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HOW MANY FUNDAMENTAL CONSTANTS ARE NECESSARY AND SUFFICIENT TO EXPRESS ALL PHYSICAL LAWS AND PARAMETERS OF THE UNIVERSE?

Abstract. *The divergent views of Duff, Okun and Veneziano on the number of fundamental constants in nature are examined from a new perspective. It is shown that the problem of the minimum number of dimensional fundamental constants can be solved by choosing the fundamental constants of the electron as the primary and independent constants. Three dimensional constants of the electron* (**m_e, r_e, t_e**) *and two dimensionless constants, the fine structure constant "***alpha***" and the large Weyl number* (**D_0 = 4.16561... x 10^42**)*, are proposed as a complete basis for independent fundamental constants. Numerous examples have shown that the fundamental constants* (**m_e, r_e, t_e, α, Do**) *are the primary basis for both physical constants and parameters of the Universe. The parameters of the Universe, physical constants, and large Weyl-Eddington-Dirac numbers originate from the primary fundamental constants* (**m_e, r_e, t_e, α, Do**)*. Five fundamental constants* (**m_e, r_e, t_e, α, Do**) *are sufficient to express the entire set of observable physical laws. Veneziano's statement about the non-fundamental status of the constants* **G, ħ** *is confirmed. Duff's statement about the non-fundamental status of all three constants* **G, ħ, c** *is confirmed. Duff's statement about the zero number of dimensional fundamental constants in Nature is not confirmed.*

Keywords: *fundamental constants; Parameters of the observable universe; large Weyl number; electron constants; cosmological constant.*

1. Introduction.

In 2002, M. J. Duff, L. B. Okun and G. Veneziano published an article in which each of the authors presented their views on the number of dimensional fundamental constants in Nature [1]. Okun believes that three dimensional constants are fundamental: Planck's constant, h, the speed of light, c, and Newton's constant, G. Duff and Veneziano disagree with him. Veneziano does not believe that G and ħ are fundamental. Veneziano believes that two constants are fundamental: the length of a string and c. Duff believes that all three dimensional constants G, ħ, c are not fundamental.

The article [1] caused a great resonance. The dispute does not cease and is mainly about the constants G, ħ, c. The magic of the constants G, ħ, c, supported by the Planck units l_P, m_P, t_P, does not allow the supporters of the three fundamental constants G, ħ, c to abandon them and take a bold step towards other constants. Sometimes the electron charge **e** appears in the trio of constants. Then their apparent fundamentality is supported by Stoney units.

The non-fundamental nature of the constants G, ħ, c is indicated by Hoyle, F. and Narlikar, J. V. [2], Jeffreys, H. [3], McCrea W. [4], Wesson, Paul S. [5]. According to Duff, M. J [6] and Wesson, Paul S. [5], the parameters c, G and h are simply artificial constants of dimensional transformation and act as coefficients. At the same time, these "only coefficients" are included in the formulas of the fundamental laws of Nature, which significantly strengthens the position of the supporters of the fundamental status of the dimensional constants G, h, c and weakens the position of their opponents.

The idea of Duff, M. J about the fundamentality of only dimensionless constants is very tempting. At the same time, the fundamentality of constants must be considered in their connection with both the fundamental laws of nature and fundamental physical objects. Will there be a place for dimensionless constants in the fundamental laws of nature? And what will the fundamental laws of nature look like, for example, Newton's law of gravity, if represented using dimensionless constants? Nobody knows. It is unconvincing to talk about the fundamentality of dimensionless constants without taking into account their connection with the fundamental laws of nature. In addition, there are a lot of dimensionless constants! Which of them can claim to be fundamental? There is no theory of fundamental constants. Tempting ideas about the fundamentality of only dimensionless constants are still at the hypothetical stage.

M. J. Duff, L. B. Okun and G. Veneziano could not agree on which constants should be considered fundamental. Over the past 22 years, the situation has not changed. The problem of fundamental constants remains unsolved.

