravitation. To reach the full force, there is not enough additional force from many other bodies in the universe. Therefore, calling the law of gravitational interaction of two bodies the law of universal gravitation is an exaggeration.

The reason for the discrepancies with observations is not the imperfection of Newton's law, but unfounded attempts to explain observations by the law of gravitational interaction of two bodies. Observations give the result of the action of the full force of universal gravitation. And Newton's law of gravitation gives only a part of the full force of universal gravitation without taking into account the force of gravitational interaction of the other bodies in the universe.

Classical Newtonian gravity does not describe all gravitational phenomena. For the part of the force of universal gravitation that Newton's law "does not see", another law of gravitation, different from Newton's law, applies.

4. The Second Law of Gravitational Interaction - the Law of Cosmological Force

The law of gravitational interaction of two bodies does not give the full force of universal gravitation. Even if it were possible to take into account the gravitational interaction of 3, 4, 5, etc. bodies, the description of gravity would be incomplete until the gravitational action of the entire universe is taken into account. It is known that the N-body problem has no analytical solution. But there is an additional gravitational force from N bodies! It must somehow be taken into account in the gravity model. The additional gravitational force manifests itself in the shift of the perihelion of the planets, its action is demonstrated by the rotation curve of the galaxies. There is an additional force, but the physical law of this additional force has not been discovered. Newtonian dynamics does not provide a solution to this problem. A new law of gravitational interaction is needed beyond Newtonian dynamics. This is the law of cosmological gravitational force [23]. The formula for the law of cosmological force is:

$$F_{Cos} = mc^2 \sqrt{\Lambda}$$

Fig. 1. The law of the cosmological force. Where: $\boldsymbol{F_{Cos}}$ is the cosmological force, **m** is the mass of the body, **c** is the speed of light in vacuum, **Λ** is the cosmological constant.

Instead of the gravitational constant **G**, the law of the cosmological force contains the cosmological constant **Λ**. The new law of gravitation shows that any body of mass m is acted upon by a cosmological force proportional to the mass of the body and the cosmological constant **Λ**. The law of the cosmological force operates beyond the applicability of Newton's law of gravitation. It is applicable to the gravitational interaction in the Universe. The cosmological force has a linear dependence on the mass of the body and does not obey the inverse square law.

On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the Newtonian force. At large distances, the main part of the force of universal gravitation is the cosmological force F_{Cos}.

The study of the equation of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2 \sqrt{\Lambda} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (2)$$

The theoretical limit of the cosmological force at **m** → $\mathbf{M_U}$ reaches the enormous value $\mathbf{c^4/G = 1.21027 \times 10^{44}\ N}$.

The law of the cosmological force of the Universe is presented by a simple formula, which is not inferior in simplicity and perfection to the formula of Newton's law of gravitation. The law of the cosmological force does not contain the constant G. Instead of the constant G, it includes the cosmological constant Λ. The cosmological constant Λ plays the role of the coupling constant in the law of the cosmological force. The law shows the force with which the body interacts with the mass of the Universe distributed in space. The main feature of the cosmological force is that it has a linear dependence on the mass of the body and does not obey the inverse square law.

5. The Third Law of Gravitational Interaction

For over 300 years, the force of gravitational interaction was represented by only one simple and mathematically perfect formula: Newton's law **F_N=GmM/r^2**. At the same time, Newton's formula (1) is not the only possible formula for the law of gravitational interaction of two bodies. In [24 - 27] it is shown that Newton's law of gravitation can be represented both with the constant G and without the constant G. In [24 - 27] it is shown that the constant G is a composite constant.

Instead of Newton's formula (1), the law of gravitation can be represented by a formula without using the constant G (Fig. 2):

$$F_K = \frac{mR^3}{T^2 r^2} \qquad (3)$$

Fig. 2. The third law of gravitation. Where: **m** is the mass of the body, **R** and **T** are orbit parameters, **r** is the distance.

