The Solution of the Nordstrom-Einstein Paradox

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Abstract

The main goal of this paper is to solve the Nordstrom-Einstein paradox in the General Theory of Relativity (GTR) and open a new perspective on understanding the generation of gravitational fields.

At present it is not exactly known how the gravitational field is generated. This field is responsible for attraction at the distance of bodies in Newton's sense and for space-time distortion in Einstein's sense.

In 1913, Nordstrom blamed Einstein's GTR that it cannot explain why the electromagnetic waves confined in a mass-less box with reflective walls have a gravitational mass while the invariant of the stress-energy tensor of the electromagnetic field is zero. This is the Nordstrom-Einstein paradox in GTR. The fact that his paradox was not solved for 111 years created a niche in GTR which blocked progress in the field of gravity.

The present paper solves the Nordstrom-Einstein paradox for the hypothetical case of a sphere with a mass-less reflective surface filled with electromagnetic waves. It demonstrates that the gravitational field of such a sphere is generated only during the reflection of electromagnetic waves by the sphere's reflective surface and not in its volume.

The methodology used is specific to GTR, establishing a new form of the stress-energy tensor for the case of the mentioned hypothetical sphere.

Solving this paradox has significant implications for understanding the gravitational field essence. It can serve as a basis for further theoretical and experimental research, with important implications for the development of Quantum Gravity, String/Superstring theory, and Unified Field theories.

Keywords: Nordstrom-Einstein paradox, General Theory of Relativity, Quantum Gravity, Gravitation, Graviton generation mechanism

1. Introduction

After Albert Einstein published GTR in 1916, the study of gravity took another direction than the classic vision of Newton's theory. Despite numerous research studies, the fundamental mechanisms underlying the phenomenon of gravity remain insufficiently understood. The specialty literature still does not offer a credible theory about the generation of gravitational fields surrounding condensed matter. Theories such as Quantum Gravity or String/ Superstrings theory do not indicate experimental tests or clear explanations of phenomena related to the generation of gravitational fields.

On the other hand, the specialists still await a credible theory that connects the gravitational interaction with the rest of the known interactions.

This paper aims to develop a new theory that explains the generating mechanism of the gravitation field starting from solving the Nordstrom-Einstein paradox. This is an unexploited niche in GTR. The solution found for the Nordstrom-Einstein paradox clarifies how the gravitational field is generated during the reflection of electromagnetic waves by the reflective surface of the sphere. This will lead to significant advances in theoretical physics and practical applications.

The methodological approach is based on conceptual analysis and mathematical modeling. This approach is detailed in the chapter 'Theoretical foundation'.

The article is organized as follows:

After 'Theoretical foundation', the chapter 'Theoretical contributions' presents in detail the original demonstration related to the unique solution of the Nordstrom-Einstein Paradox. Then, in the chapter 'Results', the consequences of the theory are presented and, in the chapter 'Discussions' it is discussed about the mass, frequency, and direction of the emitted gravitons. At 'Conclusions', essential features revealed by the solution of the Nordstrom-Einstein paradox are underlined and future research directions are indicated.

2. Theoretical foundation

The theoretical foundation is the GTR.

The main sources of information are the existing theoretical works from 1913 when this paradox was pointed out, Einstein's GTR, and other main works on gravity.

3. Theoretical contributions

The specific steps that will be followed are:

▪ A review of the Nordstrom-Einstein paradox for the case of a hypothetical sphere with a mass-less reflective surface filled with electromagnetic waves;

▪ The solution of the Nordstrom-Einstein paradox.

3.1 A review of the Nordstrom-Einstein paradox for the case of a hypothetical sphere with a mass-less reflective surface filled with electromagnetic waves

Consider Einstein's field equations in covariant form:

$$R_{ij} - \frac{1}{2} g_{ij} R = \frac{8\pi G}{c^4} T_{ij} \quad (1) \ [1]$$

, where i, j=1, 2, 3, 0 (indexes 1, 2, 3 are for space coordinates, 0 is for the time coordinate), R_{ij} is Ricci's tensor, R is Ricci's scalar curvature, g_{ij} is the metric tensor in covariant form, T_{ij} is the stress-energy tensor, G is Newton's gravitational constant, and c is the speed of light.

