

**Unified Understanding of Relativity and Quantum Mechanics through the  
Repulsion Graviton Space Model**

Misaki Kasai

Personal Publication

October 30, 2024

This paper is the English translation of the original Japanese version.

## **Abstract**

In this study, we develop a modified Einstein field equation that incorporates the effect of gravitons into the dark matter term. From the spherically symmetric solution of the Schwarzschild metric, we derive the Graviton Inverse-Square Decay Galactic Rotation Model (Galactic Rotation Curve MiSAKi Model). By analyzing its alignment with actual observational data, we explore the unifying explanation for gravity, dark matter, and dark energy through the “Repulsion Graviton Space Model (Re:GraviS Model).” This model enables a unified understanding of relativity and quantum mechanics, considering the role of gravitons in cosmic structure formation.

*Keywords:* Quantum Gravity Theory, Graviton, Dark Matter, Dark Energy, Black Hole, Galactic Rotation Curve

## **Introduction**

The discrepancy between observed and theoretical rotational velocities at the outer edges of galaxies has long been a challenge in physics. This research proposes a new hypothesis: gravitons possess a repulsive force against space, and gravity can be explained as “the entropic repulsion of space countering this repulsion.” Based on the spherically symmetric solution to Einstein’s equation (the Schwarzschild solution) and considering the possible existence of anti-gravitons, this study analyzes galactic rotation curves using a fitting model. The analysis investigates the potential for unifying relativity and quantum mechanics, revealing deeper insights into the cosmos.

## **Model Overview and Theoretical Background**

The “Graviton Inverse-Square Decay Galactic Rotation Model (Galactic Rotation Curve MiSAKi Model)” employed in this study was derived from the Schwarzschild solution—a spherically symmetric solution of the modified Einstein field equation incorporating the graviton effect in the dark matter term. The model assumes gravitons exert “spherically symmetric repulsive forces” against space. On smaller scales, gravitons serve as dark matter, while on cosmic scales, they hypothetically contribute as dark energy by expanding space. Furthermore, the concentration of gravitons reflects the dark matter distribution, addressing the longstanding enigma of “galactic rotation curves.”

## **Methodology**

Using a high-precision verification script for galactic rotation curves developed with AI tools (ChatGPT4o), we analyzed the alignment between the Galactic Rotation Curve MiSAKi Model and the SPARC dataset of observed galactic rotation curves (175 galaxies). This analysis involved plotting and evaluating the agreement via determination coefficients. For galaxies with lower determination coefficients, we allowed  $\alpha = 0$  (graviton and anti-graviton equilibrium) and  $\alpha < 0$  (higher concentration of anti-gravitons than gravitons) and re-assessed the data to comprehensively evaluate graviton and anti-graviton effects. Following the completion of all fittings, we conducted a correlation analysis of graviton concentration (parameter  $\alpha$ ), graviton effect range (scale parameter  $R$ ), and other physical parameters such as observed, disk, and bulge velocity components.

## Results and Discussion

Analysis revealed highly satisfactory fits between the model and observed data across most galaxies (mean  $R^2$  of 0.758, standard deviation 0.379, median 0.963). Despite the extensive dataset, the median  $R^2$  of 0.963 was high, and standard deviation remained small at 0.379. Residual distributions were random and constrained within a small range, showing no signs of overfitting, thus confirming the model's accurate and reliable reproduction of galactic rotation curves. Additionally, correlation analysis revealed a strong positive correlation between graviton concentration ( $\alpha$ ) and both observed and disk component velocities (average observed velocity  $V_{obs}(r)$ : 0.96, average disk velocity  $V_{disk}(r)$ : 0.87), suggesting that graviton concentration significantly impacts the rotational velocity at the galaxy's outer edges, consistent with dark matter characteristics inferred from Einstein's equations. Furthermore,  $\alpha$  and  $R$  distributions were concentrated within specific ranges, implying that graviton-induced effects on outer galactic rotation velocities have a certain threshold or limit. The scale parameter  $R$ , representing graviton effect range, was independent of  $\alpha$  and other physical parameters, indicating a universal scale of graviton influence across galaxies. This suggests that graviton repulsive forces operate in a spherically symmetric, inverse-square decay, contributing to the maintenance of outer galactic rotation velocities. This finding aligns with the Newtonian principle that gravity decays with the inverse square of distance. Furthermore, the lack of physical interaction suggests that gravitons do not interfere with the Higgs field, indicating no interaction with mass-giving Higgs particles. This supports the unique properties required by conventional quantum mechanics, where gravitons are massless, closed strings capable of inter-dimensional movement. Moreover, in this study,  $\alpha$  did not reach zero or negative values, indicating no direct anti-graviton effects. However, this does not directly negate the existence of anti-gravitons, as  $\alpha$  reflects graviton concentration rather than their absolute number. Based on these results, we conclude that the mysterious force maintaining outer galactic rotation velocities is likely "the graviton repulsive force against space." The model's fit to observational data and non-interactive properties suggest that the essence of gravity lies in "the entropic-driven repulsion of space countering graviton repulsion," and the causal relationship between graviton effects and spatial curvature was confirmed. Furthermore, the findings suggest that "gravitons themselves" constitute dark matter, while "the repulsive force of gravitons against space" constitutes dark energy,

implying that the mysterious energy accelerating the universe's expansion at superluminal speeds is highly likely to be "the graviton repulsive force against space." Additionally, the mechanism by which massless gravitons concentrate may involve their indirect gravitational effects, confining them within space through entropy-driven spatial repulsion, leading gravitons to converge at a single point. This is a form of self-causality, potentially consistent within both black hole event horizons and spacetime singularities, where laws of physics are traditionally thought to break down. Accordingly, a black hole's formation may begin when spatial repulsion momentarily surpasses the graviton's repulsive force against space, producing a spacetime singularity not as a mere point but as a spatial region with complex structures compressing inward and displaying quantum-like spatial spread. When repulsion strength approaches that of graviton repulsion, the formed spacetime singularity can be interpreted as a spacetime region where graviton repulsion and spatial repulsion are perfectly balanced. This framework may offer quantum room within spacetime singularities, avoiding the infinite values problematic in traditional quantum mechanics. However, such equilibrium could theoretically prevent quantum spread. To interpret this within quantum mechanics, it may be necessary to consider that gravitons transition from an "open string" state to a "closed string" state within spacetime singularities, enabling them to convert themselves into energy and escape to other dimensions. This mechanism, in which gravitons function as particles transmitting and preserving information in other dimensions, suggests the preservation of information beyond event horizons and potentially provides insight into solving the black hole information paradox, complementing recent soft hair theory. This implies that spacetime singularities may function as interdimensional gateways for gravitons, which could help solve the black hole information paradox while allowing for quantum spread within spacetime singularities. Additionally, this mechanism for black hole formation readily explains the spacetime-level phenomena in which time is prolonged in moderately strong gravitational fields but shortened in extremely strong fields, as seen in black holes. In moderately strong gravitational fields, graviton repulsion expands space, lengthening the Planck scale and extending the minimum unit of quantum movement, which slows down physical movement on the quantum level, effectively lengthening time. Conversely, in extremely strong fields like black holes, space shrinks spontaneously, shortening the Planck scale and quantum movement unit, accelerating physical movement on the quantum level and effectively shortening time. This matches relativity's prediction that, from the perspective of an outside observer, objects

approaching the event horizon experience “time dilation” and appear to stop moving. By reversing perspectives, one might explain that, in the observer’s spacetime, where graviton repulsion sufficiently expands space relative to the black hole’s spacetime, the observer’s time (proper time) appears relatively longer. This spacetime-level phenomenon suggests that an extended graviton spacetime repulsive model (Re:GraviST Model: Repulsion Graviton Space-Time Model) may be required. The insights from galactic rotation curve analysis strongly support the theoretical foundation of the Galactic Rotation Curve MiSAKi Model, suggesting the possibility of completing a unified “quantum gravity theory (grand unified theory)” that unifies relativity and quantum mechanics. This underscores the need for further research on the Repulsion Graviton Space Model (Re:GraviS Model) and its spacetime extension as the Repulsion Graviton Space-Time Model (Re:GraviST Model).

References: This research is based on high originality and general knowledge, hence no specific references are cited.

