Methods of measuring length lesser the than least count

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Abstract

This is a very simple paper describing some methods to measure the values especially in length dimension to a scale much less than that of the least count of the given instrument. The final aim of this paper is to trigger a thought process to get ideas , if any ,for indirect measurement to get less than Planck's length.

1 Introduction

In case of a very simple measuring instruments like a scale which is having 1 mm as the least count, one cannot accurately measure anything less than a millimetre. But with the help of a vernier callipers with coinciding vernier division methods in which 99 mm are divided into 100 parts, one can measure 1/100 mm which is the least count of the callipers. But during dividing 99 mm in to 100 , most accurate instrument having least count less than 1/100 mm must be used.
This paper highlights some methods in which indirect measurements such as calculation, estimation are used which will be actually tallying with the real length of the substance that were considering.
This paper highlights that *"it is not just putting a scale against some object to know the length as considered as only measurement, even the indirect methods of measurements are also valid ,as long as we can estimate the realistic value of the parameter considered"*.

2 Related Work till date

- Calmet, X., Graesser, M. and Hsu, S.D.H., in their work on "Minimum length from quantum mechanics and classical general relativity"[1], have discussed the limit of measurement due to the combined effects of Planck's length and Theory of Relativity .

- Chandler, W.L., Yeung, W. and Tait, J.F.,in their work titled as " A new microparticle size calibration standard for use in measuring smaller microparticles using a new flow cytometer"[2], have given a flow meter to measure small flow of fluid with high accuracy. This is practical approach and not at the level of Planck's length.

- Niedzielski, T., Migoń, P. and Placek, A.,in their work titled as " A minimum sample size required from Schmidt hammer measurements" [3] , have discussed accurate mthods of mesuring rock
- Stuchly, M.A. and Stuchly, S.S., in their work titled "Coaxial line reflection methods for measuring dielectric properties of biological substances at radio and microwave frequencies-a review"[4], they have discussed small length measurements using reflected Radio Frequencies and Microwave frequencies.

3 Methods to measure lengths smaller than the least count of the instrument

Indirect methods , calculations , statistical estimations are all a part of measurement. Many such methods are considered in this paper.

3.1 Measuring the thickness of a sheet of paper

Figure 1: books

A 2000 pages book is selected. That means 1000 sheets of paper will be there. If

Figure 2: Scale with least count of 1mm

the thickness of the book without wrapper, as measured by the scale with 1 mm as least count is say, 52mm, then we can calculate the thickness of one sheet of paper as:

$$t = \frac{52}{1000} = 0.052mm \tag{1}$$

This is much lesser than the least count of the scale!

3.2 Water level in a tank

A shallow water tank with 400 cm long and 250 cm wide for a 10 cm height is Considered. It can hold 1000 litres of water. let their be a tap and if it leaks, say one litre per hour and we measure it several times to confirm that it is uniform, then with the following calculations we can find out the change in level of water in the tank in one second interval of time. Volume of the tank =

$$V = (400 * 250 * 10) = 1000lt$$

10cm =100mm and so, each mm of height corresponds to 1000/100 =10 lt of water.
The water leaking from tap = 1 lt in one hour= 3600 seconds.
Flow of water in 1 sec = 1/3600 lt.
Reduction in height for 10 lt = 1 mm. or 0.1mm/ lt
Hence reduction of level of water in one second is =

$$(h1 - h2) = 0.1 * (1/3600) = 0.000028mm$$

where h1 = initial level and h2 = level after 1 second. This is much less than the least count of the scale .

3.3 Reduce scale 10 times

This is a simple process in which a photograph of the scale itself taken so as to capture 100 millimetre length of the scale and reduced that photo in photo editing method to 10 mm. Now we are having a photo in which 100 divisions are there in a length of 10 mm. Therefore each division is at a distance of 0.1 mm, which is 10 times less than the least count of the scale. In fact we can even make out the distance between the divisions because we can see to the accuracy of 0.1 mm. This concept is being used in case of a vernier scale where from our own eyesight we will decide the coinciding vernier division. So in that method even though interval of the smallest division is much larger we will even get 0.01 mm

3.4 Thermal expansion of steel scale

The coefficient of thermal expansion of steel is

$$\alpha = 12 * 10^{-6}$$

$$\alpha = (\delta l/l)$$

per degree rise of temperature.