Consider a hypothetical sphere with a massless reflective surface filled with electromagnetic waves with the total energy E and total mass M forming an isotropic medium with the density of energy w. Such a sphere should have around it a gravitational field in Newton's sense and warp the space-time continuum in Einstein's sense (fig.1). The accepted stress-energy tensor for such a sphere is:

$$T_{ij} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{10} \\ T_{21} & T_{22} & T_{23} & T_{20} \\ T_{31} & T_{32} & T_{33} & T_{30} \\ T_{01} & T_{02} & T_{03} & T_{00} \end{bmatrix} = \begin{bmatrix} p & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & w \end{bmatrix} = \begin{bmatrix} w/3 & 0 & 0 & 0 \\ 0 & w/3 & 0 & 0 \\ 0 & 0 & w/3 & 0 \\ 0 & 0 & 0 & w \end{bmatrix} \quad (2)\ [2]$$

, where $T_{11}=T_{22}=T_{33}=p=w/3$, p is the pressure of electromagnetic waves inside the sphere and w is the density of energy, $T_{00}=w$, $Tij=0$ for $i \neq j$.

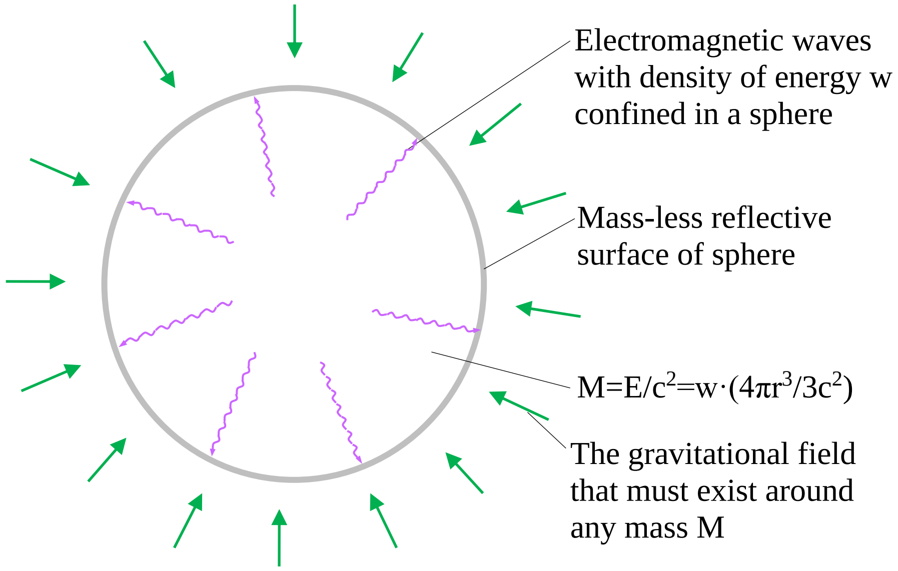

Fig.1-Gravitational field of a hypothetical sphere with massless reflective surface filled with electromagnetic waves with total mass M, total energy E, and density of energy w

If a 'contraction' of the equations (1) is done by multiplying these equations with the metric tensor expressed in counter-variant form g^{ik} (i, j=1, 2, 3, 0), followed by summation, a new form of Einstein's field equations is derived.

In the case when the mass M of electromagnetic waves that are confined in a sphere is small, the counter-variant form of metric tensor g^{ij} can be approximated with great precision with the metric tensor for the flat space:

$$g^{ij} = \begin{bmatrix} g^{11} & g^{12} & g^{13} & g^{10} \\ g^{21} & g^{22} & g^{23} & g^{20} \\ g^{31} & g^{32} & g^{33} & g^{30} \\ g^{01} & g^{02} & g^{03} & g^{00} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (3)\ [3]$$

The tensor g^{ij} is resulting from the condition:

$$g_{ji} \cdot g^{ij} = \delta^{i}_{j} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)\ [3]$$

, where i, j =1, 2, 3, 0

and $g_{ji} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ (5) [3]