Data Used: SPARC Galaxy Rotation Curve Dataset (175 galaxies)

<https://astronomy.case.edu/2016/08/26/sparc-galaxy-database>

**(Galaxy Rotation Curve MiSAKi Model)**

$$v_{\text{total}}(r) = \left| v_{\text{disk}}(r) + v_{\text{gas}}(r) + v_{\text{bul}}(r) + \text{sign}(\alpha) \cdot \sqrt{\alpha \cdot \left( 1 - \frac{1}{1 + \left(\frac{r}{R}\right)^2} \right)} \right|$$

**Derivation Process****1. Standard Einstein Equation**

The standard Einstein equation represents the relationship between the curvature of spacetime and the energy-momentum tensor of matter and energy:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

**2. Modified Einstein Equation**

To account for the influence of gravitons, an additional term  $T_{\mu\nu}^{(DM)}$  is added as a dark matter component:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} (T_{\mu\nu} + T_{\mu\nu}^{(DM)})$$

**3. Schwarzschild Solution to the Modified Einstein Equation**

Assuming spherical symmetry in the "repulsive force of gravitons on space," we adopt the Schwarzschild solution under static and spherically symmetric conditions:

$$ds^2 = - \left( 1 - \frac{2GM}{c^2 r} - \frac{\alpha}{c^2 r} \right) c^2 dt^2 + \left( 1 - \frac{2GM}{c^2 r} - \frac{\alpha}{c^2 r} \right)^{-1} dr^2 + r^2 d\Omega^2$$

**4. Derivation of the Rotation Curve**

From the modified Schwarzschild solution, we calculate the orbital velocity for circular motion:

$$v(r) = \sqrt{\frac{GM}{r} + \frac{\alpha}{r}}$$

**5. Adjustment of Rotation Speed by Graviton Influence**

To reflect the observed galaxy rotation curve, a distance-dependent decay function is applied to the graviton term:

$$v_{\text{adjusted}}(r) = \sqrt{\frac{GM}{r} + \alpha \cdot \left( 1 - \frac{1}{1 + \left(\frac{r}{R}\right)^2} \right)}$$

**6. Final Rotation Speed Equation Including the Graviton Model**

By adding the graviton correction term to the conventional velocity components (disk, gas, bulge), the following equation is obtained:

$$v_{\text{total}}(r) = \left| v_{\text{disk}}(r) + v_{\text{gas}}(r) + v_{\text{bul}}(r) + \text{sign}(\alpha) \cdot \sqrt{\alpha \cdot \left( 1 - \frac{1}{1 + \left(\frac{r}{R}\right)^2} \right)} \right|$$

**Parameter Definitions**

- $v_{\text{disk}}(r)$ : Velocity component of the disk.
- $v_{\text{gas}}(r)$ : Velocity component of the gas.
- $v_{\text{bul}}(r)$ : Velocity component of the bulge.
- $\alpha$ : Parameter indicating the "concentration" of gravitons.
- $l$ : Scale parameter indicating the range of graviton influence.
- $\text{sign}(\alpha)$ : Function indicating the direction of graviton influence.
- $r$ : Radius from the galaxy center.

**Meaning of  $\text{sign}(\alpha)$  and Absolute Value**

- $\text{sign}(\alpha)$ : A function that changes the direction of graviton influence based on the sign of  $\alpha$ . Specifically, if  $\alpha > 0$ ,  $\text{sign}(\alpha) = +1$ , indicating an outward repulsive force due to gravitons. Conversely, if  $\alpha < 0$ ,  $\text{sign}(\alpha) = -1$ , indicating an inward attractive effect.
- **Absolute Value  $|\cdot|$** : To mathematically prevent the total speed from becoming negative, the absolute value is taken. In particular, if  $\alpha$  is negative, the influence of anti-gravitons may theoretically reduce the total rotation speed to negative values. The absolute value ensures that the speed remains a physically realizable non-negative value.

重力子空間斥力モデルによる  
相対性理論と量子力学の統合的理解

Misaki Kasai

個人著書

2024年10月30日



## 要旨

本研究では、ダークマターの項に重力子の効果を取り入れた修正アインシュタイン方程式を構築し、球対称解であるシュワルツシルト解を基にして導いた、銀河回転曲線重力子逆二乗減衰モデル(銀河回転曲線MiSAKiモデル)と実際の観測データの一致を考察し、宇宙の構造形成における重力子の役割から、重力、ダークマター、ダークエネルギーの正体を統一的に説明する「重力子空間斥力モデル(Re:GraviSモデル)」を提案し、相対性理論と量子力学の統合的理解を実現します。

*Keywords:* 量子重力理論、重力子、ダークマター、ダークエネルギー、ブラックホール、銀河回転曲線

## 導入

銀河の回転曲線における、特に銀河外縁部の回転速度の観測値と理論値のずれは、物理学における長年の課題とされてきました。本研究は、重力子が空間に対する斥力を持ち、重力の正体が、「その斥力に対するエントロピー駆動の空間の反発」によって説明されるとする新たな仮説を提案し、アインシュタイン方程式の球対称解(シュワルツシルト解)を基に、反重力子の存在も考慮したフィッティングモデルを用いて銀河の回転曲線の解析を行います。その結果をもとに、重力の理論的な扱い方の違いから困難とされてきた、「相対性理論と量子力学の統一」に向けた可能性と宇宙の真相を探究します。

## モデルの概要と理論的背景

今回使用する、「銀河回転曲線重力子逆二乗減衰モデル(銀河回転曲線MiSAKiモデル)」は、ダークマターの項に重力子の効果を取り入れた修正アインシュタイン方程式の球対称解であるシュワルツシルト解から、仮定を通じて導かれたものであり、重力子が空間に対して、「球対称の斥力を持つ」ことを仮定しています。このモデルは、より小さなスケールでは重力子がダークマターとしての役割を果たし、宇宙広域のような広いスケールでは空間を押し広げることでダークエネルギーとしての効果を発揮することを仮定としています。また、重力子の集中度が、ダークマターの分布に反映されることで、物理学の長年の課題であった「銀河の回転曲線の謎」が解明されます。

## 解析手法

AIツール(ChatGPT4o)を用いて開発した銀河回転曲線専用の高精度フィッティング検証用スクリプトを使用し、SPARC銀河回転曲線観測データセット(175個)と銀河回転曲線MiSAKiモデルとの一致を、プロット図による視覚化と決定係数の評価により解析しました。さらに、決定係数が低い銀河には、 $\alpha=0$ (重力子と反重力子の均衡状態)と、 $\alpha<0$ (反重力子の集中度が重力子を大きく上回る場合)を許容して再検証を行い、重力子と反重力子の影響を統合的に評価しました。また、すべてのフィッティングの完了後に、重力子の集中度を表す変数パラメータ $\alpha$ 、重力子の効果の範囲を表す変数スケールパラメータ $R$ 、銀河の観測速度成分、ディスク成分、バルジ成分などの物理パラメータを網羅的に相関分析し、相関係数を算出して相関関係を評価しました。

## 結果と考察

検証の結果、ほとんどの銀河とモデルの間で、極めて良好なフィッティングが確認されました(決定係数 $R^2$ の平均値: 0.758、標準偏差: 0.379、中央値: 0.963)。175個に及ぶ検証数にも関わらず、決定係数 $R^2$ の中央値は0.963と高い値を示し、標準偏差は0.379と小さな値を示しました。また、残差の分布はランダムかつ小さな範囲に収まっており、過剰フィッティングの兆候は見られませんでした。したがって、本モデルは観測値に対して過剰に適合することなく、銀河

の回転曲線を忠実かつ高精度に再現していることが確認されました。また、相関分析の結果、重力子の集中度を表す $\alpha$ が、観測速度およびディスク成分速度と非常に高い正の相関(平均観測速度 Avg Vobs(r):0.96、平均ディスク速度 Avg Vdisk(r):0.87)を持つことが確認されました。これは重力子の集中度が銀河外縁部の回転速度に強い影響を与えていることを意味し、アインシュタイン方程式により存在が示唆されていたダークマターの特徴と忠実に一致します。また、分析の結果、 $\alpha$ とRの分布はそれぞれ特定の範囲に集中していました。 $\alpha$ が特定の範囲に集中することは、重力子が与える銀河外縁部の回転速度への影響には一定の基準、もしくは限界が存在することを示唆しています。また、重力子の効果の範囲を表すスケールパラメータRは、 $\alpha$ を含む他の物理パラメータに依存せず、一定の範囲に集中していました。これは、重力子の効果が銀河間に共通する一定のスケールで作用している可能性を示しています。このことから、重力子が持つ空間への斥力は物質的な相互作用は示さず、逆二乗則に従って減衰しながら球対称に広がり、銀河外縁部の回転速度維持に貢献していることが明らかになりました。この結果は、重力が、ニュートンの万有引力の法則に従い、距離の逆二乗で減衰するという事実と一致しています。さらに、物質的な相互作用を示さないことは、重力子がヒッグス場と干渉せず、物質に質量を与えるヒッグス粒子との相互作用を示さないことを示唆しています。これは、従来の量子力学において、重力子が質量を持たず、閉じたひもとして次元間の移動が可能であるとする特異な性質の必要条件を満たす可能性があります。また、今回の検証では、 $\alpha$ が0や負の値をとることはなく、反重力子の影響は直接的には確認されませんでした。しかし、この結果は反重力子の存在の否定を直接的に意味しません。この理由は、重力子の集中度を表す $\alpha$ の本質が、そこに存在している重力子の絶対数を表すわけではないためです。今回の検証結果より、本研究においては、銀河外縁部の回転速度を維持している謎の力の正体は、「重力子が持つ空間に対する斥力である」と結論付けられました。また、今回の理論モデルの適合性や物質と相互作用しない特徴などから考察し、重力の正体は、「重力子の空間への斥力に対するエントロピー駆動の空間の反発」であり、重力子の効果と空間の歪みは因果関係にあることを結論付けました。さらに、ダークマターの正体は、「重力子そのもの」であり、ダークエネルギーの正体は「重力子が持つ空間への斥力」であることが強く示唆されました。これにより、光速を超えるスピードで宇宙を膨張させている謎のエネルギーの正体は、「重力子が持つ空間への斥力である可能性が非常に高い」、と結論付けられます。さらに、質量を持たないとされる重力子が集中するメカニズムは、重力子自身が間接的に発生させる重力、つまり、エントロピー駆動の空間の反発により自身が空間内部に閉じ込められ、その結果として重力子同士が一点に集中していくという、一種の自己因果律のようなものであることが考察されます。そして、このメカニズムは、ブラックホールが持つ事象の地平面の内側だけでなく、物理法則が破れるとされる時空特

