Simplified Trigonometric Method for Distance Measurements

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Abstract

This study develops a mathematical approach to estimate the distance of a celestial object from an observer using angular measurements. The method involves capturing the subtended angles of the object from two distinct positions along a known baseline distance and employs trigonometric relationships to derive a formula for calculating the distance. Using a camera, the angular size of the object is determined based on its apparent size on the camera's sensor. The calculated angles are substituted into the derived formula to compute the distance.

Validation of the formula was performed by calculating the Moon's distance from Earth using angular measurements taken at two locations separated by 100 km. The computed distance of approximately 376,742 km closely aligns with the Moon's average distance from Earth, confirming the accuracy of the method. This straightforward approach demonstrates potential utility for amateur astronomers and educators, offering an accessible tool for celestial distance measurement using basic geometry.

1. Introduction

1.1 Background and Context

Determining the distances to celestial objects has been a cornerstone of astronomical study, offering insights into the scale and structure of the universe. Traditional methods, such as parallax measurements and radar ranging, require sophisticated instruments and significant computational resources. While effective, these techniques are often inaccessible to amateur astronomers or in educational settings, where simplicity and cost-effectiveness are critical.

1.2 Need for Simplified Methods

The resource-intensive nature of traditional methods emphasizes the need for accessible alternatives. Developing simplified approaches not only democratizes astronomical measurements but also fosters a deeper understanding of geometry and trigonometry. This is especially relevant in educational contexts, where hands-on demonstrations enhance learning.

1.3 Objective

This study aims to derive and validate a straightforward method for estimating the distance to a celestial object using minimal equipment. By capturing the object's angular size from two observer positions separated by a known distance, trigonometric principles are used to compute the distance. The methodology is tested with the Moon as an example, and the derived results are compared against its known average distance to evaluate accuracy and practicality.

2. Theory and Derivation

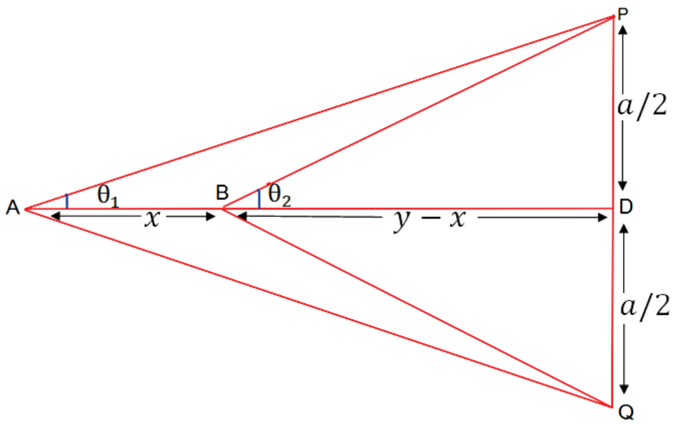


fig (1)

2.1 Problem Setup (see fig (1))

Consider an object of diameter (a) (e.g., a celestial body) located at a distance (y) from the observer. Observations are taken from two positions:
- Position (A): The initial observation point, where the angle subtended by the object's edges is (θ_1).
- Position (B): The second observation point, separated by a baseline distance (x), where the subtended angle is (θ_2).

The goal is to calculate (y), the distance from the observer at (A) to the object.

2.2 Derivation of the Formula

Using the tangent relationship:

At (A):

$$tan(\theta_1) = \frac{a}{2y}$$

Rearranging:

$$a \ = \ 2y\ tan(\theta_1) \ \text{-----------------} \ (1)$$

At (B):

$$tan(\theta_2) = \frac{a}{2(y - x)} \ \text{----------------} \ (2)$$

Substituting (a) from (1) into (2):

$$tan(\theta_2) = \frac{2y\ \ tan(\theta_1)}{2(y - x)} \ = \ \frac{y\ tan(\theta_1)}{y - x}$$

