A Generalized Tachyonic Scalar Field Lagrangian for Cosmological Phenomena

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ABSTRACT: We establish a generalized tachyonic field framework with a unique universal speed of $v_t = \sqrt{1/(\Lambda t_P^2)} \approx 5.931 \times 10^{60} c$, derived through Lorentz transformation and dimensional analysis of physical constants. Implications for quantum mechanics, particularly in the context of pilot wave theory, suggest a novel perspective on non-locality and quantum entanglement. The corresponding generalized tachyonic Lagrangian provides a unified description of inflation, dark matter, and dark energy. In the early universe, the theory predicts inflationary observables with a modified Lorentz factor; at galactic scales, it accommodates MOND phenomenology; and in the late universe, it reduces to a modified quintessence model with corrections of order c^2/v_t^2 to the dark energy equation of state. The extremely high value of v_t ensures compatibility with existing constraints while allowing for small, potentially observable effects at cosmological scales.

Contents

1 Introduction

The possibility of tachyonic fields has intrigued physicists since their theoretical proposal [1]. While physical tachyons as particles would violate causality, tachyonic fields appear naturally in various contexts of theoretical physics, from spontaneous symmetry breaking [2] to string theory [3]. Recent developments in quantum field theory have shown that tachyonic modes can be incorporated into consistent theoretical frameworks [4], particularly when treated as fields rather than particles.

Several fundamental problems in physics hint at possible roles for tachyonic fields. The observed acceleration of the universe [5] and the nature of dark energy [6] suggest the existence of fields with unusual properties. Similarly, the quantum measurement problem and non-locality [10, 11] might benefit from a deeper understanding of superluminal field modes. In cosmology, tachyonic fields have been proposed in various contexts, from inflation to dark energy [6, 24].

A key challenge has been the apparent arbitrariness of tachyonic field velocities. While standard tachyonic field theories allow for any superluminal speed, we demonstrate that Lorentz invariance, combined with quantum mechanics principles, suggests a unique, universal tachyonic speed $v_t = \sqrt{1/(\Lambda t_P^2)} \approx 5.931 \times 10^{60} c$. This speed emerges naturally from fundamental constants and provides a unified framework for understanding various phenomena.

Our approach builds on previous work in tachyonic field theory [3, 12, 13] where we propose tachyons as fundamental fields with a universal characteristic speed. This perspective allows us to connect seemingly disparate phenomena, from quantum non-locality to modified gravity.

In this paper, we present a generalized tachyonic field Lagrangian that incorporates a MOND-like interpolation function, enabling us to model dark energy as a quintessence field in the late universe, generate MOND-like behavior at galactic scales, and support inflationary dynamics in the early universe. The structure of the paper is as follows: we first derive the universal tachyonic speed from Lorentz transformation principles and dimensional analysis. We then explore implications for quantum mechanics and develop a modified tachyonic field Lagrangian that naturally reduces to quintessence, MOND-like, and inflationary behaviors in distinct cosmological contexts. Finally, we discuss implications for physics beyond the Standard Model, though many effects are highly suppressed due to the large value of v_t.

2 Unique Quantized Tachyon Speed from Lorentz Transformation

In this section, we present a derivation that supports the uniqueness of quantized superluminal speeds in tachyonic fields using Lorentz Transformation. Starting with the velocity addition formula in Special Relativity:

$$v' = \frac{v - u}{1 - \frac{vu}{c^2}} \tag{2.1}$$

Assuming quantized velocities $v = nc$ and $u = mc$, where n and m are integers, and (c) is the speed of light, we substitute into the equation:

$$v' = \frac{(n - m)c}{1 - nm} \tag{2.2}$$

To satisfy the quantization condition, the observed velocity $v' = rc$ must also be an integer multiple of c:

$$r = \frac{n - m}{1 - nm} \tag{2.3}$$

Analysis reveals that the only integer solution occurs when $|n| = |m|$, resulting in $v' = 0$ or when two of $\{|r|, |m|, |n|\}$ is 1. This means only motion of either the tachyon relative to c or between tachyons of same speed is allowed. This enforces the uniqueness of the quantized superluminal speed. In a Part II of this work [14], we give alternative derivations of this result and analysis of possible existence of various non-interacting tachyonic modes with different speeds. Next, based on dimensional analysis constrained to known physical constants, we find that the superluminal speed appearing in such setting, using values in [15, 16], is given by

$$v_t = \sqrt{\frac{1}{\Lambda t_P^2}} \approx 5.931 \times 10^{60} c \tag{2.4}$$

We give alternative estimation of v_t values in [14] each of which consistently converge around the equivalent scale of $\approx 10^{60}$ to 10^{61} times the speed of light (c). We remark that the existence of a universal tachyonic speed with such high value $v_t =\approx 5.931 \times 10^{60}c$ also provides a new framework for non-local hidden variable theories such as the Pilot-Wave (de Broglie–Bohm) theory of quantum mechanics.

