Criticizing Pauli Exclusion Principle

Jouni S. Puuronen

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Abstract

I point out that Pauli exclusion principle makes no sense, because it assumes that the wave functions of multi-electron quantum systems would separate into products of wave functions of individual electrons, while in reality the Coulomb repulsion between the electrons prevents this separation. I propose a conjecture that Pauli exclusion principle maybe somewhat works in some situations, because it approximates the Coulomb repulsion between the electrons. I also put under question whether it makes sense to demand that physical quantum systems should always have their eigenenergies bounded from below.

According to mainstream physics there exists a principle known as Pauli exclusion principle [1]. Pauli exclusion principle states that in multi-electron quantum systems two or more electrons cannot simultaneously occupy the same state. At the time of writing this article the mainstream physicists have spent almost 100 years believing that Pauli exclusion principle would be true. This is noteworthily strange, because clearly the principle is severe pseudoscientific nonsense. In order to be able to recognize the nonsensical nature of Pauli exclusion principle, it is sufficient to know some basics of Quantum Mechanics and partial differential equations (PDEs).

Suppose we are interested in a system that consists of two nonrelativistic electrons that for some reason feel some background potential

$$U : \mathbb{R}^3 \to \mathbb{R}, \quad \mathbf{x} \mapsto U(\mathbf{x}).$$

Let's call the electrons with the names "electron A" and "electron B". Suppose that at least first we want to keep the problem as simple as possible by ignoring the electron spins. If we are interested to understand the behavior of this system, one of the most relevant objectives would be to try to learn something about the solutions to the Schrödinger eigenvalue equation

$$\begin{aligned}\Big(-\frac{\hbar^2}{2m_e}\nabla_{\mathrm{A}}^2 \; + \; U(\mathbf{x}_{\mathrm{A}}) \; - \; \frac{\hbar^2}{2m_e}\nabla_{\mathrm{B}}^2 \; + \; U(\mathbf{x}_{\mathrm{B}}) \\ + \; \frac{q_e^2}{4\pi\epsilon_0}\frac{1}{\|\mathbf{x}_{\mathrm{A}} - \mathbf{x}_{\mathrm{B}}\|}\Big)\psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}}) \; = \; E\psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}}).\end{aligned} \tag{1}$$

The parameters m_e and q_e are the mass and the charge of the electron. Here the wave function is of the form

$$\psi : \mathbb{R}^6 \to \mathbb{C}.$$

The standard mathematical notation for the input vector of this kind of mapping would be $(x_1, x_2, x_3, x_4, x_5, x_6)$, but here we are using the notations $(x_1, x_2, x_3) = \mathbf{x}_\mathrm{A}$ and $(x_4, x_5, x_6) = \mathbf{x}_\mathrm{B}$ to clarify the physical interpretation of the PDE. We can also consider denoting $\mathbf{x}_\mathrm{A} = ((x_\mathrm{A})_1, (x_\mathrm{A})_2, (x_\mathrm{A})_3)$ and $\mathbf{x}_\mathrm{B} = ((x_\mathrm{B})_1, (x_\mathrm{B})_2, (x_\mathrm{B})_3)$, and emphasizing that the wave function is of the form $\psi : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{C}$. By the two Laplace operators we mean the operators

$$\nabla_\mathrm{A}^2 \;=\; \partial_{(x_\mathrm{A})_1}^2 + \partial_{(x_\mathrm{A})_2}^2 + \partial_{(x_\mathrm{A})_3}^2 \;=\; \partial_1^2 + \partial_2^2 + \partial_3^2$$

and

$$\nabla_\mathrm{B}^2 \;=\; \partial_{(x_\mathrm{B})_1}^2 + \partial_{(x_\mathrm{B})_2}^2 + \partial_{(x_\mathrm{B})_3}^2 \;=\; \partial_4^2 + \partial_5^2 + \partial_6^2.$$

If we had wanted to model the electron spins too, then the wave function should essentially be of the form $\psi : \mathbb{R}^3 \times \{0,1\} \times \mathbb{R}^3 \times \{0,1\} \to \mathbb{C}$. This modification wouldn't affect the arguments below, so we can keep the notation simpler by ignoring the spins now. Any information about the solutions to the eigenvalue equation (1) would be potentially relevant information about the two electron system in question. We encounter the immediate problem that this eigenvalue equation is a very difficult equation to study, and not much is known about its solutions. It is the presence of the repulsive Coulomb potential term that brings the significant portion of the trouble. We can make an attempt to make some progress by dividing the situation into two cases: One case is $q_e^2 = 0$, and the other case is $q_e^2 > 0$. If we assume that $q_e^2 = 0$, then the PDE we are supposed to solve becomes

$$\Big(-\frac{\hbar^2}{2m_e}\nabla_\mathrm{A}^2 + U(\mathbf{x}_\mathrm{A}) - \frac{\hbar^2}{2m_e}\nabla_\mathrm{B}^2 + U(\mathbf{x}_\mathrm{B})\Big)\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) \;=\; E\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}). \quad (2)$$

