The Relative Nonlocality or the Illusion of Superluminal Speed Due to Curvature Difference Between Different Spacetime Intervals

Rayd Majeed Al-Shammari

...The relative nonlocality in general relativity is the illusion of superluminal speed due to curvature difference between different spacetime intervals, as I will show later the relative nonlocality is very useful to bridge the gap between general relativity and quantum mechanics without the need for a new unifying theory, in fact, we could harmonize both theories if we put the relative nonlocality in our perspective, it is derived from Einstein work in 1911, as I will show that the condition of curvature difference is the master key element to solve the compatibility problem between general relativity and quantum mechanics, by following this line of work, I found that Einstein field equations are compatible with the uncertainty principle in such a way that the stress energy tensor can be extracted from the momentum uncertainty in the uncertainty principle; this happens only when we have a quantum entangled system of collective masses larger than or equal to half Planck mass as minimum requirements to bend spacetime. Then, by using a quantum entangled system with a rest mass of half the Planck mass or more, I put here the requirements for an experiment to generate an artificial gravitational singularity as a falsifiability requirement to prove or disprove my work experimentally in particles accelerators.

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Introduction:

There are two kinds of nonlocality in physics: real nonlocality and relative nonlocality, real nonlocality is a cause and effect carried unbound to spacetime since it contradicts the light cone of relativity and does not obeys the Lorentz factor; it simply involves a spontaneous information transition between two points in spacetime without crossing the distance in between these two points then the transition cannot be inside spacetime because these teleportation of informations are contradicting the light cones and as we know the light cones are the laws of causality inside spacetime^{[1],[2],[3],[4],[5],[6],[7]}.

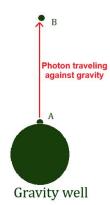
Real nonlocality could even involve quantum information traveling from the future to the past such that the reality could be change in correlation with the observer^{[8],[9],[10],[11],[12],[13],} all of which happens without any hidden variables inside spacetime, which is well verified by Bell's inequality test experiments^[14].

The relative nonlocality is different; it is an act that appears to be faster than light, but in reality, it is not, it's only appear to us in this form of illusion if we take our measurements between two spacetime intervals or more with a gravitational potential difference between these two intervals; this is an illusion^[15] of faster than light due to the shortcoming of measurements because of the difference in spacetime curvature between these two intervals^[A].

Now, let us consider two points in space (A and B) such that point (A) is very close to the surface of a gravity well and point (B)_at an infinite distance from the same gravity well. Then, for a photon at point (A) moving towards point (B), energy should be lost in this propagation between these two points due to the difference in spacetime curvature; i.e., the difference in gravitational potential and then the speed of light will be constant only for local observers, but not for a nonlocal observer, Einstein concluded that in his paper, "On the influence of gravitation on the propagation of light"^[16] in equation number (3), the speed of light would be variable in a vacuum as long as it is exclusively measured between two points with a different gravitational potential, and only for an outside observer, i.e., it's an illusion of a superluminal travel and the equation was as follows:

$$c = c_o \left(1 - \frac{MG}{r c^2} \right)$$

So many follow Einstein works of faster than light believing its real thing and not an illusion [17],[18],[19],[20],[21],[22],[23],[24],[25] and many others speculate about this phenomenon^{[26],[27],[28],[29],[30],[31],[32]}



Using this approach, Einstein predicted and calculated gravitational lensing phenomena, which was subsequently experimentally proven by Eddington in 1919^[33].

[[]A] Very important alert to remove any misconceptions may occur regards the speed of light "From now on when ever I mention a inconstant speed of light or faster than light travel or information transition faster than light, then it's all exclusively and strictly in the context of illusion of superluminal speed, and it would never be in the context of Lorentz invariance violation of speed of light such that the principle of the speed of light constancy will always hold well regardless of these illusions of measurements that leading to the mirage of superluminal speed and as I will prove later all this illusions of superluminal speed is due to curvature difference between two spacetime intervals or more"

It was a solid framework with exceptional experimental proof; however, it was only an approximation and not the whole truth. Schwarzschild corrected Einstein's work with his famous metric^{[34],} and he came to very close but not quite the same results.

Because this equation indicates that the velocity of light varies within a special sphere, depending on the position, the sphere collapses at a constant mass; however, with increasing density, it transitions to a smaller radius than before and emits radiant energy. The radiation emitted from the surface of the massive sphere is red-shifted according to the gravitational redshift formula^[35] as follows:

$$\lambda_{\text{redshift}} = \lambda \left(1 - \frac{r_s}{r}\right)^{-1/2}$$

The idea of relative nonlocality was used to explore the false difference in the speed of light^[36] between the interior and exterior of black holes^[37] and in the warp drive by Miguel Alcubierre^[38] such that the speed of light inside a spacetime hypersurface will always be equal to (c); however, because the spacetime hypersurface itself moves faster than light, then for an outside observer, it will appear that the person inside the hypersurface is moving faster than light, whereas its speed and whats happening is nothing more than that of spacetime itself is expanding behind the hypersurface and constricting in front of the hypersurface, and we should not forget that the hypersurface itself is nothing but rather a spacetime arrangement under the influence of the energy density distribution.

1. Gravitational blueshift effect on the electric permittivity of free space (ϵ_{\circ}) & Schwarzschild's gravitational singularity::

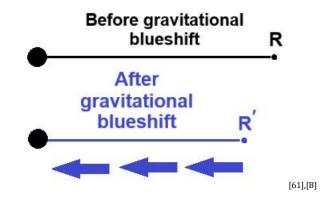
If we have a point such as a nonmoving gravity well and a photon with wavelength (λ) falling in this gravity well from a fixed point in space with a distance of (R) from the center of the same gravity well such that ($\lambda = R$), then the photon should have a gravitational blueshift in which the inverse of the gravitational redshift occurs, as follows^{[39],[40]}.

$$\lambda_{\text{redshift}} = \lambda \left(1 - \frac{r_s}{r}\right)^{-1/2} \& \lambda_{\text{blueshift}} = \lambda \left(1 - \frac{r_s}{r}\right)^{1/2} \dots^{[41],[42]}$$

∵ (R) is a real point in space separated from the point such as nonmoving gravity well by a real distance, and there is no relative movement between the source and gravity well; however, despite this, the photon suffers a gravitational blueshift^{[43],} but since the photon path is the world line for light cones^{[44],} it cannot experience any change in proper time since this will lead to causal contradictions^{[45],} i.e., photons experience no time^{[46],[47]} whatsoever^{[48],[49],[50]} then the effects of gravitational time dilation cannot be responsible for the changes in photon wave length^{[51],[52]} and lead to a very unique outcome^{[53],[54],[55]}.

The space itself is shortened by a factor of $\left(\sqrt{1-\frac{r_s}{r}}\right)$ because of the difference in the gravitational potential between the two points. ^{[56],[57],[58],[59],[60]}

$$\therefore \lambda_{\text{blueshift}} = \lambda \left(1 - \frac{r_s}{r}\right)^{1/2} \therefore \Rightarrow \left(R = R \sqrt{1 - \frac{r_s}{r}}\right)$$



where (R) is a real fixed point in space as measured by a local observer, i.e., at (R) or from a point that has the same gravitational potential, and where (R) is the same real point in space as measured by a nonlocal observer, i.e., an observer at a point with less gravitational potential^{[C][62]}.

If we have an electric charge at the center of this point, such as a non-moving gravity well in an empty space, then owing to the influence of gravity and because the imaginary photon of the electric field of this electric charge is affected by gravity, as we know from general relativity ^[63], the electric field will occupy a smaller space owing to a shortening in its radius only with respect to the nonlocal observer, such that it will change the electric flux only with respect to the nonlocal observer as follows:

$$:: (\Phi_{\rm E}) = {\rm E}4\pi {\rm R}^2 : \Rightarrow \Phi_{\rm E}' = \frac{{\rm E}4\pi {\rm r_o}^2}{\left(1 - \frac{{\rm r_o}}{{\rm r_o}}\right)} [^{\rm D}]$$

Because the electric charge here is conserved, for a nonlocal observer, it affects the electric permittivity of free space (ε_{o}) as follows:

$$\approx \epsilon_{\circ} = \frac{q}{\Phi_{E}} = \frac{q}{E4\pi R^{2}} \approx \text{ under gravity} \Rightarrow \epsilon_{\circ}' = \frac{q}{\frac{E4\pi R_{\circ}^{2}}{\left(1 - \frac{r_{s}}{r}\right)}} [^{E}]$$

$$\Rightarrow \varepsilon_{\circ}' = \varepsilon_{\circ} \left(1 - \frac{r_{s}}{r} \right) :: r_{s} < r : \Rightarrow \varepsilon_{\circ}' < \varepsilon_{\circ}$$

; (ϵ_{\circ} ') \equiv Vacuum permittivity under gravity as only observed from flat spacetime

This does not apply to the magnetic permeability of free space because it is a fully geometrically characterized entity ^{[64],[65]}.