Formula (3) does not contain the gravitational constant **G** and the large mass **M**. The formula includes the Kepler ratio $\mathbf{R^3/T^2}$ and the inverse square law. It is known that mass is not included in Kepler's laws. Formula (3) follows directly from Kepler's third law. The third law of gravitation (Fig. 2) compares favorably with Newton's law of gravitation (1). In the Universe, determining the values of the masses of bodies is a complex task. Parameters such as distances and periods of revolution of bodies are known much more accurately.

Formula (3) calls into question the universality of the constant G. The fundamental status of the gravitational constant G has long been questioned [28].

Formula (3) has the same shortcomings as Newton's law of gravitation. Being a two-body law, this formula "does not see" the cosmological force. It does not take into account the force of gravitational interaction of bodies with the mass of the Universe. Formula (3) is just an equivalent of Newton's law of gravitation. It is known that Hooke in a letter to Newton in 1680 pointed out the need to take into account elliptical orbits in the law of gravitation. In fact, Hooke gave Newton a hint to use Kepler's law in the law of gravitation. But Newton did not use this hint from Hooke. The orbital parameters from Kepler's law were not included in Newton's formula.

6. The three laws of gravitation.

Newtonian dynamics, supplemented by two new laws of gravitation (AuND), provides a complete and consistent description of gravitational interaction in the Universe. The three laws of gravitation (Fig. 3) make Newtonian dynamics a complete model of gravitation.

$$F_N = G\frac{Mm}{r^2} \qquad F_K = \frac{mR^3}{T^2 r^2} \qquad F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 3. Three laws of gravitation, which make Newtonian dynamics complete.

Two laws of gravitational interaction of two bodies (**F_N=GmM/r^2** and **F_K =mR^3/T^2*r^2**) are equivalent. The second law of gravitation - the law of cosmological force (**F_{Cos}=(mc^2)√Λ**) gives the value of the additional force to the force of gravitational interaction of two bodies. Each of the three laws of gravitation separately does not give the value of the total force of universal gravitation. And Newton's law, and the second law of gravitation, and the third law of gravitation separately give only a part of the total force of gravitational interaction in the Universe. Newton's law of gravitation and the third law of gravitation "do not see" the cosmological force and do not take into account the influence of other bodies in the Universe. In turn, the law of cosmological force "does not see" the force of gravitational interaction of two bodies. Only a combination of the two laws gives the value of the total force of universal gravitation. These are two combinations of the laws of gravity:

- Newton's law of gravity + the law of cosmological force (**F_U = GmM/r^2 + (mc^2)√Λ**);
- the law of cosmological force + the third law of gravity (**F_U = (mc^2)√Λ + mR^3/T^2*r^2**).

Therefore, the law of universal gravitation has two equivalent forms of representation (Fig. 4).

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases} \qquad \begin{cases} F_K = \dfrac{mR^3}{T^2 r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

Fig. 4. Two equivalent forms of representation of the law of universal gravitation.

Newton's law of gravity is not sufficient to fully describe gravity. Gravity is represented by three fundamental physical laws and combinations of these laws (Fig. 5). Newton's law of gravity together with the two new laws of gravity provide a complete description of gravitational interactions in Nature.