Now it is a simple thing to put forward a logical claim that if the equations

$$\Big(-\frac{\hbar^2}{2m_e}\nabla_\mathrm{A}^2 + U(\mathbf{x}_\mathrm{A})\Big)\psi_\mathrm{A}(\mathbf{x}_\mathrm{A}) \;=\; E_\mathrm{A}\psi_\mathrm{A}(\mathbf{x}_\mathrm{A}),$$

$$\Big(-\frac{\hbar^2}{2m_e}\nabla_\mathrm{B}^2 + U(\mathbf{x}_\mathrm{B})\Big)\psi_\mathrm{B}(\mathbf{x}_\mathrm{B}) \;=\; E_\mathrm{B}\psi_\mathrm{B}(\mathbf{x}_\mathrm{B}),$$

$$\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) \;=\; \psi_\mathrm{A}(\mathbf{x}_\mathrm{A})\psi_\mathrm{B}(\mathbf{x}_\mathrm{B}) \qquad \text{and} \qquad E \;=\; E_\mathrm{A} + E_\mathrm{B}$$

are true, then also the PDE in Equation (2) is true. This means that the separation ansatz reduces the original 6-dimensional PDE into a simpler 3-dimensional PDE. Suppose we have found a large set of solutions

$$(\psi_0(\mathbf{x}), E_0),\ (\psi_1(\mathbf{x}), E_1),\ (\psi_2(\mathbf{x}), E_2), \ldots$$

that satisfy the 3-dimensional eigenvalue equation

$$\Big(-\frac{\hbar^2}{2m_e}\nabla^2 \ + \ U(\mathbf{x})\Big)\psi_n(\mathbf{x}) \ = \ E_n\psi_n(\mathbf{x}) \qquad \forall\, n \in \{0,1,2,\ldots\}.$$

Then we can use these as building blocks to construct a new set of wave functions and energies

$$\psi_{n_\mathrm{A},n_\mathrm{B}}(\mathbf{x}_\mathrm{A},\mathbf{x}_\mathrm{B}) \ = \ \psi_{n_\mathrm{A}}(\mathbf{x}_\mathrm{A})\psi_{n_\mathrm{B}}(\mathbf{x}_\mathrm{B}) \qquad \text{and} \qquad E_{n_\mathrm{A},n_\mathrm{B}} \ = \ E_{n_\mathrm{A}} + E_{n_\mathrm{B}}$$

where the index is $(n_\mathrm{A}, n_\mathrm{B}) \in \{0,1,2,\ldots\}\times\{0,1,2,\ldots\}$. These are solutions to the PDE in Equation (2), and now it is possible to speak about the relations $n_\mathrm{A} = n_\mathrm{B}$ and $n_\mathrm{A} \neq n_\mathrm{B}$. We can say that if $n_\mathrm{A} = n_\mathrm{B}$, then according to the wave function $\psi_{n_\mathrm{A},n_\mathrm{B}}$ the electrons A and B occupy the same state, and that if $n_\mathrm{A} \neq n_\mathrm{B}$, then the electrons A and B occupy different states. If somebody speaks about Pauli exclusion principle that states that the solutions, where $n_\mathrm{A} = n_\mathrm{B}$, are forbidden, and the solutions, where $n_\mathrm{A} \neq n_\mathrm{B}$, are allowed, it sounds like the speech somewhat makes sense. The principle sounds like something that can be true or false.

If we assume that $q_e^2 > 0$, then the separation ansatz does not work. This means that in this case the factors $\psi_\mathrm{A}(\mathbf{x}_\mathrm{A})$ and $\psi_\mathrm{B}(\mathbf{x}_\mathrm{B})$ do not exist. Because the factors do not even exist, it doesn't make sense to speak about the factors being the same or different. Consequently, in this case it doesn't make sense to speak about the electrons occupying the same or different states. If somebody in this case too speaks about Pauli exclusion principle, now the principle sounds like kind of nonsense that almost maybe can be neither true or false.

Carefully pay attention to these logical steps and facts. One step goes like this: The electrons occupying the same state means that the factors $\psi_\mathrm{A}(\mathbf{x}_\mathrm{A})$ and $\psi_\mathrm{B}(\mathbf{x}_\mathrm{B})$ are the same. The electrons occupying different states means that the factors $\psi_\mathrm{A}(\mathbf{x}_\mathrm{A})$ and $\psi_\mathrm{B}(\mathbf{x}_\mathrm{B})$ are different. Second step goes like this: If the factors exist, it makes sense to speak about the factors being the same or different. If the factors do not exist, then it does not make sense to speak about the factors being the same or different. Third step goes like this: If $q_e^2 = 0$, then the factors exist. If $q_e^2 > 0$, then the factors do not exist. An important fact looks like this: When q_e is the charge of the electron, the relation $q_e^2 = 0$ is false, and the relation $q_e^2 > 0$ is true. When we put these facts and logical steps together, we see that in reality it does not make sense to speak about the electrons occupying the same or different states. The Coulomb repulsion between the electrons prevents the separation ansatz from working, and consequently the Coulomb repulsion prevents us from speaking about the electrons occupying the same or different states.

One idea that may surface is that maybe we can use a relation $q_e^2 \approx 0$ as an approximation. Then it would make sense to speak about Pauli exclusion principle under this approximation, and maybe we could consider using some

perturbation series with respect to q_e^2 later. The answer to this idea is that if the Coulomb repulsion between the electrons was relatively weak, then we could consider ignoring it as an approximation. However, in the case of atoms and small molecules, the Coulomb repulsion between the electrons is actually quite strong, so in these relevant cases it doesn't make sense to try to use the approximation $q_e^2 \approx 0$. In other words the factors $\psi_{\mathrm{A}}(\mathbf{x}_{\mathrm{A}})$ and $\psi_{\mathrm{B}}(\mathbf{x}_{\mathrm{B}})$ are nowhere near being in existence even approximatively.

