

Fundamental Physics and the I Ching

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The eightfold way is an organizational scheme for a class of subatomic particles known as hadrons leading to the development of the quark model. Murray Gell-Mann proposed the idea in 1961 alluding to the Noble Eightfold Path of Buddhism.

However, the concept goes deeper; applying not only to hadrons (triplets of RGB quarks) but to the building blocks of all the fermions (as this analysis demonstrates: i.e.: e(h)-earth, v(h)-wind, u(h)-fire, d(h)-water.

The I Ching (Yijing) (Zhou yi) (Book of Changes) is an ancient Chinese divination text that is manual in the Western Zhou period (1000–750 BC) traditionally ascribed to King Wen of Zhou and the Duke of Zhou, and also associated with the legendary Fuxi; originating around 5000 years ago.

The I Ching is formulated using hexagrams, in-turn made-up of trigrams made from the yin and yang. yin and yang are represented by broken and solid lines: the yin broken (- -) and yang solid (—) forming a binary system. trigrams are the eight triplets of yin/yang elements stacked vertically; forming a modulo seven reduced residue system.



(乾 qián) = (天) heaven = ☰ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 0 & 0 & 0 \end{matrix} = 0$

(兌 duì) = (澤) lake = ☱ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 1 & 0 & 0 \end{matrix} = 4$

(巽 xùn) = (風) wind = ☴ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 0 & 0 & 1 \end{matrix} = 1$

(坎 kān) = (水) water = ☵ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 1 & 0 & 1 \end{matrix} = 5$

(離 lí) radiance = (火) fire = ☲ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 0 & 1 & 0 \end{matrix} = 2$

(震 zhèn) = (雷) thunder = ☳ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 1 & 1 & 0 \end{matrix} = 6$

(艮 gèn) = (山) mountain = ☶ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 0 & 1 & 1 \end{matrix} = 3$

(坤 kūn) = (地) earth = ☷ = ||| = $\begin{matrix} 2^2 & 2^1 & 2^0 \\ 1 & 1 & 1 \end{matrix} = 7$

Note that adding 1 to a trigram yields the next trigram (mod 7).

In a four-dimensional space-time which includes the above electromagnetic–nuclear basis the **E** & **B** field strengths may be associated and constructed [] as follows:

In fact, these base vectors and dot and cross products may be constructed from the four 4 × 4 permutation matrices, four 4 × 4 weight matrices, and the weighted matrix product [].

For this electromagnetic algebra the weights for the four 4 × 4 permutation matrices as basis are []:

$$\mathbf{u}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathbf{u}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \mathbf{u}_{-1} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

$\mathbf{u}_i \circ \mathbf{u}_j$ is weighted matrix multiplication []: $(a_{ij})(b_{ij}) = \left(\sum_h \Phi_{hij} a_{ih} b_{hj} \right)$ with weights:

$$\Phi_{0ij} \equiv \begin{pmatrix} -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 \end{pmatrix}, \Phi_{1ij} \equiv \begin{pmatrix} -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 \end{pmatrix}$$

$$\Phi_{2ij} \equiv \begin{pmatrix} -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \end{pmatrix}, \Phi_{3ij} \equiv \begin{pmatrix} -1 & -1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

Just as the Dirac equation is a factorization of the Klein-Gordon equation, the d'Alembertian operator may be factored by two 4×4 matrices:

$$\square \mathbf{f} = \begin{pmatrix} -i\partial_0 & \partial_3 & -\partial_2 & -\partial_1 \\ -\partial_3 & -i\partial_0 & \partial_1 & -\partial_2 \\ \partial_2 & -\partial_1 & -i\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & i\partial_0 \end{pmatrix} \begin{pmatrix} -i\partial_0 & -\partial_3 & \partial_2 & -\partial_1 \\ \partial_3 & -i\partial_0 & -\partial_1 & -\partial_2 \\ -\partial_2 & \partial_1 & -i\partial_0 & -\partial_3 \\ -\partial_1 & -\partial_2 & -\partial_3 & i\partial_0 \end{pmatrix} \mathbf{f}$$