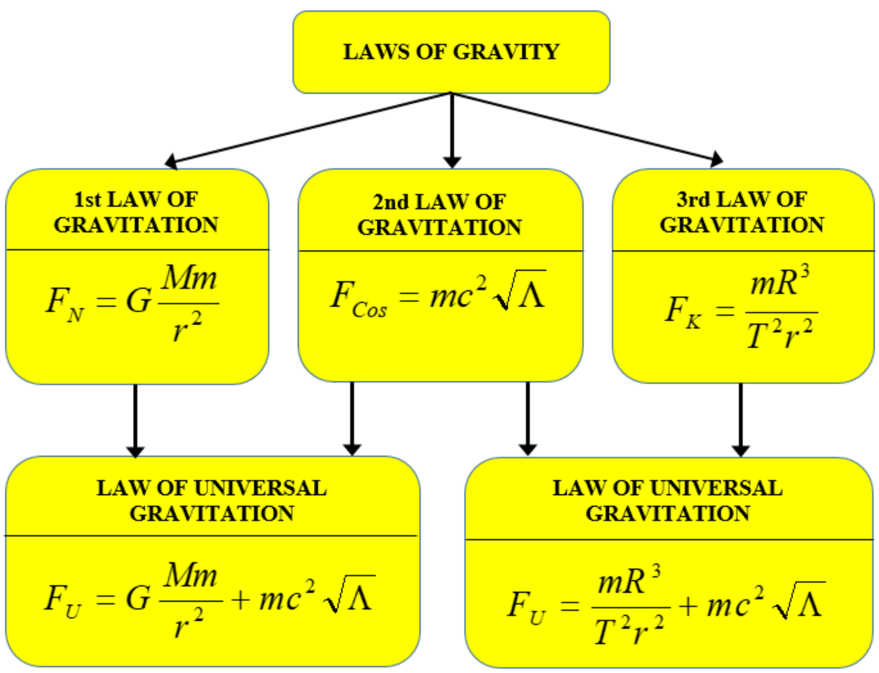

Fig. 5. Laws of gravity.

The law of universal gravitation turned out to be much more complicated than Newton claimed. Newton's law of gravity F_N=GmM/r^2 is only a part of the law of universal gravitation. The missing part of the law of universal gravitation is represented by the law of cosmological force. The additional law of cosmological force F_{Cos}=(mc^2)√Λ makes Newtonian dynamics a complete model of gravity. Fig. 5 shows two equivalent formulas of the law of universal gravitation.

7. Newton's law of gravitation + law of cosmological force = law of universal gravitation

The law of cosmological force (**F_{Cos}=(mc^2)√Λ**) leads to a new mathematical model of gravitation. This allows us to represent the law of universal gravitation in a new form. The law of universal gravitation includes two laws: Newton's law of gravitation and the law of cosmological force (Fig. 6).

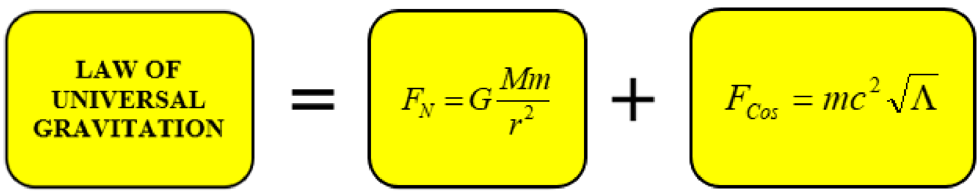

Fig. 6. Law of universal gravitation = Newton's law of gravity + law of cosmological force.

The force of universal gravitation is represented by the vector sum of two forces: the Newtonian gravitational force **F_N** and the cosmological gravitational force **F_{Cos}**:

$$\vec{F}_U = \vec{F}_N + \vec{F}_{Cos} \qquad (4)$$

The value of the resulting force of universal gravitation is in the range of values from **F_U = GmM/r^2 - (mc^2)√Λ** to **F_U = GmM/r^2 + (mc^2)√Λ** (Fig. 7).

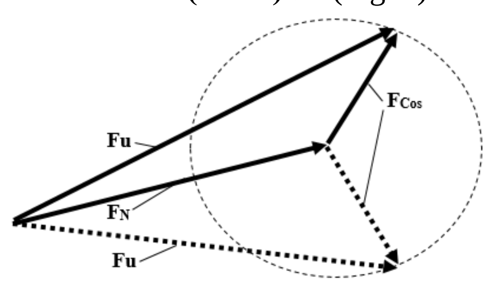

Fig. 7. The force of universal gravitation F_U as a vector sum of two forces: F_N and F_{Co}s.

8. The third law of gravitation + the law of cosmological force = the law of universal gravitation.

The law of universal gravitation can be represented by an equivalent formula. The equivalent formula of the law of universal gravitation includes the third law of gravitation and the law of cosmological force (Fig. 8).