We could now consider the issue to have been dealt with. We explained what's wrong with Pauli exclusion principle, and the principle has now been debunked. Nevertheless, considering the length of time the mainstream physicists have already spent in believing in Pauli exclusion principle, and anticipating the resistance that will eventually come at this debunking, we could still consider elaborating the debunking in a more pedagogical way. There is a psychological phenomenon that humans usually understand what they see better than what gets explained to them in words. So a question arises that could it be possible to somehow show the incorrectness of Pauli exclusion principle. There is an obvious challenge that it is difficult to plot any graphs of wave functions $\psi : \mathbb{R}^6 \to \mathbb{C}$ visible for humans to see. One idea that might work is that we replace the Schrödinger equation with a simpler equation that is still sufficiently similar to the Schrödinger equation, so that we can study the used arguments, while simultaneously also being able to look at the solutions. Let's try to achieve this by switching to studying the PDE

$$\Big(-\partial_x^2 \; - \; \partial_y^2 \; + \; \frac{q_e^2}{|x-y|} \Big) u(x,y) \; = \; E u(x,y) \tag{3}$$

where we want to solve a function of the form

$$u : [0,1] \times [0,1] \to \mathbb{R},$$

while also imposing the boundary conditions

$$u(0,y) = 0 \quad \text{and} \quad u(1,y) = 0 \quad \forall \; y \in [0,1]$$

and

$$u(x,0) = 0 \quad \text{and} \quad u(x,1) = 0 \quad \forall \; x \in [0,1].$$

So we replaced the 3-dimensional spatial space $\mathbb{R}^3$ with a bounded 1-dimensional interval $[0,1]$. By demanding that the wave functions' values must be real, we get wave functions $u(x,y)$ that can be plotted in 3D-figures and inspected by human eyes. The differential operators $-\partial_x^2$ and $-\partial_y^2$ are like the kinetic energy operators $-\frac{\hbar^2}{2m_e}\nabla_{\mathrm{A}}^2$ and $-\frac{\hbar^2}{2m_e}\nabla_{\mathrm{B}}^2$ from the ordinary Schrödinger equation. The repulsive Coulomb potential term $\frac{q_e^2}{4\pi\epsilon_0}\frac{1}{\|\mathbf{x}_{\mathrm{A}}-\mathbf{x}_{\mathrm{B}}\|}$ has been replaced with a similar term $\frac{q_e^2}{|x-y|}$. No background potential is visible in the PDE itself, but the boundary conditions are equivalent to the

background potential being ∞ in the regions $x < 0$, $x > 1$, $y < 0$ or $y > 1$, and 0 in the interior $]0,1[\times]0,1[$. The situation with the separation ansatz strategy with the PDE in Equation (3) is the same as it was with the PDE in Equation (1): If we assume that $q_e^2 = 0$, the separation ansatz works, and if we assume that $q_e^2 > 0$, the separation ansatz does not work.

The separation ansatz in the case $q_e^2 = 0$ leads us easily to the solutions

$$u_{n_{\mathrm{A}},n_{\mathrm{B}}}(x,y) \;=\; \sin(n_{\mathrm{A}}\pi x)\sin(n_{\mathrm{B}}\pi y) \qquad \text{and} \qquad E_{n_{\mathrm{A}},n_{\mathrm{B}}} \;=\; (n_{\mathrm{A}}^2 + n_{\mathrm{B}}^2)\pi^2$$

where the index is $(n_{\mathrm{A}}, n_{\mathrm{B}}) \in \{1,2,3,\ldots\} \times \{1,2,3,\ldots\}$. Again it makes sense to speak about the relations $n_{\mathrm{A}} = n_{\mathrm{B}}$ and $n_{\mathrm{A}} \neq n_{\mathrm{B}}$, so in this $q_e^2 = 0$ case we can speak about the two electrons occupying the same or different states.

If we assume that $q_e^2 > 0$, then Equation (3) is too difficult, and there are no known simple solution formulas. However, handling functions of the form $u : [0,1] \times [0,1] \to \mathbb{R}$ with float arrays on an ordinary computer is not tremendously challenging, at least if not much precision is demanded, and it is possible to write computer programs that estimate the solutions to this PDE numerically even in the $q_e^2 > 0$ case. I wrote a such program, and generated some graphs of the solutions for this article, so now it is possible for us to see these solutions too.

Figure 1 contains some graphs of some example solutions to Equation (3). The left most column and the center column contain solutions in the $q_e^2 = 0$ case. These solutions are based on the solution formula $u_{n_{\mathrm{A}},n_{\mathrm{B}}}(x,y) = \sin(n_{\mathrm{A}}\pi x)\sin(n_{\mathrm{B}}\pi y)$. The right most column in Figure 1 contains solutions in the $q_e^2 > 0$ case. These solutions have been produced by numerical iterations that handle arbitrary float arrays, and there is no known simple formula for these. So we got some bumps to behold. Next we have to pay attention to what these bumps are doing. It is possible to see that the bumps on the left most column and the center column form some kind of grid structures that have some simple relation to the directions of the x-axis and y-axis. This is what it looks like when the wave function separates into a product $u(x,y) = u_{\mathrm{A}}(x)u_{\mathrm{B}}(y)$. Although bumps are present on the right most column too, there the bumps do not form the grid structures that would have the simple relation to the directions of the x-axis and y-axis. This is what it looks like when the wave function does not separate into a product of two factors. So staring at Figure 1 is an opportunity for humans to see what an separation ansatz looks like. This also an opportunity to learn that a wave function that does not separate into a product of nontrivial factors is distinctly different kind of wave function than a wave function that does separate.

Let's pay attention to the numbers of the bumps. On the left most column and the center column it is possible to perform a counting procedure, where we fix y, and count how many bumps get encountered as x traverses