$$= \begin{pmatrix} i\partial_0 & \partial_3 & -\partial_2 & \partial_1 \\ -\partial_3 & i\partial_0 & \partial_1 & \partial_2 \\ \partial_2 & -\partial_1 & i\partial_0 & \partial_3 \\ \partial_1 & \partial_2 & \partial_3 & -i\partial_0 \end{pmatrix} \begin{pmatrix} i\partial_0 & -\partial_3 & \partial_2 & \partial_1 \\ \partial_3 & i\partial_0 & -\partial_1 & \partial_2 \\ -\partial_2 & \partial_1 & i\partial_0 & \partial_3 \\ \partial_1 & \partial_2 & \partial_3 & -i\partial_0 \end{pmatrix} \mathbf{f}$$

This d'Alembertian operator factorization may be generalized, using two 8×8 matrices, operating on 4-vector doublets:

$$(\square - |m|^2) \mathbf{f} = \begin{pmatrix} -D_0 & D_3^{\leftrightarrow} & -D_2^{\leftrightarrow} & -D_1 \\ -D_3^{\leftrightarrow} & -D_0 & D_1^{\leftrightarrow} & -D_2 \\ D_2^{\leftrightarrow} & -D_1^{\leftrightarrow} & -D_0 & -D_3 \\ -D_1^{\updownarrow} & -D_2^{\updownarrow} & -D_3^{\updownarrow} & D_0 \end{pmatrix} \begin{pmatrix} -D_0^{\updownarrow} & -D_3^{\leftrightarrow} & D_2^{\leftrightarrow} & -D_1 \\ D_3^{\leftrightarrow} & -D_0^{\updownarrow} & -D_1^{\leftrightarrow} & -D_2 \\ -D_2^{\leftrightarrow} & D_1^{\leftrightarrow} & -D_0^{\updownarrow} & -D_3 \\ -D_1^{\updownarrow} & -D_2^{\updownarrow} & -D_3^{\updownarrow} & D_0 \end{pmatrix} \mathbf{f}$$

$$= \begin{pmatrix} D_0 & D_3^{\leftrightarrow} & -D_2^{\leftrightarrow} & D_1 \\ -D_3^{\leftrightarrow} & D_0 & D_1^{\leftrightarrow} & D_2 \\ D_2^{\leftrightarrow} & -D_1^{\leftrightarrow} & D_0 & D_3 \\ D_1^{\updownarrow} & D_2^{\updownarrow} & D_3^{\updownarrow} & -D_0^{\updownarrow} \end{pmatrix} \begin{pmatrix} D_0^{\updownarrow} & -D_3^{\leftrightarrow} & D_2^{\leftrightarrow} & D_1 \\ D_3^{\leftrightarrow} & D_0^{\updownarrow} & -D_1^{\leftrightarrow} & D_2 \\ -D_2^{\leftrightarrow} & D_1^{\leftrightarrow} & D_0^{\updownarrow} & D_3 \\ D_1^{\updownarrow} & D_2^{\updownarrow} & D_3^{\updownarrow} & -D_0^{\updownarrow} \end{pmatrix} \mathbf{f}$$

where:

$$D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} \quad , \quad D_i^{\updownarrow} \equiv \begin{pmatrix} D_i^- & 0 \\ 0 & D_i^+ \end{pmatrix} \quad , \quad D_i^{\leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} \quad , \quad D_i^{\leftrightarrow\updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_i^- & 0 \end{pmatrix}$$

In this case, in general: $\mathbf{f} = \begin{pmatrix} \begin{pmatrix} f_1^1 \\ f_2^1 \end{pmatrix} \\ \begin{pmatrix} f_1^2 \\ f_2^2 \end{pmatrix} \\ \begin{pmatrix} f_1^3 \\ f_2^3 \end{pmatrix} \\ \begin{pmatrix} f_1^0 \\ f_2^0 \end{pmatrix} \end{pmatrix}$

In terms of algebraic basis, the basis corresponding to the electromagnetic-nuclear basis is []:

$$\mathbf{w}_j \equiv \mathbf{u}_j e^{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{m} \cdot \mathbf{r}} \quad , \quad (\mathbf{m}^* \cdot \mathbf{r} \equiv -m_0 x^0 + m_1 x^1 + m_2 x^2 + m_3 x^3)$$

This accounts for the Yukawa potential [B].