3 Implications for Cosmological Physics

We propose a modified tachyonic field Lagrangian that incorporates the universal speed v_t. This formulation extends previous work on tachyonic fields [3, 6] while providing a reduction to quintessence models [17]. In this section, we put $c = 1$. The fundamental Lagrangian for our tachyonic field takes the form:

$$\mathcal{L}_t = -V(\phi)\sqrt{1 - \frac{h^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi}{v_t^2}} \tag{3.1}$$

where:

- $V(\phi)$ is the tachyonic potential
- $h^{\alpha\beta} = \mu\left(\frac{|\nabla\phi|}{a_0}\right) g^{\alpha\beta}$ where $\mu\left(\frac{|\nabla\phi|}{a_0}\right)$ can be MOND-like interpolating function [7].

This form, which is similar to the Dirac-Born-Infeld (DBI) action used in inflationary cosmology [8], ensures Lorentz invariance while incorporating the characteristic speed scale. It also provides a flexible framework that can be interpreted differently depending on the functional form of $V(\phi)$, the MOND-like interpolation function μ, and the role of the tachyonic speed v_t. We explore three main reductions of this model: a quintessence-like behavior in the late universe, a MOND-like behavior for galactic dynamics, and an inflationary phase in the early universe.

3.1 Reduction to Quintessence

In the limit where the field derivatives $\partial_\alpha\phi$ are small compared to v_t and $\mu \approx 1$, the Lagrangian can be expanded around small kinetic terms:

$$\mathcal{L}_{\phi,\mathrm{DE}} \approx -V(\phi)\sqrt{1 - \frac{\partial_\alpha\phi\,\partial^\alpha\phi}{v_t^2}}. \tag{3.2}$$

For $|\partial_\alpha\phi|^2 \ll v_t^2$, we can expand the square root term and the full Lagrangian can be written in the form:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\left(1 - \frac{\partial_\mu\phi\partial^\mu\phi}{2v_t^2}\right) \tag{3.3}$$

The first two terms correspond to the standard quintessence Lagrangian [17, 18], while the third term represents a high-energy correction characterized by the universal speed v_t. Varying the action with respect to ϕ yields the field equation:

$$\Box\phi + \frac{dV}{d\phi}\left(1 - \frac{\partial_\mu\phi\partial^\mu\phi}{2v_t^2}\right) - \frac{V(\phi)}{v_t^2}\Box\phi = 0 \tag{3.4}$$

In a cosmological context, assuming a Friedmann-Robertson-Walker metric, this reduces to:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi}\left(1 - \frac{\dot{\phi}^2}{2v_t^2}\right) - \frac{V(\phi)}{v_t^2}(\ddot{\phi} + 3H\dot{\phi}) = 0 \tag{3.5}$$

where H is the Hubble parameter. The energy-momentum tensor for the field is:

$$T_{\mu\nu} = \frac{\partial_\mu\phi\partial_\nu\phi}{\sqrt{1 - \frac{\partial_\alpha\phi\partial^\alpha\phi}{v_t^2}}} - g_{\mu\nu}V(\phi)\sqrt{1 - \frac{\partial_\alpha\phi\partial^\alpha\phi}{v_t^2}} \tag{3.6}$$

For a homogeneous field in FRW spacetime, this gives the energy density and pressure:

$$\rho = \frac{V(\phi)}{\sqrt{1 - \frac{\dot{\phi}^2}{v_t^2}}} \tag{3.7}$$

$$p = -V(\phi)\sqrt{1 - \frac{\dot{\phi}^2}{v_t^2}} \tag{3.8}$$

In the limit where $|\dot{\phi}| \ll v_t$, the equation of state parameter becomes:

$$w = \frac{p}{\rho} \approx -1 + \frac{\dot{\phi}^2}{v_t^2} \tag{3.9}$$

This aligns with the observed equation of state of dark energy [5, 26] while predicting small deviations from $w = -1$.