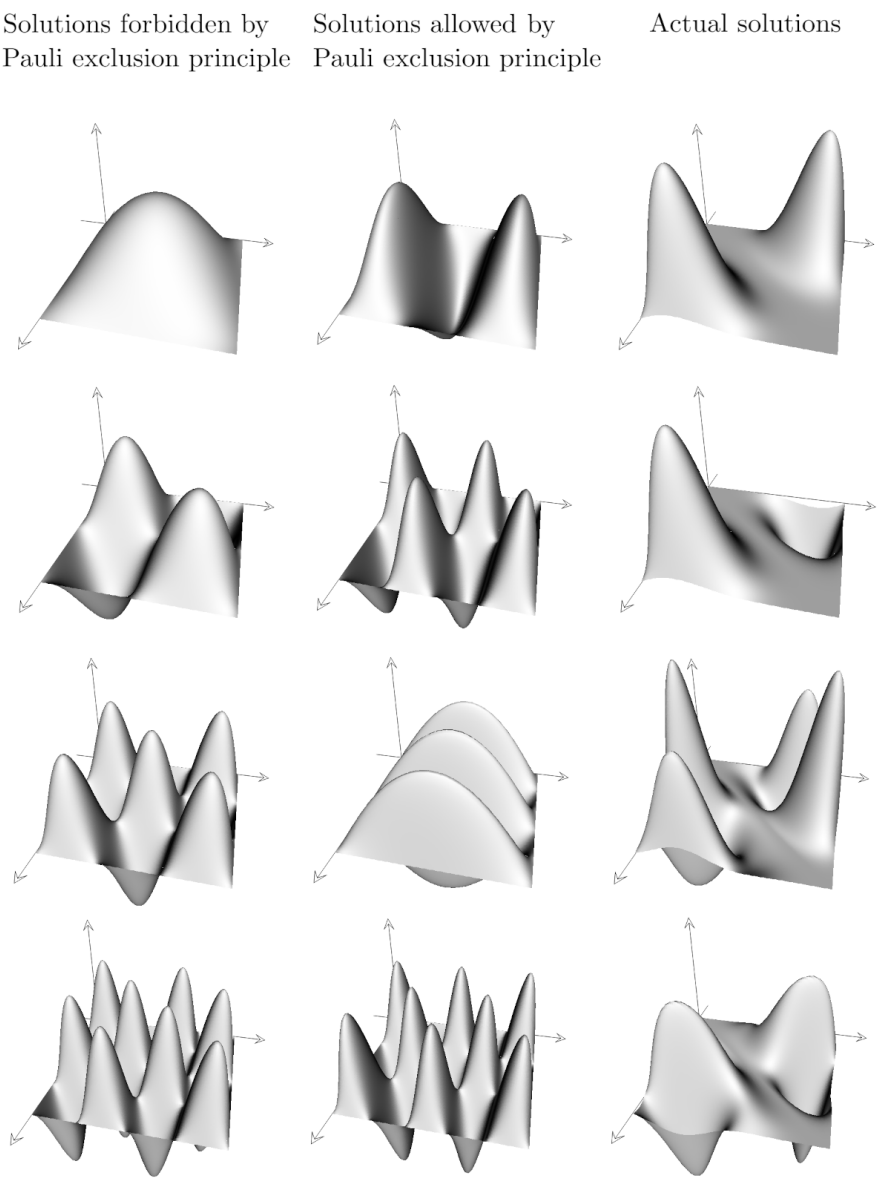

Figure 1: The left most column and the center column contain example solutions to the PDE (3) in the case $q_e^2 = 0$. The left most column contains example solutions where $n_{\mathrm{A}} = n_{\mathrm{B}}$, and the center column contains example solutions where $n_{\mathrm{A}} \neq n_{\mathrm{B}}$. The right most column contains example solutions to the PDE (3) in the case $q_e^2 > 0$, and in this case there are no indices n_{A} and n_{B}.

through the interval $[0, 1]$. Here we should count both the local maxima and the local minima as bumps. Then it is also possible to fix x, and count how

many bumps get encountered as y traverses through the interval $[0, 1]$. Then the total number of bumps on the set $[0, 1] \times [0, 1]$ is the product of those two numbers. This a consequence of the fact that the wave function has separated into a product $u(x, y) = u_{\mathrm{A}}(x)u_{\mathrm{B}}(y)$. The number of bumps on the right most column cannot be interpreted as a product of two numbers similarly, so here again we see that a wave function that does not separate into a product of nontrivial factors is distinctly different kind of wave function than a wave function that does separate.

On the left most column the number of bumps in the x-direction is always the same as the number of bumps in the y-direction. This is because these wave functions have been defined with indices that satisfy the relation $n_{\mathrm{A}} = n_{\mathrm{B}}$. This is what it looks like when electrons A and B occupy the same state. On the center column the number of bumps in the x-direction is always different from the number of bumps in the y-direction. This is because these wave functions have been defined with indices that satisfy the relation $n_{\mathrm{A}} \neq n_{\mathrm{B}}$. This is what it looks like when electrons A and B occupy different states. The wave functions on the right most column do not look the same as the wave functions on the left most column. This means that the wave functions on the right most column do not represent system states where the two electrons would occupy the same state. The wave functions on the right most column do not look the same as the wave functions on the center column either. This means that the wave functions on the right most column do not represent system states where the two electrons would occupy different states. Pay attention to this conclusion: On the right most column the electrons are neither occupying the same state or different states.

Although humans do not have an ability to convincingly imagine things in a 6-dimensional vector space, it is possible to attempt to combine feats of imagination and logic to gain some insight. Suppose $\psi : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{C}$, $(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}}) \mapsto \psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}})$ is a solution to Equation (2), and suppose it has separated into a product $\psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}}) = \psi_{\mathrm{A}}(\mathbf{x}_{\mathrm{A}})\psi_{\mathrm{B}}(\mathbf{x}_{\mathrm{B}})$. It is possible to imagine that if we fix $\mathbf{x}_{\mathrm{B}}$, and let $\mathbf{x}_{\mathrm{A}}$ wander around in the space $\mathbb{R}^3$, a certain number of bumps in $\psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}})$ can be encountered. A certain number of bumps can be encountered also if we fix $\mathbf{x}_{\mathrm{A}}$ and let $\mathbf{x}_{\mathrm{B}}$ wander around in the space $\mathbb{R}^3$. Then it is possible to understand that the total number of bumps that can be found in the space $\mathbb{R}^6$ is the product of those two numbers. So we can understand and believe that something similar can happen in $\mathbb{R}^6$ as happens in $[0, 1] \times [0, 1]$ in Figure 1. Then it is possible to understand that if $\psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}})$ describes a system state where the electrons occupy the same state, the bumps that can be found by letting $\mathbf{x}_{\mathrm{A}}$ wander around are somehow the same as the bumps that can be found by letting $\mathbf{x}_{\mathrm{B}}$ wander around. If $\psi(\mathbf{x}_{\mathrm{A}}, \mathbf{x}_{\mathrm{B}})$ describes a system state where the electrons occupy different states, the bumps that can be found by letting $\mathbf{x}_{\mathrm{A}}$ wander around are somehow different from the bumps that can be found by letting $\mathbf{x}_{\mathrm{B}}$

wander around.