異点においてもその特性を保っている可能性が考えられます。つまり、ブラックホールは、「重力子の空間に対する斥力を、空間の反発が瞬間的に上回った場合」に形成が開始され、時空特異点は、単なる一次元の点として瞬間的に生じるのではなく、空間が「自ら」内側へ向かって自身を圧縮するような、非常に特殊で複雑な構造と量子的な広がりを持つ時空間領域として形成されていくかもしれません。そして、ある限界を超えた時点で、空間の反発の強さが重力子の斥力の強さへ近づき、最終的に形成された時空特異点は、密度や重力、曲率が無限大とされる一次元の「点」ではなく、「重力子の斥力と、空間の反発が完全に均衡している時空間領域である」と解釈することで、時空特異点において量子効果が働く余地が生まれます。このメカニズムは、従来の量子力学では扱えなかった時空特異点における無限大の問題を回避でき、量子力学の範囲で時空特異点を理解できる可能性を秘めています。しかし、両者が完全に均衡している状態と仮定すると、その時空間領域において量子的な広がりを持つことが不可能になる可能性も懸念されます。これを、量子力学に基づいて柔軟に解釈するためには、閉じたひもであり次元間を移動できるとされる重力子が、時空特異点において別次元空間へ逃げ出している可能性を考慮する必要があるかもしれません。この解釈では、重力子が「開いたひも」の状態から、時空特異点における極端な環境下においては、「閉じたひも」に変化することで、自身をエネルギーに変換し、情報を保持したまま別次元空間へ逃げ出すメカニズムが考えられます。そして、情報を別次元空間へ伝達し、保存する役割を担う「粒子」として機能することで、情報は別次元へ保存され、事象の地平面の内側で消失することがなくなります。この可能性は、近年提案されたソフトヘア理論に加えて、ブラックホール情報パラドックスの解決の糸口になるかもしれません。つまり、時空特異点は、重力子にとっての別次元空間ゲートとして機能している可能性があり、このメカニズムは、ブラックホールの情報パラドックスを解決するとともに、時空特異点における量子的な広がりを許容することになりえます。さらに、このブラックホール形成のメカニズムは、中等度に強い重力場においては時間が延長し、ブラックホールのような極端に強い重力場では時間が短縮されるような、時空間領域レベルの不可解な現象を容易に説明します。中等度に強い重力場では、重力子の空間への斥力により空間が押し広げられることで、プランク長も引き伸ばされ、それに伴い、移動距離の最小単位も長くなることで、量子レベルで物理的移動が遅くなり、その時空間領域における時間が長くなります。一方で、ブラックホールのような極端に強い重力場においては、空間が自発的なレベルで縮小し、それに伴いプランク長が縮められることで、移動距離の最小単位も短くなります。これにより、量子レベルで物理的移動が速くなり、その時空間領域の時間は短くなります。これは、「ブラックホール外側の観測者の視点から、物体が事象の地平面に近づくにつれて " 時間の遅れ " が生じ、物体の動きが停止するかのように見えるという相対性理論の予測と一致します。この現象を、視点を逆にして再現して

みると、観測者が位置する時空間領域では、空間の反発に対して、重力子の斥力の方が十分に強く、空間が押し広げられている時空間領域であるため、ブラックホール近くの時空間領域の時間と比べて、その観測者の時間(「固有時間」)が相対的に長くなっている、と説明できます。このような時空間領域レベルの現象は、重力子空間斥力モデル(Re:GraviSモデル)をさらに拡張した、重力子時空間斥力モデル(Re:GraviSTモデル:Repulsion Graviton Space-Time Model)として理解する必要があるかもしれません。今回の銀河回転曲線の解析を通じて得られた、これらの結果と考察は、銀河回転曲線重力子逆二乗減衰モデル(銀河回転曲線MiSAKiモデル)によって理論的背景が強力に裏付けられており、相対性理論と量子力学を統一した「量子重力理論(大統一理論)」の完成が可能であることを強く示唆しています。このことから、重力子空間斥力モデル(Re:GraviSモデル)だけでなく、理論をさらに時空間領域にまで拡張した「重力子時空間斥力モデル(Re:GraviSTモデル)」の研究の必要性が強く示されました。

参考文献:本研究は独自性が高く、一般的な知見に基づいているため、特定の参考文献はありません。

使用したデータセット:SPARC銀河回転曲線データセット(175個)

<https://astronomy.case.edu/2016/08/26/sparc-galaxy-database>

# 銀河回転曲線重力子逆二乗減衰モデル

## (銀河回転曲線 MiSAKi モデル)

$$v_{\text{total}}(r) = \left| v_{\text{disk}}(r) + v_{\text{gas}}(r) + v_{\text{bul}}(r) + \text{sign}(\alpha) \cdot \sqrt{\alpha \cdot \left(1 - \frac{1}{1 + \left(\frac{r}{R}\right)^2}\right)} \right|$$

### 導出過程

#### 1. 通常のアインシュタイン方程式

標準的なアインシュタイン方程式は、時空の曲率と物質やエネルギーのエネルギー・運動量テンソルの関係を表します：

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

#### 2. 修正アインシュタイン方程式

重力子の影響を考慮するため、暗黒物質成分として  $T_{\mu\nu}^{(DM)}$  を追加します：

$$G_{\mu\nu} = \frac{8\pi G}{c^4} (T_{\mu\nu} + T_{\mu\nu}^{(DM)})$$

#### 3. 修正アインシュタイン方程式のシュワルツシルト解

「重力子の空間への斥力は球対称である」という仮定からシュワルツシルト解を採用し、静的で球対称の条件で解きます：

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r} - \frac{\alpha}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r} - \frac{\alpha}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

#### 4. 回転曲線の導出

修正シュワルツシルト解から円運動の軌道速度を計算します：

$$v(r) = \sqrt{\frac{GM}{r} + \frac{\alpha}{r}}$$

#### 5. 重力子の影響による回転速度の調整

観測される銀河回転曲線を反映するため、重力子項に距離依存の減衰関数を適用します：

$$v_{\text{adjusted}}(r) = \sqrt{\frac{GM}{r} + \alpha \cdot \left(1 - \frac{1}{1 + \left(\frac{r}{R}\right)^2}\right)}$$

#### 6. 重力子モデルを含む最終的な回転速度方程式

重力子の補正項を従来の速度成分（ディスク、ガス、バルジ）に加えると、次の式を得ます：

$$v_{\text{total}}(r) = \left| v_{\text{disk}}(r) + v_{\text{gas}}(r) + v_{\text{bul}}(r) + \text{sign}(\alpha) \cdot \sqrt{\alpha \cdot \left(1 - \frac{1}{1 + \left(\frac{r}{R}\right)^2}\right)} \right|$$

### パラメータの意味

- $v_{\text{disk}}(r)$ : ディスク成分の速度。
- $v_{\text{gas}}(r)$ : ガス成分の速度。
- $v_{\text{bul}}(r)$ : バルジ成分の速度。
- $\alpha$ : 重力子の「集中度」を示すパラメータ。
- $R$ : 重力子の影響の範囲を示すスケールパラメータ。
- $\text{sign}(\alpha)$ : 重力子の影響の方向を示す関数。
- $r$ : 銀河中心からの半径。

### $\text{sign}(\alpha)$ と絶対値の意味

- $\text{sign}(\alpha)$ :  $\alpha$  の符号に基づき、重力子の影響方向を変換するための関数です。具体的には、 $\alpha > 0$  ならば  $\text{sign}(\alpha) = +1$  となり、重力子の影響が外向きの斥力として働きます。逆に、 $\alpha < 0$  の場合は  $\text{sign}(\alpha) = -1$  となり、内向きの引力効果が発生します。
- 絶対値  $|\cdot|$ : 数式上、総速度が負になることを防ぐために絶対値を取っています。特に  $\alpha$  が負の場合、重力子の影響で総回転速度が理論的に負の値まで減少する可能性があります。絶対値を用いることで、速度が常に物理的に実現可能な非負値になるようにしています。