So, defining:

$$\mathbf{E} \equiv \frac{1}{2} \left\{ \frac{1}{2} \left(\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}(\mathbf{x}) \} \right] \right) + \left[\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}(\mathbf{x}) \} \right] \right]^* \right\}$$

$$- \frac{1}{2} \left\{ \frac{1}{2} \left(\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}^*(\mathbf{x}) \} \right] \right) + \left[\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}^*(\mathbf{x}) \} \right] \right]^* \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} \left[\left(\overset{m}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m}{\nabla} \circ \mathbf{f} \right)^* \right] - \frac{1}{2} \left[\left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right)^* \right] \right\}$$

$$= \frac{1}{2} \left\{ \left(\overset{m^*}{\nabla} \times \mathbf{f} \right) - \left(\overset{m}{\nabla} \times \mathbf{f}^* \right) \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) - \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) \right] + \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) - \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) \right]^* \right\}$$

$$\mathbf{B} \equiv \frac{1}{2} \left\{ \frac{1}{2} \left(\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}(\mathbf{x}) \} \right] \right) + \left[\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}(\mathbf{x}) \} \right] \right]^* \right\}$$

$$+ \frac{1}{2} \left\{ \frac{1}{2} \left(\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}^*(\mathbf{x}) \} \right] \right) + \left[\sum_{m=1}^4 \left[\lim_{\substack{\delta x^m \rightarrow 0 \\ \delta x^h=0, (h \neq m)}} \{ (\delta \mathbf{x})_R^{-1} \circ \delta \mathbf{f}^*(\mathbf{x}) \} \right] \right]^* \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m^*}{\nabla} \circ \mathbf{f} \right)^* \right] + \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m^*}{\nabla} \circ \mathbf{f} \right)^* \right]^* \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) \right] + \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) \right]^* \right\}$$

$$= \frac{1}{2} \left\{ \left(\overset{m^*}{\nabla} \times \mathbf{f} \right) + \left(\overset{m}{\nabla} \times \mathbf{f}^* \right) \right\}$$

$$= \frac{1}{2} \left\{ \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) \right] + \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \circ \mathbf{f} \right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^* \right) \right]^* \right\}$$

are basis independent definitions of \mathbf{E} and \mathbf{B} .

So:

$$\begin{aligned} \mathbf{E} &= \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \times \mathbf{f} \right) - \left(\overset{m}{\nabla} \times \mathbf{f}^* \right) \right] \\ &= \mathbf{w}^{4;1} (-f_{;0}^1 - f_{;1}^0) + \mathbf{w}^{4;2} (-f_{;0}^2 - f_{;2}^0) + \mathbf{w}^{4;3} (-f_{;0}^3 - f_{;3}^0) \\ &= \mathbf{w}^{4;1} \left[- \begin{pmatrix} (\partial_0 - m_0) \\ (\partial_0 + m_0) \end{pmatrix} f^1 - \begin{pmatrix} (\partial_1 + m_1) \\ (\partial_1 - m_1) \end{pmatrix} f^0 \right] + \\ &+ \mathbf{w}^{4;2} \left[- \begin{pmatrix} (\partial_0 - m_0) \\ (\partial_0 + m_0) \end{pmatrix} f^2 - \begin{pmatrix} (\partial_2 + m_2) \\ (\partial_2 - m_2) \end{pmatrix} f^0 \right] + \\ &+ \mathbf{w}^{4;3} \left[- \begin{pmatrix} (\partial_0 - m_0) \\ (\partial_0 + m_0) \end{pmatrix} f^3 - \begin{pmatrix} (\partial_3 + m_3) \\ (\partial_3 - m_3) \end{pmatrix} f^0 \right], \end{aligned}$$

and:

$$\begin{aligned} \mathbf{B} &= \frac{1}{2} \left[\left(\overset{m^*}{\nabla} \times \mathbf{f} \right) + \left(\overset{m}{\nabla} \times \mathbf{f}^* \right) \right] \\ &= \mathbf{w}^{4;1} (f_{;2}^3 - f_{;3}^2) + \mathbf{w}^{4;2} (-f_{;1}^3 + f_{;3}^1) + \mathbf{w}^{4;3} (f_{;1}^2 - f_{;2}^1) \\ &= \mathbf{w}^{4;1} \left[\begin{pmatrix} (\partial_2 + m_2) \\ (\partial_2 - m_2) \end{pmatrix} f^3 - \begin{pmatrix} (\partial_3 + m_3) \\ (\partial_3 - m_3) \end{pmatrix} f^2 \right] + \\ &+ \mathbf{w}^{4;2} \left[- \begin{pmatrix} (\partial_1 + m_1) \\ (\partial_1 - m_1) \end{pmatrix} f^3 + \begin{pmatrix} (\partial_3 + m_3) \\ (\partial_3 - m_3) \end{pmatrix} f^1 \right] + \\ &+ \mathbf{w}^{4;3} \left[\begin{pmatrix} (\partial_1 + m_1) \\ (\partial_1 - m_1) \end{pmatrix} f^2 - \begin{pmatrix} (\partial_2 + m_2) \\ (\partial_2 - m_2) \end{pmatrix} f^1 \right]. \end{aligned}$$

In terms of algebraic basis, the basis corresponding to the electromagnetic \mathbf{E} & \mathbf{B} field strengths are: (corresponding to Maxwell's equations and the d'Alembertian and d'Alembertian operator factoring)

$$\begin{aligned} \mathbf{E} &= \mathbf{u}_1 (-\partial_0 f^1 - \partial_1 f^0) + \mathbf{u}_2 (-\partial_0 f^2 - \partial_2 f^0) + \mathbf{u}_{-1} (-\partial_0 f^3 - \partial_3 f^0) = -\partial_0 \mathbf{f}^* - \nabla f^0 \\ \mathbf{B} &= \mathbf{u}_1 (\partial_2 f^3 - \partial_3 f^2) + \mathbf{u}_2 (-\partial_1 f^3 + \partial_3 f^1) + \mathbf{u}_{-1} (\partial_1 f^2 - \partial_2 f^1) = \nabla \times \mathbf{f} \end{aligned}$$

In terms of algebraic basis, the basis corresponding to the electromagnetic-nuclear \mathbf{E} & \mathbf{B} field strengths are: (corresponding to Maxwell's equations and the Helmholtzian and Helmholtzian operator factorization)

$$\begin{aligned} \mathbf{E} &= \mathbf{w}^{4;1} (-D_0^\dagger f^1 - D_1 f^0) + \mathbf{w}^{4;2} (-D_0^\dagger f^2 - D_2 f^0) + \mathbf{w}^{4;3} (-D_0^\dagger f^3 - D_3 f^0) \\ \mathbf{B} &= \mathbf{w}^{4;1} (D_2 f^3 - D_3 f^2) + \mathbf{w}^{4;2} (-D_1 f^3 + D_3 f^1) + \mathbf{w}^{4;3} (D_1 f^2 - D_2 f^1) \end{aligned}$$

From these the fermions may be constructed and tabulated as:

| | | |
|--|--|--|
| $e^- = e(1) = \overline{(E^1, E^2, E^3)}_1$ | $\mu^- = e(2) = \overline{(E^1, E^2, E^3)}_2$ | $\tau^- = e(3) = \overline{(E^1, E^2, E^3)}_3$ |
| $\nu_e = \nu(1) = (B^1, B^2, B^3)_1$ | $\nu_\mu = \nu(2) = (B^1, B^2, B^3)_2$ | $\nu_\tau = \nu(3) = (B^1, B^2, B^3)_3$ |
| $u_R = u_1(1) = (B^1, E^2, E^3)_1$ | $c_R = u_1(2) = (B^1, E^2, E^3)_2$ | $t_R = u_1(3) = (B^1, E^2, E^3)_3$ |
| $u_G = u_0(1) = (E^1, B^2, E^3)_1$ | $c_G = u_0(2) = (E^1, B^2, E^3)_2$ | $t_G = u_0(3) = (E^1, B^2, E^3)_3$ |
| $u_B = u_{-1}(1) = (E^1, E^2, B^3)_1$ | $c_B = u_{-1}(2) = (E^1, E^2, B^3)_2$ | $t_B = u_{-1}(3) = (E^1, E^2, B^3)_3$ |
| $d_R = d_1(1) = \overline{(E^1, B^2, B^3)}_1$ | $s_R = d_1(2) = \overline{(E^1, B^2, B^3)}_2$ | $b_R = d_1(3) = \overline{(E^1, B^2, B^3)}_3$ |
| $d_G = d_0(1) = \overline{(B^1, E^2, B^3)}_1$ | $s_G = d_0(2) = \overline{(B^1, E^2, B^3)}_2$ | $b_G = d_0(3) = \overline{(B^1, E^2, B^3)}_3$ |
| $d_B = d_{-1}(1) = \overline{(B^1, B^2, E^3)}_1$ | $s_B = d_{-1}(2) = \overline{(B^1, B^2, E^3)}_2$ | $b_B = d_{-1}(3) = \overline{(B^1, B^2, E^3)}_3$ |

All the possible quark triplets are given by:

| | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $u_1(h) : u_{-1}(j) : u_0(k)$ | $u_1(h) : u_{-1}(j) : d_0(k)$ | $u_1(h) : d_{-1}(j) : u_0(k)$ | $u_1(h) : d_{-1}(j) : d_0(k)$ |
| $d_1(h) : d_{-1}(j) : u_0(k)$ | $d_1(h) : u_{-1}(j) : d_0(k)$ | $d_1(h) : d_{-1}(j) : u_0(k)$ | $d_1(h) : u_{-1}(j) : d_0(k)$ |

| | | |
|-------------------|--------------------|---------------------|
| $e^- = e(1)$ | $\mu^- = e(2)$ | $\tau^- = e(3)$ |
| $\nu_e = \nu(1)$ | $\nu_\mu = \nu(2)$ | $\nu_\tau = \nu(3)$ |
| $u_R = u_1(1)$ | $c_R = u_1(2)$ | $t_R = u_1(3)$ |
| $u_G = u_0(1)$ | $c_G = u_0(2)$ | $t_G = u_0(3)$ |
| $u_B = u_{-1}(1)$ | $c_B = u_{-1}(2)$ | $t_B = u_{-1}(3)$ |
| $d_R = d_1(1)$ | $s_R = d_1(2)$ | $b_R = d_1(3)$ |
| $d_G = d_0(1)$ | $s_G = d_0(2)$ | $b_G = d_0(3)$ |
| $d_B = d_{-1}(1)$ | $s_B = d_{-1}(2)$ | $b_B = d_{-1}(3)$ |

(and their anti-matter counterparts)

Denoting quark types (u, d) , colors $(1, 0, -1)$ & flavours $(1, 2, 3)$.

(The associated anti-fermion has negative charge & color of it's counterpart.)

But this may be simplified into a purely mathematical data structure (especially since Left & Right neutrinos have different characteristics):