One of the features that we can see in the graphs on the right most column in Figure 1, is that $u(x, y)$ tends to be close to zero near the diagonal defined by the equation $x = y$. This is a consequence of the Coulomb repulsion term, and the presence of this feature is related to the fact that these wave functions $u(x, y)$ aren't separating into nontrivial products. Suppose $\psi : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{C}$, $(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) \mapsto \psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$ is a solution to Equation (1) when $q_e^2 > 0$. It is possible to imagine that $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$ is close to zero near the 3-dimensional subspace of $\mathbb{R}^6$ defined by the equation $\mathbf{x}_\mathrm{A} = \mathbf{x}_\mathrm{B}$. This again is a consequence of the Coulomb repulsion term. It is also possible to imagine that some bumps are present in the space $\mathbb{R}^6$, but the number of them cannot naturally be interpreted as a product of two numbers. These things that we can imagine using the power of imagination and logic hopefully help us understand what it means that in the $q_e^2 > 0$ case the wave function $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$ isn't separating into nontrivial factors, and that consequently it will not make sense to speak about the electrons occupying the same or different states.

If somebody puts forward a claim:

"Electrons A and B must occupy different states, because a law of nature forbids them from occupying the same state."

It is the same thing as putting forward a claim:

"The graphs on the right most column must look like the graphs on the center column, because a law of nature forbids them from looking like the graphs on the left most column."

Then we must respond to this by noting that:

"No, that is wrong. The graphs on the right most column do not look like the graphs on the center column."

Some people might say that this debunking of Pauli exclusion principle has been invalid, because the essence of Pauli exclusion principle is not only that the electrons would have to occupy different states, but instead that the wave function must have the antisymmetricity property $\psi(\mathbf{x}_\mathrm{B}, \mathbf{x}_\mathrm{A}) = -\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$. A response to this is that this thing with antisymmetricity is a minor complication that doesn't essentially affect the arguments used in the debunking. It could be a good idea to clarify the meaning of some language used in this topic. Suppose ψ_{n_1} and ψ_{n_2} are different wave functions, and suppose that a wave function ψ has been defined by the formula

$$\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) \;=\; \psi_{n_1}(\mathbf{x}_\mathrm{A})\psi_{n_2}(\mathbf{x}_\mathrm{B}) \;-\; \psi_{n_2}(\mathbf{x}_\mathrm{A})\psi_{n_1}(\mathbf{x}_\mathrm{B}).$$

One issue we should clarify is that is it correct to say that according to ψ one electron is now occupying the state n_1, and another electron is occupying the state n_2? It is a justified answer that no, one electron is not occupying the state n_1. If electron A was occupying the state n_1, and electron B was occupying the state n_2, the wave function should be $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) = \psi_{n_1}(\mathbf{x}_\mathrm{A})\psi_{n_2}(\mathbf{x}_\mathrm{B})$ without the antisymmetricity property. If the wave function has been made antisymmetric, then the both electrons are occupying the both states; but not in an arbitrary way, but instead in the certain engtangled way. Then we may ask that if the english statement that one electron is occupying the state n_1, and another electron is occupying the state n_2, is wrong, what english statement would be a correct one? There does not exist a good answer, because the human languages have not evolved to be used to describe these type of things. If we are smart, at this point we could consider getting more understanding and tolerant towards the attempts to use our limited human languages to describe these things. We could agree that the statement that one electron is occupying the state n_1, and another electron is occupying the state n_2, is valid after all. The idea behind this agreement would be that if everybody knows that the electrons are supposed to have the indistinguishability property, then everybody will understand that the statement that one electron is occupying the state n_1, and another electron is occupying the state n_2, is just code language that means that the wave function is $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) = \psi_{n_1}(\mathbf{x}_\mathrm{A})\psi_{n_2}(\mathbf{x}_\mathrm{B}) - \psi_{n_2}(\mathbf{x}_\mathrm{A})\psi_{n_1}(\mathbf{x}_\mathrm{B})$. The reason for why we should agree to this is that after this agreement we can avoid distracting pedantry in the subsequent debate.

If somebody puts forward a claim that a wave function $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$ must simultaneously satisfy the conditions that it satisfies the PDE in Equation (1) and also satisfies the antisymmetricity property $\psi(\mathbf{x}_\mathrm{B}, \mathbf{x}_\mathrm{A}) = -\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$, we should agree that this claim makes sense. It is a kind of claim that can be true or false. The claim is not that kind of nonsense that it could be neither true of false. However, carefully pay attention to these logical facts: If the separation ansatz works, then the antisymmetricity property implies that the wave function must be of the form $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) = \psi_{n_1}(\mathbf{x}_\mathrm{A})\psi_{n_2}(\mathbf{x}_\mathrm{B}) - \psi_{n_2}(\mathbf{x}_\mathrm{A})\psi_{n_1}(\mathbf{x}_\mathrm{B})$, and then we can speak about the electrons occupying different states (because above we agreed that the meaning of the english words can be interpreted like this), but if the separation ansatz does not work, then the antisymmetricity property does not imply that the wave function would be of the form $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) = \psi_{n_1}(\mathbf{x}_\mathrm{A})\psi_{n_2}(\mathbf{x}_\mathrm{B}) - \psi_{n_2}(\mathbf{x}_\mathrm{A})\psi_{n_1}(\mathbf{x}_\mathrm{B})$, and it will genuinely not make sense to speak about the electrons occupying different states. Since the relation $q_e^2 > 0$ is the true relation, and since the separation ansatz isn't working in this case, it will still not make sense to speak about the electrons occupying different states, because the factors ψ_{n_1} and ψ_{n_2} don't even exist. You can impose a demand that the wave function must have the antisymmetricity property, but the factors ψ_{n_1} and ψ_{n_2} still won't

exist.