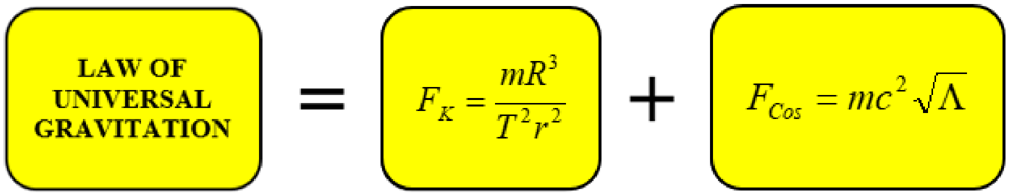

Fig. 8. Law of universal gravitation = third law of gravity + law of cosmological force.

The force of universal gravitation is represented by the vector sum of two forces: the force $\boldsymbol{F_K}$ and the cosmological gravitational force $\boldsymbol{F_{Cos}}$:

$$\vec{F}_U = \vec{F}_K + \vec{F}_{Cos} \qquad (5)$$

The value of the resulting force of universal gravitation is in the range from **F_U = mR^3/(T*r)^2 - (mc^2)√Λ** to **F_U = mR^3/(T*r)^2+ (mc^2)√Λ**.

9. Contribution of the cosmological force F_{Cos} = (mc^2)√Λ to the law of universal gravitation

The share of the cosmological force F_{Cos} in the new formulas of the law of universal gravitation (Fig. 4) depends on the scale of gravitational interaction. On small scales, the additional cosmological force F_{Cos} is much less than the Newtonian force. For example, on Earth, F_{Cos} is ~**10^10** times less than the Newtonian force. As we can see, within the solar system, Newton's law of gravity has a high accuracy. At small distances, the main part of the universal gravitational force is the Newtonian force F_N. With an increase in the mass of interacting bodies and the distance, the share of the Newtonian force in the law of universal gravitation decreases, and the share of the cosmological force increases.

On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the Newtonian force. The cosmological force F_{Cos} has a theoretical limit equal to the Planck force **F_P = c^4/G = 1.21027 • 10^44 N**. At large distances, the main part of the universal gravitational force is the cosmological force **F_{Cos}.**

On the scale of galaxies, the cosmological force Fcos becomes commensurate with the Newtonian force. This occurs at the corresponding value of the ratio **M/r^2**:

$$\mathbf{M/r^2 = (c^2\sqrt{\Lambda})/G = 15.720...\ kg/m^2} \qquad (6)$$

With the value M/r^2 = 15.720... kg/m2 the cosmological force (F_{Cos} = (mc^2)√Λ) is equal to the Newtonian force (F_N = GMm/r^2). It is noteworthy that the same constant (15.7202... kg/m^2), but with much greater accuracy, is given by the following combination of fundamental physical constants of the electron:

$$\mathbf{m_e/\alpha r_e^2 = 15.7202729...\ kg/m^2} \qquad (7)$$

The constant (15.7202...kg/m^2) corresponds to a mass density of $18.3520 \cdot 10^{-26}$ kg/m^3.

10. Augmented Newtonian Dynamics (AuND)

Newtonian dynamics lacks a law of gravity that takes into account the gravitational effect of the mass of the universe. This means that Newtonian dynamics is incomplete. Newton's law of gravity gives the value of the gravitational force of interaction between two bodies. It does not give a complete description of the gravitational interaction in the universe. Newton's law of gravity "sees" only a part of the force of universal gravitation. Namely: it "sees" the force of gravitational interaction between two bodies. The additional force with which many other bodies in the universe act on the test body obeys the law of cosmological force. The Newtonian force and the additional cosmological force together give the value of the total force of universal gravitation. Separately, both the Newtonian force and the cosmological force are only a part of the force of universal gravitation. Granting Newton's law of gravitation the status of the law of universal gravitation turned out to be an exaggeration. This exaggeration created the illusion that Newton's formula gave a complete description of gravity in the universe. That this was not the case was revealed only many years later. This was indicated by the observation of the shift in the perihelion of Mercury and the rotation curves of galaxies. An additional force, not taken into account by Newton's law (the Pioneer anomaly), was discovered in the Pioneer effect. These three examples revealed the incompleteness of Newtonian dynamics. Exaggerated expectations of Newton's formula had negative consequences. Inflated expectations led to the emergence of the concept of dark matter, which is not confirmed by observations.