2. Five primary fundamental constants from which the physical constants and parameters of the Universe originate.

According to Okun, the number of dimensional fundamental physical constants should be equal to the number of basic physical units. This requirement is clearly not sufficient for the correct choice of specific fundamental physical constants. Four additional requirements must be met:

1. Physical constants with fundamental status must be constants (parameters) of a fundamental physical object. The constants G, ħ, c do not satisfy this requirement.

2. Physical constants with fundamental status must not have complex dimensions. Ideally, they should have dimensions that coincide with the basic physical units. The constants G, ħ, c do not satisfy this requirement.

3. Secondary physical constants must originate from physical constants with fundamental status.

4. Fundamental constants must be included in fundamental physical laws.

These additional requirements significantly limit the number of candidates for fundamental status. The Electron claims the role of a fundamental physical object, the owner of fundamental physical constants. The constants of the electron do not have a complex dimension. Among the constants of the electron, it is easy to select a number of constants that coincides with the number of basic physical units. The following three constants of the electron satisfy all five requirements:

$$
\begin{aligned}
m_e &= 9.1093837139\ \ldots \bullet 10^{-31}\,kg\\
r_e &= 2.8179403205\ \ldots \bullet 10^{-15}\,m\\
t_e &= 0.9399637133\ \ldots \bullet 10^{-23}\,s
\end{aligned}
$$

Fig.1. Three dimensional fundamental constants. Where: m_e is the electron mass; r_e is the classical electron radius; t_e is the characteristic time of the electron (the time during which light travels the distance r_e.)

We consider these three constants m_e, r_e, t_e as the minimum number of dimensional fundamental physical constants. These physical constants are primary and independent. At the same time, only these three fundamental constants are not enough to express the entire set of observed physical laws. For this, an additional minimal basis of dimensionless constants is needed.

The following dimensionless constants satisfy four additional requirements:

$$
\begin{aligned}
\alpha &= 7.2973525643\ \ldots \bullet 10^{-3}\\
D_0 &= 4.16561 \ldots \bullet 10^{42}
\end{aligned}
$$

Fig.2. Two dimensionless fundamental constants. Where: α is a fine-structure constant; D_0 is a large Weyl number.

The complete group of independent fundamental constants (Fig. 3) contains three constants m_e, r_e, t_e and an additional subgroup of two dimensionless fundamental constants α and D_0. These five fundamental constants are sufficient to obtain other physical constants and parameters of the Universe. These five constants are sufficient to express the entire set of observed physical laws.

$$
\begin{aligned}
&1.\quad m_e = 9.1093837139\ \ldots \bullet 10^{-31}\,kg\\
&2.\quad r_e = 2.8179403205\ \ldots \bullet 10^{-15}\,m\\
&3.\quad t_e = 0.9399637133\ \ldots \bullet 10^{-23}\,s\\
&4.\quad \alpha = 7.2973525643\ \ldots \bullet 10^{-3}\\
&5.\quad D_0 = 4.16561 \ldots \bullet 10^{42}
\end{aligned}
$$

Fig.3. Five primary fundamental constants.

3. The large Weyl number ($D_0 = 4.16561...x\ 10^{42}$) as a dimensionless fundamental physical constant.

We introduce the dimensionless constant $D_0 = 4.16561...x\ 10^{42}$ in a new rank into the constant basis. I consider it as a fundamental constant. The constant $D_0 = 4.16561...x\ 10^{42}$ is not in the CODATA list. Nevertheless, the large number D_0 is a long-known constant of the electron. This is the ratio of the electrostatic Coulomb force to the gravitational force. H. Weyl was the first to draw attention to this large number more than 100 years ago. He also drew attention to the incredibly large number of coincidences of large numbers [7 - 13]. H. Weyl obtained the number 4 × 10^42 as the ratio of the electric force to the gravitational force between two electrons. H. Weyl put the number 4×10^{42} in importance on a par with the fine structure constant "alpha". Theorists paid little attention to this number. They underestimated the importance of the large Weyl number as a dimensionless constant. The connection of the large number $D_0 = 4.16561...x\ 10^{42}$ with other large numbers remained undisclosed for a long time. For more than 100 years, this dimensionless fundamental constant undeservedly remained "on the outskirts" of physics.