One extremely interesting question is that does there exist a law of nature that states that a wave function describing two electrons must have the antisymmetricity property $\psi(\mathbf{x}_\mathrm{B}, \mathbf{x}_\mathrm{A}) = -\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$? A smart way to approach this thing is to ask that does there exist empirical evidence that would support this hypothesis? If one tries to find information about this thing, the explanations always start the same way: The start is that firstly we have to understand that the antisymmetricity implies the equation $\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B}) = \psi_{n_1}(\mathbf{x}_\mathrm{A})\psi_{n_2}(\mathbf{x}_\mathrm{B}) - \psi_{n_2}(\mathbf{x}_\mathrm{A})\psi_{n_1}(\mathbf{x}_\mathrm{B})$ and that the electrons must occupy different states, and then the explanations continue somewhere from there. This nonsensical start exposes the fact that there is no empirical evidence supporting the antisymmetricity hypothesis. Consequently, it doesn't make sense to believe in the antisymmetricity.

Another relevant question is that does there exist reasons to believe that the wave functions of multi-electron systems have any special symmetry properties at all? If we interpret the elementary particles as excitations of quantum fields, this approach does naturally lead to the indistinguishability property of the particles, and subsequently the symmetricity property $\psi(\mathbf{x}_\mathrm{B}, \mathbf{x}_\mathrm{A}) = +\psi(\mathbf{x}_\mathrm{A}, \mathbf{x}_\mathrm{B})$ too. This means that we should consider the hypothesis that all elementary particles are bosons, and that there are no fermions at all. Particles with half-integer spins most apparently do exist of course though, so the new hypothesis is that also particles with half-integer spins are bosons.

Some people might say that this debunking of Pauli exclusion principle has been invalid, because here we have forgotten that the electron spin functions as a degree of freedom that must be taken into account in the application of Pauli exclusion principle. A response to this is that yes this mistake has been present in the debunking, but fixing this mistake doesn't essentially change how the debunking still works. We can take a closer look at the relevance of the electron spin for the antisymmetricity thing. Suppose we are interested in a system that consists of three nonrelativistic electrons that for some reason feel some background potential. This means that we are interested in a 9-dimensional PDE. Let's assume that $q_e^2 = 0$, and that the separation ansatz can be used to reduce the 9-dimensional PDE into a 3-dimensional PDE. Let's assume that we have obtained a large set of solutions to the 3-dimensional PDE, and denote these solutions as $(|\psi_0\rangle, E_0), (|\psi_1\rangle, E_1), (|\psi_2\rangle, E_2), \ldots$. These abstract vectors mean that if $\psi_n(\mathbf{x})$ is a solution to the PDE in an ordinary way, then a relation $\psi_n(\mathbf{x}) = \langle \mathbf{x}|\psi_n\rangle$ is true. Suppose we don't want to simplify the model by ignoring the electron spins. However, let's keep things simple by assuming that the Hamiltonian operator doesn't really do anything with the spins. So whether or not the Schrödinger equation is satisfied will depend only on whether or not the spatial PDE is satisfied, and the spin degrees of freedom

just hang around. These means that we could use notations $|\psi_n\rangle \otimes |\uparrow\rangle$ and $|\psi_n\rangle \otimes |\downarrow\rangle$ to denote some possible states of individual electrons. Then a question arises that how do we denote the spin state of the whole three electron system? There are several equivalent ways of accomplishing this, and one way is to use the abstract vectors $|\uparrow\uparrow\uparrow\rangle$, $|\downarrow\uparrow\uparrow\rangle$, $|\uparrow\downarrow\uparrow\rangle$, $|\downarrow\downarrow\uparrow\rangle$, $|\uparrow\uparrow\downarrow\rangle$, $|\downarrow\uparrow\downarrow\rangle$, $|\uparrow\downarrow\downarrow\rangle$ and $|\downarrow\downarrow\downarrow\rangle$. The idea is that the first arrow denotes electron A's state, the second arrow denotes electron B's state, and the third arrow denotes electron C's state. Suppose that an abstract vector $|\psi\rangle$ is supposed to denote the whole system's state. What kind of values can this abstract vector assume? An example of a possible value could be

$$|\psi\rangle \;=\; |\psi_{n_{\rm A}}\rangle \otimes |\psi_{n_{\rm B}}\rangle \otimes |\psi_{n_{\rm C}}\rangle \otimes |s\rangle$$

where $n_{\rm A}, n_{\rm B}, n_{\rm C} \in \{0, 1, 2, \ldots\}$ and the possible values of $|s\rangle$ are $|\uparrow\uparrow\uparrow\rangle$, $|\downarrow\uparrow\uparrow\rangle$, $\ldots$, $|\downarrow\downarrow\downarrow\rangle$. However, these are not all the possible values of $|\psi\rangle$, because more can be found by constructing linear combinations of these basis solutions. We'll see what is essential about Pauli exclusion principle by studying the example vector

$$\begin{aligned}
|\psi\rangle \;=\;& |\psi_0\rangle \otimes |\psi_0\rangle \otimes |\psi_1\rangle \otimes |\uparrow\downarrow\uparrow\rangle \\
+\;& |\psi_0\rangle \otimes |\psi_1\rangle \otimes |\psi_0\rangle \otimes |\downarrow\uparrow\uparrow\rangle \\
+\;& |\psi_1\rangle \otimes |\psi_0\rangle \otimes |\psi_0\rangle \otimes |\uparrow\uparrow\downarrow\rangle \\
-\;& |\psi_1\rangle \otimes |\psi_0\rangle \otimes |\psi_0\rangle \otimes |\uparrow\downarrow\uparrow\rangle \\
-\;& |\psi_0\rangle \otimes |\psi_0\rangle \otimes |\psi_1\rangle \otimes |\downarrow\uparrow\uparrow\rangle \\
-\;& |\psi_0\rangle \otimes |\psi_1\rangle \otimes |\psi_0\rangle \otimes |\uparrow\uparrow\downarrow\rangle.
\end{aligned}$$