Incomplete Newtonian dynamics does not mean that it is wrong. It provides a correct description of the gravitational interaction of two bodies. At the same time, it is not complete. It does not present all the laws of gravitational interaction. In addition to the forces described by Newtonian dynamics, there is a gravitational force in the Universe, which is observed in experiments, but does not follow from Newton's law of gravitation. Outside the applicability of Newton's law of gravitation, the law of cosmological force operates. Newtonian dynamics, supplemented by the law of cosmological force, becomes a complete model of gravitation. Augmented Newtonian dynamics (AuND) covers both the gravitational interaction of two bodies and the gravitational interaction of bodies with the mass of the Universe. AuND directly leads to a new law of universal gravitation. The law of universal gravitation in AuND contains two particular laws of gravitation: Newton's law for the gravitational interaction of two bodies and the law of cosmological force of the Universe. Traditional Newtonian dynamics is an integral part of AuND. Augmented Newtonian Dynamics (AuND) opens up new possibilities for the classical gravity model. AuND provides a solution to the following problems of astrophysics and cosmology:

- gives a new mathematical formula for the law of universal gravitation;
- explains the rotation curves of galaxies;
- explains the gravitational nature of the Pioneer anomaly;
- solves the problem of dark matter;
- solves the problem of the cosmological constant Λ ($\mathbf{\Lambda = 1.36285 \ldots \cdot 10^{-52}\ m^{-2}}$);
- introduces a new parameter of the Universe - the cosmological acceleration A_0 ($\mathbf{A_0 = 10.4922 \ldots \cdot 10^{-10}\ ms^{-2}}$).

11. The problem of dark matter, rotation curves of galaxies

The cosmological force has a linear dependence on the mass of the body (Fig. 9) and does not obey the law of inverse squares. Fig. 9 conventionally shows the contribution of the cosmological force to the Galaxy rotation curve.

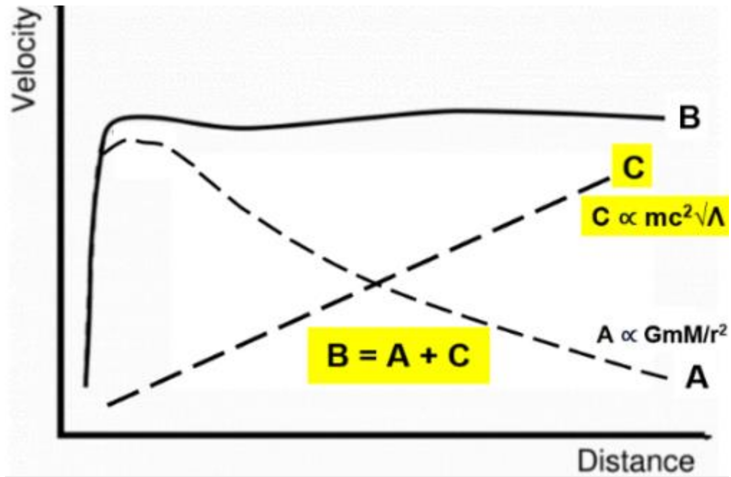

Fig. 9. Galaxy rotation curve (B) as a result of two forces: the contribution of the Newtonian force (A) and the contribution of the cosmological force (C).