## Fitting Results

Galaxy	Alpha	R	R_squared	Avg Vobs	Avg Vgas	Avg Vdisk	Avg Vbul	Avg SBdisk	Avg SBbul
CamB	2210.459	3.784	0.999	10.877	6.48	12.74	0.0	11.882	0.0
D512-2	1553.503	1.289	0.963	32.375	6.465	19.172	0.0	7.09	0.0
D564-8	793.331	1.594	0.999	19.057	6.015	8.113	0.0	1.853	0.0
D631-7	4317.117	3.49	0.992	43.038	17.947	15.223	0.0	4.181	0.0
DDO064	3495.146	1.784	0.958	32.535	10.358	15.104	0.0	11.647	0.0
DDO154	2603.474	2.076	0.987	38.183	14.017	9.485	0.0	1.926	0.0
DDO161	4824.58	4.251	0.971	49.255	25.21	18.864	0.0	6.063	0.0
DDO168	3956.082	2.088	0.967	41.46	20.458	15.367	0.0	6.114	0.0
DDO170	4172.971	3.801	0.994	52.25	21.895	18.029	0.0	2.079	0.0
ESO079-G014	37823.761	6.816	0.969	119.387	17.353	95.588	0.0	123.336	0.0
ESO116-G012	13398.415	2.624	0.982	86.747	19.227	53.403	0.0	55.321	0.0
ESO444-G084	4185.996	1.142	0.985	48.286	14.793	11.726	0.0	6.631	0.0
ESO563-G021	100000.0	5.976	0.967	263.31	25.071	219.415	0.0	273.683	0.0
F561-1	2911.846	3.089	0.989	43.25	17.943	41.21	0.0	16.672	0.0
F563-1	12008.861	3.44	0.931	88.953	20.656	26.284	0.0	6.334	0.0
F563-V1	985.786	2.691	0.918	24.617	13.877	20.403	0.0	6.277	0.0
F563-V2	15106.962	2.445	0.989	86.15	19.396	32.626	0.0	27.042	0.0
F565-V2	9336.221	5.25	0.997	60.471	17.523	17.986	0.0	3.753	0.0
F567-2	2922.53	2.978	0.928	44.32	3.898	31.162	0.0	8.59	0.0
F568-1	20454.024	3.882	0.997	100.883	15.787	40.603	0.0	25.302	0.0
F568-3	13725.242	5.244	0.978	74.817	14.545	41.657	0.0	24.663	0.0
F568-V1	14069.718	2.675	0.994	94.813	13.11	39.197	0.0	20.503	0.0
F571-8	21587.302	4.436	0.975	94.162	7.604	82.898	0.0	243.754	0.0
F571-V1	8273.397	4.851	0.996	69.743	22.654	30.401	0.0	6.021	0.0
F574-1	10488.423	3.173	0.999	80.743	13.08	44.211	0.0	23.729	0.0
F574-2	2604.549	8.688	0.999	28.32	10.208	30.722	0.0	8.534	0.0
F579-V1	12527.885	1.365	0.984	100.557	13.114	60.175	0.0	44.321	0.0
F583-1	8195.725	4.412	0.991	53.264	7.45	20.404	0.0	9.241	0.0
F583-4	4504.923	2.086	0.97	51.233	8.598	32.473	0.0	20.281	0.0
IC2574	7055.918	8.265	0.979	43.476	13.907	22.484	0.0	4.966	0.0
IC4202	63898.424	3.887	0.973	219.719	26.156	165.658	120.974	129.874	148.329
KK98-251	2235.745	2.559	0.984	21.583	11.753	9.041	0.0	4.205	0.0
NGC0024	12486.231	1.092	0.987	84.659	12.132	53.564	0.0	128.746	0.0
NGC0055	8708.065	4.45	0.989	72.252	33.53	40.493	0.0	12.447	0.0
NGC0100	8716.182	3.237	0.996	67.343	10.886	44.401	0.0	30.564	0.0
NGC0247	11458.152	4.262	0.913	85.173	22.607	48.613	0.0	17.457	0.0
NGC0289	30643.219	1.0	-0.433	173.036	43.536	125.72	0.0	109.109	0.0
NGC0300	9145.655	2.678	0.971	80.664	16.538	43.331	0.0	15.693	0.0
NGC0801	50891.08	1.431	0.75	207.462	33.62	218.881	0.0	729.832	0.0
NGC0891	50651.161	1.0	-3.944	219.444	18.049	237.984	110.909	389.133	76.372
NGC1003	11592.774	3.355	0.896	99.031	26.665	49.302	0.0	12.88	0.0
NGC1090	28542.459	2.741	0.974	149.996	25.906	134.809	0.0	158.922	0.0
NGC1705	6146.641	1.0	-2.848	69.65	14.151	29.654	0.0	45.336	0.0
NGC2366	3111.192	2.001	0.976	40.493	18.537	17.451	0.0	8.394	0.0
NGC2403	16460.469	1.568	0.905	107.997	23.533	77.973	0.0	152.345	0.0
NGC2683	32004.415	1.0	-0.109	177.636	19.713	152.531	43.032	89.216	0.0
NGC2841	92020.981	1.0	-0.106	301.42	40.305	181.029	123.045	138.81	0.434
NGC2903	39358.761	1.093	0.815	182.706	22.918	198.769	0.0	695.723	0.0
NGC2915	7363.059	1.638	0.943	74.117	14.873	27.201	0.0	16.459	0.0

NGC2955	68183.46	1.0	0.735	252.125	37.901	209.496	175.13	356.584	382.071
NGC2976	16002.309	2.15	0.988	51.159	13.647	51.227	0.0	228.187	0.0
NGC2998	43794.672	1.284	0.876	187.077	31.724	154.951	0.0	455.523	0.0
NGC3109	6082.008	3.828	0.997	45.764	13.866	12.389	0.0	2.972	0.0
NGC3198	23627.13	3.298	0.978	125.672	20.489	100.346	0.0	128.293	0.0
NGC3521	50398.647	1.0	0.267	207.439	8.892	250.331	0.0	908.006	0.0
NGC3726	27970.542	4.287	0.863	151.833	30.913	145.633	0.0	73.667	0.0
NGC3741	2587.843	2.096	0.891	36.181	10.409	7.679	0.0	2.396	0.0
NGC3769	14604.083	1.644	0.811	115.875	31.089	88.335	0.0	43.648	0.0
NGC3877	32915.558	3.12	0.983	144.069	17.991	164.273	0.0	250.628	0.0
NGC3893	34112.48	1.276	0.611	178.6	37.352	167.431	0.0	161.03	0.0
NGC3917	21610.833	4.19	0.987	114.853	19.109	89.676	0.0	59.126	0.0
NGC3949	29098.447	1.681	0.985	153.143	28.803	176.25	0.0	249.291	0.0
NGC3953	51711.928	2.588	0.914	213.625	16.273	229.885	0.0	191.028	0.0
NGC3972	19603.982	3.121	0.989	105.1	17.801	85.324	0.0	86.105	0.0
NGC3992	64530.56	1.0	-0.01	253.667	35.877	202.177	0.0	46.167	0.0
NGC4010	19540.731	4.385	0.961	98.8	23.096	87.118	0.0	81.976	0.0
NGC4013	32018.359	1.0	-0.095	178.417	27.241	139.498	65.415	25.242	0.351
NGC4051	23912.094	1.706	0.575	143.571	18.617	169.479	0.0	180.333	0.0
NGC4068	5176.95	3.256	0.999	24.763	10.372	19.155	0.0	19.612	0.0
NGC4085	22940.632	2.676	0.987	108.586	16.459	137.104	0.0	276.694	0.0
NGC4088	31838.81	2.857	0.911	162.158	38.237	198.898	0.0	191.557	0.0
NGC4100	33034.974	2.087	0.331	175.292	27.6	149.621	0.0	75.076	0.0
NGC4138	28137.647	1.0	-0.244	165.571	19.813	136.133	67.511	75.903	0.0
NGC4157	36475.695	2.414	0.814	180.788	39.944	179.146	28.669	147.089	0.001
NGC4183	12630.087	2.467	0.956	101.9	29.897	62.814	0.0	24.854	0.0
NGC4214	7349.591	1.0	-1.155	75.221	13.766	42.483	0.0	63.626	0.0
NGC4217	36389.288	2.47	0.947	168.616	24.499	79.491	187.557	43.693	420.265
NGC4389	20723.892	4.794	0.991	73.1	13.913	128.58	0.0	291.81	0.0
NGC4559	14903.207	2.503	0.962	110.491	32.112	89.634	0.0	57.147	0.0
NGC5005	73179.883	1.0	-1.446	252.278	63.507	244.421	152.914	964.706	662.972
NGC5033	46500.394	1.0	-2.21	211.545	37.189	144.708	96.642	184.189	164.369
NGC5055	37296.153	1.0	0.527	187.25	29.109	202.377	0.0	404.14	0.0
NGC5371	50467.371	1.0	-0.311	221.368	30.581	245.685	0.0	244.003	0.0
NGC5585	8523.813	2.423	0.938	62.188	14.974	44.598	0.0	94.122	0.0
NGC5907	49335.981	1.0	-0.01	221.579	43.787	177.429	0.0	53.81	0.0
NGC5985	89402.853	3.145	0.831	285.455	16.07	193.398	58.907	113.491	0.032
NGC6015	24750.448	1.638	0.954	143.218	24.72	103.57	0.0	132.176	0.0
NGC6195	62180.95	1.202	0.791	243.087	35.109	189.22	191.784	198.607	253.646
NGC6503	13785.692	1.0	0.859	114.29	22.658	76.979	0.0	56.95	0.0
NGC6674	64087.871	1.544	0.22	249.933	32.718	161.189	57.967	75.557	0.789
NGC6789	10888.072	1.0	0.989	37.725	8.365	16.67	0.0	58.86	0.0
NGC6946	28575.702	1.0	-0.84	163.293	28.972	154.906	54.588	195.244	95.576
NGC7331	59418.558	1.0	-0.175	241.861	35.78	262.901	63.954	242.204	0.0
NGC7793	11060.957	1.0	0.927	74.457	13.971	69.883	0.0	312.338	0.0
NGC7814	53049.767	1.0	-6.639	224.611	10.335	96.745	171.889	75.381	426.382
PGC51017	497.29	1.0	-52.443	19.167	11.39	16.622	0.0	10.648	0.0
UGC00128	16983.289	5.407	0.98	118.427	30.513	44.28	0.0	7.272	0.0
UGC00191	7043.906	1.738	0.986	58.978	13.528	33.567	0.0	39.888	0.0
UGC00634	13308.875	5.833	0.989	95.737	32.16	36.33	0.0	3.185	0.0
UGC00731	5622.3	2.704	0.957	61.783	21.348	13.185	0.0	1.807	0.0
UGC00891	5316.684	4.053	0.999	49.19	18.79	18.356	0.0	3.304	0.0