| | | |
|-------------------------------------|------------------------------------|------------------------------------|
| $v_{e_R} = v(1) = f(0, 1, -1, -1)$ | $v_{\mu} = v(2) = f(0, 1, -1, 0)$ | $v_{\tau} = v(3) = f(0, 1, -1, 1)$ |
| $e^- = e(1) = f(0, -1, 0, -1)$ | $\mu^- = e(2) = f(0, -1, 0, 0)$ | $\tau^- = e(3) = f(0, -1, 0, 1)$ |
| $v_{e_L} = v(1) = f(0, 1, 1, -1)$ | $v_{\mu} = v(2) = f(0, 1, 1, 0)$ | $v_{\tau} = v(3) = f(0, 1, 1, 1)$ |
| $u_R = u_1(1) = f(1, -1, -1, -1)$ | $c_R = u_1(2) = f(1, -1, -1, 0)$ | $t_R = u_1(3) = f(1, -1, -1, 1)$ |
| $u_G = u_0(1) = f(1, -1, 0, -1)$ | $c_G = u_0(2) = f(1, -1, 0, 0)$ | $t_G = u_0(3) = f(1, -1, 0, 1)$ |
| $u_B = u_{-1}(1) = f(1, -1, 1, -1)$ | $c_B = u_{-1}(2) = f(1, -1, 1, 0)$ | $t_B = u_{-1}(3) = f(1, -1, 1, 1)$ |
| $d_R = d_1(1) = f(1, 1, -1, -1)$ | $s_R = d_1(2) = f(1, 1, -1, 0)$ | $b_R = d_1(3) = f(1, 1, -1, 1)$ |
| $d_G = d_0(1) = f(1, 1, 0, -1)$ | $s_G = d_0(2) = f(1, 1, 0, 0)$ | $b_G = d_0(3) = f(1, 1, 0, 1)$ |
| $d_B = d_{-1}(1) = f(1, 1, 1, -1)$ | $s_B = d_{-1}(2) = f(1, 1, 1, 0)$ | $b_B = d_{-1}(3) = f(1, 1, 1, 1)$ |

i.e.: $\boxed{e(h) \mid u_1(h) \mid u_0(h) \mid u_{-1}(h) \mid d_1(h) \mid d_0(h) \mid d_{-1}(h) \mid v(h)} \quad h \in \{1, 2, 3\}$

For: $f(x_1, x_2, x_3, x_4)$:

| | |
|--|--|
| $x_1 = \begin{cases} 0 : \text{lepton} \\ 1 : \text{quark} \end{cases}$ | $x_2 = \begin{cases} -1 : \text{up} \\ 1 : \text{down} \end{cases}$ |
| $x_3 = \text{color} = \begin{cases} -1 : \text{R} \\ 0 : \text{G} \\ 1 : \text{B} \end{cases}$ | $x_4 = \text{generation} = \begin{cases} -1 : \\ 0 : \\ 1 : \end{cases}$ |

Mesons are quark pairs/doublets. Baryons are quark triplets.

Meson & baryon color indices must add up to 0 .

(thus, meson pairs may only be composed of a quark & an anti-quark of the same-negated color.

The color doublet operation is simply a permutation operation on the two quarks

(color indices adding to 0), two at a time (i.e.: flipping with each other) :

So, all the possible quark doublets are given by:

| | | |
|-------------------------|-------------------------|-------------------------------|
| $u_0(h) : \bar{u}_0(j)$ | $u_1(h) : \bar{u}_1(j)$ | $u_{-1}(h) : \bar{u}_{-1}(j)$ |
| $d_0(h) : \bar{d}_0(j)$ | $d_1(h) : \bar{d}_1(j)$ | $d_{-1}(h) : \bar{d}_{-1}(j)$ |

||

$$\boxed{f(1, x_2, x_3, x_4) : \overline{f(1, x_2, -x_3, x_4)}}$$

As with the color doublet operation, the color triplet operation is simply a permutation operation on the three quarks

(color indices adding to 0), two at a time :

So, all the possible quark triplets are given by:

| | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $u_1(h) : u_0(j) : u_{-1}(k)$ | $u_1(h) : u_0(j) : d_{-1}(k)$ | $u_1(h) : d_0(j) : u_{-1}(k)$ | $u_1(h) : d_0(j) : d_{-1}(k)$ |
| $d_1(h) : d_0(j) : d_{-1}(k)$ | $d_1(h) : d_0(j) : u_{-1}(k)$ | $d_1(h) : u_0(j) : d_{-1}(k)$ | $d_1(h) : u_0(j) : u_{-1}(k)$ |

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| | |
|-------------------------------|-------------------------------|
| $u_1(h) : u_0(j) : u_{-1}(k)$ | $d_1(h) : d_0(j) : d_{-1}(k)$ |
| $u_1(h) : u_0(j) : d_{-1}(k)$ | $d_1(h) : d_0(j) : u_{-1}(k)$ |
| $u_1(h) : d_0(j) : u_{-1}(k)$ | $d_1(h) : u_0(j) : d_{-1}(k)$ |
| $u_1(h) : d_0(j) : d_{-1}(k)$ | $d_1(h) : u_0(j) : u_{-1}(k)$ |