^[B] The gravitational lensing of the Cosmic Horseshoe is the best example for gravitational blue and redshift, i.e., i.e., some photons enter in a shorter path of spacetime and some others enter a longer path of spacetime for no reason other than the differnce in shortining spacetime between two regions of spacetime with differnt gravity potential

^[C] If the photon was falling in the gravity well then we use gravitational blueshift but if the photon was leaving the gravity well then we use gravitational redshift , here we have a falling photon in a gravity well then we use gravitational blueshift.

^[D] Don't let the common sense deceive you, gravity will compress more spacetime inside a smaller space such that under gravity for the same space you will have more spacetime inside space than what apparently there without gravity then you have to use factor of $\left(\frac{1}{(1-\frac{r_s}{r})}\right)$ and not $\left(1-\frac{r_s}{r}\right)$

^[E]Under gravity we will have a much crowded space with virtual photon than without gravity and since the electric charge is conserved then this will affect the electric permittivity of the free space and will create the illusion of superluminal speed in which a direct indicator for the spacetime curvature difference between the two points of measurements.

$$\therefore \quad \mu_{\circ} = \frac{B_{H}}{H} \quad \therefore H = \frac{B_{H}}{\mu_{\circ}} \quad \therefore \Rightarrow H = \frac{\left(\frac{B_{H}}{\left(1 - \frac{r_{s}}{r}\right)}\right)}{\mu_{\circ}}$$
$$\therefore \Rightarrow \quad \mu_{\circ}` = \frac{\left(\frac{B_{H}}{\left(1 - \frac{r_{s}}{r}\right)}\right)}{\left(\frac{B_{H}}{\left(1 - \frac{r_{s}}{r}\right)}\right)} \quad \therefore \Rightarrow \quad \mu_{\circ}` = \mu_{\circ} \frac{\left(\frac{B_{H}}{\left(1 - \frac{r_{s}}{r}\right)}\right)}{\left(\frac{B_{H}}{\left(1 - \frac{r_{s}}{r}\right)}\right)} \quad \therefore \Rightarrow \quad \mu_{\circ}` = \mu_{\circ}$$

Because the speed of light is not a vector quantity and is a scalar quantity that is independent of the direction of the moving source or observer, it is dependent only on the nature of the empty space itself^[66]:

$$\because c = \frac{1}{\sqrt{\varepsilon_{\circ}\mu_{\circ}}} \therefore \Rightarrow c` = \frac{1}{\sqrt{\varepsilon`\mu_{\circ}}}$$

From a low gravitational potential to a high gravitational potential from a spacetime perspective,

$$\Rightarrow c' = \frac{1}{\sqrt{\epsilon_{\circ}\mu_{\circ}\left(1 - \frac{r_{s}}{r}\right)}} \therefore \Rightarrow c' = c_{\circ}\left(1 - \frac{r_{s}}{r}\right)^{-1/2} \dots \boxed{1.1}$$

Additionally, from a high gravitational potential to a low gravitational potential from a space-time perspective,

$$\Rightarrow c` = \sqrt{\varepsilon_{\circ}\mu_{\circ}\left(1 - \frac{r_{s}}{r}\right)} \Rightarrow c` = c_{\circ}\left(1 - \frac{r_{s}}{r}\right)^{1/2} \dots \boxed{1.2}$$

This equation is an illusion of superluminal speed^[67], and it is a direct useful indicator of spacetime curvature difference between two intervals, such that the principle of the speed of light constancy holds well; however, unlike what usually underlies the special theory of relativity, this illusion leads to a serious attempt to find a new solution by upgrading spacetime to include these faster than light instead of accepting the fact that it is only an illusion of faster than light and not a real thing^{[68],[69],[70],[71],[72],[73],[74],[75],[76],[77],[78],[79],[80],[81],[82],[83],[84],[85],[86],[87],[88].}

The idea of the illusion of being faster than light is due to the shortcomings of these measurements because of the difference in spacetime curvature between the two intervals. This approach has been used in many forms and approaches in well-respectful mainstream physics. This idea of the illusion of being faster than light, which does not contradict special relativity, is also a useful indicator for the difference in spacetime curvature between two intervals^{[89],[90],[91],[92],[93],[94],[95],[96],[97],[98],[99].}

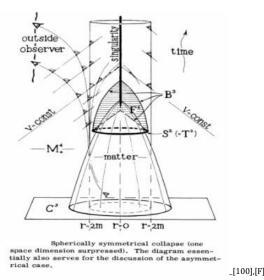
Now, let me explain the illusion of being faster than light in a way that will clear any suspicions or confusion about the speed of light constancy.

Let us consider two photons, photon (A) and photon (B), where photon (A) propagates in a flat spacetime, while photon (B) propagates under the influence of gravity from point (x_1) to point (x_2) such that (x_1) is in flat

spacetime and (x₂) is a Schwarzschild black hole.Then, as shown in equation 1, we have the illusion of superluminal speed as follows:

$$\Rightarrow c = c_{\circ} \left(1 - \frac{r_{s}}{r}\right)^{-1/2} \dots \boxed{1.1}$$

When photon (B) approaches the event horizon, let us say point between (x_1) and (x_2) ; let us name it $(x_{1/2})$. Then, the event horizon itself will run away from photon(B) in the same ratio, and this chasing will continue to singularity to the point at which spacetime itself collapses into a gravitational singularity with a radius equal to zero.



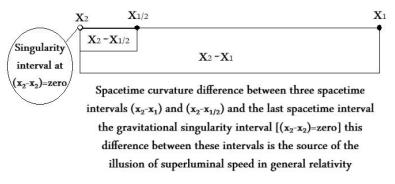
This is the illusion and not the reality; in reality, when a photon (B) reaches a near-event horizon at point $(x_{1/2})$ will experience less gravitational curvature difference than its original state when it is at point (x_1) because at point (x_1) , the difference in curvature is equal to the curvature ratio at point (x_2) minus the curvature ratio at point (x_1) ; however, because point (x_1) is flat spacetime and point $(x_{1/2})$ has greater gravitational potential, that is, a greater curvature, then the difference in this situation is less.

When the photon reach point $(x_{1/2})$ is associated with less potential difference (i.e., less curvature difference since some of the curvature is right now in and behind point $(x_{1/2})$), it is just like measuring acceleration due to gravity at sea level and then take it again at the bottom of the Mariana Trench^{[101],[G]} it will experience less gravity. In addition, when the photon approaches a Schwarzschild black hole, owing to the change in curvature difference between the different spacetime intervals in the photon path, there will be a smaller curvature difference in front of the photon, and the Schwarzschild radius will be shorter than the falling photon and will gradually decrease until it will vanishes in the end. Additionally, we will always have (c) a constant and the mass of the black hole will be conserved but distributed differently than before, and the curvature difference will gradually change to smaller values until it becomes zero at the singularity, which gives us the illusion of superluminal speed^{[102],[103],[104]}; Thus, nothing can ever cross the event horizon.