4. The formula for Planck's constant "ħ", represented by fundamental constants.

Let us show that the constants **m_e, r_e, t_e, α, Do** are primary. All other constants of physics and cosmology come from them. The formula for Planck's constant can be represented using 4 fundamental constants m_e, r_e, t_e, α:

$$\hbar = \frac{m_e r_e^2}{t_e \alpha} = 1.054571817... \bullet 10^{-34}\, Js \qquad (1)$$

5. The formula of the gravitational constant G, represented by fundamental constants

The formula of the gravitational constant G can be represented using 4 fundamental constants me, re, te, D0:

$$G = \frac{r_e^3}{t_e^2 m_e D_0} = 6.67430... \bullet 10^{-11} kg^{-1} m^3 s^{-2} \qquad (2)$$

In addition, the three constants G, ħ, c are interdependent:

$$G = \frac{\hbar c \alpha}{m_e^2 D_0} = 6.67430... \bullet 10^{-11} kg^{-1} m^3 s^{-2} \qquad (3)$$

The functional dependence of "G" on other constants clearly indicates its non-fundamental nature. The fact that one of the three constants "G", "c" or "h" can be excluded from the number of fundamental constants is noted by the authors in [14].

Additional confirmation that the gravitational constant G is not fundamental is the new form of the law of gravitational interaction. The new law of gravitation does not contain the constant G Fig.4.:

$$F_K = \frac{mR^3}{T^2 r^2}$$

Fig. 4. The law of gravitational interaction without the gravitational constant G. Where: m is the mass of the body, R and T are orbit parameters, r is the distance.

For more than 300 years, the force of gravitational interaction was represented by a single physical law - Newton's formula $F_N = GmM/r^2$. The law of gravitation without any alternative gave rise to the illusion of the fundamental status of the constant G. The formula of the law of gravitation (Fig. 4) does not include the gravitational constant G and the large mass M. The formula includes the Kepler ratio R^3/T^2. At the same time, this law of gravitation is a complete equivalent of Newton's law of gravitation.

On the scale of the Universe, determining the values of the masses of bodies is a difficult task. Such parameters as distances and periods of revolution of bodies are known much more accurately. This is the advantage of the formula (Fig. 4) compared to Newton's formula $F_N = GmM/r^2$.

The formulas given above demonstrate the secondary status of the constants G, ħ, c. This confirms the statement of Duff and Venezivno that G and ħ are not fundamental constants [1].

6. Formula of elementary electric charge e, represented by fundamental constants

According to Wesson, Paul S. charge e is not a fundamental constant [5]. The five-constant basis confirms this. The formula of elementary electric charge e can be represented by 3 fundamental constants m_e, r_e, t_e:

$$e = \pm\sqrt{4\pi\varepsilon_0 m_e r_e^3 / t_e^2} = 1.602176634... \bullet 10^{-19} C \qquad (4)$$

where: e is the electric charge of the electron, m_e is the electron mass, r_e is the classical radius of the electron, t_e is time, $4\pi\varepsilon_0$ is the coefficient for representing the electric charge in the SI system.

The electric charge and mass of the electron are related to each other by the Kepler ratio. Formula (4) includes the Kepler ratio for the electron, represented as r_e^3/t_e^2. The signs "±" in front of the square root give two types of electric charge equal in magnitude: positive charge and negative charge. The value of the elementary charge with the "-" sign is the charge of the electron, the value with the "+" sign is the charge of the positron. Formula (4) confirms the existence of a positive electron (positron) without a complex physical interpretation of the negative energy in the Dirac equation [15].