3.2 Reduction to MOND-Like Behavior

In the modified tachyonic Lagrangian incorporating the MOND interpolating function, the low-gradient limit ($|\partial_\mu\phi| \ll v_t$) gives

$$\mathcal{L}_{\phi,\mathrm{DM}} \approx -V(\phi) + \frac{V(\phi)}{2v_t^2}\,\mu\left(\frac{|\partial_\alpha\phi|}{a_0}\right)\, g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi. \tag{3.10}$$

For this Lagrangian, the modified energy density ρ and pressure p are given by:

$$\rho \approx V(\phi)\left(1 + \frac{\mu\left(\frac{|\nabla\phi|}{a_0}\right)\, g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi}{2v_t^2}\right), \tag{3.11}$$

$$p \approx -V(\phi)\left(1 - \frac{\mu\left(\frac{|\nabla\phi|}{a_0}\right)\, g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi}{2v_t^2}\right). \tag{3.12}$$

The equation of state parameter $w = \frac{p}{\rho}$ in this case is:

$$w \approx -1 + \frac{\mu\left(\frac{|\nabla\phi|}{a_0}\right)\, g^{\alpha\beta}\,\partial_\alpha\phi\,\partial_\beta\phi}{v_t^2}. \tag{3.13}$$

This modified equation of state parameter includes the MOND interpolating function $\mu\left(\frac{|\nabla\phi|}{a_0}\right)$, which adjusts the influence of $|\nabla\phi|$ depending on the acceleration regime [7, 19].

For small values of $|\nabla\phi|$, the MOND interpolating function $\mu \approx 1$, and we recover the original tachyonic equation of state $w \approx -1 + \frac{\dot{\phi}^2}{v_t^2}$. This implies that, in the regime where ϕ is nearly homogeneous, the field behaves like dark energy with $w \approx -1$, explaining the observed cosmic acceleration [5, 26]. In regions of higher field gradient, where $\mu\left(\frac{|\nabla\phi|}{a_0}\right)$ modulates the effect, the equation of state deviates more significantly from -1. This dependence on the MOND interpolating function suggests that the field behaves in a manner resembling dark matter by producing effective gravitational effects [9].

3.3 Reduction to Inflationary Dynamics

In the early universe, inflation can be realized by choosing a potential $V(\phi)$ that supports a slow-roll phase. Suitable forms include:

$$V(\phi) = V_0 \exp(-\lambda\phi) \quad \text{or} \quad V(\phi) = V_0\left(1 - \frac{\phi^p}{\mu^p}\right). \tag{3.14}$$

Assuming the MOND interpolation $\mu \approx 1$ (i.e., $h^{\alpha\beta} = g^{\alpha\beta}$), the Lagrangian reduces to a DBI-like form:

$$\mathcal{L}_{\phi,\text{inflation}} = -V(\phi)\sqrt{1 - \frac{\partial_\alpha\phi\,\partial^\alpha\phi}{v_t^2}} \tag{3.15}$$

.

This induces a speed limit on the field, enforcing a slow-roll even on steep potentials [3, 25]. The Lorentz factor γ in this case is:

$$\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}^2}{v_t^2}}}. \tag{3.16}$$

The slow-roll parameters ϵ and η [24] are then modified by γ as follows:

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2\frac{1}{\gamma}, \quad \eta = M_{\rm Pl}^2\frac{V''(\phi)}{V(\phi)}\frac{1}{\gamma}. \tag{3.17}$$

When $\gamma \approx 1$, these conditions replicate standard slow-roll inflation. The extremely high value of v_t ensures $\gamma \approx 1$ during most of inflation, however, deviations in inflationary observables, such as the spectral index and tensor-to-scalar, due to the presence of the universal speed v_t and the non-canonical kinetic term, could be tested by Cosmic Microwave Background (CMB) experiments [20] and large-scale structure surveys that probe primordial non-Gaussianity.

The generalized DBI-inspired Lagrangian provides a unified framework that accommodates quintessence-like dark energy, MOND-like dark matter dynamics, and inflationary expansion in the early universe. By varying $V(\phi)$ and the MOND interpolation μ at different cosmological epochs, the field ϕ evolves through inflation, mimics dark matter behavior at galactic scales, and stabilizes as dark energy in the late universe.

4 Summary and Outlook

We have presented a comprehensive framework based on a universal tachyonic speed $v_t = \sqrt{1/(\Lambda t_P^2)} \approx 5.931 \times 10^{60} c$ and a generalized Lagrangian incorporating a MOND-like interpolating function. This formalism provides several key insights: (1) quantization of superluminal velocities in field theories, emerging from basic principles of special relativity; (2) a novel framework for understanding quantum non-locality in the context of pilot wave theory; and (3) a unified description of dark energy, modified gravity, and inflation through different limits of the same Lagrangian. The framework makes specific predictions in different regimes: at cosmological scales, it predicts small deviations from $w = -1$ in the dark energy equation of state; while at galactic scales, it accommodates MOND phenomenology. In the early universe, it modifies inflationary observables through the modified Lorentz factor and DBI-like kinetic term.

The extremely high value of v_t ensures compatibility with existing constraints while allowing for small, potentially observable effects at cosmological scales. Future work will focus on developing explicit observational tests, particularly in cosmological contexts where the effects are most likely to be detectable, such as the observed CMB polarization anomalies [21] and large-angle correlations, as well as cosmic dipole anisotropy [22].

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