This vector has three interesting properties. One is that it is nonzero, so it hasn't vanished due to possible cancellations. Second interesting property is that if we swap the places of electrons A and B, the vector changes its sign. Similarly, if we swap the places of electrons A and C, or electrons B and C, then too the vector changes its sign. So the vector changes its sign in all possible swaps of electrons. Third interesting property is that under a reasonable interpretation of english language we can say that now there are two electrons occupying the state $|\psi_0\rangle$, and one electron occupying the state $|\psi_1\rangle$. According to Pauli exclusion principle having two electrons occupying the same state wouldn't have been possible without the spin, so we see that when focusing on the spatial part of the representations, the presence of spin does change the effect of this principle. If one tries to construct a state $|\psi\rangle$, where the all three electrons would simultaneously occupy the same state $|\psi_0\rangle$, then the state will vanish due to the cancellations, so a such state cannot exist. This is how Pauli exclusion principle is supposed to work according to the mainstream physics.

Next, carefully pay attention to these logical steps and facts: If $q_e^2 = 0$, the separation ansatz works, the solutions $|\psi_0\rangle$, $|\psi_1\rangle$, $|\psi_2\rangle$,... exist, it is

possible to construct solutions to the three electron system as products $|\psi_{n_\mathrm{A}}\rangle \otimes |\psi_{n_\mathrm{B}}\rangle \otimes |\psi_{n_\mathrm{C}}\rangle$, it makes sense to speak about the electrons occupying the same or different states, and by using the vectors $|\uparrow\uparrow\uparrow\rangle$, $|\downarrow\uparrow\uparrow\rangle$, ..., $|\downarrow\downarrow\downarrow\rangle$ it is possible to derive the result that when focusing on the spatial part of the representations, two electrons can occupy the same state, but three cannot. If instead $q_e^2 > 0$, the separation ansatz does not work, the solutions $|\psi_0\rangle$, $|\psi_1\rangle$, $|\psi_2\rangle$,... do not exist, it is not possible to construct solutions to the three electron system as products $|\psi_{n_\mathrm{A}}\rangle \otimes |\psi_{n_\mathrm{B}}\rangle \otimes |\psi_{n_\mathrm{C}}\rangle$, it does not make sense to speak about the electrons occupying the same or different states, and it is not possible by using the vectors $|\uparrow\uparrow\uparrow\rangle$, $|\downarrow\uparrow\uparrow\rangle$, ..., $|\downarrow\downarrow\downarrow\rangle$ to derive the result that when focusing on the spatial part of the representations, two electrons could occupy the same state, and three could not. So Pauli exclusion principle is still nonsense for the reason explained earlier in this article, and the vectors $|\uparrow\rangle$ and $|\downarrow\rangle$ are only a minor complication that do not turn the nonsensical nature of Pauli exclusion principle into sensical. The truth is that the presence of the repulsive Coulomb potential term prevents us from speaking about the electrons occupying the same or different states, and this truth is not about to go away easily.

One psychological phenomenon that may be relevant here is that humans often end up committing the fallacy that if a human being has put effort into something, then the human deserves something in return. Here with antisymmetric wave functions humans have to put some effort into studying the applications of permutations and their signs. It may feel an unpleasant possibility, that the effort that was put into studying the permutations and their signs was in vain. We should remind people that the fact that the permutations and their signs themselves are a legitimate topic doesn't mean that every attempt to apply the permutations and their signs would be legitimate. It is possible that you end up wasting your time with permutations and their signs.

A big mystery related to Pauli exclusion principle is that how is it possible that the mainstream physicists have spent almost 100 years believing that there would be empirical evidence supporting this principle, while the truth is that the principle is so nonsensical that it doesn't really mean anything, and consequently it is not possible for the supporting evidence to exist? One possible speculation and conjecture is that if we construct a multi-electron model in a such way that first we ignore the repulsive Coulomb potential term, and then add Pauli exclusion principle into the model, maybe the model can tend to give qualitatively right kind of results, because Pauli exclusion principle somehow approximates the effects from the repulsive Coulomb potential term? Let's recall what we saw in Figure 1. One of the features of the graphs on the right most column was that there $u(x, y)$ tended to be close to zero near the diagonal $x = y$. If we start with some $u(x, y)$ from the center column, and project the $u(x, y)$ into an anti-

symmetric form with the operation $u(x,y) \leftarrow u(x,y) - u(y,x)$, this also will produce $u(x,y)$ that is close to zero near the diagonal $x = y$. So we found a function $u(x,y)$ that is close to an actual solution near the diagonal $x = y$, because there it is close to zero as it is supposed to be, and that is close to an actual solution far away from the diagonal $x = y$ too, because there the Coulomb repulsion is weak. So we found a function $u(x,y)$ that maybe is close to an actual solution for all (x,y). We are not getting the legitimate solutions to the full PDE this way, but maybe the antisymmetrized wave functions do resemble the solutions in some ways?

It can be foreseen that many people will not like the explanation that Pauli exclusion principle would be merely approximating the effects from the repulsive Coulomb potential term, because this explanation doesn't have the same feel of an elegant mathematical principle as Spin-statistics theorem has. I would like that ask these people that what do they think is going on with the repulsive Coulomb potential term then? The Coulomb repulsion between electrons exists whether humans want it or not, and it doesn't make sense to try to ignore it. The effects from the Coulomb repulsion do not care whether humans perceive them as elegant or inelegant.