On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force is enormous. For large distances it exceeds the Newtonian force and has a theoretical limit equal to the Planck force **F_P = c^4/G = 1.21027•10^44 N**. The total force of universal gravitation, which acts on a body, consists of two forces. This is the Newtonian force of gravitational interaction of two bodies and the additional cosmological force of gravitational interaction of a body with the mass of the Universe. Accordingly, the Galaxy rotation curve (B) (Fig. 9) is represented by the sum B = A + C. Combining the two laws of forces F_N and F_{Cos} gives the law of universal gravitation. At small distances, the main share of the force of universal gravitation is represented by the Newtonian force F_N. At large distances, the cosmological force F_{Cos} represents the major fraction of the universal gravitational force. As a result of the two forces, the velocity in the graph (Fig. 9) is represented by curve (B). There is no need to invoke the dark matter hypothesis to explain the galaxy rotation curve. Instead of a dark matter halo, the gravitational force of the "light matter" of the visible Universe is at work.

The assumption about the influence of the rest of the visible Universe on the rotation curves of galaxies was first made by Philip D. Mannheim [29].

12. The Pioneer anomaly is a cosmological gravitational effect.

In the Pioneer effect, an unknown force was experimentally discovered that did not follow from Newton's law of gravity [30 - 32]. The Pioneer effect still has no convincing explanation. The value of the force that follows from the law of cosmological force (Fig. 1) turned out to be surprisingly close to the Pioneer anomaly.

The value of cosmological acceleration that follows from the law of cosmological force:

$$c^2\sqrt{\Lambda} = 10.4922... \bullet 10^{-10} m/s^2 \qquad (8)$$

Meaning of the cosmological force:

$$F_{Cos} = m \bullet (10.4922 \bullet 10^{-10}) N \qquad (9)$$

Meaning of the unknown force discovered in the pioneer effect:

$$F_{Pioneer} = m \bullet (8.74 \pm 1.33) \bullet 10^{-10} N \qquad (10)$$

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [33 - 36].

Meaning of unknown force for Galileo:

$$F_{Galileo} = m \bullet (8 \pm 3) \bullet 10^{-10} N \qquad (11)$$

Meaning of Unknown Force for Ulysses:

$$F_{Ulysses} = m \bullet (12 \pm 3) \bullet 10^{-10} N \qquad (12)$$

The law of cosmological force gives a force value very close to both the Pioneer anomaly value and the values for Galileo and Ulysses. The force value F = m(10.4922... × 10^{-10})N is very close to the experimental values F = m((8.74 ± 1.33) x 10^{-10})N, F = m((8±3) x 10^{-10})N, F = m((12±3) x 10^{-10})N. The coincidence of the force values casts doubt on attempts to explain the pioneer anomaly by the temperature effect. The pioneer anomaly is explained by the cosmological gravitational effect. The effect discovered by the Pioneers was essentially an experimental discovery of cosmological acceleration (**A_0 = 10.4922... × 10^{-10} m/s^2**) as an additional parameter of the Universe.

13. Parameters of the Universe as constants in the laws of augmented Newtonian dynamics (AuND)

In [37], cosmological equations were obtained, composed of the parameters of the Universe and fundamental physical constants. Systems of equations were composed of cosmological equations. Fig. 10 shows four systems of cosmological equations.

$$\begin{cases} G\hbar/r_e^3A_0 = c^1 \\ 1/T_U^2\Lambda = c^2 \\ M_UA_0G = c^4 \\ M_UR_UA_0G/T_U = c^5 \\ M_UR_UA_0^2G = c^6 \end{cases}$$

a)

$$\begin{cases} M_U\Lambda G = A_0 \\ c^5r_e^3/M_UG^2 = \hbar \\ M_UGT_u^2 = R_u^3 \\ GM_U = c^2R_U \\ M_U\Lambda GT_u^2 = R_U \end{cases}$$

b)

$$\begin{cases} \dfrac{c^5r_e^3}{M_UG^2} = \hbar \\ M_U\Lambda cr_e^3 = \hbar \\ \dfrac{R_U\Lambda c^3r_e^3}{G} = \hbar \\ \dfrac{cr_e^3A_0}{G} = \hbar \\ \dfrac{M_Ur_e^3}{R_UT_U} = \hbar \end{cases}$$

