THE GOLDBACH CONJECTURE PROOF

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ABSTRACT

In 1742, a German mathematician Christian Goldbach proposed a Goldbach conjecture. The conjecture states that every even integer greater than 2 can be expressed as a sum of two prime numbers. The conjecture has been verified up to 4×10^{18} and no counter example has been given up to date. Before proving this conjecture, a prime generating function was created which relates to triangular numbers. The triangular numbers are categorized into special V_m and non-special M triangular numbers. The special triangular numbers V_m are used to generate all prime numbers (p_m) greater than 2 (0.375 is included among V_m to generate 2) using the function $p_m = \sqrt{8V_m + 1}$. The V_m is obtained from$T_n \cap M'$. The proof holds true for all even integers greater than 2. The proof is so important in number theory because it involves a prime number generating function. This function may help us solve many prime number related problems.

Keywords: Triangular numbers, special, non-special, Goldbach conjecture, prime numbers

INTRODUCTION

The conjecture which was proposed by a German mathematician in a letter to Swiss Mathematician Leonhard Euler in 1742 states that every integer greater than 2 can be expressed as a sum of two prime numbers. Below are few numerical examples of the conjecture.

Prime numbers	Even integers
2+2	4
3+3	6
3+5	8
5+5, 3+7	10
5+7	12
7+7, 3+11,	14
3+13, 5+11	16

The conjecture motivates mathematicians to understand how prime numbers are distributed among the natural numbers and how they can produce these even integers. There have been a lot of attempts by many mathematicians to try and prove the conjecture but to no avail. Computational evidence verifies that this conjecture is true. This paper introduces a new relationship of triangular numbers with prime numbers. The main challenge in proving Goldbach conjecture lies in the understanding of distribution of prime numbers and general formula that generates prime numbers.

METHODOLOGY

Before we can look at the proof of Goldbach conjecture, it is important that we look at how the formula that generates **ALL** prime numbers is derived.

The set of triangular numbers Tn is used. The numbers in this set are categorized in two groups namely special triangular numbers Vm and non-special triangular numbers M.

$T_n = V_m + M = V_m \cup M$

$V_m \cap M = \emptyset$

$V_m = T_n - M = T_n \cap M'$

Where

$T_n = \frac{n(n+1)}{2}$, n∈natural numbers

$M = \frac{(ab)^2-1}{8}$, a, b∈ odd integers> 1.

Therefore, the formula for calculating all prime numbers is:

$p_m = \sqrt{8V_m + 1}$

Where $V_m = T_n - M = T_n \cap M'$

The numbers that belong to set M forms a non-special triangular numbers multiplicative table. Below is a small portion of non-special triangular numbers multiplicative table:

$\frac{(ab)^2 - 1}{8}$	3	5	7	9	11	13	15
3	10	28	55	91	136	190	253
5	28	78	153	253	378	528	703
7	55	153	300	496	741	1035	1378
9	91	253	496	820	1225	1711	2278
11	136	378	741	1225	1830	2556	3403
13	190	528	1035	1711	2556	3570	4753

15	253	703	1378	2278	3403	4753	6328

IMPORTANT RULES ON THE USE OF M AND Vm

1. Make sure that the largest Vm input in $\sqrt{8Vm + 1}$ is less than the largest M in second row or column in the non-special triangular multiplicative table above. This means that you need to list set M in ascending order to largest M in second row or column in the non-special triangular multiplicative table above (Vm∩M) = ∅
2. 0.375 is included in special triangular numbers Vm in order to get a 2

Example

Find all the prime numbers using set Tn below:

Tn = $\{1,3,10,15, 21, 28, 36, 45, 55, 66, 78, 91, 105,120,136,153, 171, 190\}$

Solutions

M = $\{10, 28, 55, 78, 91, 136, 153, 190\}$

Vm = Tn – Vm = $\{1, 3, 6, 15, 21, 36, 45, 66, 105, 120, 171\}$

Solutions

$$p = \sqrt{8Vm + 1}$$

In order to include 2 among prime numbers, 0.375 from $\frac{3}{8}$ must be included in set Vm.