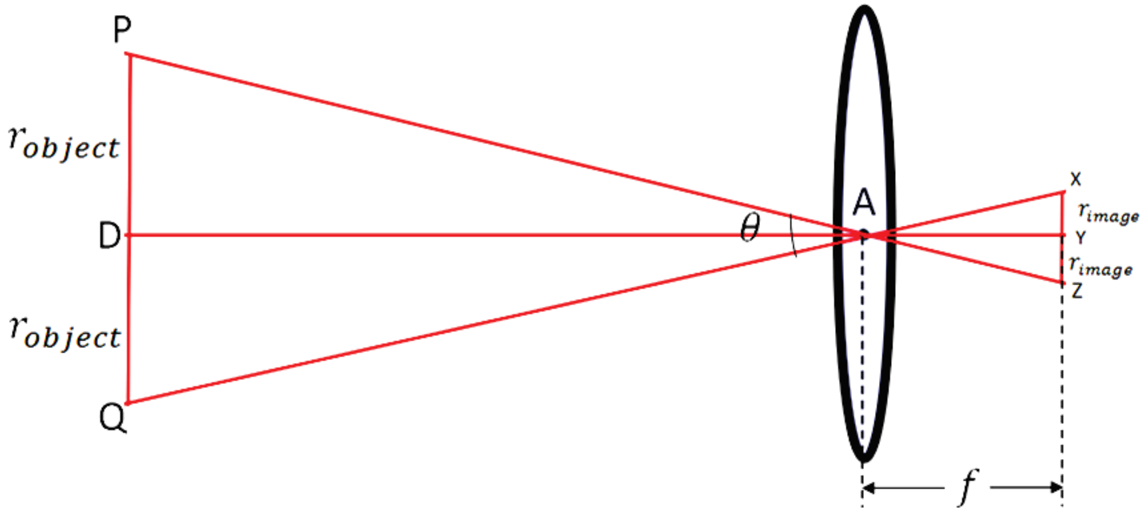
Simplifying:

$$y\ (tan(\theta_2)\ -\ tan(\theta_1))\ =\ x\ tan(\theta_2)$$

Solving for (y):

$$y \ = \frac{x\ tan(\theta_2)}{tan(\theta_2) - \ tan(\theta_1)}$$

3. Methodology



3.1 Setup and Measurements (see fig (2))

- Observer's Positions: Choose two positions, (A) and (B), separated by a known baseline (x).
- Object of Observation: Select a celestial object with a measurable angular size, such as the Moon.
- Equipment: Use a digital camera with a known focal length (f) to capture images of the object from (A) and (B).

3.2 Procedure

1. Image Capture:
   - At (A), capture an image of the celestial object.
   - Move a distance (x) to (B) and capture a second image.
2. Calculate Angular Size:

   Using the apparent radius of the object's image (r_{image}) on the sensor:

   $$tan\left(\frac{\theta}{2}\right) = \frac{r_{image}}{f}$$

   Determine (θ_1) and (θ_2) for positions (A) and (B).
3. Calculate Distance:

   Substitute the measured angles into the formula:

   $$y = \frac{x\ tan(\theta_2)}{tan(\theta_2) - tan(\theta_1)}$$

4. Results and Discussion

4.1 Results

For the Moon:
- Baseline: ($x = 100$ km)
- Focal length: ($f = 0.001$ km)
- Apparent radii:
  - $(r_{image})_1 = 0.0000045197\ km$ at (A)
  - $(r_{image})_2 = 0.0000045209\ km$ at (B)

Computed angles:

$$\theta_1 = \frac{(\mathrm{r_{image}})_1}{\mathrm{f}} = \frac{0.0000045197}{0.001}\,\mathrm{rad} = 0.0045197\ \mathrm{rad}$$

$$\theta_2 = \frac{(\mathrm{r_{image}})_2}{\mathrm{f}} = \frac{0.0000045209}{0.001}\,\mathrm{rad} = 0.0045209\ \mathrm{rad}$$

$$\tan\theta_1 = \tan(0.0045197) \approx 0.0045197$$

$$\tan\theta_2 = \tan(0.0045197) \approx 0.0045197$$

$$\therefore \text{distance of moon from observer} = y = \frac{x\tan\theta_2}{\tan\theta_2 - \tan\theta_1}$$

$$= \frac{100 \times 0.0045209}{0.0045209 - 0.0045197} \approx 376741.666667\ \mathrm{km}$$

Result:
$y \approx 376741.666667$ km

4.2 Discussion

The calculated distance aligns closely with the Moon's average distance (384,400 km). Minor discrepancies arise from measurement precision and environmental factors. The simplicity of this method makes it ideal for amateur and educational applications.

5. Conclusion

This study presents a practical and accessible method for calculating celestial distances using simple trigonometric relationships. The successful validation of the formula with the Moon underscores its potential for educational and observational purposes. Future work could explore enhancements for measuring more distant objects.