c)

$$\begin{cases} \dfrac{M_UG^2m_e}{c^4r_e^2} = \alpha, \\ \dfrac{m_e}{M_U\Lambda r_e^2} = \alpha, \\ \dfrac{Gm_e}{R_U\Lambda c^2r_e^2} = \alpha, \\ \dfrac{Gm_e}{r_e^2A_0} = \alpha, \\ \dfrac{Gm_eT_U}{r_e^2c} = \alpha. \end{cases}$$

d)

Fig. 10. Systems of cosmological equations for calculating the parameters of the observed Universe. Where : α - fine-structure constant, ħ - Planck constant, M_U - mass of the observable

Universe, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, A_0 - cosmological acceleration, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass.

All systems of cosmological equations presented in Fig. 10 give the same values of the parameters of the Universe, close to the accuracy of the gravitational constant G.

$$M_U = 1.15348... \bullet 10^{53} kg$$
$$R_U = 0.856594... \bullet 10^{26} m$$
$$T_U = 2.85729... \bullet 10^{17} s$$
$$\Lambda = 1.36285... \bullet 10^{-52} m^{-2}$$
$$A_0 = 10.4922... \bullet 10^{-10} m/s^2$$

Fig. 11. Values of the parameters of the Universe obtained from the systems of cosmological equations.

The cosmological constant Λ is a fundamental parameter of the Universe. The constant Λ together with the gravitational constant G are included in the extended Newtonian dynamics. It is included in the law of cosmological force (Fig. 1) and in the law of universal gravitation (Fig. 4) as a fundamental constant.

From the systems of cosmological equations, the constant of cosmological acceleration (**A_0 = 10.4922... × 10^{-10} m/s^2**) was obtained as an additional parameter of the Universe.

14. Tully–Fisher relation for the Universe.

The empirical baryonic Tully–Fisher relation [38] is known, which gives the relationship between the baryon mass and the asymptotic rotation velocity in the form: M ∝ V4. Here we will show that the Tully–Fisher relation is satisfied for the Universe. In the MOND theory [9, 10], the equation is obtained:

$$\mathbf{V = (GMa_0)^{\wedge}1/4} \quad (13)$$

Where: M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, V is the speed, a_0 is the acceleration.

The Tully–Fisher relation is universal. It is valid not only for galaxies, but also for the Universe as a whole. For **M** → **M_U** the velocity **V** → **c**:

$$\mathbf{V^4 = GM_UA_0 = c^4} \quad (14)$$

Where: V is the speed, M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, A_0 is the cosmological acceleration, c is the speed of light in vacuum.

Equation (5) is one of the cosmological equations in the system of equations of the Universe in Fig. 1. The Tully–Fisher baryon relation for the Universe has a power law index of exactly 4..

From this cosmological equation (5) follows the well-known formula of the Planck force:

$$\mathbf{M_UA_0 = c^4/G = 1.21027 \bullet 10^{44}\ N} \quad (15)$$

From this cosmological equation (5) follows the well-known formula of A. E. H. Bleksley [39]:

$$\mathbf{G = c^2 R_U/M_U} \qquad (16)$$

From this cosmological equation (5) follows the Kepler constant for the Universe:

$$\mathbf{GM_U = R_U^3/T_U^2 = c^2 R_U = c^4/A_0 = 7.69868 \bullet 10^{42}\ m^3/s^2} \qquad (17)$$

Where: M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, A_0 is the cosmological acceleration, c is the speed of light in vacuum.

The Baryonic Tully–Fisher relation for a Universe with power-law index exactly 4 is a direct consequence of the law of cosmological force.