⁽F) This is [figer-1] in Penrose paper, Phys. Rev. Lett. 14, 57 – 18 January 1965, Gravitational Collapse and Space-Time Singularities. It's proves that spacetime at (r=0) will collapse in to nothing and this is my exact argument here, i.e., [(x₂) – (x₂) = 0] is the singularity interval with zero curvature difference(r=0) https://link.aps.org/doi/10.1103/PhysRevLett.14.57

^[G] This paper (Geophysical tests of the gravitational redshift and ether drift) is comparing the gravitational redshift with ether by comparing gravitational redshift between sea level and the Mariana trench bottom approximately 11 km depth.

$$(x_2) - (x_1) > (x_2) - (x_{1/2}) > [(x_2) - (x_2) = 0] \Rightarrow r_s = \sqrt{\frac{2MG}{c^2} \left(1 - \frac{r_s}{r}\right)} \dots \boxed{1.3}^{[105, H]}$$



[106],[107],[108],[109]

Spacetime curvature difference between three spacetime intervals^[110] (x2-x1) and (x2-x1/2) and the last spacetime interval the gravitational singularity interval [(x2-x2)=zero] this difference between these intervals is the source of the illusion of superluminal speed in general relativity

The term $\left(1 - \frac{r_s}{r}\right)$ does not change the speed of light, nor does the mass represent the change in the effect of the spacetime curvature difference^[111] on photons that fall in a Schwarzschild black hole^{[112],[113],[114]}, which is the true source of the superluminal speed illusion and how to calculate it. It is a highly valuable and significant way to deal with and understand spacetime curvature.

In a less accurate way, photon (B) is similar to a man walking on a travelator^{[1],[115],[116],[117]}, in reality, the speed is exactly the same^{[118],[119]} as the same man walking on an ordinary sidewalk ^{[120],} that is, its just-illusion of superluminal speed because its casual disconnection from the outside hypersurface can produce such effects^[121].

To understand a Schwarzschild black hole, we need to address it from two perspectives: first from an illusionfree perspective, that is, a falling_photon perspective, and then from a spacetime perspective. We can then solve the gravitational singularity without having indeterminate forms of dividing by zero because dividing by zero is an indication of mathematical failure^[122].

First, the falling photon perspective is very useful for calculating the ratio of the shortening of spacetime coordinates due to gravity by calculating the illusion of superluminal speed due to the change in curvature difference between two spacetime intervals.

We can make things strictly basic by considering an empty universe with nothing in it except for a single photon and a Schwarzschild black hole, and letting the photon fall from infinity toward the Schwarzschild black hole; additionally, we take the measurements for the photon from infinity to the event horizon of the non-rotating black hole.

The spacetime interval for the Schwarzschild metric is as follows:

^[H] Equation number (3) $\left[\lambda_{redshift} = \frac{GM_e}{c^2} \left(\frac{1}{R_e} - \frac{1}{r}\right) - \frac{1}{2} \left(\frac{w_e}{c}\right)^2\right]$ in this paper \Rightarrow (An orbiting clock experiment to determine the gravitational redshift. *Astrophys Space*) is very close to my argument here its show the curvature difference between two spacetime intervals. <u>https://articles.adsabs.harvard.edu/cgi-bin/nph-iarticle_query?1970Ap%26SS...6..13K&defaultprint=YES&page_ind=2&filetype=.pdf</u>

US7861843B2, Inventor: Esko AulankoJorma MustalahtiMarc Ossendorf, Travelator and method for controlling the operation of a travelator 2006, https://patents.google.com/patent/EP1915314A1/en

$$\therefore ds^2 = -\left(1 - \frac{r_s}{r}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

As I proved earlier, owing to the change in curvature difference,

$$(x_2) - (x_1) > (x_2) - (x_{1/2}) > [(x_2) - (x_2) = 0] \Rightarrow r_s = \sqrt{\frac{2MG}{c^2} \left(1 - \frac{r_s}{r}\right)} \dots \boxed{1.3}$$

Because this photon falls within the Schwarzschild black hole, the Schwarzschild radius continues to shrink until it reaches zero, that is, the spacetime interval is equal to zero. Then, we have a radial null geodesic, and the Schwarzschild metric should be as follows:

$$\therefore ds^{2} = 0 = -\left(1 - \frac{r_{s}}{r}\right)c^{2}dt^{2} + \frac{dr^{2}}{\left(1 - \frac{r_{s}}{r}\right)}[^{123}], [^{124}]$$

However, since we first consider the photon perspective and as we know photons experience no time^{[125], [126]}, the photon will experience no difference regardless of its path; in fact, photons are stranded in time and experience nothing until it is observed by another field force or observer ^{[127], [128]}, which will lead to a local observer effect, i.e., the photon will not experience the illusion of superluminal speeds, whereas the photon itself is the messenger of causality, i.e., for photons (c'=c), i.e., photons are illusion free; then, at the event horizon, we can safely neglect the line element for time, and we pretend that it does not exist then for photons at the event horizon and at singularity and for all in between, the time line element equals zero.

: time line element =
$$-\left(1 - \frac{r_s}{r}\right)c^2dt^2$$
 : $r_s = r \implies$ time line element = zero

This is a photon perspective of the curvature of spacetime, such that time in spacetime is present and not absent, but photons do not interact with the temporal dimension in spacetime; photons do not feel time in spacetime, and it is fair to remove it from the perspective of the photon.

Owing to the change in the curvature difference in the path of the falling photon in the Schwarzschild black hole from a photon perspective, the spacetime interval should be as follows:

$$ds^2 = \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)}$$

At the event horizon, space time is represented by a spherical surface because it holds a coordinate singularity, that is, from the photon perspective, the spacetime interval at the event horizon should be as follows:

$$\frac{\mathrm{d}r^2}{\left(1-\frac{\mathrm{r_s}}{\mathrm{r}}\right)} = 4\pi \ \mathrm{r_s}^2$$

Previously, we concluded that

$$(x_2) - (x_1) > (x_2) - (x_{1/2}) > [(x_2) - (x_2) = 0] \Rightarrow r_s = \sqrt{\frac{2MG}{c^2} \left(1 - \frac{r_s}{r}\right) \dots 1.3}$$

Because the event horizon collapses from (r_s) to zero owing to the change in the curvature difference between two spacetime intervals, and because the event horizon of a Schwarzschild black hole is a perfect sphere surface, then

$$\therefore \Rightarrow \text{ at } (r = r_s) \Rightarrow r_s` = 0 \therefore \Rightarrow 0 < r_s` < (r = r_s) \therefore \Rightarrow 0 \le ds^2 \le 4\pi r_s^2 \therefore \Rightarrow ds^2 = 4\pi r_s^2^2$$
$$\therefore \Rightarrow \frac{dr^2}{\left(1 - \frac{r_s}{r_s}\right)} = 4\pi r_s^2 \ldots \boxed{2.1}$$
$$; r_s` = \sqrt{\frac{2MG}{c^2} \left(1 - \frac{r_s}{r}\right)} ; 0 < r_s` < (r = r_s), \text{ i.e., } r, r_s \& r_s` \text{ are like a steps counter}$$

This approach is not unprecedented because it is somewhat similar to previous ideas based on the use of Kruskal–Szekeres coordinates and De Sitter, among others, to solve the problem of coordinate singularity at event horizons^{[129],[130],[131],[132]}.

$$\because (\mathrm{dr_s}^2) = \mathrm{dr_s} \cdot \mathrm{dr_s} \quad \therefore \Longrightarrow \frac{\mathrm{dr_s} \cdot \mathrm{dr_s}}{\left(1 - \frac{\mathrm{r_s}}{\mathrm{r_s}}\right)} = 4\pi (\mathrm{r_s})^2 \quad \therefore \Longrightarrow \frac{\mathrm{dr_s} \cdot \mathrm{dr_s}}{\left(1 - \frac{\mathrm{r_s}}{\mathrm{r_s}}\right)4\pi(\mathrm{r_s})^2} = 1$$