Contraction of the right side of Einstein's field equations (1):
The stress-energy tensor T_{ij} (equation (2)) is multiplied with the metric tensor g^{ij} (equation (3)) followed by summation. These operations lead to the invariant T of the stress-energy tensor which in the present case is a null scalar:

$T = g^{ij} \cdot T_{ij} = 1 \cdot w/3 + 1 \cdot w/3 + 1 \cdot w/3 - 1 \cdot w = 0$ (6)

Contraction of the left side of Einstein's field equations (1):
Multiplying the left side of Einstein's field equations (1) with g^{ij} followed by summation leads to:

$$g^{ij}(R_{ij} - \frac{1}{2} g_{ij} R) = g^{ij} R_{ij} - \frac{1}{2} g^{ij} g_{ij} R = R - \frac{1}{2} 4R = -R \quad (7)$$

because,

$g^{ij} \cdot R_{ij} = R$ [4]

and,

$$g^{ij} \cdot g_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + (-1) \cdot (-1) = 4$$

In this way, the scalar expression of Einstein's field equations is obtained:

$$R = -\frac{8\pi G}{c^4} T \quad (8)$$

This equation establishes the simplest relation between the stress-energy tensor's invariant, T (a scalar), and space-time curvature, R (Ricci's scalar).
According to expression (6), T=0. So, according to (8), Ricci's scalar curvature R must be 0, too. This is the Nordstrom-Einstein paradox:
According to Newton, a mass M must be surrounded by a gravitational field, and according to Einstein, it must warp space-time.
Today, it is considered that if a mass M is composed of electromagnetic waves, it cannot warp space-time because the R=T=0. This paradox remained unexplained for 111 years since 1913 when it was exposed for the first time during the debates between Nordstrom and Einstein [5].
The single explanation given by Einstein at that time was that: "…the electromagnetic radiation enclosed within a mass-less box having reflective walls would not acquire a gravitational mass, although that radiation would exert pressure on the walls of the box. These walls would become stressed and, simply because of this stress, the walls would acquire a gravitational mass" [5]. Today, it is very clear to everybody that such a mass-less wall cannot be stressed simply because in the absence of a mass, such stress can have no physical sense.
Einstein did not mention the existence of this paradox in his theory from 1916.
For this reason, the paradox remained unexplained until our days.

3.2 The solution of the Nordstrom-Einstein paradox

Nobody can admit today that a spherical mass M of condensed matter generates a gravitational field and a sphere with a mass-less reflective surface, filled with electromagnetic waves with the same mass M does not generate a gravitational field.

As the scalar curvature R of the sphere filled in with electromagnetic waves is a null scalar in any point of the sphere's interior because the stress-energy invariant T is a null scalar as specified by equation (8), something important must happen on the sphere's surface. While a sphere of condensed matter with mass M (e.g. a sphere filled with dust) generates a gravitational field in every point of its volume, a sphere with a mass-less reflective surface, filled with electromagnetic waves with the same mass M, must generate its gravitational field only on its surface. This must happen obviously during the reflection of electromagnetic waves by the sphere's surface (fig.2). The paradox is solved if the stress-energy tensor is completed with an additional tensor for the sphere's surface, T_{ij-ss}. In this case, the stress-energy tensor from Einstein field equations must be:

$$T'_{ij} = T_{ij-sv} + T_{ij-ss} = \begin{bmatrix} w/3 & 0 & 0 & 0 \\ 0 & w/3 & 0 & 0 \\ 0 & 0 & w/3 & 0 \\ 0 & 0 & 0 & w \end{bmatrix}_{sv} + \begin{bmatrix} 2w/3 & 0 & 0 & 0 \\ 0 & 2w/3 & 0 & 0 \\ 0 & 0 & 2w/3 & 0 \\ 0 & 0 & 0 & w \end{bmatrix}_{ss} \quad (9)$$

, where T_{ij-sv} is the classic stress-energy tensor for the sphere's volume (equation (3)).