15. Conclusion.

Newtonian dynamics remained an incomplete model of gravity for over 300 years. Newton's law of gravity (**F_N=GmM/r^2**) gives the force of gravitational interaction between two bodies. Observations give the result of the action of the total force of universal gravitation from all bodies in the Universe. Newton's law of gravity shows only a part of the force of universal gravitation. Newton's law of gravity is only one of three laws of gravitational interaction in the Universe.

The law of universal gravitation applicable to the Universe is presented in a new form. The law of universal gravitation applicable to the Universe contains Newton's law and the law of cosmological force. Two equivalent formulas of the law of universal gravitation are obtained: **F_U=GmM/r^2+(mc^2)√Λ; F_U=mR^3/T^2*r^2+(mc^2)√Λ.**

The impossibility of applying Newton's law of universal gravitation on the scale of the Universe and the discrepancy between observations and the prediction of Newton's law of gravitation led to the unfounded opinion about the erroneousness of Newtonian dynamics. Such a statement was based only on the single law of Newton's gravitation. Incomplete Newtonian dynamics does not mean that it is wrong. Newtonian dynamics gives the correct value of the force of gravitational interaction between two bodies. This is the correct and necessary part of the force of universal gravitation. The difficulties of Newton's gravity model served as a reason to consider classical gravity an imperfect model. Classical gravity has been the subject of much criticism, including unfounded criticism. It is too early to write off classical gravity and rely on quantum gravity. With the advent of two new law of universal gravitation, classical gravity is capable of revealing the secrets of the Universe. I emphasize the need to study gravity in its classical sense - as an external force, starting with Kepler and Newton. There are good prospects here. Two new laws of gravitation remove the difficulties of Newton's gravity model. The augmented Newtonian dynamics (AuND) gives predictions that agree with observations.

16. Conclusions

1. For over 300 years, Newtonian dynamics was an incomplete model of gravity. The difficulties of Newton's gravity model gave reason to consider classical gravity an imperfect model. Newton's law (**F_N=GmM/r^2**) gives the force of gravitational interaction between two bodies. This is only part of the force of universal gravitation.

2. Newton's law (**F_N=GmM/r^2**) is not sufficient to describe gravitational effects in the universe. Newton's law of universal gravitation is one of the three laws of gravitational interaction in the universe.

3. Law of gravity ***F_{Cos}*****=(mc^2)√Λ** revive classical gravity and complement Newtonian dynamics (AuND).

4. Augmented Newtonian dynamics (AuND) is a complete model of gravity. AuND elegantly and simply provides a solution to the problems of cosmology without going beyond the framework of classical gravity.

5. The law of universal gravitation applicable to the Universe is presented in a new form. It is presented by two equivalent formulas: **F_U=GmM/r^2+(mc^2)√Λ; F_U=mR^3/T^2*r^2+(mc^2)√Λ.**

6. Two equivalent laws of universal gravitation (**F_U=GmM/r^2+(mc^2)√Λ; F_U=mR^3/T^2*r^2+(mc^2)√Λ**) provide a complete description of gravitational interaction in the Universe.

7. The Pioneer anomaly is a cosmological gravitational effect. The effect discovered by the Pioneers was an experimental discovery of cosmological acceleration (**A_0 = 10.4922... × 10^{-10} m/s^2**), as an additional parameter of the Universe.

8. The Tully–Fisher relation is universal. This relation is valid not only for galaxies, but for the Universe as a whole (**$V^4 = GM_UA_0 = c^4$**).

9. From the Tully–Fisher relation for the Universe (**$V^4 = GM_UA_0 = c^4$**) follows the value of cosmological acceleration **Ao = 10.4922•10^-10 m/s^2**.

10. The problem of galaxy rotation curves has a solution without involving the dark matter hypothesis.

11. Classical gravity has been the subject of much criticism, including unfounded criticism. It is shown here that classical gravity has a high heuristic potential, and its problems stem from its incompleteness. I emphasize the need to study gravity in its classical sense - as an external force, starting with Kepler and Newton. Classical gravity should not be written off. With the advent of two new laws, classical gravity is capable of revealing the secrets of the Universe.

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