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| | |
|---|--|
| $f(1, -1, -1, h) : f(1, -1, 0, j) : f(1, -1, 1, k)$ | $f(1, 1, -1, h) : f(1, 1, 0, j) : f(1, 1, 1, k)$ |
| $f(1, -1, -1, h) : f(1, -1, -1, h) : f(1, 1, 1, k)$ | $f(1, 1, -1, h) : f(1, 1, 0, j) : f(1, -1, 1, k)$ |
| $f(1, -1, -1, h) : f(1, 1, 0, j) : f(1, -1, 1, k)$ | $f(1, 1, -1, h) : f(1, -1, 0, j) : f(1, 1, 1, k)$ |
| $f(1, -1, -1, h) : f(1, 1, 0, j) : f(1, 1, 1, k)$ | $f(1, 1, -1, h) : f(1, -1, 0, j) : f(1, -1, 1, k)$ |

$$\left(\begin{array}{cc} u_1(h) = f(1, -1, -1, h) & d_1(h) = f(1, 1, -1, h) \\ u_0(j) = f(1, -1, 0, j) & d_0(j) = f(1, 1, 0, j) \\ u_{-1}(k) = f(1, -1, 1, k) & d_{-1}(k) = f(1, 1, 1, k) \end{array} \right)$$

i.e.: $\boxed{d_R : d_B : d_G \mid u_R : d_B : d_G \mid d_R : u_B : d_G \mid d_R : d_B : u_G \mid d_R : u_B : u_G \mid u_R : d_B : u_G \mid u_R : u_B : d_G \mid u_R : u_B : u_G}$

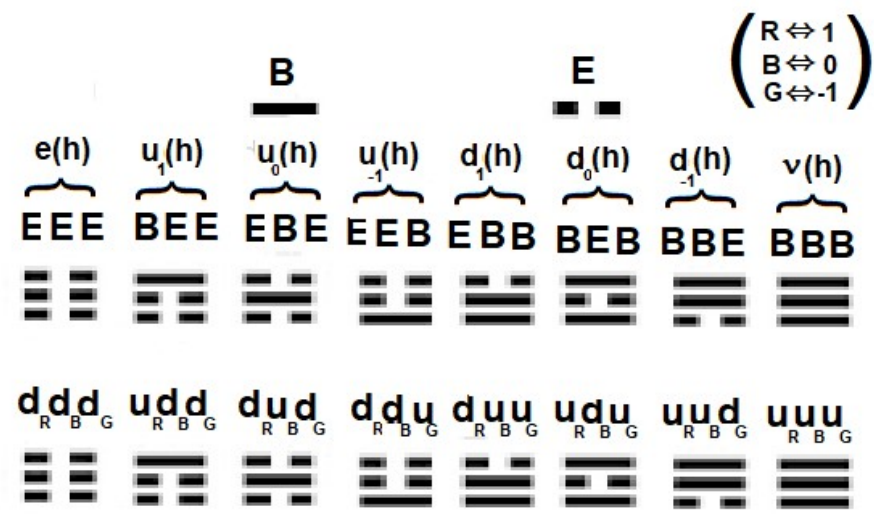
In the future, with numbers becoming increasingly predominant - at least in part due to increasing reliance on computers

the notation/nomenclature will undoubtedly vary (thought equivocally)

The eightfold way is an organizational scheme for a class of subatomic particles known as hadrons leading to the development of the quark model. Murray Gell-Mann proposed the idea in 1961 alluding to the Noble Eightfold Path of Buddhism.

However, the concept goes deeper; applying not only to hadrons (triplets of RGB quarks) but to the building blocks of all the fermions (as the above analysis demonstrates).

Referring to above equivalent fermion table confirms this (Quod Erat Demonstrandum)).



Thus, was it "enlightened" who taught the ancient teachers that earth, wind, fire, and water were the elements making up the universe? And were they merely referring to tangible things their students could comprehend?