Modification of the complex scalar field Lagrangian in FLRW space-time and its cosmological outcomes.

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The most popular approach of understanding the cosmic evolution is Einstein's General Relativity (GR). GR is the classical gateway to obtain the dynamics of the cosmic fluid and it can at best describe the macroscopic evolution of the Universe. In this work, we have accomplished an alternative tool to examine the microscopic dynamics of the cosmic fluid particles via quantum field theory (QFT). We have proposed a cosmic fluid model with complex scalar field Lagrangian corresponding to the modified Klein-Gordon equation (KG equation) of the scalar field in FLRW space-time. Following this Lagrangian, the cosmic fluid system has been quantized under certain restrictions on the parameters. This restrictions are used to determine the cosmic evolution pattern in the macroscopic level.
Keywords : Evolution of the Universe, Quantum field theory, Cosmology.

The mechanism behind the origin and evolution of the Universe is still an unsolved topic for the cosmologists. Although General relativity (GR) is the most widely accepted framework in the perspective of the cosmology, yet it has some serious theoretical issues [1–4]. GR is a completely classical tool and, hence it should not be applied in the case of very small length scales during the cosmic origin[5]. Therefore some alternative quantum models can be formulated to handle such situations. In some previous works[6–8], S.Maity along with different collaborators attempted to obtain the cosmic evolution pattern from the microscopic dynamics of the cosmic fluid. Canonical Quantization of the different modified cosmic fluid models has been prescribed and various aspects have been explored. In this letter we are demonstrating another such alternative model to obtain the cosmic evolution pattern from the quantum field dynamics of the cosmic fluid.

The Klein-Gordon (KG) equation of a real scalar Boson cosmic fluid system in the curved space-time is given by

$$\left(\nabla_\mu \nabla^\mu + m^2\right)\phi(X). = 0 \tag{1}$$

For flat FLRW space-time, KG equation leads to the following.

$$\ddot{\phi} + 3H\dot{\phi} - {\nabla'}^2\phi + m^2\phi = 0. \tag{2}$$

$H = \frac{\dot{a}}{a}$, the Hubble parameter. $\vec{\nabla}' = \frac{1}{a}\vec{\nabla}$, the gradient operator in the comoving frame. a is the scale factor of the Universe. The term $3H\dot{\phi}$ causes the dissipation of energy from the real scalar field similar to a damped oscillator.

Universe as an open quantum system :

In this work, we propose a pair of KG equations of a two - component scalar field (complex scalar field) such that one dissipates energy from the system while another gains energy .

$$\ddot{\phi}_0^\dagger + 3H\dot{\phi}_0^\dagger - {\nabla'}^2\phi_0^\dagger + m^2\phi_0^\dagger = -\frac{3}{2}\dot{H}\phi_0^\dagger \tag{3}$$

$$\ddot{\phi}_0 - 3H\dot{\phi}_0 - {\nabla'}^2\phi_0 + m^2\phi_0 = \frac{3}{2}\dot{H}\phi_0. \tag{4}$$

The net dissipation of the fields is given by $3H(\dot{\phi}_0 - \dot{\phi}_0^\dagger) - \frac{3}{2}\dot{H}(\phi_0^\dagger - \phi_0) = 2i[3H im(\dot{\phi}_0) - \frac{3}{2}\dot{H} im(\phi_0)]$. The net real dissipation is 0. Hence, in classical argument, the Universe is an isolated system. However, it is still an open quantum system [9–11] with an imaginary dissipation of the fields.

The equivalent Lagrangian of the cosmic fluid in Minkowski space-time and its canonical quantization:

This pair of KG equations can be obtained from a canonical Lagrangian density as

$$\mathcal{L} = \partial_\mu\phi_+^\dagger\partial^\mu\phi_- - \left(m^2 - \frac{9}{4}H^2\right)\phi_+^\dagger\phi_- \tag{5}$$

under Minkowski space-time. Here $\phi_\pm = \phi_0\exp\left(\pm\frac{3}{2}\int H dt\right)$. Notably the equations (3) and (4) can be written as $(\Box^2 + (m^2 - \frac{9}{4}H^2))\phi_\pm = 0$ where $\Box^2 \equiv \partial^\mu\partial_\mu$ in the Minkowski space-time. The key

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feature in this proposition is that the signature of the gravity has been included in the modified Lagrangian. The modified Lagrangian in the comoving Minkowski space-time $(t, -ax, -ay, -az)$ describes the KG equation identical to that of the general Lagrangian in the FLRW metric.