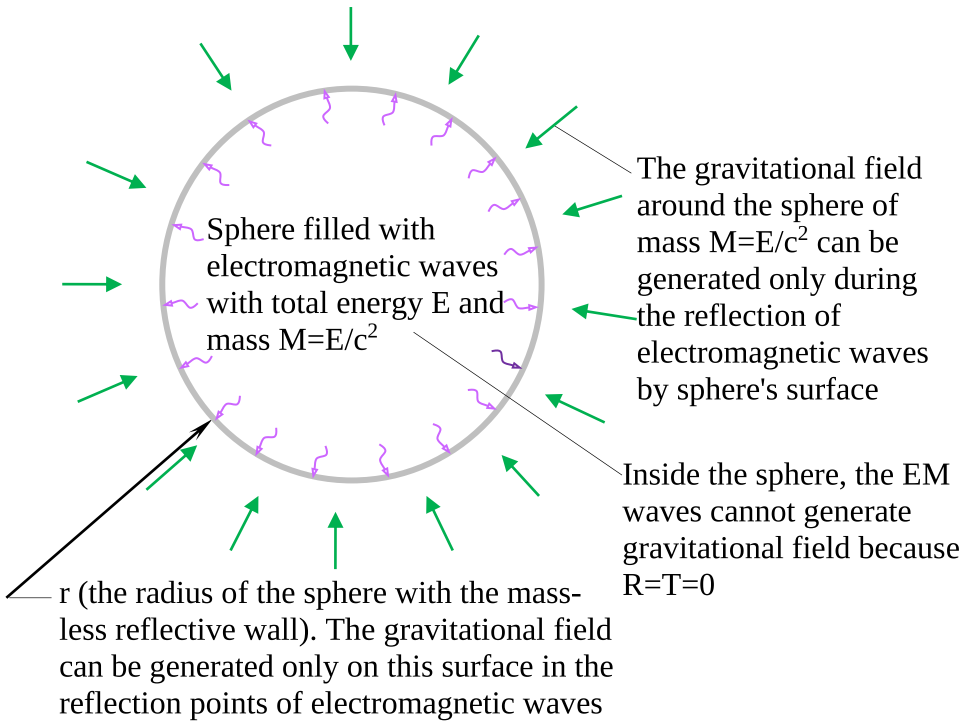

Fig.2-Generation of the gravitational field of a hypothetical sphere with a mass-less reflective wall filled in with electromagnetic waves with total mass $M=E/c^2$

The terms of the second part of the stress-energy tensor T_{ij-ss} are $T_{11-ss} = T_{22-ss} = T_{33-ss} = p_{ss}=2w/3$ and $T_{00-ss} = w$, where p_{ss} is the pressure of light on the sphere's surface and w is the density of energy. One can observe the exceptional fact that the pressure of light on the sphere's surface $p_{ss} = 2w/3$ is two times higher than the pressure $p_{sv} = w/3$ inside the sphere's volume. This is happening because, near the sphere's wall, there are both the incident and reflected rays [6].

Doing the contraction of the new stress-energy tensor T'_{ij} as before, the new invariant T' is:

$T'=g^{ij}\cdot T'_{ij}=(1\cdot w/3+1\cdot w/3+1\cdot w/3-1\cdot w)_{sv}+(1\cdot 2w/3+1\cdot 2w/3+1\cdot 2w/3-1\cdot w)_{ss}=0+(2w-w)=w$ (10)

In this case, the scalar form of Einstein's field equations becomes:

$$R=-\frac{8\pi G}{c^4}w=-\frac{8\pi G}{c^2}\rho \quad (10)$$

, where $\rho=w/c^2$ is the density of matter.
The invariant of the stress-energy tensor, R, is the same as in the case of a sphere filled in with dust of density ρ and the total mass M where Einstein's field equations admit Schwarzschild solution for the space element ds. [7] The sphere filled with electromagnetic waves will have around it the same gravitational field as a sphere composed of condensed matter (dust) and the paradox is definitively solved in this way.

Results

The consequences of solving the Nordstrom Einstein paradox

The first consequence:

In at least one hypothetical scenario - a sphere with a massless reflective surface filled with electromagnetic waves the hypothetical case of a sphere with a mass-less reflective surface filled with electromagnetic waves) the gravitational field is generated only during the reflection of electromagnetic waves by the sphere’s reflective surface and not in its volume.