7. Formulas Planck length l_P, Planck mass m_P, Planck time t_P, represented by fundamental constants.

Fundamental constants m_e, r_e, t_e, α, Do confirm the dependent status of Constants in the category "Universal constants" [16]. For example, this applies to Planck units:

$$m_P = \sqrt{\frac{\hbar c}{G}} = 2.176434... \bullet 10^{-8} kg \quad (5)$$

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616255... \bullet 10^{-35} m \quad (6)$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.391247... \bullet 10^{-44} s \quad (7)$$

The fundamental constants **m_e, r_e, t_e, α, Do** allow us to represent Planck units in the following form:

$$m_P = m_e \sqrt{\frac{D_0}{\alpha}} = 2.176434... \bullet 10^{-8} kg \quad , \quad (8)$$

$$l_P = \frac{r_e}{\sqrt{\alpha D_0}} = 1.616255... \bullet 10^{-35} m \quad , \quad (9)$$

$$t_P = \frac{t_e}{\sqrt{\alpha D_0}} = 5.391247... \bullet 10^{-44} s \quad . \quad (10)$$

Formulas (5), (6), (7) and formulas (8), (9), (10) are equivalent.

8. Stoney units formulas (m_S, l_S, t_S) represented by fundamental constants

Stoney units are formed by a combination of constants G, e, c [17]:

$$m_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} = 1.8592... \bullet 10^{-9} kg \quad (11)$$

$$l_S = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^4}} = 1.3807... \bullet 10^{-36} m \quad (12)$$

$$t_S = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^6}} = 4.6054... \bullet 10^{-45} s \quad (13)$$

The fundamental constants m_e, r_e, t_e, Do allow us to represent the Stoney units in the following form:

$$m_S = m_e \sqrt{D_0} = 1.8592 ... \bullet 10^{-9} kg \quad , \quad (14)$$

$$l_S = \frac{r_e}{\sqrt{D_0}} = 1.3807... \bullet 10^{-36} m \quad , \quad (15)$$

$$t_S = \frac{t_e}{\sqrt{D_0}} = 4.6054...\bullet 10^{-45} s$$.(16)

From equations (8) - (16) it follows that Planck units and Stoney units are related to each other via the fine structure constant alpha (√α).

$$\frac{m_S}{m_P} = \sqrt{\alpha}\,, \quad \frac{l_S}{l_P} = \sqrt{\alpha}\,, \quad \frac{t_S}{t_P} = \sqrt{\alpha}\,. \qquad (17)$$

9. Formulas of Rydberg constants, von Klitzing constants, characteristic impedance of vacuum, represented by fundamental constants.

Using the five-constant fundamental basis, the Rydberg constant is reduced to the formula:

$$R_\infty = \alpha^2 m_e c / 2h = \alpha^3 / 4\pi r_e \qquad (18)$$

Using the five-constant fundamental basis, the von Klitzing constant is reduced to the formula:

$$R_K = h / e^2 = t_e / 2\alpha\varepsilon_0 r_e \qquad (19)$$

The characteristic impedance of vacuum has the following formula:

$$Z_0 = t_e / \varepsilon_0 r_e \qquad (20)$$

10. Coulomb's law for the force of electrostatic interaction between two electrons, represented by fundamental constants

Coulomb's law for the force of electromagnetic interaction between two electrons can be represented using 3 fundamental constants m_e, r_e, t_e:

$$F_e = m_e r_e^3 / t_e^2 r^2 \qquad (21)$$

Formula (21) includes the constant of the force of electromagnetic interaction:

$$F_0 = m_e r_e \;/ t_e^2 = 29.0535101\, N \qquad (22)$$

11. Newton's law of gravitation, represented by fundamental constants

The formula for Newton's law of gravitation can be represented using 4 fundamental constants m_e, r_e, t_e, D_0:

$$F_N = \frac{r_e^3 m M}{t_e^2 m_e D_0 r^2} \qquad (23)$$

From equation (23) it follows that the formula for Newton's law of gravitation can be represented using dimensionless parameters:

$$F_N = F_0\left(\frac{\frac{m}{m_e}\bullet\frac{M}{m_e}}{\frac{r^2}{r_e^2}\bullet D_0}\right) = 29.0535101\bullet\left(\frac{k_m k_M}{k_r^2 D_0}\right) \qquad (24)$$