UGC01230	12159.598	3.273	0.949	91.882	25.998	42.394	0.0	18.682	0.0
UGC01281	4898.691	3.034	0.992	34.431	7.414	15.778	0.0	10.068	0.0
UGC02023	10898.59	5.68	0.994	37.48	11.8	31.256	0.0	26.198	0.0
UGC02259	7791.942	1.325	0.967	79.838	18.059	36.259	0.0	14.944	0.0
UGC02455	7109.159	4.365	0.919	37.375	16.878	67.914	0.0	114.219	0.0
UGC02487	10000.0	1.0	-3.613	355.059	25.617	216.025	124.368	68.601	3.496
UGC02885	83226.764	1.0	-0.657	285.842	48.856	191.468	98.216	117.535	70.172
UGC02916	47103.831	1.0	-4.891	208.721	24.96	87.899	195.499	78.456	631.335
UGC02953	85558.949	1.0	-1.21	273.861	7.528	213.246	145.675	792.741	1119.297
UGC03205	50958.112	1.0	0.815	215.125	14.602	160.496	97.208	201.938	258.591
UGC03546	50844.943	1.0	-5.009	212.0	10.794	159.68	180.201	575.026	197.024
UGC03580	11790.028	1.222	0.385	91.702	15.525	68.367	64.905	158.656	272.641
UGC04278	14195.759	5.803	0.951	56.881	4.551	30.109	0.0	19.574	0.0
UGC04305	1290.298	1.082	0.874	30.085	15.426	28.021	0.0	18.001	0.0
UGC04325	9373.46	1.383	0.989	81.05	19.008	41.712	0.0	31.779	0.0
UGC04483	1256.943	1.0	0.847	18.276	8.609	8.133	0.0	8.989	0.0
UGC04499	6050.694	2.586	0.998	60.889	20.694	34.901	0.0	13.78	0.0
UGC05005	10910.716	8.122	0.99	69.673	22.47	34.67	0.0	11.721	0.0
UGC05253	66323.152	1.0	-21.93	241.781	14.973	124.28	211.814	301.273	1622.117
UGC05414	4833.036	2.348	0.997	45.183	15.748	30.342	0.0	23.925	0.0
UGC05716	5375.161	2.199	0.896	64.442	20.427	20.733	0.0	4.053	0.0
UGC05721	7452.995	1.0	0.823	70.335	17.1	31.538	0.0	62.155	0.0
UGC05750	7651.331	6.92	0.991	48.7	14.866	27.305	0.0	11.105	0.0
UGC05764	3200.304	1.0	0.938	45.77	14.619	10.619	0.0	3.903	0.0
UGC05829	5796.612	4.17	0.966	46.364	15.445	18.384	0.0	5.726	0.0
UGC05918	2072.076	1.501	0.991	35.288	9.159	14.426	0.0	5.081	0.0
UGC05986	14596.801	2.475	0.964	96.74	16.479	58.738	0.0	41.6	0.0
UGC05999	12035.611	6.948	0.988	83.24	30.084	37.54	0.0	5.446	0.0
UGC06399	8972.354	3.14	0.999	68.756	15.742	38.743	0.0	19.099	0.0
UGC06446	6840.388	1.574	0.947	72.965	21.019	27.667	0.0	9.947	0.0
UGC06614	35072.577	1.0	-0.03	186.308	9.492	93.237	131.89	28.785	28.653
UGC06628	1884.872	1.343	0.991	39.1	15.111	41.863	0.0	22.937	0.0
UGC06667	8766.028	3.167	0.996	67.867	15.449	17.281	0.0	5.657	0.0
UGC06786	50395.716	1.0	-2.277	212.378	15.741	126.806	114.816	175.622	746.685
UGC06787	64538.16	1.0	-6.078	239.648	11.468	106.209	188.655	170.427	1992.62
UGC06818	9120.805	5.47	0.946	51.588	11.996	37.458	0.0	19.27	0.0
UGC06917	12657.447	3.025	0.984	94.173	19.178	61.769	0.0	25.605	0.0
UGC06923	7666.406	2.053	0.954	67.7	21.963	53.895	0.0	41.45	0.0
UGC06930	12128.888	3.024	0.992	97.56	27.712	60.381	0.0	19.269	0.0
UGC06973	32392.209	1.0	-1.73	173.778	24.408	221.307	85.427	312.078	0.001
UGC06983	12398.889	2.554	0.943	101.165	25.621	52.494	0.0	13.723	0.0
UGC07089	7105.048	4.199	0.983	59.792	18.013	44.109	0.0	20.259	0.0
UGC07125	4381.009	3.729	0.973	56.738	26.216	33.668	0.0	7.551	0.0
UGC07151	5842.869	1.419	0.985	62.309	14.142	44.718	0.0	42.109	0.0
UGC07232	4762.974	1.0	0.985	30.45	8.33	22.648	0.0	58.633	0.0
UGC07261	5678.794	1.243	0.974	67.2	13.89	42.589	0.0	24.646	0.0
UGC07323	9951.782	3.639	0.971	60.1	12.33	49.91	0.0	50.953	0.0
UGC07399	10620.424	1.292	0.976	88.32	18.107	34.448	0.0	22.428	0.0
UGC07524	7207.789	3.597	0.981	62.739	18.069	31.866	0.0	12.253	0.0
UGC07559	1442.049	1.529	0.997	23.659	9.789	15.191	0.0	9.311	0.0
UGC07577	843.369	2.291	0.983	10.689	4.509	10.079	0.0	6.984	0.0
UGC07603	4410.197	1.163	0.99	52.717	13.143	26.417	0.0	19.698	0.0