We could optimistically try to hope that maybe an elegant way of handling the effects from the Coulomb repulsion could eventually be discovered? Maybe humans just haven't succeeded in this yet, because not much effort has been put into trying? Maybe we should show Figure 1 to young talented students of our time, and request them to try to come up with new theories for explaining what's happening on the right most column?

According to mainstream physics Pauli exclusion principle is a consequence of Spin-statistics theorem [2]. On the surface this might seem to enhance and validate the status of Pauli exclusion principle. However, one of the silly and noteworthy issues physics students of our time have to face is that while everybody knows that Spin-statistics theorem exists, nobody knows its proof. All pedagogical materials always omit the proof as too difficult. The reason for this is probably that the proof is so nonsensical that nobody understands it, and consequently nobody wants to embarrass themselves by attempting to recite it. We should recognize that this hidden nature of the proof certainly is not positive for the status of Pauli exclusion principle. Nevertheless, some information about the proof is in circulation. According rumours the proof has something to do with the demand that eigenenergies of physical quantum systems must be bounded from below. If one tries to quantize a half-integer spin field as a boson field, one gets eigenenergies that can reach arbitrarily low values (approach $-\infty$), and then we can proceed to the conclusion that half-integer spin particles must be fermions. Apparently the proof uses an axiom that all particles must be either bosons or fermions, and that no other type of particle could exist.

Let's think more carefully about the demand that eigenenergies must be

bounded from below. Is it true that they must be bounded from below? Why is it so? It is possible to come up with some answers to this question. For example, suppose there was a special kind of atom whose discrete energy levels were not bounded from below. How would that atom behave? The atom could then emit an infinite amount of photons by successively jumping to lower energy levels indefinitely, which would be nonsensical. In other words the state of the atom would diverge in an unphysical way. So if somebody puts forward a claim that the eigenenergies of a physical quantum system must be bounded from below, apparently the claim makes sense? Let's think more carefully about what forbidding unphysical divergence means. There exist example systems, where the eigenenergies are bounded from below, and where the state does not diverge in an unphysical way. There exist example systems, where the eigenenergies are not bounded from below, and where the state does diverge in an unphysical way. Strictly speaking, the eigenenergies being not bounded from below as such is not a problem for us. What is a problem for us is the unphysical divergence of the state of the system. If there is a correlation phenomenon that eigenenergies being not bounded from below always leads to unphysical divergence, this would be a legitimate reason to demand that the eigenenergies must always be bounded from below then. It is true that some example systems somewhat support the presence of this correlation phenomenon, but strictly speaking, there is no reason to assume that this correlation would always hold. The time evolution of a state of a system is not solely dictated by the available eigenenergies, because there are other mechanisms that affect the time evolution too. Therefore, it is possible that there exist systems that do not have their eigenenergies bounded from below, while simultaneously having the property that their states never diverge in an unphysical way, because it is possible that some mechanism prevents the unphysical divergence. There is no reason to immediately forbid these type of systems as unphysical.

Next, I'm going to debunk the proof of Spin-statistics theorem. One of the ideas in modern physics is that in the same way as photons are excitations of quantized electromagnetic field, electrons are excitations of quantized Dirac field. One of the issues with Dirac field is that the energy density of Dirac field $\mathrm{Re}(\overline{\psi}(-i\boldsymbol{\gamma}\cdot\nabla+m)\psi)$ is not bounded from below. This has a consequence that when Dirac field is quantized according to the principles of ordinary Quantum Mechanics, the eigenenergies of the quantized field will not be bounded from below either. According to mainstream physics this is an issue that has to be solved somehow. The mainstream solution is that we must not quantize Dirac field in the ordinary way, but instead the field must be quantized with the ad hoc canonical quantization that is completely pulled out of a hat in a such way that the eigenenergies reaching arbitrarily low values (approaching $-\infty$) vanish. This is supposed to be how Spin-statistics theorem manifests in this situation. At this point I'm going to tell

people something that almost everybody (in this field) already knows, but that almost nobody yet has recognized. Both of these two claims are simultaneously true: One claim is that the energy density of Dirac field can reach arbitrarily low values. Another claim is that Dirac field ψ never diverges in an unphysical way. It is likely that some people will disagree with this, and insist that if the energy density of Dirac field can reach arbitrarily low values, surely that will imply that Dirac field ψ could diverge in an unphysical way. My response is that no that is not true. The truth is that the energy density of Dirac field can reach arbitrarily low values, and still Dirac field ψ never diverges in an unphysical way. The justification of this claim is that Dirac field has a conserved current $\overline{\psi}\gamma^{\mu}\psi$ known as Dirac current. There is a technical result that if Dirac equation $(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$ is satisfied, then also the continuity equation $D_{\mu}(\overline{\psi}\gamma^{\mu}\psi) = 0$ is satisfied. This means that Dirac current is conserved. If Dirac field diverged in an unphysical way, also Dirac current would diverge in an unphysical way. We know that Dirac current doesn't diverge in an unphysical way, because it is conserved. Therefore we know that Dirac field doesn't diverge in an unphysical way. Dirac field doesn't need a lower bound for its energy density, because the conservation of Dirac current is sufficient to prevent an unphysical divergence. When Dirac field is quantized in the ordinary way, we get eigenenergies that reach arbitrarily low values, and we also get a quantized Dirac current that is still conserved. The new knowledge, that there is no need to impose an ad hoc procedure to the eigenenergies, because the conserved quantized Dirac current will prevent the quantized system from diverging in an unphysical way even in the presence of the eigenenergies that reach arbitrarily low values, nullifies the proof of Spin-statistics theorem.

References

[1] https://en.wikipedia.org/wiki/Pauli_exclusion_principle

[2] https://en.wikipedia.org/wiki/Spin-statistics_theorem