The dimensionless parameters k_m, k_M, k_r are represented by the ratios of the parameters m, r, M to the fundamental constants m_e, r_e. This is another form of representing Newton's law of gravitation. I draw attention to the fact that the constant in the law of gravitation is not the gravitational constant G, but the constant of electromagnetic nature (Fo = 29.0535101 N). Coulomb's law has the same form. The only difference is in the constant D_0:

$$F_{Coulomb} = F_0\left(\frac{\frac{q_1}{e}\bullet\frac{q_2}{e}}{\frac{r^2}{r_e^2}}\right) = 29.0535101\bullet\left(\frac{k_1 k_2}{k_r^2}\right) \qquad (25)$$

Formulas (24) and (25) demonstrate the deep connection between electromagnetism and gravitation. The formulas for the gravitational force and the electromagnetic force differ only in the scale factor D_0 = 4.16561... x 10^42.

The constant of the gravitational force is represented by 4 fundamental constants m_e, r_e, t_e, D_0:

$$F_{ps} = \frac{Gm_e^2}{r_e^2} = \frac{r_e\, m_e}{t_e^2 D_0} = \frac{F_0}{D_0} = 6.97461...\bullet 10^{-42}\ N \qquad (26)$$

12. The law of cosmological force represented by fundamental constants

The additional cosmological force, which does not follow from Newton's law of gravity, can be represented using the fundamental constants m_e, r_e, t_e, α, Do:

$$F_{Cos} = mc^2\sqrt{\Lambda} = \frac{m\ r_e}{t_e^2\alpha D_0} \qquad (27)$$

For an electron, the cosmological force is represented by the constant:

$$F_{Cos(e)} = \frac{m_e r_e}{t_e^2\alpha D_0} = 9.55773...\bullet 10^{-40}\ N \qquad (28)$$

For the Universe, the cosmological force is equal to the Planck force:

$$F_{Cos(U)} = \frac{M_U c^2}{r_e\ \alpha D_0} = 1.21027...\bullet 10^{44}\ N \qquad (29)$$

In formula (29), the combination of constants c^2/reαDo is the cosmological acceleration. The cosmological acceleration constant Ao is very close to the acceleration value obtained in the MOND theory [18]:

$$A_0 = \frac{r_e}{t_e^2 \alpha D_0} = 10.4922... \bullet 10^{-10} m/s^2 \qquad (30)$$

13. Large Numbers Represented by Dimensionless Fundamental Constants

The history of large numbers began with two large numbers 10^{40} and 10^{42}. H. Weyl was the first to pay attention to these numbers. He obtained the ratio of the radius of the Universe to the radius of the electron ($R_U/r_e \approx 10^{40}$), leading to a large number of the order of 10^{40} [8 - 13]. The number 4 × 10^42 was obtained as the ratio of the electric force to the gravitational force between two electrons. After H. Weyl, many prominent scientists (A. S. Eddington, P. A. M. Dirac, Stewart J., O, S. Weinberg, Rice J., E. Teller) paid attention to large numbers of other scales. Muradyan, R. M. gives ratios of dimensional quantities that give large numbers of the scale of 10^{60}, 10^{120} [19]. Pierre-Henri Chavanis showed that the ratios of the masses of macroobjects and microobjects to the Planck mass yield large numbers of the order of 10^±20, 10^±30 10^±40 10^±60 [20].

The family of large numbers is not limited to the scales listed above. The coincidences of large numbers show that the family of large numbers must be extended to scales of 10^{100}, 10^{140}, 10^{160}, and 10^{180}. As a result, the family of large numbers covers the range of scales from 10^{20} to 10^{180}. The large number 10^{180} is formed by the ratio of the volume of the Universe to the Planck volume. Large numbers for scales from 10^{20} to 10^{180} and formulas for their calculation are shown in Fig. 5. All large numbers are functionally dependent on two fundamental constants: the fine structure constant "alpha" and the large Weyl number Do.

$$(\sqrt{\alpha D_0})^0 = 1$$
$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349... \bullet 10^{20}$$
$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979... \bullet 10^{40}$$
$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987... \bullet 10^{60}$$
$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80}$$
$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100}$$
$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120}$$
$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140}$$
$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160}$$
$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180}$$

Fig. 5. Large numbers and formulas for their calculation.