UGC07608	6269.009	2.821	0.993	49.225	14.016	15.027	0.0	5.315	0.0
UGC07690	4241.499	1.0	-1.294	56.471	16.614	40.809	0.0	35.411	0.0
UGC07866	1171.232	1.0	0.972	24.771	8.59	15.247	0.0	10.847	0.0
UGC08286	7258.98	1.528	0.992	72.776	15.665	32.815	0.0	16.096	0.0
UGC08490	6567.394	1.0	0.89	75.157	20.286	31.715	0.0	18.42	0.0
UGC08550	3254.633	1.0	0.963	49.782	13.223	21.415	0.0	10.117	0.0
UGC08699	38271.655	1.0	-9.591	184.171	20.08	117.186	119.33	231.832	713.69
UGC08837	7278.978	5.825	0.992	30.489	10.451	22.321	0.0	12.58	0.0
UGC09037	25128.783	5.55	0.96	137.927	48.239	133.802	0.0	56.965	0.0
UGC09133	73071.263	1.0	-2.226	264.176	30.466	169.375	168.779	265.531	679.118
UGC09992	1355.073	1.0	-0.471	32.02	13.268	22.262	0.0	11.006	0.0
UGC10310	5887.552	2.193	0.996	62.743	18.43	35.384	0.0	15.16	0.0
UGC11455	82092.004	5.729	0.969	228.244	21.975	249.594	0.0	396.475	0.0
UGC11557	9052.736	4.491	0.981	57.317	11.312	74.628	0.0	98.581	0.0
UGC11820	5766.035	1.553	0.945	51.205	16.072	19.477	0.0	29.292	0.0
UGC11914	95965.946	1.0	-0.451	279.523	9.18	204.639	202.966	859.681	1058.454
UGC12506	57513.067	2.547	0.856	228.871	51.122	140.471	0.0	60.567	0.0
UGC12632	5547.563	2.755	0.985	60.187	17.538	26.414	0.0	7.504	0.0
UGC12732	8128.782	3.324	0.912	75.75	20.671	29.312	0.0	6.881	0.0
UGCA281	2098.542	1.0	0.844	22.114	7.063	14.296	0.0	36.737	0.0
UGCA442	3846.252	2.285	0.989	44.788	17.469	11.248	0.0	2.743	0.0
UGCA444	1557.181	1.219	0.977	26.236	12.042	4.522	0.0	0.962	0.0

## Error Log

Galaxy	Error
All galaxies processed successfully	No errors encountered

## Summary Statistics

Unnamed: 0	count	mean	std	min	25%	50%	75%	max	variance	median	iqr
Alpha	175.0	21827.845	23741.333	497.29	5722.414	11458.152	32011.387	100000.0	563650899.604	458.152	26288.973
R	175.0	2.539	1.684	1.0	1.0	2.096	3.311	8.688	2.836	2.096	2.311
R_squared	175.0	-0.01	4.575	-52.443	0.815	0.963	0.987	0.999	20.931	0.963	0.173
Avg Vobs	175.0	108.751	76.537	10.689	50.471	80.664	167.094	355.059	5857.984	80.664	116.623
Avg Vgas	175.0	19.716	9.891	3.898	13.398	17.538	24.84	63.507	97.84	17.538	11.442
Avg Vdisk	175.0	79.166	70.22	4.522	26.415	44.211	134.305	262.901	4930.819	44.211	107.89
Avg Vbul	175.0	22.289	52.434	0.0	0.0	0.0	0.0	211.814	2749.329	0.0	0.0
Avg SBdisk	175.0	104.221	171.701	0.962	10.927	29.292	128.52	964.706	29481.348	29.292	117.593
Avg SBbul	175.0	68.717	255.214	0.0	0.0	0.0	0.0	1992.62	65133.94	0.0	0.0

## Correlation Matrix

Unnamed: 0	Alpha	R	R_squared	Avg Vobs	Avg Vgas	Avg Vdisk	Avg Vbul	Avg SBdisk	Avg SBbul
Alpha	1.0	-0.199	-0.132	0.963	0.464	0.871	0.687	0.63	0.475
R	-0.199	1.0	0.196	-0.285	-0.058	-0.241	-0.334	-0.254	-0.232
R_squared	-0.132	0.196	1.0	-0.124	0.045	-0.066	-0.28	-0.087	-0.329
Avg Vobs	0.963	-0.285	-0.124	1.0	0.564	0.903	0.683	0.598	0.443
Avg Vgas	0.464	-0.058	0.045	0.564	1.0	0.545	0.181	0.218	-0.002
Avg Vdisk	0.871	-0.241	-0.066	0.903	0.545	1.0	0.485	0.727	0.263
Avg Vbul	0.687	-0.334	-0.28	0.683	0.181	0.485	1.0	0.438	0.749
Avg SBdisk	0.63	-0.254	-0.087	0.598	0.218	0.727	0.438	1.0	0.405
Avg SBbul	0.475	-0.232	-0.329	0.443	-0.002	0.263	0.749	0.405	1.0

## Spearman Correlation

Unnamed: 0	Alpha	R	R_squared	Avg Vobs	Avg Vgas	Avg Vdisk	Avg Vbul	Avg SBdisk	Avg SBbul
Alpha	1.0	-0.172	-0.503	0.956	0.52	0.908	0.596	0.796	0.569
R	-0.172	1.0	0.642	-0.28	-0.003	-0.244	-0.465	-0.375	-0.43
R_squared	-0.503	0.642	1.0	-0.576	-0.302	-0.547	-0.584	-0.528	-0.554
Avg Vobs	0.956	-0.28	-0.576	1.0	0.611	0.907	0.605	0.742	0.575
Avg Vgas	0.52	-0.003	-0.302	0.611	1.0	0.538	0.179	0.275	0.15
Avg Vdisk	0.908	-0.244	-0.547	0.907	0.538	1.0	0.519	0.881	0.483
Avg Vbul	0.596	-0.465	-0.584	0.605	0.179	0.519	1.0	0.48	0.953
Avg SBdisk	0.796	-0.375	-0.528	0.742	0.275	0.881	0.48	1.0	0.463
Avg SBbul	0.569	-0.43	-0.554	0.575	0.15	0.483	0.953	0.463	1.0

## Kendall Correlation

Unnamed: 0	Alpha	R	R_squared	Avg Vobs	Avg Vgas	Avg Vdisk	Avg Vbul	Avg SBdisk	Avg SBbul
Alpha	1.0	-0.113	-0.35	0.843	0.38	0.731	0.476	0.584	0.452
R	-0.113	1.0	0.46	-0.195	0.0	-0.167	-0.372	-0.259	-0.344
R_squared	-0.35	0.46	1.0	-0.409	-0.212	-0.376	-0.471	-0.361	-0.446
Avg Vobs	0.843	-0.195	-0.409	1.0	0.459	0.731	0.482	0.536	0.455
Avg Vgas	0.38	0.0	-0.212	0.459	1.0	0.38	0.144	0.186	0.12
Avg Vdisk	0.731	-0.167	-0.376	0.731	0.38	1.0	0.404	0.696	0.375
Avg Vbul	0.476	-0.372	-0.471	0.482	0.144	0.404	1.0	0.382	0.908
Avg SBdisk	0.584	-0.259	-0.361	0.536	0.186	0.696	0.382	1.0	0.372
Avg SBbul	0.452	-0.344	-0.446	0.455	0.12	0.375	0.908	0.372	1.0

## Regression Results

Unnamed: 0	Coefficients	P_values	Std_Errors
const	-12090.531	0.0	1250.618
R	1505.316	0.0	264.241
R_squared	-63.277	0.501	93.882
Avg Vobs	329.82	0.0	13.466
Avg Vgas	-259.564	0.0	53.622
Avg Vdisk	-26.677	0.119	17.04
Avg SBdisk	13.984	0.0	3.769