TABLE I. Dynamics of the scalar fields in Minkowski and FLRW metric

	Real scalar field	Complex scalar field	Nature of the oscillation of the scalar field
Minkowski	$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2,$ $(\Box^2 + m^2)\phi_0 = 0$ $\ddot{\phi}_0 - \nabla^2\phi_0 + m^2\phi_0 = 0$	$\mathcal{L}_0 = \partial_\mu\phi_0^\dagger\partial^\mu\phi_0 - m^2\phi_0^\dagger\phi_0,$. $(\Box^2 + m^2)\phi_0^\dagger = 0$,$(\Box^2 + m^2)\phi_0 = 0$ $\ddot{\phi}_0^\dagger - \nabla^2\phi_0^\dagger + m^2\phi_0^\dagger = 0,$ $\ddot{\phi}_0 - \nabla^2\phi_0 + m^2\phi_0 = 0$	Free oscillator
FLRW	$\mathcal{L} = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m^2\phi^2$ $\left(\nabla_\mu\nabla^\mu + m^2\right)\phi = 0$ $\ddot{\phi}^\dagger + 3H\dot{\phi} - {\nabla'}^2\phi + m^2\phi = 0$	$\mathcal{L} = \nabla_\mu\phi^\dagger\nabla^\mu\phi - m^2\phi^\dagger\phi$ $\left(\nabla_\mu\nabla^\mu + m^2\right)\phi^\dagger = 0,\quad \left(\nabla_\mu\nabla^\mu + m^2\right)\phi = 0$ $\ddot{\phi}^\dagger + 3H\dot{\phi}^\dagger - {\nabla'}^2\phi^\dagger + m^2\phi^\dagger = 0,\quad \ddot{\phi} + 3H\dot{\phi} - {\nabla'}^2\phi + m^2\phi = 0$	Damped oscillator

TABLE II. Modification of the KG equation and the corresponding modified Lagrangian

Modified KG equation of $\phi_0^\dagger$	Modified KG equation of ϕ_0	Effective Lagrangian for the modified KG equations
$\ddot{\phi}_0^\dagger + 3H\dot{\phi}_0^\dagger - {\nabla'}^2\phi_0^\dagger + m^2\phi_0^\dagger = -\frac{3}{2}\phi_0^\dagger\dot{H},$ Dissipation of energy with an in homogeneous term $-\frac{3}{2}\dot{H}\phi_0^\dagger$	$\ddot{\phi}_0 - 3H\dot{\phi}_0 - {\nabla'}^2\phi_0 + m^2\phi_0 = +\frac{3}{2}\dot{\phi}_0\dot{H},$. Gain of energy with an in homogeneous term $+\frac{3}{2}\dot{H}\phi_0$.	$\mathcal{L} = \partial_\mu\phi_+^\dagger\partial^\mu\phi_- - \left(m^2 - \frac{9}{4}H^2\right)\phi_+^\dagger\phi_-$ $\phi_\pm = \phi_0\exp\left(\pm\frac{3}{2}\int Hdt\right)$

The solutions of the KG equations can be written as

$$\hat{\phi}_0(X) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\frac{1}{\sqrt{2\omega}}d^3k'\left[\hat{a}(\vec{k}')e^{-i\int KdX} + \hat{b}^\dagger(\vec{k}')e^{+i\int KdX}\right], \tag{6}$$

$$\hat{\phi}_0^\dagger(X) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int\frac{1}{\sqrt{2\omega}}d^3k'\left[\hat{a}^\dagger(\vec{k}')e^{+i\int KdX} + \hat{b}(\vec{k}')e^{-i\int KdX}\right]. \tag{7}$$

Here $\int KdX = \int \omega(t)dt - \vec{k}.\vec{x}$, $\omega(t) = \sqrt{|\omega_0^2 - \frac{9}{4}H^2|}$,

$\omega_0 = \sqrt{k'^2 + m^2}$. One may introduce $\vec{x} \to \vec{x}' = a\vec{x}, \vec{k} \to \vec{k}' = \frac{\vec{k}}{a}$. $\vec{x}', \vec{k}'$ are the comoving space-coordinate and momenta respectively. Eventually $\vec{k}.\vec{x} = \vec{k}'.\vec{x}'$.