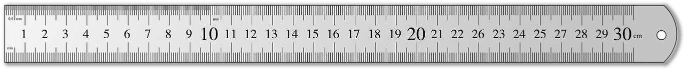

Figure 3: Expansion of steel scale

So, the increase in length of the scale of 30 cm, heated through 1 deg Centigrade is :

$$\delta l = 12 * 10^{-6} * 30$$

= 0.00036 cm = 0.0036mm.

3.5 Microscope with 1000 time magnification

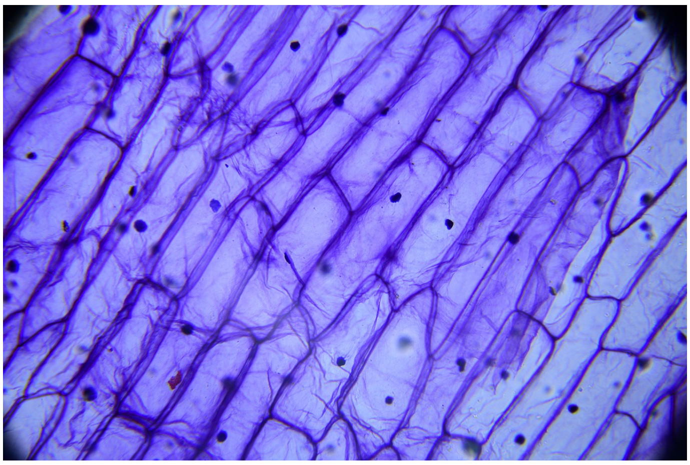

Figure 4: Plant cell under a microscope

Considering a microscope with 1000 times magnification ,casting image on to a screen, one can measure the width of the plant cell. If it is say, 1 mm on the screen,then the actual width of the cell will be 1/1000 mm = 0.001 mm.

3.6 Lorentz contraction method

There is actually no limit for measuring the smallest length that can be measured indirectly from Relativistic contraction or Laurence contraction but the limit is actually at Planck's length.
The Lorentz equation for Relativistic contraction is :

$$L = Li * \sqrt{1 - (\frac{v^2}{c^2})} \tag{2}$$

Considering again the 300 mm scale, when it is moving at $2m/sec$,
$v = 2m/sec, c = 3 * 10^8 m/sec, Li = 300mm$

$$L = 300 * \sqrt{1 - (\frac{2^2}{300000000^2})} \tag{3}$$

L= 299.999999 mm. Change in length = 300 -299.999999 =0.000001 mm.

4 Discussion and Conclusion

- In case of measurement of thickness of the paper we have assumed that all the pages are equality equality thick . But anyway, even they if there is a slight variation, we have measured the thickness to a value which is much less than the least count of the scale
- Relativity , according to some scientists, will not work at Planck's length but still it is to be verified.
- In thermal expansion of the scale, uniform temperature throughout the scale is assumed. It will be more or less so and our estimated increase in length will be much smaller than the least count of the scale.
- In case of marking units much smaller than least count of the scale, a uniform magnification is assumed throughout the length of 100 mm and the magnification is exactly 10 times. Anyway the resulting image will have 0.1 mm markings provided the pixels are much smaller than that and the camera is accurate.
- This is an attempt not just measure less than 1 mm, but it is to trigger a thought process to get measurements much smaller than the Planck's length and time by indirect methods.

References

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[4] Stuchly, M.A. and Stuchly, S.S., 1980. Coaxial line reflection methods for measuring dielectric properties of biological substances at radio and microwave frequencies-a review. IEEE Transactions on instrumentation and measurement, 29(3), pp.176-183.