; $r_{s}^{'} \equiv$ Schwarzschild upgraded radius due to nonlocality i.e., illusion of VSL^{[133],[134],[135]}

$$\therefore \Rightarrow \frac{\mathrm{dr}_{\mathrm{s}}}{2\sqrt{\pi} \, \mathrm{r}_{\mathrm{s}}^{\,\,\cdot} \, \sqrt{\left(1 - \frac{\mathrm{r}_{\mathrm{s}}^{\,\,\cdot}}{\mathrm{r}_{\mathrm{s}}^{\,\,\cdot}}\right)}} = 1 \ \Rightarrow \mathrm{dr}_{\mathrm{s}} = 2\sqrt{\pi} \, \mathrm{r}_{\mathrm{s}}^{\,\,\cdot} \, \sqrt{\left(1 - \frac{\mathrm{r}_{\mathrm{s}}^{\,\,\cdot}}{\mathrm{r}_{\mathrm{s}}^{\,\,\cdot}}\right)} \ \Rightarrow \mathrm{dr}_{\mathrm{s}} \equiv \mathrm{line \ element}$$

$$, \mathrm{let} \, \mathrm{r}_{\mathrm{s}} = \mathrm{x} \ \& \, \mathrm{r}_{\mathrm{s}}^{\,\,\cdot} = \mathrm{y}$$

$$\therefore \Rightarrow \frac{\mathrm{dx}}{2\sqrt{\pi} \, \mathrm{y} \, \sqrt{\left(1 - \frac{\mathrm{y}}{\mathrm{x}}\right)}} = 1$$

$$\mathrm{By \ integration}, \Rightarrow \frac{\mathrm{y \ ln}\left(\sqrt{\left(1 - \frac{\mathrm{y}}{\mathrm{x}}\right) + 1}\right) + 2\mathrm{x}\left(\sqrt{\left(1 - \frac{\mathrm{y}}{\mathrm{x}}\right)}\right) - \mathrm{y \ ln}\left(\left|\sqrt{\left(1 - \frac{\mathrm{y}}{\mathrm{x}}\right) - 1}\right|\right)}{\frac{4\sqrt{\pi}}} + \mathrm{C} = \mathrm{x} + \mathrm{D}$$

Here, we have two possibilities, $C \neq D \& C=D$; then, if $C \neq D$, then this is inconvenient, so we overlook it, and we take the other less likely possibility because it is much easier to work with, that is, C=D.

$$\therefore \Rightarrow y \ln\left(\sqrt{\left(1 - \frac{y}{x}\right)} + 1\right) - y \ln\left(\left|\sqrt{\left(1 - \frac{y}{x}\right)} - 1\right|\right) = 4\sqrt{\pi} x - 2x \left(\sqrt{\left(1 - \frac{y}{x}\right)}\right)$$
$$\therefore \Rightarrow e^{y} e^{-y} \frac{\left(\sqrt{\left(1 - \frac{y}{x}\right)} + 1\right)}{\left(\left|\sqrt{\left(1 - \frac{y}{x}\right)} - 1\right|\right)} = e^{\left(4\sqrt{\pi} r_{s} - 2x \left(\sqrt{\left(1 - \frac{y}{x}\right)}\right)\right)}$$

We substitute for x&y by($r_s = x \& r_s = y$)

$$\therefore \Longrightarrow \frac{\left(\sqrt{1 - \frac{r_{s}}{r_{s}}} + 1\right)}{\pm \left(\sqrt{\left(1 - \frac{r_{s}}{r_{s}}\right)} - 1\right)} = e^{\left(4\sqrt{\pi}r_{s} - 2r_{s}\left(\sqrt{\left(1 - \frac{r_{s}}{r_{s}}\right)}\right)\right)}$$

At the singularity, each time a photon reaches the event horizon owing to the change in curvature difference, as previously described 1.3; here, we have a step counter $(r_s \& r_s)$, so when the photons reach (r_s) , it becomes the new (r_s) until the collapsing steps reach the center of the black hole.

At the center of the Schwarzschild black hole for the local observer, we obtain the following:

$$(\mathbf{r}_{s} = 0) \therefore \left(\mathbf{c}^{`} = \mathbf{c} \left(1 - \frac{0}{\mathbf{r}}\right)^{-\frac{1}{2}}\right) \therefore \mathbf{c}^{`} = \mathbf{c} \dots \boxed{2.1}$$
$$\therefore \mathbf{c}^{`} = \mathbf{c} \implies \mathbf{r}_{s}^{`} = 0 \therefore \implies \left(1 - \frac{0}{\mathbf{r}_{s}}\right) = 1 \dots \boxed{2.2}$$

The singularity for local observer ($r_s = r_s$) $\Rightarrow \left(1 - \frac{r_s}{r_s}\right) = 0 \dots \boxed{2.3}$

$$:: \frac{\left(\sqrt{1 - \frac{r_{s}}{r_{s}}} + 1\right)}{\pm \left(\sqrt{\left(1 - \frac{r_{s}}{r_{s}}\right)} - 1\right)} = e^{\left(4\sqrt{\pi}r_{s} - 2r_{s}\left(\sqrt{\left(1 - \frac{r_{s}}{r_{s}}\right)}\right)\right)}$$

at singularity
$$\Rightarrow \Rightarrow \frac{\left(\sqrt{(1-1)}+1\right)}{\pm \left(\sqrt{(1-1)}-1\right)} = e^{\left(4\sqrt{\pi}r_s - 2r_s\left(\sqrt{(1-1)}\right)\right)}$$

(for non–local observers only) $\therefore \Rightarrow \pm 1 = e^{(4\sqrt{\pi}r_s)}$

$$\therefore \Rightarrow \text{ from Euler's identity} \begin{cases} (1 = e^{2i\pi}) \quad \therefore \Rightarrow e^{2i\pi} = e^{4(\sqrt{\pi})r_s} \quad \therefore \Rightarrow r_s = i\frac{\sqrt{\pi}}{2} \\ \text{or} \\ (-1 = e^{i\pi}) \quad \therefore \Rightarrow e^{i\pi} = e^{4(\sqrt{\pi})r_s} \quad \therefore \Rightarrow r_s = i\frac{\sqrt{\pi}}{4} \end{cases}$$

$$\because r_s > r_s` \therefore \Rightarrow r_s = i\frac{\sqrt{\pi}}{2} , , , r_s` = i\frac{\sqrt{\pi}}{4} ; i\frac{\sqrt{\pi}}{2}\&i\frac{\sqrt{\pi}}{4} \equiv ratio radii i.e. line element,$$

I refer to the short ratio radius as (r_T)

$$\left(r_{\mathrm{T}} = \mathrm{i} \frac{\sqrt{\pi}}{4} \equiv \mathrm{dr}\right) \boxed{2.4}$$

; $\mathbf{r}_{\mathrm{T}} \equiv$ length element at the singularity i. e. singularity ratio radius

$$\therefore r_{s} > r_{s}` \therefore \Rightarrow r_{s} = i\frac{\sqrt{\pi}}{2}, ..., r_{s}` = i\frac{\sqrt{\pi}}{4} \therefore \Rightarrow \frac{r_{s}`}{r_{s}} = \frac{i\frac{\sqrt{\pi}}{4}}{i\frac{\sqrt{\pi}}{2}} = \frac{1}{2}$$

Now, we take a new approach from the spacetime perspective, as was proven earlier, owing to the change in the difference in curvature.

$$(x_2) - (x_1) > (x_2) - (x_{1/2}) > (x_2) - (x_2) \Rightarrow r_s = \sqrt{\frac{2MG}{c^2} \left(1 - \frac{r_s}{r}\right) \dots 1.3}$$

Because the Schwarzschild radius continues to shrink until it reaches zero, we have here a radial null geodesic, and the Schwarzschild metric should be as follows ^[136]:

$$\therefore ds^{2} = 0 = -\left(1 - \frac{r_{s}}{r}\right)c^{2}dt^{2} + \frac{dr^{2}}{\left(1 - \frac{r_{s}}{r}\right)} \\ \therefore \frac{r_{s}}{r_{s}} = \frac{i\frac{\sqrt{\pi}}{4}}{i\frac{\sqrt{\pi}}{2}} = \frac{1}{2} \\ \& \\ r_{T} = i\frac{\sqrt{\pi}}{4} \\ \equiv dr$$

$$\therefore \Rightarrow ds^{2} = -\left(1 - \frac{1}{2}\right)c^{2}dt^{2} + \frac{\left(i\frac{\sqrt{\pi}}{4}\right)^{2}}{\left(1 - \frac{1}{2}\right)} \\ = 0 \\ \therefore \Rightarrow \left(1 - \frac{1}{2}\right)dt_{s}^{2} = \frac{\left(i\frac{\sqrt{\pi}}{4}\right)^{2}}{c^{2}\left(1 - \frac{1}{2}\right)} \\ \therefore \Rightarrow dt_{s}^{2} = \frac{\left(i\frac{\sqrt{\pi}}{4}\right)^{2}}{c^{2}\left(\frac{1}{2}\right)^{2}} \\ = \frac{4\left(i\frac{\sqrt{\pi}}{4}\right)^{2}}{c^{2}} \\ = -\frac{\pi}{c^{2}4} \\ \therefore \Rightarrow ds^{2} = -\left(\frac{1}{2}\right)c^{2}\left(-\frac{\pi}{c^{2}4}\right) + \frac{\left(\frac{i\sqrt{\pi}}{4}\right)^{2}}{\left(\frac{1}{2}\right)} \\ \therefore \Rightarrow ds^{2} = -\left(\frac{\pi}{8}\right) - \left(\frac{\pi}{8}\right) \\ = 0 \\ \dots \\ \boxed{2.5} \\ \begin{bmatrix} 137 \end{bmatrix}, \begin{bmatrix} 137 \end{bmatrix}, \begin{bmatrix} 137 \end{bmatrix}, \begin{bmatrix} 138 \end{bmatrix} \\ \end{bmatrix}$$