14. Formulas for the parameters of the Universe, represented by fundamental constants

Five primary fundamental constants (**m_e, r_e, t_e, α, Do**) allow us to represent the parameters of the Universe with beautiful formulas:

$$M_U = m_e \alpha D_0^2 = 1.15348... \bullet 10^{53}\ kg$$

$$R_U = r_e \alpha D_0 = 0.856594... \bullet 10^{26}\ m$$

$$T_U = t_e \alpha D_0 = 2.85729\ ... \bullet 10^{17}\ s$$

$$\Lambda = \frac{1}{r_e^2 \alpha^2 D_0^2} = 1.36285\ ... \bullet 10^{-52}\ m^{-2}$$

$$A_0 = \frac{r_e}{t_e^2 \alpha D_0} = 10.4922\ ... \bullet 10^{-10}\ m/s^2$$

Fig. 6. Parameters of the Universe, represented by fundamental constants **m_e, r_e, t_e, α, Do**. Where: M_U is the mass of the Universe; R_U is the radius of the Universe; T_U is the time of the Universe; Λ is the cosmological constant; A_0 is the cosmological acceleration; m_e is the electron mass; r_e is the classical electron radius; t_e is the characteristic time of the electron; α is the fine-structure constant; D_0 is the large Weyl number.

15. The dimensionless parameter of the Standard Model GhΛ/c^3 ≈10^{-120}, represented by fundamental constants

The dimensionless Cosmological parameter GhΛ/c^3 is known, which gives a large number of the order of 10^{-120} [6, 21, 22]. This dimensionless cosmological parameter, represented by the dimensionless fundamental constants, leads to the exact value of the large number of scale 10^{120}:

$$\frac{c^3}{G\hbar\Lambda} = \alpha^3 D_0^3 = 28.088... \bullet 10^{120} \qquad (31)$$

The value of the cosmological constant:

$$\Lambda = 1.36285... \bullet 10^{-52} m^{-2} \qquad (32)$$

There are many other combinations of the parameters of the Universe that yield dimensionless cosmological parameters. For example, the combination of constants G, c, M_U, Λ yields the large number of scale 10^{160}. This dimensionless cosmological parameter is also represented by the dimensionless fundamental constants α and D_0:

$$\frac{M_U c^2 \alpha^2}{\sqrt{\Lambda} \bullet G m_e^2} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160} \qquad (33)$$

The value of the mass of the Universe:

$$M_U = 1.15348...\bullet 10^{53} kg \tag{34}$$

The combination of constants M_U, Λ, m_e, α, r_e gives a large scale number 10^{40}:

$$\frac{M_U \alpha\ r_e \sqrt{\Lambda}}{m_e} = (\sqrt{\alpha D_0})^2 = 3.03979...\bullet 10^{40} \tag{35}$$

16. Rice formula, represented by fundamental constants

The approximate Rice cosmological equation [23] has the form:

$$\frac{4\pi}{\alpha} \approx \frac{r_e^2 c^2}{6 R_U G m_e} \tag{36}$$

The exact Rice equation, represented by fundamental constants, has the form:

$$\frac{r_e^4}{t_e^2 R_U G m_e} = \frac{1}{\alpha} \tag{37}$$

17. Planck units formulas represented by the parameters of the Universe.

To demonstrate the secondary status of the Planck units, here I show a third version of the representation of Planck units. This time, the Planck units are obtained using the parameters of the Universe:

$$m_P = \frac{M_U}{\sqrt{\alpha^3 D_0^3}} = \frac{c^2}{G\sqrt{\Lambda}\sqrt{\alpha^3 D_0^3}} = 2.176434...\bullet 10^{-8} kg\,, \tag{38}$$