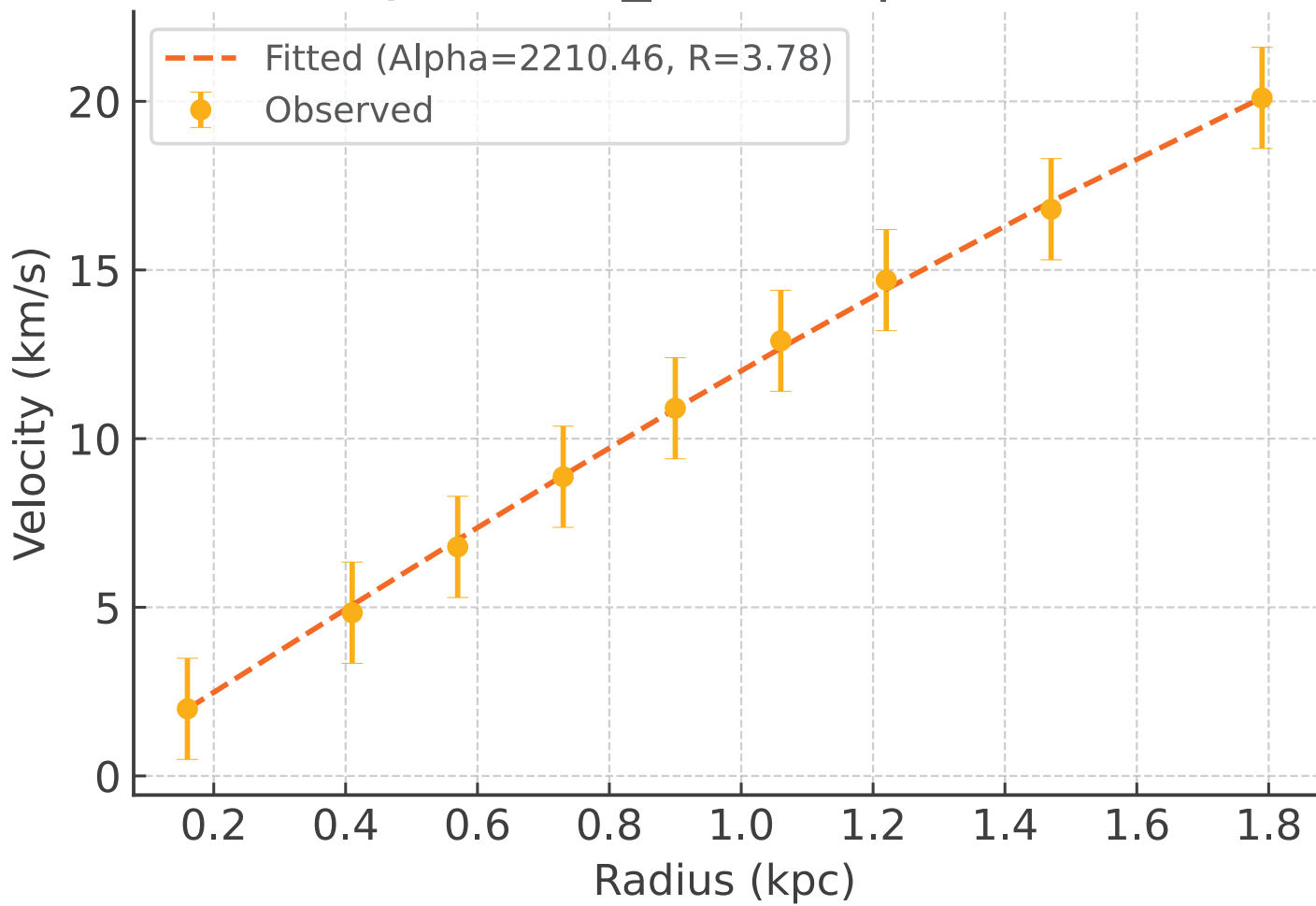
## Evaluation Metrics

Unnamed: 0	MAE	MSE	RMSE
0	4127.102	28763414.1	5363.153

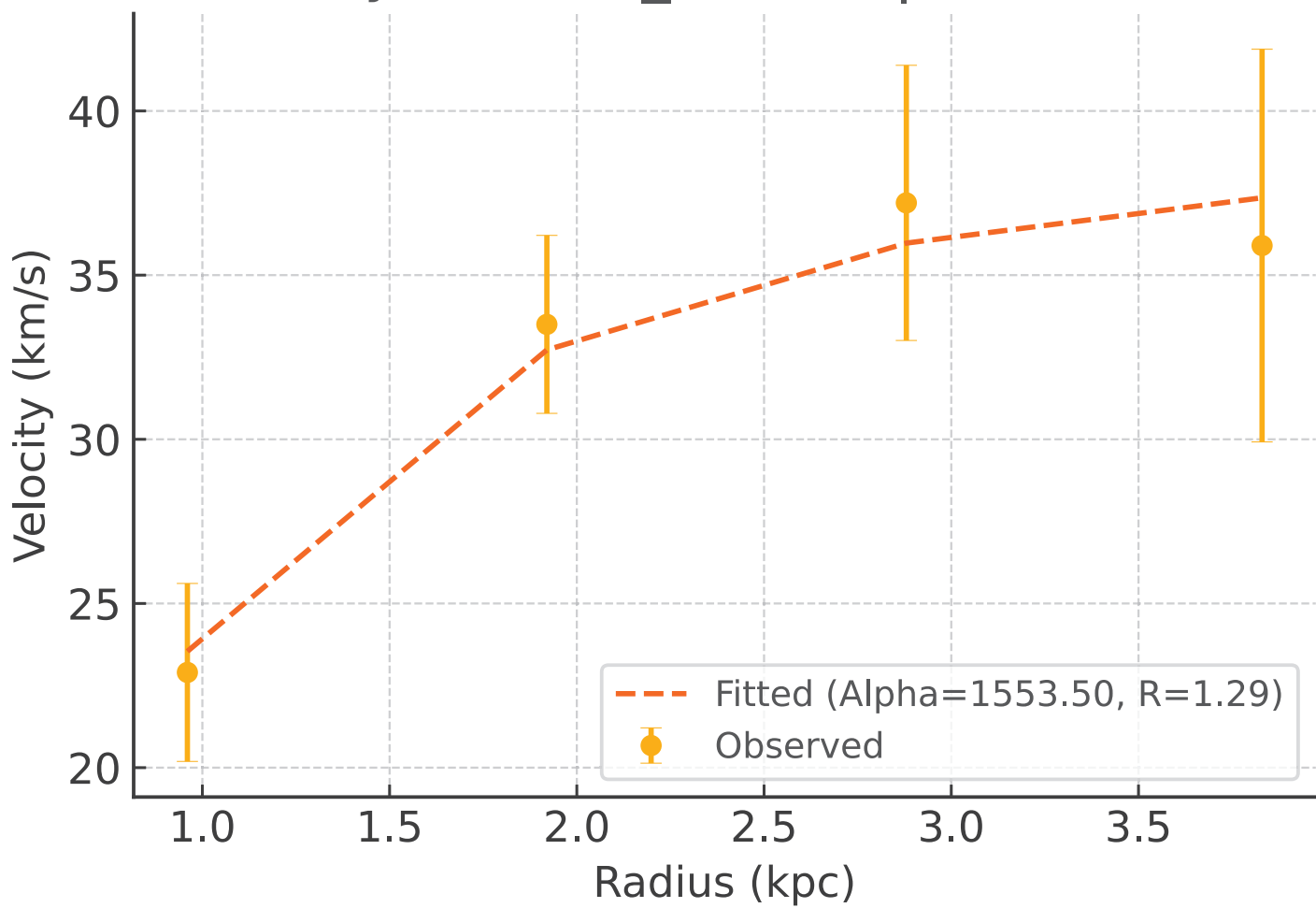
## Normality Test Results

Variable	W-Statistic	p-value
Alpha	0.786	0.0
R	0.848	0.0
R_squared	0.206	0.0
Avg Vobs	0.886	0.0
Avg Vgas	0.915	0.0
Avg Vdisk	0.841	0.0
Avg Vbul	0.485	0.0
Avg SBdisk	0.607	0.0
Avg SBbul	0.302	0.0