 \therefore at singularity \Rightarrow ds² = 0 \equiv the real spacetime interval at singularity

since
$$r > r_s > 0 \therefore \Rightarrow r_s - r_s \neq 0 \therefore \Rightarrow \Delta r_s \neq 0$$

$$\therefore c' = \frac{c}{\sqrt{\left(1 - \frac{r_s'}{r_s}\right)}}; r_s > r_s'$$

i.e., (r_s) will always be larger than (r_s)

 $: 0 < \frac{r_s}{r_s} < 1 \therefore \Rightarrow \text{chaing in position} \neq 0 \therefore \Rightarrow r - r_s' \neq 0 \equiv \text{uncertainty in position}$

^[]] Here singularity is equal to zero and not a broken mathematics of dividing by zero, it is the end of spacetime this result is completely compatible with Penrose findings in his paper "Gravitational Collapse and Spacetime Singularities (1965)", he solved the gravitational singularity to be equal to zero and not a mathematical singularity of dividing by zero, i.e., no broken mathematics involve, but he solved it with Penrose–Carter diagrams and not with Einstein field equations.

Because we have a mass with an uncertain position between zero and one, $\left(0 < \frac{r_s}{r} < 1\right)$, this could result from a quantum measurement problem^{[139],[140],[141],[142],} and if this occurs only under the Heisenberg uncertainty principle^{[143],[144],[K],[145],[L],[146],[147],[148],[149]};

$$\therefore \Longrightarrow \bigtriangleup \mathbf{r}_{\mathrm{s}} \bigtriangleup \mathbf{P}_{\mathrm{s}} \ge \frac{\hbar}{2}$$

This is reasonable because we are reaching such a tiny scale; then, most certainly^{[150],[M]} we will hit quantum effects^{[151],[152],[153]}.

when reaching singularity
$$\Rightarrow \left(r_{s} = r_{T} = i\frac{\sqrt{\pi}}{4}\right) \therefore i\frac{\sqrt{\pi}}{4} = \frac{2MG}{c^{2}}$$

 $\therefore \Rightarrow M = ic^{2}\frac{\sqrt{\pi}}{8G}$; for a local observer at singularity $\Rightarrow c^{*} = c$
when $r_{s}^{*} \rightarrow 0 \therefore \Rightarrow i\frac{\sqrt{\pi}}{4} \cdot ic^{2}\frac{\sqrt{\pi}}{8G}c \ge \frac{\hbar}{2}$; $\left(r_{s}Mc = n\frac{\hbar}{2}\right)$
 $\therefore \Rightarrow i\frac{\sqrt{\pi}}{4} \cdot ic^{2}\frac{\sqrt{\pi}}{8G}c = n\frac{\hbar}{2} \therefore \Rightarrow \frac{c^{3}}{\hbar G}i\frac{\sqrt{\pi}}{4} \cdot i\frac{\sqrt{\pi}}{4} = n$



The idea of the illusion of superluminal speed is not new; it was previously approached as a mirage observation in relativistic jets of supermassive black holes^[154]

at
$$n = 1 \therefore \Longrightarrow \frac{c^3}{\hbar G} \left(i \frac{\sqrt{\pi}}{4} \right)^2 = 1$$

$$\therefore \Longrightarrow \frac{\left(i\frac{\sqrt{\pi}}{4}\right)^2}{l_p^2} = 1 \therefore \Longrightarrow i\frac{\sqrt{\pi}}{4} = l_p; l_p \equiv \text{Planck length} \cdots \boxed{2.6} [N], [155], [156], [157]$$
$$\therefore \Longrightarrow n = \frac{r_s}{l_p} \text{at } n = 1 \therefore \Longrightarrow \frac{r_s}{l_p} = 1 \therefore \Longrightarrow \frac{2GM}{c^2 l_p} = 1$$
$$\frac{2GM}{c^2 l_p} = 1 \therefore M = \frac{c^2}{2G} \sqrt{\frac{G\hbar}{c^3}} = \frac{1}{2} \sqrt{\frac{c\hbar}{G}} \therefore \Longrightarrow M = \frac{m_p}{2}; m_p \equiv \text{Planck mass}, [158]$$

[[]K] professor Penrose in his paper (On Gravity's role in Quantum State Reduction. 1996) rehighlighted the idea that uncertainty in the energy is proportional to the gravitational self-energy, in which is very close to my point here. ^[1] professor Jonathan Oppenheim, in his paper (A Postquantum Theory of Classical Gravity?2023) take very close approach to mine he deal with energy to be

quantum but spacetime to be classical ^[M] Hawking in the abstract of this paper highlighted this idea and I quote his words here " quantum gravitational effects become important. This would not be expected to happen until the radius of curvature of spacetime became approximately 10^{A-14}c.m" end of quote, well we are reaching here infinity near zero then it's very likely that quantum effects is relevant here.

[[]N] Since this is a quantized energy then it's most certainly it would obey Heisenberg uncertainty principle and as we approach the lower limits of Heisenberg uncertainty principle then most certainly we will reach Planck length because there is no energy or momentum could exist in time and length lower than Planck length nor time because Heisenberg uncertainty principle forbid that.

 $\therefore \Longrightarrow \frac{m_p}{2} \text{ is the least required mass to form a black hole}^{[159,0]}$ $\therefore \Longrightarrow \frac{m_p}{2} \text{ is the least mass considered as a gravity well}$

since energy is quantized^{[160][161,P]}

$$\therefore \Rightarrow M = n \frac{m_p}{2} ; n = 1, 2, 3 \dots \& \therefore \Rightarrow \frac{2M}{m_p} \equiv \frac{r_s}{l_p} \cdots \boxed{2.7}$$

The half_Planck mass is the least mass condition required to curve spacetime, and as a consequence, to form a black hole if condensed in the smallest area possible, that is, the area of Planck length. This condition is hereafter referred to as the (T) condition.

The heaviest particle in the standard model was the top quark at approximately 172 $(GeV/c_^2)^{[162]}$ to 176 (G. $eV/c_^2)^{[163]}$. This value is 17 orders of magnitude less than the Planck mass; thus, gravity does not work without quantum entanglement because quantum entanglement causes a group of particles to behave as collective masses of the half Planck mass or more, acting as one mass because it has the same wavefunction. Then, breaking quantum entanglement and making the wavefunction collapse for any amount of mass less than the (T) condition, that is, a half_Planck mass sequence, will make the total mass incapable of bending spacetime; thus, the gravity of such a system will vanish, that is, in theory, switching off quantum entanglement will switch off gravity.

The illusion of the superluminal speed at singularity for an observer at infinity is as follows:

$$\therefore \text{ c. } (T) = \frac{c}{\left(\sqrt{1 - \frac{1}{2}}\right)^{\frac{2M}{m_p}}} = \frac{c}{\left(\frac{1}{\sqrt{2}}\right)^{\frac{2M}{m_p}}}; (T) = \left(\sqrt{2}\right)^{\frac{2M}{m_p}} \equiv (T) = \left(\sqrt{2}\right)^{\frac{r_s}{l_p}}$$
$$\therefore \Rightarrow c_T = c\left(\sqrt{2}\right)^{\frac{2M}{m_p}} \equiv c_T = c\left(\sqrt{2}\right)^{\frac{r_s}{l_p}} \dots \boxed{2.8}$$

 \Rightarrow A black hole is any spacetime curvature that increases the speed of light on its outer surface by at least a factor of $\sqrt{2}$.

A gravity well is defined as a mass equal to or greater than half of the Planck mass.