The corresponding momenta density of the fields of the Lagrangian density (5) will be $\pi_+ = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_+^\dagger} = \dot{\phi}_-$ and $\pi_- = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_-} = \dot{\phi}_+^\dagger$. Hence the corresponding Hamiltonian $h = \int d^3x' \left[\pi_+ \dot{\phi}_+^\dagger + \pi_- \dot{\phi}_- - \mathcal{L}\right]$ can be found in the form

$$h = \int d^3x' \left[\dot{\phi}_+^\dagger \dot{\phi}_- + \vec{\nabla}\phi_+^\dagger . \vec{\nabla}\phi_- + (m^2 - \frac{9}{4}H^2)\phi_+^\dagger \phi_-\right] \tag{9}$$

Now putting the value of $\phi_+^\dagger$ and ϕ_- in the expression of h, one finds the explicit form of the normal ordered Hamiltonian as,

$$: \hat{h} := \int d^3k' \frac{1}{2\omega} \left[\left(2\omega^2 - \frac{9}{4}H^2 - 3iH\omega \cosh\left(\frac{3}{2}\int Hdt\right)\right) \hat{a}_{\vec{k}'}^\dagger \hat{a}_{\vec{k}'} + \left(2\omega^2 - \frac{9}{4}H^2 + 3iH\omega \cosh\left(\frac{3}{2}\int Hdt\right)\right) \hat{b}_{\vec{k}'}^\dagger \hat{b}_{\vec{k}'}\right]$$
$$+ i\frac{3}{2}H \int d^3k' \sinh\left(\frac{3}{2}\int Hdt\right) \left(\hat{a}_{\vec{k}'}^\dagger \hat{b}_{-\vec{k}'}^\dagger e^{2i\int \omega dt} - \hat{a}_{\vec{k}'} \hat{b}_{-\vec{k}'} e^{-2i\int \omega dt}\right).$$

The above Hamiltonian is not hermitian. It can be expressed as the sum of a hermitian and an anti-hermitian Hamiltonian. $: \hat{h} := \hat{h}_+ + \hat{h}_-$ with

$$\hat{h}_+ = \int \frac{1}{2\omega} d^3k' \left[\left(2\omega^2 - \frac{9}{4}H^2\right)\left(\hat{a}_{\vec{k}'}^\dagger \hat{a}_{\vec{k}'} + \hat{b}_{\vec{k}'}^\dagger \hat{b}_{\vec{k}'}\right) + i3H\omega \sinh\left(\frac{3}{2}\int Hdt\right)\left(\hat{a}_{\vec{k}'}^\dagger \hat{b}_{-\vec{k}'}^\dagger e^{2i\int \omega dt} - \hat{a}_{\vec{k}'} \hat{b}_{-\vec{k}'} e^{-2i\int \omega dt}\right)\right]$$

and $\hat{h}_- = i\frac{3}{2}H \cosh\left(\frac{3}{2}\int Hdt\right) \int d^3k' \left[\hat{b}_{\vec{k}'}^\dagger \hat{b}_{\vec{k}'} - \hat{a}_{\vec{k}'}^\dagger \hat{a}_{\vec{k}'}\right]$.

This outcome is expected as the cosmic fluid system is taken as an open quantum system[11].

At the static phase ($H = 0$), the anti-hermitian part vanishes and the hermitian part leads to the quantised complex scalar field Hamiltonian in the Minkowski space-time. But at the other cosmic evolutionary phase (when $H \neq 0$), the anti-hermitian part become significant. Although the anti-hermitian part is quantised at any arbitrary condition, yet the hermitian part can be quantised only under certain conditions on the parameters involved in the cosmic evolution.

For hermitian part, one may introduce a Bogoliubov transformation $\hat{O}_{\vec{k}'} = \alpha \hat{a}_{\vec{k}'} \exp\left(-i\int \omega dt\right) + \beta^* \hat{b}_{-\vec{k}'}^\dagger \exp\left(+i\int \omega dt\right)$ and $\hat{O}_{\vec{k}'}^\dagger = \alpha^* \hat{a}_{\vec{k}'}^\dagger \exp\left(+i\int \omega dt\right) + \beta \hat{b}_{-\vec{k}'} \exp\left(-i\int \omega dt\right)$. Hence the hermitian part of the Hamiltonian can be written in the quantised form as,

$$\hat{h}_+ = \int d^3k' \; \omega \; \hat{O}_{\vec{k}'}^\dagger \hat{O}_{\vec{k}'}, \tag{10}$$

with $|\alpha|^2 = -|\beta|^2 = \frac{2\omega^2 - \frac{9}{4}H^2}{2\omega^2}$, $\alpha^*\beta^* = i3H \exp\left(2i\int \omega dt\right) \sinh\left(\frac{3}{2}\int Hdt\right)$ $\alpha\beta = -i3H \exp\left(-2i\int \omega dt\right) \sinh\left(\frac{3}{2}\int Hdt\right)$. The new creation and annihilation operator $\hat{O}_{\vec{k}'}^\dagger, \hat{O}_{\vec{k}'}$ follow the canonical transformation $[\hat{O}, \hat{O}^\dagger] = 1$. This condition essentially implies $|\alpha|^2 - |\beta|^2 = 1$.