The second consequence:

In at least one hypothetical scenario - a sphere with a massless reflective surface filled with electromagnetic waves - the gravitational field has an electromagnetic origin i.e. it is generated during the reflection of a photon.

The third consequence:

In at least one hypothetical scenario - a sphere with a massless reflective surface filled with electromagnetic waves - the frequency of the radiated gravitational waves forming the sphere's gravitational field must correspond to the frequency of the incident/ reflecting electromagnetic waves. This must happen because, the generation process of the gravitational wave must start in the moment of reflection of electromagnetic wave on the sphere’s surface and stop when the reflection is finished i.e., after one period of time $T=1/\nu$, where ν is the frequency of the electromagnetic waves confined in the sphere. The detailed demonstration is given below at Discussions.

Discussions

Solving the Nordstrom-Einstein Paradox has significant implications because it provides a new perspective on the generation of the gravitational field.
At present, no gravitation theory explains the gravitational field's origin. In GTR, the gravitational waves are considered ‘ripples in the space-time continuum’ which are generated only at cosmic scale. e.g. during a head-on collision or the last phase of in-spiral evolution of neutron stars. The nature of those gravitational waves should be the same as in the case of the gravitational waves generated by the electromagnetic waves confined in the above-mentioned sphere except for the frequency. Really, in the case of cosmic bodies, the frequency of gravitational waves is around several thousands of Hz whilst in the case of gravitational waves radiated by the sphere, the frequency can be very high, about equal to the frequency of electromagnetic waves.
In addition, it can be specified that the gravitational waves as ripples in the existing space-time continuum are perturbations in the existing gravitational fields. In the case of the

electromagnetic waves confined in a sphere, the generated gravitational waves are primordial, constituting even the gravitational field surrounding the sphere.
According to point 3.2, the gravitational field of the above-mentioned sphere must be generated only on the sphere's surface and not in the sphere's volume.
Two questions arise here:
- The first question is related to the direction of radiated graviton (gravitational wave). When the direction of the incident electromagnetic wave is perpendicular on the reflective surface, the direction of the emitted gravitational wave (graviton) must be opposite to the incident electromagnetic wave, i.e. it coincides with the direction of the reflected electromagnetic wave.
In a way, this counters our intuition but can be easily demonstrated with the law of momentum conservation:

$$\mathrm{m_{iph}} \cdot \mathrm{c} + \mathrm{m_{rph}} \cdot (-\mathrm{c}) + \mathrm{m_g} \cdot (-\mathrm{c}) = 0 \quad (11)$$

, where m_{iph} is the mass of the incident photon, m_{rph} is the mass of the reflected photon and m_g is the mass of the emitted graviton, c is the speed of the photon and graviton.
In this case, the mass of the graviton is given by the difference between the mass of the incident and the reflected photon:

$$\mathrm{m_g} = \mathrm{m_{iph}} - \mathrm{m_{rph}} \quad (12)$$

When the incident ray has the angle θ relative to normal, the law of momentum conservation projected on the reflection normal is:

$$\mathrm{m_{iph}} \cdot \mathrm{c} \cdot \cos\theta + \mathrm{m_{rph}} \cdot (-\mathrm{c}) \cdot \cos\theta + \mathrm{m_g} \cdot (-\mathrm{c}) = 0 \quad (13)$$

In this case, the mass of the graviton is given by:

$$\mathrm{m_g} = (\mathrm{m_{iph}} - \mathrm{m_{rph}}) \cdot \cos\theta \quad (14)$$

From the equation of energy conservation for the case of perpendicular reflection,

$$\mathrm{m_g} \cdot \mathrm{c}^2 = \mathrm{m_{iph}} \cdot \mathrm{c}^2 - \mathrm{m_{rph}} \cdot \mathrm{c}^2 \quad (15)$$

, one can find again that:

$$\mathrm{m_g} = \mathrm{m_{iph}} - \mathrm{m_{rph}} \quad (16)$$

This means that the mass of the photon decreases after reflection with the mass of graviton m_g.
- The second question is related to the frequency of the reflected photon and the emitted graviton.
As the photon loses a quantity of mass equally to m_g, the frequency of the reflected photon must be smaller than the frequency of the incident photon:

$$\nu_{\mathrm{rph}} = \mathrm{m_{rph}} \cdot \frac{\mathrm{c}^2}{h} = (\mathrm{m_{iph}} - \mathrm{m_g})\mathrm{c}^2/h \quad (17)$$