The gravitational time dilation for a black hole is as follows:

$$\Rightarrow t_g = t_{ob} (\sqrt{2})^{\frac{2M}{m_p}} \dots \boxed{2.9}$$

; t_g is proper time at event horizon from non – local perspective

; t_{ob} is proper time for observer at flat spacetime perspective

The gravitational time dilation in GR is as follows:

^[0] Stephen Hawking predict that the least mass required to form a black hole is Planck mass because he did not take in to his considerations the effects of relative nonlocality, in fact his results were defected because when you have a black hole then the mass should be in center then due to Heisenberg uncertainty principle the mass will occupy a Planck length, i.e., half Planck length as a radius by default

^[P] This paper (Emergence of cosmic space and minimal length in quantum gravity: a large class of spacetimes, equations of state, and minimal length approaches) show that energy is bounded by the uncertainty principle such that we need to modify it to over come the big bang singularity.

$$t_{g} = t_{ob} \sqrt{1 - \frac{r_{s}}{r}}$$

This is necessary because, at the event horizon, if we take it to be like a time dilation of special relativity, then we will have an indeterminate form due to dividing by zero, and this puts real doubts about the existence of time since it would be zero in nonzero intervals of spacetime, such as event horizons. However, since I introduced the relative nonlocality, it is no longer a problem because, owing to the relative nonlocality, you cannot cross the event horizon and eventually become singular at the center of the black hole.

This equation tells us that time is real. i. e. , $t_g = t_{ob} \big(\sqrt{2}\big)^{\frac{2M}{m_p}} [9]$

2. Acceleration due to gravity:

To drive acceleration due to gravity, two measurements must be taken for the speed of light, first, from a local perspective on the surface of the gravity well, that is, (c = c) then we measured from a nonlocal perspective, that is, measuring the speed of light at the surface of the gravity well from an infinite distance point in space, that is, measuring the speed of light from a flat space_time to a curved space_time, that is,

$$\mathbf{c}`=\mathbf{c}\left(\sqrt{1-\frac{r_s}{r}}\right)$$

The first is the speed of light from a local perspective, that is, on the surface of the gravity well, and the second is the speed of light from a nonlocal perspective, that is, from flat space time, that is, far from the gravity well, measuring the speed of light at the gravity well.

local perspective
$$(c = c)$$

nonlocal perspective c' = c
$$\left(\sqrt{1-\frac{r_s}{r}}\right)$$

The acceleration is the difference between two velocities in time:

$$\therefore \Rightarrow g = \frac{(c - c`)}{t}$$

$$t = \frac{r}{\overline{v}}; \overline{v} = \text{average velocity} = \frac{(c+c^{`})}{2} \therefore \Rightarrow t = \frac{2r}{(c+c^{`})}$$
$$\Rightarrow g = \frac{(c-c^{`})}{\frac{2r}{(c+c^{`})}} \Rightarrow g = \frac{(c+c^{`})(c-c^{`})}{2r} \therefore c^{`} = c\left(\sqrt{1-\frac{r_s}{r}}\right)$$
$$\therefore \Rightarrow g = \frac{\left(c+c\left(\sqrt{1-\frac{r_s}{r}}\right)\right)\left(c-c\left(\sqrt{1-\frac{r_s}{r}}\right)\right)}{2r}$$

$$\therefore \Rightarrow g = \frac{c^2}{2r} \left(1 + \left(\sqrt{1 - \frac{r_s}{r}} \right) \right) \left(1 - \left(\sqrt{1 - \frac{r_s}{r}} \right) \right)$$
$$\therefore \Rightarrow g = \frac{c^2}{2r} \left(1 - \left(1 - \frac{r_s}{r} \right) \right) \therefore \Rightarrow g = \frac{c^2}{2r} \left(\frac{r_s}{r} \right)$$
$$\therefore \Rightarrow g = \frac{c^2 r_s}{2(r)^2} = \frac{c^2}{2(r)^2} \frac{2GM}{c^2}; M = n \frac{m_p}{2} \therefore \Rightarrow g = n \frac{m_p}{2} \frac{G}{r^2} \dots \boxed{3.1}$$

For a black hole, we do not have a fixed point as a surface-to-event horizon, as in the ordinary gravity well.

Then, to describe the local and nonlocal perspectives, we have only the Schwarzschild radius because it is a non-reachable region because, when it is reached, the speed of light will be greater, and the Schwarzschild radius will be smaller.

To solve this problem, we take measurements between two Schwarzschild radii: the first is the Schwarzschild radius measured from a flat spacetime, and the second is from a curved spacetime.

$$g = \frac{c}{\underline{\Delta(r_s)}} \Rightarrow g = \frac{c^2}{\Delta(r_s)}; \Delta(r_s) = \frac{2GM}{c^2} - \frac{2GM}{\left(\frac{c}{\sqrt{2}}\right)^2} = r_s(1-2) = -r_s$$
$$\therefore \Rightarrow g = -\frac{c^2}{r_s} = -\frac{c^4}{2GM} = \dots \boxed{3.1}$$

Here, I predict twice the surface gravity from what is known about textbooks because, in textbooks, we have

$$\left(K = \frac{1}{4M} \equiv \frac{c^4}{4GM} = \frac{c^2}{2 r_s} \right) [^{164}]$$

Now, if we integrate the acceleration over time, we should obtain the speed of light, and our time interval is calculated from constants, that is, the speed of the light gravitation constant and black hole mass, so it ranges from $\left(\frac{r_s}{c}\right)$ to (0).

$$\int_{\frac{r_s}{c}}^{0} g dt = \int_{\frac{r_s}{c}}^{0} -\frac{c^2}{r_s} dt = \frac{r_s}{c} \frac{c^2}{r_s} = c$$

3-The Hoofing effects:

We previously established that the lowest mass-to-curve spacetime is the half Planck mass, such that a lower mass would be unable to curve spacetime; however, there is a problem in this vision. All elementary particles were under this limit by several orders of magnitude. For example, the heaviest elementary particle is the top quark, which is less than this limit by approximately 17th order of magnitude. However, owing to the effect of quantum entanglement, we will have a group of elementary particles that are entangled in the same wavefunction; then, the system as a whole will act as a unit of the total mass of the system under the (T)

condition, that is, half the Planck mass, and its multiplication will be sufficient to curve spacetime; moreover, any relativistic mass effects could be added to the total mass in the direction of the movement, I name it the hoofing effect because the shape of the electromagnetic fields around the elementary particles will take the shape of the mule or donkey hoof.

For a local observer in an ordinary gravity well, for any mass that exceeds the $\left(\frac{2M}{m_p}\right)$ condition and moves at a relativistic speed, the relativistic mass is added to the total mass as follows:

$$\therefore c_{Br} = c \frac{1}{\sqrt{\left(1 - \frac{r_{Br}}{r}\right)}}; r_{Br} = \frac{2GM_{Br}}{c^2}$$

; $M_{Br} = M \gamma \cos(B)$; $0 \le B \le \pi$; $M = n \frac{m_P}{2}$; n = 1,2,3...n

;
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$
; at B = 0; v = velocity of the gravity well

$$:: g_{Br} = \frac{GM_{Br}}{r^2} :: \Rightarrow g_{Br} = \frac{G}{r^2} n \frac{m_P}{2} \frac{1}{\sqrt{1 - v^2/c^2}} \dots \boxed{4.2}$$

We will have a different rate of gravitational time dilation that varies with respect to the angle, which will create a gravity difference around the accelerated mass; artificial gravity or artificial antigravity will enable us to move particles via the hoofing effect, which is similar to a warp drive but without the need for negative energy or negative mass.