The cosmic evolution scenario :

Accumulating the conditions for a valid quantisation, one obtains

$$|\alpha|^2 = -|\beta|^2 = \frac{1}{2} \tag{11}$$

$$\omega = \pm \frac{3}{2}H \tag{12}$$

$$\sinh\left(\frac{3}{2}\int Hdt\right) = \frac{1}{6H}. \tag{13}$$

The vacuum expectation value of the number operator $\langle 0| \hat{O}_{\vec{k}'}^\dagger \hat{O}_{\vec{k}'} |0\rangle = |\alpha|^2 = \frac{1}{2}$ i.e. nonzero.

The equation (13) yields the solution for Hubble parameter and scale factor as,

$$H(t) = \pm\frac{1}{6}\frac{1}{\sqrt{[\frac{1}{4}(t-t_0)+\sqrt{1+\frac{1}{36H_0^2}}]^2-1}}, \tag{14}$$

$$a(t) = a_0\left[\frac{\left|\left(t-t_0+\frac{2}{3H_0}\sqrt{1+36H_0^2}\right)+\sqrt{\left(t-t_0+\frac{2}{3H_0}\sqrt{1+36H_0^2}\right)^2-16}\right|}{\left|\frac{2}{3H_0}\sqrt{1+36H_0^2}+\sqrt{\frac{4}{9H_0^2}(1+36H_0^2)-16}\right|}\right]^{\pm\frac{2}{3}}. \tag{15}$$

Here t_0 is an arbitrary reference epoch of time with $H(t_0) = H_0$ and $a(t_0) = a_0$. Here we take t_0 as a free parameter and choose $t_0 = \frac{2}{3H_0}\sqrt{1+36H_0^2}$ for simplicity of the expressions. Thus the cosmological solutions in the expanding Universe lead to

$$H(t) = \frac{1}{6}\frac{1}{\sqrt{(\frac{t}{4})^2-1}} \tag{16}$$

and

$$a(t) = a_0\left[\frac{\left|t+\sqrt{t^2-16}\right|}{\left|t_0+\sqrt{t_0^2-16}\right|}\right]^{\frac{2}{3}} \tag{17}$$

Eventually at the limit $t >> 4$, the scenario approaches to the Einstein de -Sitter Universe[12] ($a \sim t^{\frac{2}{3}}, H \sim \frac{1}{t}$).

Duality invariant cosmic evolution and the complete scenario :

We have described an expanding Universe with scale factor $a(t) = \left[\frac{\left|t+\sqrt{t^2-16}\right|}{4}\right]^{\frac{2}{3}}$, Hubble parameter $H(t) = \frac{1}{6}\frac{1}{\sqrt{(\frac{t}{4})^2-1}}$ with the choice $a_0 = \left[\frac{\left|t_0+\sqrt{t_0^2-16}\right|}{4}\right]^{\frac{2}{3}}$. While expansion ($H > 0$), the energy eigen value of the Hamiltonian over a single particle state is $\omega(H) = +\frac{3}{2}H$ i.e. $\hat{h}_+\left|H>0\right\rangle = +\frac{3}{2}H\left|H>0\right\rangle$.

Besides we can observe that transformation of the system under $t \to -t$ yields $a \xrightarrow{t\to -t} \frac{1}{a}, H \xrightarrow{t\to -t} -H$ and the energy eigen value $\omega(H) = -\frac{3}{2}H$. $\hat{h}_+\left|H<0\right\rangle = -\frac{3}{2}H\left|H<0\right\rangle$. Hence this present model exhibits a duality invariant cosmic evolutionary scenario.

On the other hand, this cosmic evolution pattern is not valid within the time epoch $|t| \leq 4$. During this period ($-4 \leq t \leq +4$), the Universe consists of a free complex scalar field under flat minkowski space-time with $H = 0$. But nearing $|t| > 4$, a global Noether $U(1)$ symmetry breaks up and the Universe changes the evolution pattern as the above mentioned form (equations (16),(17)).

