Pseudo-Isolated Numbers (PIN): A New Class of Composite Numbers:

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Summary :

This article introduces a research program to study a new class of composite numbers, called Pseudo-Isolated Numbers (PIN), defined by their unique position between two consecutive prime numbers. where does it go to present its formal definition. This research program can open new perspectives in number theory and suggests potential applications in cryptography and algorithmic theory,...,etc,...

Keywords :

Number theory, Composite numbers, Distribution of prime numbers, Factorization, Cryptography.

I- Introduction:

Number theory continues to be a fertile source of mathematical discoveries. The study of the relationships between prime numbers and compounds regularly reveals new structures and properties. In this context, we introduce Pseudo-Isolated Numbers (PIN), a particular class of composite numbers characterized by their position between two consecutive prime numbers. NPIs represent an emerging class of numbers in number theory, which, although not prime, exhibit unique and interesting characteristics. A pseudo-isolated number is a composite number, but which is framed by two successive prime numbers, creating a particular factorial structure. This unique position between two primes attracts attention because it could lead to important discoveries about the distribution of primes and prime factors across the set of integers. Although the concept is still in the exploration phase, it opens promising avenues for understanding the distribution of integers and the properties of prime factors. This research program provides a general introduction to this concept and explores its implications, its connections with other classes of numbers and its potential applications.

A. Historical Context:

The study of the distribution of prime numbers and the structures surrounding them constitutes a fundamental area of mathematics. Classic work on gaps between prime numbers (Erdős, 1940; Goldston et al., 2009) paved the way for exploring the properties of composite numbers in these intervals.

B. Motivation:

The unique position of NPIs between two consecutive primes suggests potentially significant structural properties. This research program aims to establish the theoretical foundations of this new class of numbers and explore their implications.

C. Interest in the Study of Pseudo-Isolated Numbers:

The study of NPIs is particularly interesting because it opens the way to a new class of numbers whose statistical and algebraic properties and analytical properties are still largely unexplored. These numbers could offer new insights into the distribution of prime factors, including how composite integers behave near prime numbers. Identifying hidden regularities in the distribution of NPI prime factors could enrich the understanding of fundamental mathematical phenomena. Additionally, NPIs could play a role in improving cryptography and algorithms associated with factoring large numbers, an area in which prime numbers play a central role.

II. Definitions and Fundamental Properties:

A.Origin Of Term:

The term "pseudo-isolated number" is chosen to reflect the distinct position and apparent singularity of this type of number among the integers. The word "pseudo" means "falsely" or "appearing to be", indicating that although these numbers lie in intervals where one might imagine some isolation, they are not actually isolated in the strict sense of prime numbers, but occupy a unique position with respect to neighboring prime numbers. The word "isolated" is used because these composite numbers are framed by prime numbers, as if they were isolated between these two prime integers. However, unlike prime numbers, they are not "autonomous" in the sense of being indivisible, but are compound integers, having factors other than 1 and themselves.

B. Formal Definition:

A pseudo-isolated number is a composite number, but which can possess a particular factorial structure, distinguishing itself from other classical composite integers. A number n is a Pseudo-Isolated Number if and only if:

1. n is a composite number
2. $\exists\ p_1, p_2 \in P$ (set of prime numbers) such that:
   - $p_1 < n < p_2$
   - $\nexists\ p \in P$ such that: $p_1 < p < p_2$

     Exp: 4, 6, 12, 18, 30, 42, 60, 72, 102,...

   This characteristic places NPIs in a unique position, between two primes, while exhibiting a factorial structure that merits further exploration.

C. The First Factor of Pseudo-Isolated Numbers:

The prime factors of the NPIs have a particular structure. Indeed, unlike most other composite integers, NPIs seem to exhibit a regular distribution of prime factors. Although not prime themselves, NPIs are often factored into products of powers of small primes, leading to speculation that further study of these factors could reveal recurring patterns. Understanding the first factors of NPIs could pave the way for generalizations or new conjectures in number theory.

D. Relationship to Other Numbers :

NPIs can be compared to other well-established classes of numbers, such as twin numbers, semi-primes and Chen numbers. Twin numbers are pairs of primes that differ by 2 (e.g., 11 and 13, 17 and 19, etc.), and semi-primes are products of two primes, while a Chen number is a prime number p such that p + 2 is either a prime number (thus forming a pair of twin prime numbers), or the product of two prime numbers (a semi-prime). NPIs share characteristics with these three types of numbers: just like twin numbers, NPIs are often framed by successive primes, and just like semi-primes, they are composite numbers whose factorial structure can offer patterns interesting, and although the two concepts NPI and Chen numbers seem different at first glance, they share a common interest: the study of the relationships between prime numbers and composite numbers. Studying these relationships could provide a framework to better understand the behavior of NPIs and their place in the hierarchy of integers.