With respect to the superluminal speed, there are two parties: the Lorentz invariance party (L.I.), ^{[168], [169], [170],} ^{[171], [172], [173], [173], [174]} and the Lorentz invariance violation party (L.I.V.), ^{[175], [176], [177], [178], [179], [180], [181], [182], [183], [184]}. Fortunately, we have an experimental way to settle the debate in this paper by accelerating two beams of protons with (T) conditions, that is, $\left(M = n \frac{m_P}{2}\right)$ condition to relativistic velocities and colliding them head to head. If the Lorentz invariance violation party is correct, the electric field of these protons will travel at a speed higher than the speed of light, and the electric field will be accelerated as follows:

$$g_{Br} = \frac{GM_{Br}}{r^2} = \frac{GM}{r^2} \frac{1}{\sqrt{1 - v^2/c^2}}$$
; $M = n\frac{m_P}{2}$

$$g_{Br} = \frac{(c'-c)}{t} \div g_{Br}t + c = c'$$
$$\Rightarrow c' = \left(\frac{G\left(n\frac{m_{P}}{2}\right)}{r^{2}}t\gamma + c\right)$$

Each electric field collides at a speed higher than that of light, which creates a gravitational singularity with a temperature very close to the Planck temperature^[185] as follows^{[186],[187]:}

$$K = \frac{\hbar c(c^2)}{8\pi G M k_B} = \frac{(m_P)^2 (c^2)}{8\pi M k_B}$$

at $M = n \frac{m_P}{2} \therefore \Rightarrow K = \frac{m_P (c^2)}{4\pi n k_B} \therefore \Rightarrow K = \frac{K_P}{4\pi n} [Q]$

This temperature is very close to the Planck temperature and is sufficient to sustain continuous nuclear fusion^[188]

However, if the two colliding beams with (T) conditions did not produce a singularity, then the Lorentz invariance party (L.I.) is correct.

We could test this in the LHC at CERN or in the Fermi National Accelerator Laboratory (Fermilab), and the setting of the experiment should be as follows:

Producing two beams of protons should have two main conditions:

I. First condition:

The beam passes through a unified barrier of electromagnetic fields to provide quantum entanglement between the particles, thus creating a unified wavefunction for all particles in the beam.

II. Second condition:

The particle beam has a collective mass that should not be less than half the Planck mass per beam crosssectional area; that is, a mole of protons per cross-sectional area should be sufficient to fulfil this condition.

As a test, let us consider the following values:

B = 0 degree,
$$0.9c \le v \le 0.99c \Rightarrow 2.29 \ge \gamma \ge 7 \Rightarrow g_{Br} = \frac{GM}{r^2}\gamma; M = n\frac{m_P}{2}$$

at $r = 10^{-6}$ m, M = 1mole protons $\therefore \Rightarrow M \cong 0.001$ k.g

$$\Rightarrow g_{Br} = 6.6743 \times 10^{-2} \times 2.29 = 0.1528 \text{ m. s}^{-2}$$

^Q Hawking's work in black hole thermodynamics is an approximation and not the full scope of reality because he did not take into account the relative nonlocality, but despite this, his work is acceptable approximation

This is not a usual acceleration; this is an acceleration due to gravity, and then, its velocities could be added to each other. In reality, if we collide with another beam of particles under the same exact conditions, then their velocities would be added to each other.

In theory, the collision does not obey special relativity because it is not a real velocity, which is an illusion of superluminal speed because of the difference in curvature between two spacetime intervals.

In reality, these two beams of particles under acceleration owing to gravity induced further by a relativistic mass create a gravitational collapse area at the impact point, which collapses the colliding masses into gravitational singularities.

Because this gravitational acceleration is not bound by the speed of the light limit, it will not exceed the speed of light; rather, it will appear to us as a superluminal speed due to movement of spacetime itself, which should, in theory, achieve a gravitational singularity.

The LHC in CERN^[R] or the Fermi National Accelerator Laboratory (Fermilab)^[S] are good candidates for testing this theory.

Most likely, the Lorentz invariance party (L.I.) will prevail and the previous experimental setup will fail. However, in theory, we may still produce a gravitational singularity if the collision reaches the following required condition:

$$c_{\rm T} = c\sqrt{2}^{[189],[190],[191]}$$

$$\therefore \Rightarrow v = g_{Br}t = \frac{GM}{r^2} \frac{1}{\sqrt{1 - v^2/c^2}} t ; t \equiv time....[4.3]$$

at
$$\frac{2M}{m_p} = 1 \implies v + c = c\sqrt{2} \implies v + c = c\sqrt{2} \implies v = c(\sqrt{2} - 1) \implies v = c(0.41421356)$$

$$\therefore \Rightarrow g_{Br}t = (0.41421356) = \frac{dW}{r^2} \frac{1}{\sqrt{1 - V^2/c^2}}t$$

The hoofing effect is not a warp drive because the warp drive depends on exotic matter distributed unevenly through spacetime, whereas the hoofing effect depends on adding relativistic mass to the total mass through the use of the (T) condition, that is, the half Planck mass. Because the relativistic mass depends on the direction of movement, we create artificial gravity that can be used in superluminal travel and in creating artificial gravitational singularities that can be used in nuclear fusion, and it's a very convenient solution to the missing mass of the proton.

^R https://cds.cern.ch/record/1606826/files/Poster-2013-302.pdf

^s Brown, Bruce. "Current and Future High Power Operation of Fermilab Main Injector". Researchgate. 2009.

https://www.researchgate.net/publication/239886364

4. Experimental results

All experiments were performed using a Michelson interferometer (PHYWE 08557-00) ^{[192].}

The speed of light is independent of the direction of the moving source or observer, that is, it is dependent only on the nature of the empty space itself:

$$c=\frac{1}{\sqrt{\epsilon_{\circ}\,\mu_{\circ}}}\; ; \epsilon_{\circ}=\frac{q}{\Phi_{E}}=\frac{q}{E4\pi r^{2}}\hat{r}\; ; \;\; \mu_{\circ}=\frac{B_{H}}{H}$$

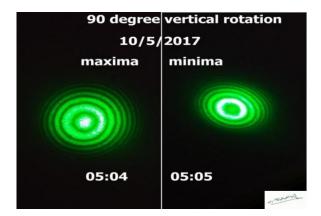
Then, changing the distance from a large gravity well will change the nature of the empty space owing to gravitational blueshift; thus, we should detect a notable interference pattern.

We could detect this by setting up a vertical Michelson–Morley experiment relative to Earth and not (parallel or horizontal) to Earth. In this way, when we rotate Michelson's interferometer by 90 °, we should observe a significant change due to gravitational **redshift** and **blueshift**, which respond to the change in the speed of light, as follows:

$$c^{`} = \frac{1}{\sqrt{\epsilon_{\circ} \mu_{\circ}}} = \frac{1}{\sqrt{\epsilon_{\circ} \mu_{\circ} \left(1 - \frac{r_{s}}{r}\right)}} \Rightarrow c^{`} = c \left(1 - \frac{r_{s}}{r}\right)^{-1/2}$$

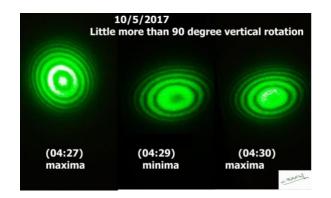
This is not a new thing that was made before in the Pound-Rebka experiment.

For the 90° rotation, I confirmed a positive change in the central interference pattern from the maxima to the minima^{[193],[T]} as in the next images.



For rotations greater than 90° rotations, there was a positive change in the central interference pattern from the maxima to the minima to the maxima.

^T In fact, there is a german physics enthusiastic his name is Mr. Martin Grusenick he made the second working vertical moving Michelson–Morley experiment in 2009 after (Professor C. Y. Lo) in 2003 and both works of Mr. Martin Grusenick and Professor C. Y. Lo should be noticed but both of them could not figure it out Mr. Martin Grusenick even put a full demonstration and documentation on YouTube for his experiment with full results but his work was used by pseudoscience on the internet a lot, the video name is "Extended Michelson–Morley Interferometer experiment. English version" (https://youtu.be/7T0d7o8X2-E), for Professor C. Y. Lo his work was published in Chinese Journal of Physics(https://www.sciencedirect.com/journal/chinese-journal-of-physics) the magazine is now owned by Elsevier Group since 2016, so all previous issues are not available thanks God the professor's work was on his page on researchgate (https://www.researchgate.net/publication/252315461 Space Contractions Local Light Speeds and the Question of Gauge in General Relativity).



We can perform an ordinary working horizontal Michelson–Morley experiment; however, next to a large mountain chain, the mass of the mountain chain will likely act like a runaway gravity well, and we will still experience a positive change in the interference pattern.

However, detecting the spacetime hoofing effect is much more difficult because it depends on the movement of the gravity well; therefore, I use a vertical nonrotating interferometer in which the horizontal arm is oriented north or south to eliminate the Sagnac effect; this setup should be sufficient.