Symmetry breaking at the termination of the static phase :

The proposed Lagrangian (5) is the canonical part of the whole Lagrangian density of the cosmic fluid. Considering a self interaction of the scalar field, one may take a complete Lagrangian as,

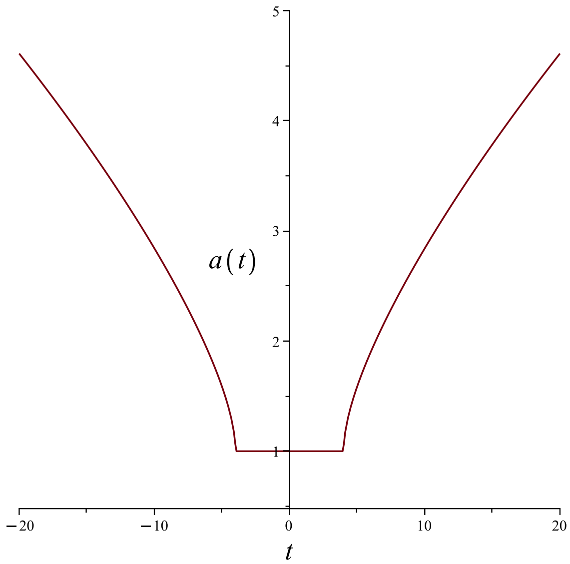
$$\mathcal{L}_{\text{total}} = \mathcal{L} + \xi H^4(\phi_+^\dagger\phi_-)^2, \tag{18}$$

ξ is a free parameter. The effective potential of this Lagrangian is $V(\phi_+^\dagger, \phi_-, H) = (m^2 - \frac{9}{4}H^2)\phi_+^\dagger\phi_- - \xi H^4(\phi_+^\dagger\phi_-)^2$.

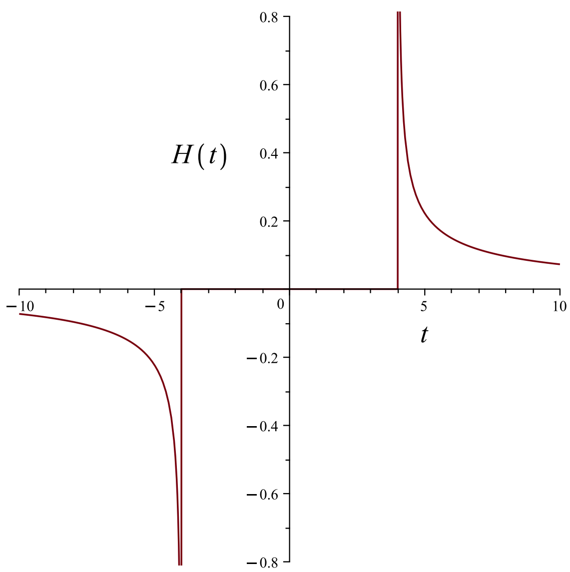
Such self interaction can be termed as the gravitation mediated self interaction.

However this Lagrangian holds a global $U(1)$ symmetry under the transformation $\phi_- \to e^{iq}\phi_-, \phi_+^\dagger \to e^{-iq}\phi_+^\dagger$. q is independent of the space-time.

At the static phase, $\phi_+^\dagger = \phi_0^\dagger,\ \ \phi_- = \phi_0$ and the cosmic fluid system behaves as a free canonical Lagrangian system. In this case, the ground state of the system is found at $\phi_0 = 0$ and the $U(1)$ symmetry holds for any arbitrary condition. we have proposed earlier that this static phase ($H = 0$) prevails during the time epoch $|t| \leq 4$. But beyond that epoch, the self interacting term grows (when $H \neq 0$) and the ground state of the system is shifted to $\phi_+^\dagger\phi_- = |\phi_0|^2 = \frac{m^2-\frac{9}{4}H^2}{2\xi H^4} = \mathcal{J}^2$(say). Hence the vacuum state field is given by $\psi_0 = \mathcal{J}e^{il}$ with l, another real constant. Now near the ground state, the nature of the field $\phi_0 = \frac{1}{\sqrt{2}}(\mathcal{J}+\Delta\mathcal{J})e^{i\frac{1}{\sqrt{2}\mathcal{J}}(l+\Delta l)}$, $\Delta\mathcal{J}$ and Δl are the small variations of $\mathcal{J}$ and l near the vacuum state. Hence the potential near the ground state takes the form



(a)



(b)

FIG. 1. A duality invariant complete evolution of the Universe : (a)Variation of Scale factor : a with time t (top) (b) Variation of Hubble parameter : $H$with t (bottom) .

$$V = V_0 + \frac{1}{2}(4\xi H^4 \mathcal{J}^2)(\Delta\mathcal{J})^2 + \sqrt{2}\xi H^4 \mathcal{J}(\Delta\mathcal{J})^3 + \frac{\xi H^4}{4}(\Delta\mathcal{J})^4, \tag{19}$$

V_0 is a constant.