III- Study approaches*:*

A. Statistical Approach to the Study of NPIs:

The study of NPIs benefits from a statistical approach which makes it possible to test hypotheses on their distribution and their structure which can be used to compare the distribution of the prime factors of the NPIs to classical laws or the normal law. Statistical tests can also be used to explore the distribution of successive gaps between NPIs. This analysis could reveal whether these deviations follow specific distributions, such as that of prime numbers, and could thus strengthen or refine conjectures regarding the distribution of NPIs**.**

B. Algebraic Approach:

The algebraic approach to the study of NPIs relies on examining their properties in extensions of fields and analyzing their prime factors in a broader framework, such as algebraic number theory. One avenue of research could be to study NPIs in the context of extensions of cyclic fields, where the prime factors of NPIs are considered in an abstract algebra framework. Furthermore, the Galois groups associated with NPIs could offer information on the symmetry and internal structure of these numbers, thus shedding light on how NPIs behave in more general systems of algebraic integers..

C. Analytical Approach:

An analytical approach to the study of NPIs involves the use of techniques from real and complex analysis to understand their properties. This includes exploring the counting functions associated with these numbers, as well as studying their densities and asymptotic distributions. Analytical methods can be used to study the discrepancies between pseudo-isolated numbers and

successive prime numbers. In particular, the analysis of the average deviations and the distribution of NPIs could make it possible to develop conjectures on their behavior on a large scale. Tools such as the analysis of series summations or the use of zeta functions could be used to obtain more precise results on their relative frequency in the set of integers. The approach analytical could also involve the method of exponential sums, used to study the distribution of prime numbers and their neighbors, and which could find applications in the study of NPIs**.**

D. Relationship with other Conjectures:

Pseudo-isolated numbers can be related to several famous conjectures in number theory, particularly those concerning the distribution of prime numbers and composite numbers. A first conjecture to explore is the twin prime conjecture, which states that there are infinitely many pairs of prime numbers that differ by 2. NPIs, as composite numbers located between two successive primes, can offer insights on the structure of pairs of neighboring prime numbers. NPIs could also be explored within the framework of the Goldbach conjecture, which proposes that every even number greater than 2 can be expressed as the sum of two prime numbers. The NPIs, being composed of prime factors and located between two successive prime numbers, could offer interesting links with results relating to this conjecture, particularly for integers close to these numbers. Additionally, NPIs could be studied from the perspective of the distribution of semiprimes (products of two primes), which is related to many conjectures and theorems in number theory. The relationships between NPIs and semi-primes could make it possible to propose new conjectures on the distribution of prime products in the set of integers.

IV. Potential Applications and Implications:

The study of NPIs has potential applications in several areas of applied mathematics, notably in cryptography and computer security. In modern cryptography, the difficulty of factoring large numbers composed of two prime factors is essential to ensuring the security of communication systems. A better understanding of NPIs could lead to improvements in factoring algorithms, or even the discovery of new methods for testing primality or factoring composite numbers more efficiently. Additionally, the study of NPIs could have applications in pure mathematics, notably in number theory, where regularities detected in prime factors could shed light on open questions, such as the distribution of primes and other composite numbers.

V. Conclusion and Research Perspectives:

Pseudo-isolated numbers constitute a promising research topic in number theory. Although this field is still in its infancy, it could offer new insights into the distribution of prime factors and the internal structure of composite numbers. Exploring the relationships between NPIs and other classes of numbers, such as twin numbers or semiprimes, could open avenues for future conjectures and theories. Furthermore, studying NPIs in an algebraic framework could reveal hidden properties that enrich our understanding of the underlying structures of integers. Thus, NPI research could not only have practical applications in cryptography, but also contribute to the expansion of our knowledge of number theory and other branches of mathematics.

Summary of Key Concepts:

● Definition of NPI: A pseudo-isolated number is a composite number, framed by two successive prime numbers.

● Relationship to other numbers: NPIs have similarities with twin numbers, semi-primes and Chen numbers .

● Potential applications: NPIs could have applications in cryptography, particularly in primality testing and factorization algorithms,...,...

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