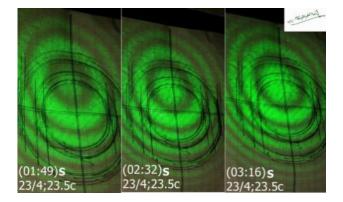
The justification for this is as follows:

The earth is a gravity well, and because it revolves around the sun, it should gain a relative mass in the direction of movement. According to my work, the gravitational potential should be different for a fixed observer; however, because the Earth revolves around itself, the velocity rate of this movement is related to an observer on the surface of the Earth, as it changes with this rotation as follows:.

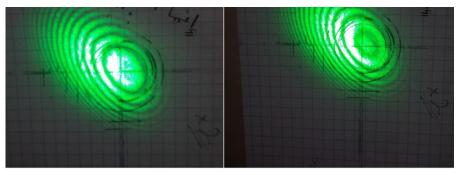
$$g_{B} = \frac{GM_{B}}{r^{2}}; M_{B} = M(\gamma \cos{(b)}); M \ge \frac{m_{P}}{2} \therefore M = \frac{m_{P}}{2}, n = 1, 2, 3 \cdots; \gamma = \frac{1}{\sqrt{1 - \frac{v^{2}}{c^{2}}}}; 0 \le b \le \pi$$

I obtained many results considering the same temperature and minimum elapsed time.

The following are some of these results that occurred on 23/4/2017, and I put the horizontal arm orientation to the south. The three images show a gradual change in the interference pattern from the minima to the maxima over a time period of 87 min at a steady temperature of $23.5^{\circ C}$.



I made a different setup to detect spacetime hoofing by placing the interferometer in a V orientation such that both arms were fixed at 45°: the first arm was oriented to the north and the second arm was oriented to the south; that is, there was no Sagnac effect, and I detected only the effects of spacetime hoofing on the following images on 2017/05/19. The time and temperature of each image were recorded. The first image shows the maximum interference, and the second image shows a minimum interference time period of 40 min and a temperature change of 0.4c.



^{19:05 ; 26.3}c°

19:45 ; 25.9c°

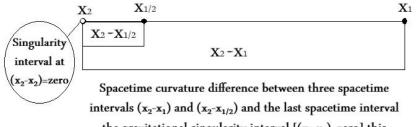
5. Conclusions

1. Gravity is not a force; in fact, gravity is the difference in spacetime curvature between two points in space due to differences in energy density distribution through this interval only and only if the collective masses are equal to or greater than half the Planck mass, such a mass cannot be produced without quantum entanglement, such that the half Planck mass is the minimum requirement for creating any curvature in spacetime fabric.

2. The (T) condition, that is, the effects of the half_Planck mass and its multiples, affects only the difference in spacetime curvature between two intervals, since physical constants are measured only under symmetric conditions and are considered constants in the first place, that is, physical constants according to the definitions are not affected by the (T) condition, that is, the effects of the half_Planck mass and its multiples because these constants are measured from a local perspective, that is, there is no difference in spacetime curvature between the observer and the point of measurement, that is, both points are in flat spacetime relative to each other, and this is the meaning of the local observer or local perspective and the nonlocal perspective or nonlocal observer.

For example, a photon falling towards a black hole will always have a constant speed of (c), but there will be an illusion for some observers of a superluminal speed or a faster than light photons^[194] because spacetime itself collapses towards the singularity of the black hole itself, that is, it is not the speed of the photon, but it is the speed of space time itself being mistaken by a nonlocal observer as the speed of this photon, that is, it is an illusion.

$$(x_2) - (x_1) > (x_2) - (x_{1/2}) > [(x_2) - (x_2) = 0] \Rightarrow r_s = \sqrt{\frac{2MG}{c^2} \left(1 - \frac{r_s}{r}\right) \dots 1.3}$$



tervals (x₂-x₁) and (x₂-x_{1/2}) and the last spacetime interval the gravitational singularity interval [(x₂-x₂)=zero] this difference between these intervals is the source of the illusion of superluminal speed in general relativity

[195],[196],[197],[198],[199],[200],[201]

$$\because \mathbf{c} = \mathbf{c} \left(1 - \frac{\mathbf{r}_{s}}{\mathbf{r}}\right)^{-1/2}$$

When both the observer and the event are in flat spacetime in relative to each other

i.e.,
$$r_s = zero \Rightarrow c = c \left(1 - \frac{zero}{r}\right)^{-1/2}$$

 $\Rightarrow c = c$

All the physical constants are consistent as long as the symmetry is unbroken; in fact, the (T) condition is an excellent example for Noether's theorem of the "Invariant Variations problem" ^{[202],[203],[204]} since the effect of the (T) condition will lead to broken symmetry as long measured or observed from a point with a different spacetime curvature. ^{[205],[206],[207],[208],[209],[210]}

Thus, when the difference in the spacetime curvature between the Schwarzschild radius and the dimensions of the mass in question, when the difference between the Schwarzschild radius and the dimensions of the mass in question, and when the difference between the Schwarzschild radius and the dimensions of the mass in question becomes small, the gravity effect becomes increasingly large.

Because elementary particles do not meet the (T) condition, that is, half Planck mass cannot affect spacetime, but a group of quantum entangled elementary particles with a collective mass greater than or equal to half the Planck mass will curve spacetime; that is, if there is no quantum entanglement, there is no gravity, that is, in principle, we could switch off the gravity of any mass if we break the entanglement of each particle of that mass until we reach less than the (T) condition. This is why we have the missing mass problem of the proton because we never accelerated any mass that obeys the (T) condition, that is, quantum entangled masses equal to half Planck mass or more.

In other words, an electron travelling through a double-slit experiment does not affect spacetime; instead, a cluster of elementary particles with a mass equal to or greater than half the Planck mass bonded by quantum entanglement will bend spacetime as it travels through space. When it passes through the double-slit experiment, its wavefunction will change, and its gravitational effect will also change, obeying the difference in the energy density distribution through spacetime.

3. The hooving effect occurs when a mass with a (T) condition moves at a relative speed, and the relativistic mass gained from this speed will receive additional gravitational potential in the direction of the velocity, which

will create controlled artificial gravity and can be used to create gravitational singularities for nuclear fusion and pseudo_superluminal speed travel as follows:

$$g_{Br} = \frac{GM_{Br}}{r^2}$$
; $M_{Br} = M(\gamma \cos{(B)})$; $M \ge \frac{m_P}{2} \therefore M = \frac{m_P}{2}$, $n = 1, 2, 3 \cdots$; $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$; $0 \le B \le \pi$

 $v \equiv$ the velocity of the gravity well

$$\therefore \Rightarrow c` = \left(\frac{GM}{r^2}t \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\cos(B) + c}\right); t \equiv \text{acceleration time}$$
$$at B = 0 \therefore \Rightarrow c` = \left(\frac{GM}{r^2}t\gamma + c\right)$$

However, if the two colliding beams with (T) conditions did not produce a singularity, then the Lorentz invariance party (L.I.) is correct.

We will have a different rate of gravitational time dilation that varies with respect to the angle, which will create gravity around the accelerated mass, that is, artificial antigravity. The hoofing effect is not a warp drive because warp drives depend on exotic matter distributed unevenly through spacetime, whereas the hoofing effect depends on adding relativistic mass to the total mass through the use of the (T) condition, that is, the half-Planck mass, because the relativistic mass depends on the direction of movement, we create artificial gravity that can be used in superluminal travel and in creating artificial gravitational singularities that can be used in nuclear fusion; quantum interaction is limited by one of two factors, either causal speed limits, that is, the speed of light or any theoretical exotic causal connection, such as the tachyonic field^{[211],[212]}, or through quantum entanglement, which is causally unbound to spacetime because of its ability to disobey and break the light cones and even transfer information from the future to the past^{[213],[214],[215],[216],[217],[218]}

This is not the case for gravitational effects; in fact, for any gravitational effect, we have spacetime expanding on a universal scale faster than light on a specific gradually increasing ratio^{[219],[220]}.

In the event horizon of black holes, spacetime collapses with the speed of light^{[221],} and as a consequence, because spacetime collapses below the event horizon faster than light^[222] with a gradually increasing ratio, we have images of the event horizon for some of these black holes^{[223],[U]} we have to accept that quantum gravity and string theories are disproved by the superluminal experimental observation that spacetime is a real entity and not just a mathematical perspective; it is a physically continuous non-discrete unquantized fabric of four dimensions.

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^[U] In fact the touching outside of event horizon but still this is an undeniable experimental observations

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