Evidently the global $U(1)$ symmetry of the Lagrangian breaks spontaneously and it yields the masses of the fields as $m_{\Delta\mathcal{J}} = \sqrt{2(m^2 - \frac{9}{4}H^2)}$ and $m_{\Delta i} = 0$. The masslessness of Δi is obvious because there is still a hidden symmetry under $\frac{\Delta i}{\sqrt{2}\mathcal{J}} \rightarrow \frac{\Delta i}{\sqrt{2}\mathcal{J}} + \gamma$ with γ ,a constant. Such massless Bosons are similar to Nambu-Goldstone Bosons[13]. The other massive Bossons $\Delta\mathcal{J}$ contributes in the canonical quantization process of the Hamiltonian.

The role of the anti-hermitian part of the Hamiltonian :

For an open quantum system, the energy eigen value is not stable due to dissipation. This effect is contained within the expression of the Hamiltonian of an open system. The complete Hamiltonian of the system consists of a hermitian part $\hat{h}_+$ which delivers the instantaneous energy eigen value and an anti-hermitian part $\hat{h}_-$ which may be a signature of the dissipation of energy from the system. For a well defined representation, both operators can be simultaneously measurable. Hence, $\hat{h}_+$ and $\hat{h}_-$ have simultaneous eigen states i.e. $[\hat{h}_+, \hat{h}_-] = 0$. In our model, this condition holds if $\hat{a}_{\vec{k}'}\hat{b}_{-\vec{k}'}e^{-2i\int \omega dt}$ be an anti-hermitian quantity. Allowing this condition, we can introduce a hermitian operator[11] $\hat{\Gamma} = i\hat{h}_-$ which will serve as a decay or dissipation rate operator with $[\hat{h}_+, \hat{\Gamma}] = 0$. Hence one can rewrite the total Hamiltonian as

$$: \hat{h} := \hat{h}_+ - i\hat{\Gamma}. \tag{20}$$

Discussion :

In this letter, we have proposed a modification of the KG equations of a complex scalar field system in the flat FLRW metric. Here we have considered some dissipation in both the components of the scalar field. Notably the net real dissipation is zero although there is still some imaginary dissipation. For such an open quantum system, we have obtained an effective Lagrangian which can be carried out under flat Minkowski space-time in a co-moving frame and corresponds to the modified KG equations in FLRW metric. Thus we can study the microscopic dynamics of the cosmic fluid in a comoving frame via the quantum field theory instead of following any gravity theory (including GR).

However in this model, the amplitudes of the scalar fields become time varying. one becomes damped oscillator, another appears to be time growing. This change is one of the effects of the gravity on the scalar field. The corresponding Hamiltonian consists of a hermitian and an anti-hermitian terms. The canonical quantization of the hermitian part determines the cosmic evolution pattern. On the other hand, the anti-hermitian part represents the dissipation of energy from or to the system. At the static phase, there is no dissipation but under a spontaneous symmetry breaking mechanism the static phase terminates and the dissipating term grows. The symmetry breaking process occurs due to a self interaction of the field mediated by gravity. This is another effect of the gravity on the scalar field system. There is also a correspondence between the dissipation of energy and the Noether charge of this system. The Noether charge corresponding to the $U(1)$ symmetry is $\hat{q} = \int d^3k' \left[\hat{a}^\dagger_{\vec{k}'}\hat{a}_{\vec{k}'} - \hat{b}^\dagger_{\vec{k}'}\hat{b}_{\vec{k}'}\right]$ and hence $\hat{\Gamma} = \frac{3}{2}H\cosh\left(\frac{3}{2}\int H dt\right)\hat{q}$.

In a nutshell, we conclude that the dynamics of a complex scalar field in curved space-time (here in FLRW) can be studied in a co-moving Minkowski metric via quantum field theory. The role of gravity in such scenario is to produce dissipation in the scalar field system. Another role of gravity is in the symmetry breaking mechanism which occurs due to a self interaction of the scalar field. The authors are hopeful that this letter may open an alternative way of GR to study the evolution of the Universe in the small scale.

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