$$l_P = \frac{GM_U}{c^2\sqrt{\alpha^3 D_0^3}} = \sqrt{\frac{GM_U}{A_0 \alpha^3 D_0^3}} = \frac{\sqrt{\alpha^9 D_0^9}\Lambda r_e^6}{GM_U T_U^2} = \frac{GM_U^2\sqrt{\Lambda}\ r_e \alpha}{c^2 m_e \sqrt{\alpha^5 D_0^5}} = 1.616255...\bullet 10^{-35} m\,, \tag{39}$$

$$t_P = \frac{GM_U}{c^3\sqrt{\alpha^3 D_0^3}} = \sqrt{\frac{GM_U}{A_0^2 R_U \alpha^3 D_0^3}} = \frac{\sqrt{\alpha^9 D_0^9}\Lambda r_e^6}{GM_U T_U^2 c} = \frac{GM_U^2\sqrt{\Lambda}\ r_e \alpha}{c^3 m_e \sqrt{\alpha^5 D_0^5}} = 5.391247...\bullet 10^{-44} s \tag{40}$$

The Planck units are not independent. For this reason, they cannot claim fundamental status.

18. Conclusion

Okun' correctly defined the minimum number of dimensional fundamental constants. But he made a mistake in choosing their personal composition. None of the constants G, ħ, c proposed by Okun' is fundamental. It is shown that all three dimensional constants proposed by Okun' are not independent. For this reason, they cannot claim fundamental status. The Planck units or Stoney constants are also not fundamental. The personal composition of fundamental constants is outside the Constants in the category "Universal constants" [16].

Dimensional fundamental constants alone are not enough to express the entire set of observed physical laws. The complete group of independent fundamental constants additionally contains a subgroup of two dimensionless fundamental constants. The following five constants claim fundamental status: m_e, r_e, t_e α and D_0. These are constants of the fundamental physical object - the electron. They have simple dimensions. The number of dimensional fundamental constants is equal to the number of basic physical units. Secondary physical constants derive from the five constants m_e, r_e, t_e α and D_0. The parameters of the Universe originate from the five constants m_e, r_e, t_e α and D_0. The formulas of fundamental physical laws can be represented by the constants m_e, r_e, t_e α and D_0.

Of the five fundamental constants, only three (m_e, r_e, α) are included in the CODATA list. Two fundamental constants t_e and D_0 are not included in the CODATA list. I draw attention to the underestimated and unreasonably forgotten large Weyl number (D_0 = 4.16561... x 10^42). For more than 100 years, this dimensionless fundamental constant has unfairly remained "on the margins" of physics. The numerous examples above demonstrate the key role of the large Weyl number D_0 .

19. Conclusions

1. None of the constants G, ħ, c proposed by Okun are fundamental.

2. The complete group of independent fundamental constants contains a subgroup of three dimensional fundamental constants and a subgroup of two dimensionless fundamental constants.

3. The independent fundamental constants are the three constants of the electron (me, re, te) and two dimensionless constants: the fine structure constant "**alpha**" and the large Weyl number (**D_0=4.16561... x 10^{42}**).

4. Only three dimensional fundamental constants are not enough to express the entire set of observed physical laws.

5. Only two dimensionless fundamental constants are not enough to express the entire set of observed physical laws.

6 The five primary fundamental constants **m_e, r_e, t_e, α, Do** are the primary basis of physical constants and parameters of the Universe.

7. All dimensional physical constants that do not have a fundamental status are functionally dependent on the primary five-constant basis. They originate from the fundamental constants **m_e, r_e, t_e, α, Do**.

8. All parameters of the Universe are functionally dependent on the primary five-constant basis. They originate from the fundamental constants **m_e, r_e, t_e, α, Do**.

9. All dimensionless physical constants and dimensionless parameters of the Universe are a function of two dimensionless constants from the primary five-constant basis. They originate from the constant "**alpha**" and the large Weyl number **Do**.

10. All large Weyl-Eddington-Dirac numbers are a function of two dimensionless constants from the primary five-constant basis. They originate from the constant "**alpha**" and the large Weyl number **Do**.

11. The statements of Duff and Veneziano about the non-fundamental status of the constants **G, ħ, c** are confirmed.

12. Duff's assertion about the zero number of dimensional fundamental constants in Nature is not confirmed.

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