The mass of the graviton must be much lower than the reflecting photon's mass. This must happen because the gravitational field of any elementary particle (e.g. electron, proton) is much weaker than its electrostatic field.
For this reason, the frequency of the reflected photon is very close to the frequency of the incident photon and about equal to the frequency of graviton emitted during the photon's reflection:

$$\nu_{\mathrm{rph}} \cong \nu_{\mathrm{iph}} \cong \nu_{\mathrm{g}} \quad (18)$$

Conclusions

▪ This paper solves the Nordstrom-Einstein paradox which was not solved for 111 years from 1913.

▪ The paradox solution shows that the stress-energy tensor of a hypothetical sphere with a massless surface filled with electromagnetic waves is composed of two tensors: the classic stress-energy tensor $T_{ij\text{-}sv}$ with the terms T_{11sv}=T_{22sv}=T_{33sv} =w/3, T_{00sv}=w in the sphere's volume and stress-energy tensor on sphere's surface $T_{ij\text{-}ss}$ with the terms T_{11ss}=T_{22ss}=T_{33ss} =2w/3, T_{00sv}=w (for i≠j all the terms of $T_{ij\text{-}SV}$ and $T_{ij\text{-}ss}$ are zero).

▪ While the invariant of the stress-energy tensor for the sphere's volume T_{sv} is a null scalar and Ricci's scalar curvature R_{sv} is a null scalar, too, the invariant of stress-energy tensor for the sphere's surface T_{ss}=w≠0, Ricci's scalar curvature $R_{sv} = -\frac{8\pi G}{c^4} w$ and Einstein's field equations admit the Schwarzschild's solution. This definitively solves the Nordstrom-Einstein paradox.

▪ The consequences are important for the development of physics:
- The gravitational field can be generated by the reflection of electromagnetic waves on reflective surfaces. The gravitational field existing around the given sphere in Newton's and Einstein's sense must have a discrete structure composed of elementary quanta of energy which can be assimilated with gravitons/gravitational waves.
- The frequency of the radiated gravitational waves forming the sphere's gravitational field must correspond to the frequency of the reflecting electromagnetic waves confined in the sphere.

▪ The solution of the Nordstrom-Einstein paradox must be considered by Quantum Gravity and Strings/Superstrings theory as a basis for future development because it defines the physical process of generation of gravitons/gravitational waves in the mentioned particular case.

▪ In addition to its importance in theoretical physics, the solution of the Nordstrom Einstein paradox has multiple technological applications which will be presented in a future paper.

Data Availability Statement: No data are associated with the manuscript

Reference

1. A. Einstein, The Foundation of the General Theory of Relativity (translated from 'Die Grundlage der allgemeinen Relativitatstheorie', Annalen der Physik, 49, 1916), The Principle of Relativity, Dover Publications Inc., ISBN 0-486-60081-5;

2. L.D. Landau, E.M. Lifsit, The Classical Theory of Fields, Third revised English edition, Pergamon Press (1971);

3. V. Fock, The Theory of Space, Time and Gravitation, Pergamon Press (1959);

4. C.W. Misner, K.S. Thorne, J.A. Wheeler, Gravitation, W. H. Freeman and Company (1973);

5. J. Earman, M. Janssen, J. Norton, 993, Einstein Studies, Volume 5, Part.1-Disputes with Einstein, Einstein and Nordstrom-Some Lesser Known Thought Experiments in Gravitation , pages 17-18, Birkhauser (1993);

6. A.V. Masalov, First Experiments on Measuring Light Pressure (Pyotr Nikolaevich Lebedev), Springer Series in Optical Sciences (2019), DOI: 10.1007/978-3-319-98402-5_12;

7. K. Schwarzschild, Uber das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, Sitzungsberichte der Preussischen Akademie der Wissenschaften (1916);

Statements and Declarations

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No financial interest is directly or indirectly related to the work submitted for publication.