P1 = $\sqrt{8(0.375) + 1} = \sqrt{4} = 2$

P2 = $\sqrt{8(1) + 1} = \sqrt{9} = 3$

P3 = $\sqrt{8(3) + 1} = \sqrt{25} = 5$

P4 = $\sqrt{8(6) + 1} = \sqrt{49} = 7$

P5 = $\sqrt{8(15) + 1} = \sqrt{121} = 11$

$P6 = \sqrt{8(21) + 1} = \sqrt{169} = 13$

$P7 = \sqrt{8(36) + 1} = \sqrt{289} = 17$

$P8 = \sqrt{8(45) + 1} = \sqrt{361} = 19$

$P9 = \sqrt{8(66) + 1} = \sqrt{529} = 23$

$P10 = \sqrt{8(105) + 1} = \sqrt{841} = 29$

$P11 = \sqrt{8(120) + 1} = \sqrt{961} = 31$

$P12 = \sqrt{8(171) + 1} = \sqrt{1369} = 37$

THE PROOF

Now that the formula for calculating ALL prime numbers has been created, the Goldbach conjecture can be proven. The following are extra points to take note of before proving the Goldbach conjecture.

- The Goldbach conjecture states that every even integer greater than 2 can be expressed as a sum of two prime numbers.
- $P_1 + P_2 = 2m$ ($m \geq 2$, m is an integer)
- $P_1, P_2 \in \sqrt{8V_m + 1}$. For every Vm there exist a prime number P_n such that $P_n = \sqrt{8V_m + 1}$
- $V_1, V_2 \in V_m$

By using $P_1 = \sqrt{8V_1 + 1}$ and $P_2 = \sqrt{8V_2 + 1}$ where $V_m = T_n - M = T_n \cap M'$ show that

$P_1 + P_2 = 2m$ ($m \geq 2$)

Using LHS (left hand side of equation)

$$P_1 + P_2 = 2m$$

$$\sqrt{8V_1 + 1} + \sqrt{8V_2 + 1} = 2m$$

$$\sqrt{4(2V_1 + \frac{1}{4})} + \sqrt{4(2V_2 + \frac{1}{4})} = 2m$$

$$\sqrt{4}\sqrt{(2V_1 + \frac{1}{4})} + \sqrt{4}\sqrt{(2V_2 + \frac{1}{4})} = 2m$$

$$2\sqrt{(2V_1 + \frac{1}{4})} + 2\sqrt{(2V_2 + \frac{1}{4})} = 2m$$

$$2\left[\sqrt{(2V_1 + \frac{1}{4})} + \sqrt{(2V_2 + \frac{1}{4})}\right] = 2m$$

$$2\left[\sqrt{\frac{8V_1 + 1}{4}} + \sqrt{\frac{8V_2 + 1}{4}}\right] = 2m$$

$$2\left[\frac{\sqrt{8V_1 + 1}}{\sqrt{4}} + \frac{\sqrt{8V_2 + 1}}{\sqrt{4}}\right] = 2m$$

$$2\left[\frac{\sqrt{8V_1 + 1}}{2} + \frac{\sqrt{8V_2 + 1}}{2}\right] = 2m$$

$$2\left[\frac{\sqrt{8V_1 + 1} + \sqrt{8V_2 + 1}}{2}\right] = 2m$$

$$2\left[\frac{P_1 + P_2}{2}\right] = 2m$$

$$2m = P_1 + P_2$$

$$2\left[\frac{2m}{2}\right] = 2m$$

Hence 2m = 2m proven.

CONCLUSION

The Goldbach conjecture has successfully been proven generally true that every even integer greater than 2 can be expressed as a sum of two prime numbers. This proof may will help the mathematics community solve other prime related problems in mathematics. The proof has also some implications for other areas of analytic number theory. The only limitation to the special triangular numbers Vm that are used to find the prime numbers is lack of an integer that generates a prime number 2. However, 0.375 is included deliberately in order to obtain 2.

REFERENCES

1. Burton, D. M. (2011). *Elementary Number Theory* (7th ed.) McGraw-Hill.
2. Conway, J. H., & Guy, R. K. 91996). *The Book of Numbers*. Copernicus.
3. Hardy, G. H., & Wright, E. M. (2008). *An Introduction to the Number Theory of Numbers* (6th ed.) Oxford University Press.