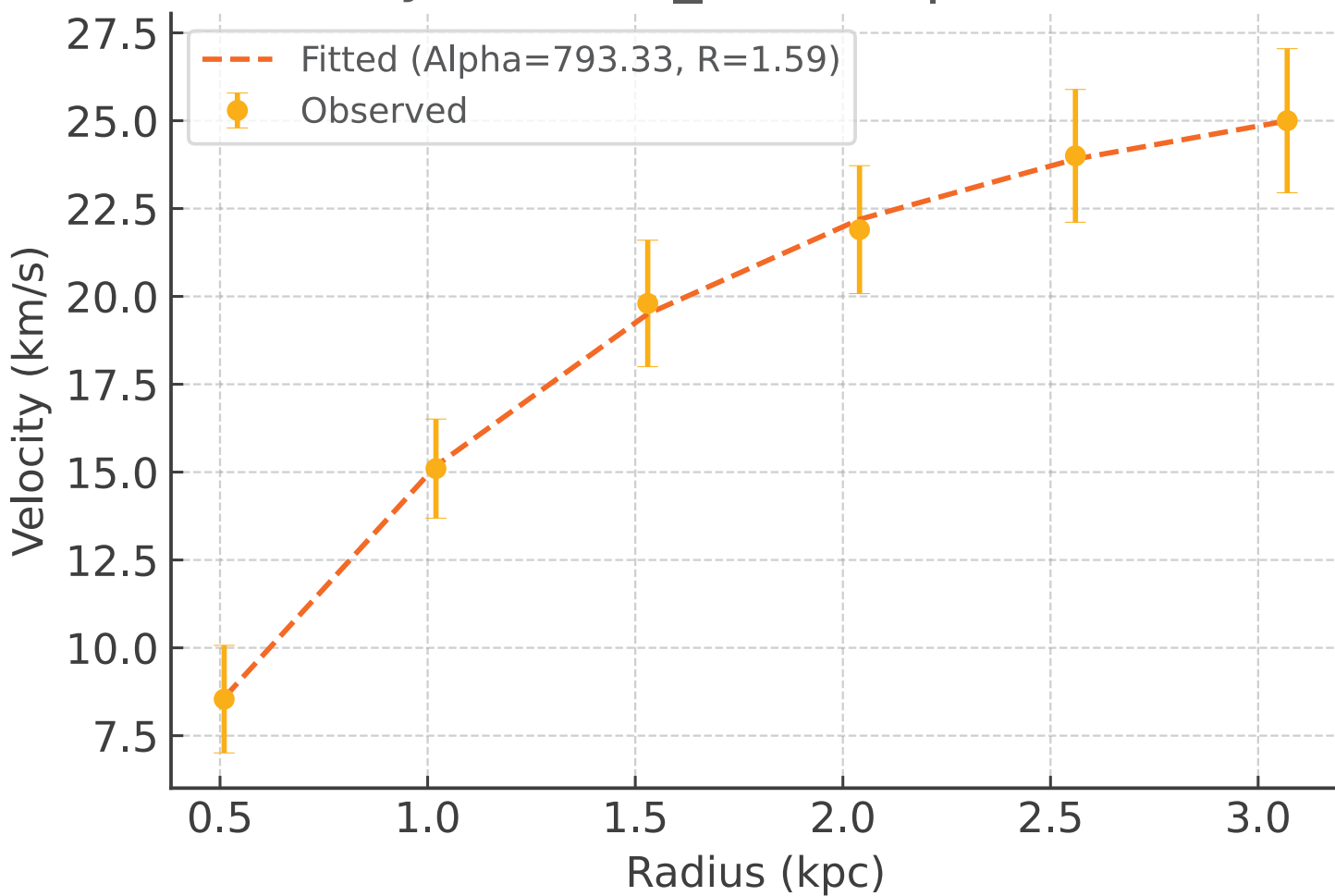
# Galaxy: CamB\_rotmod | $R^2 = 0.999$



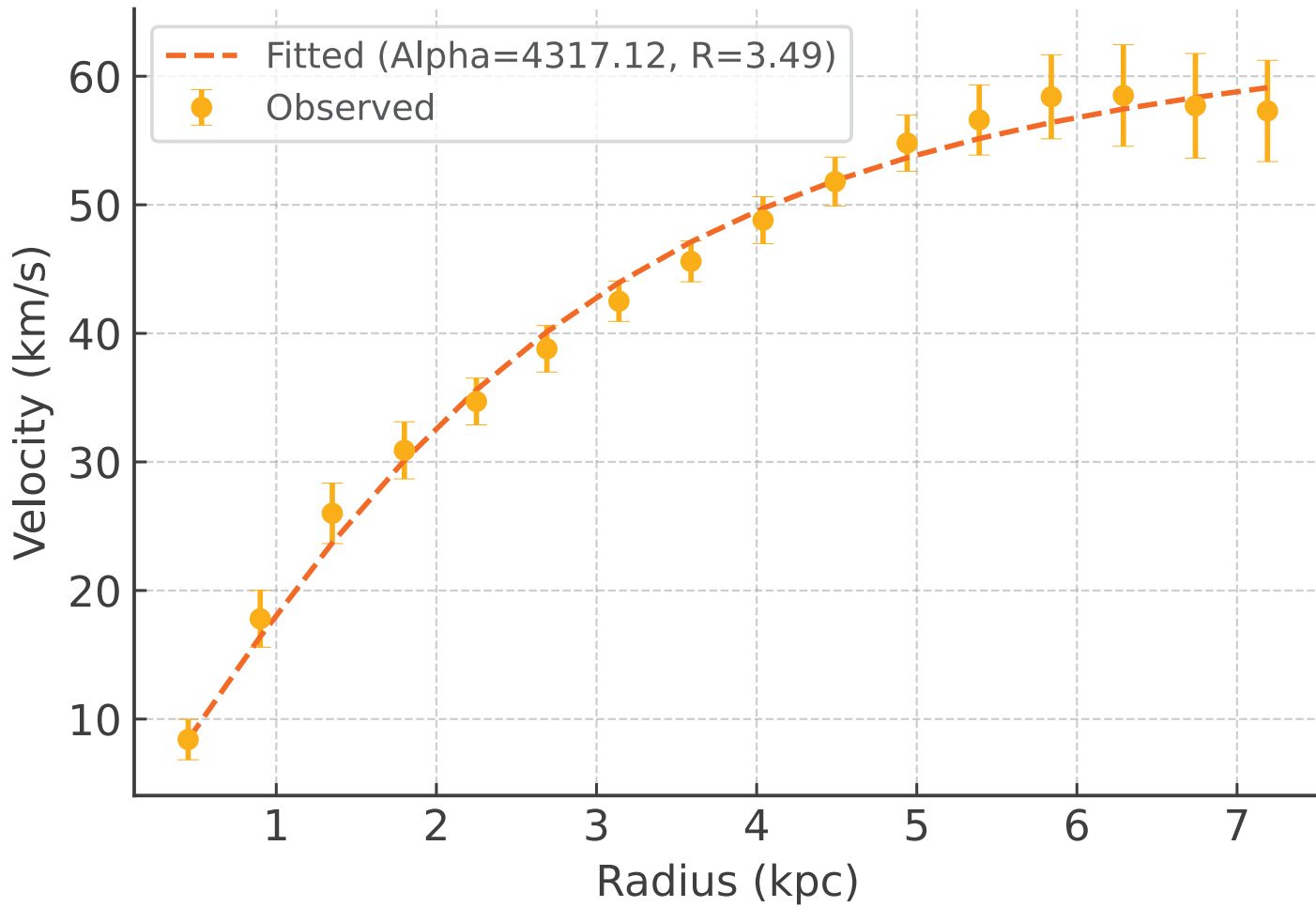
Galaxy: D512-2\_rotmod |  $R^2 = 0.963$



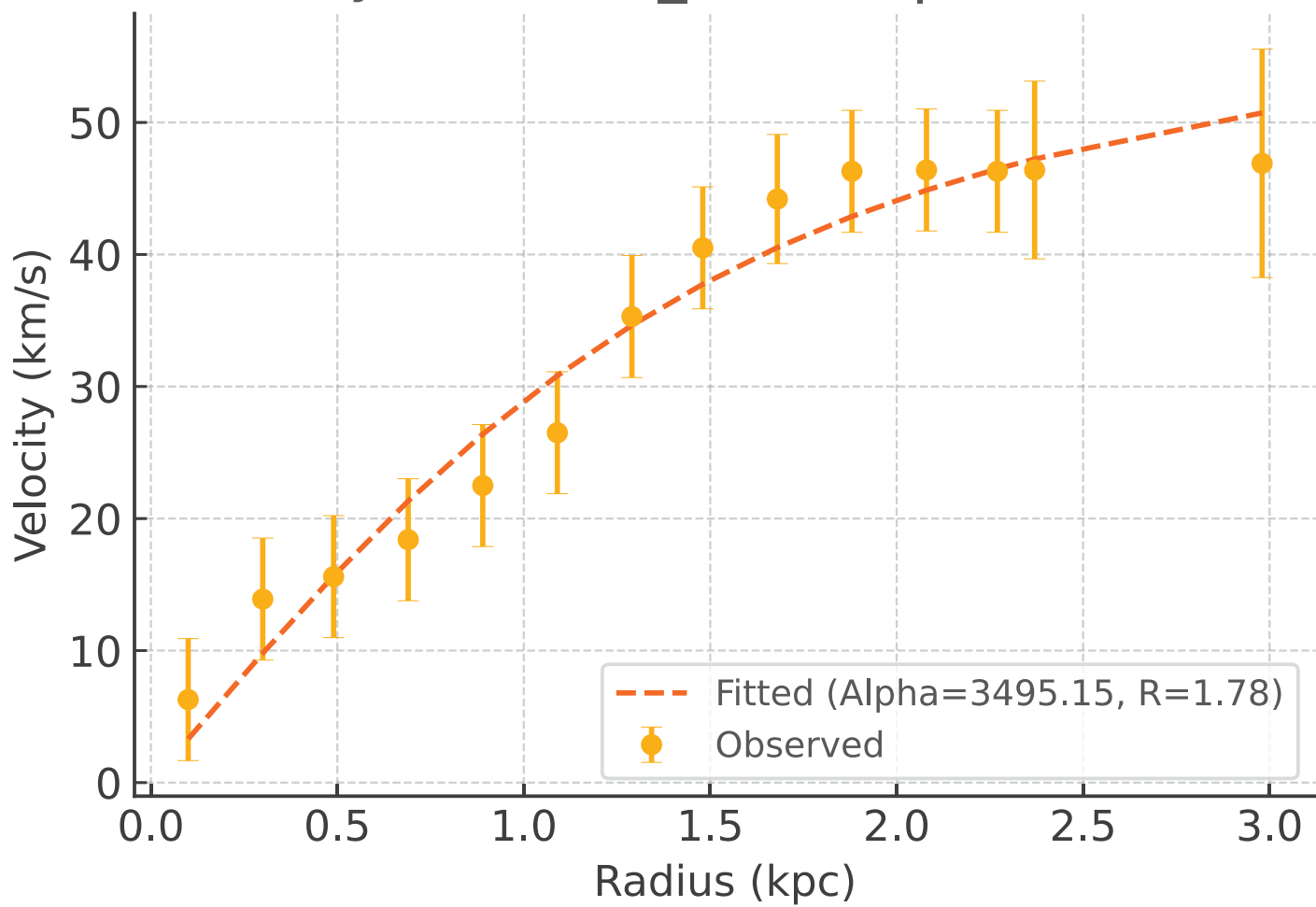
# Galaxy: D564-8\_rotmod | $R^2 = 0.999$



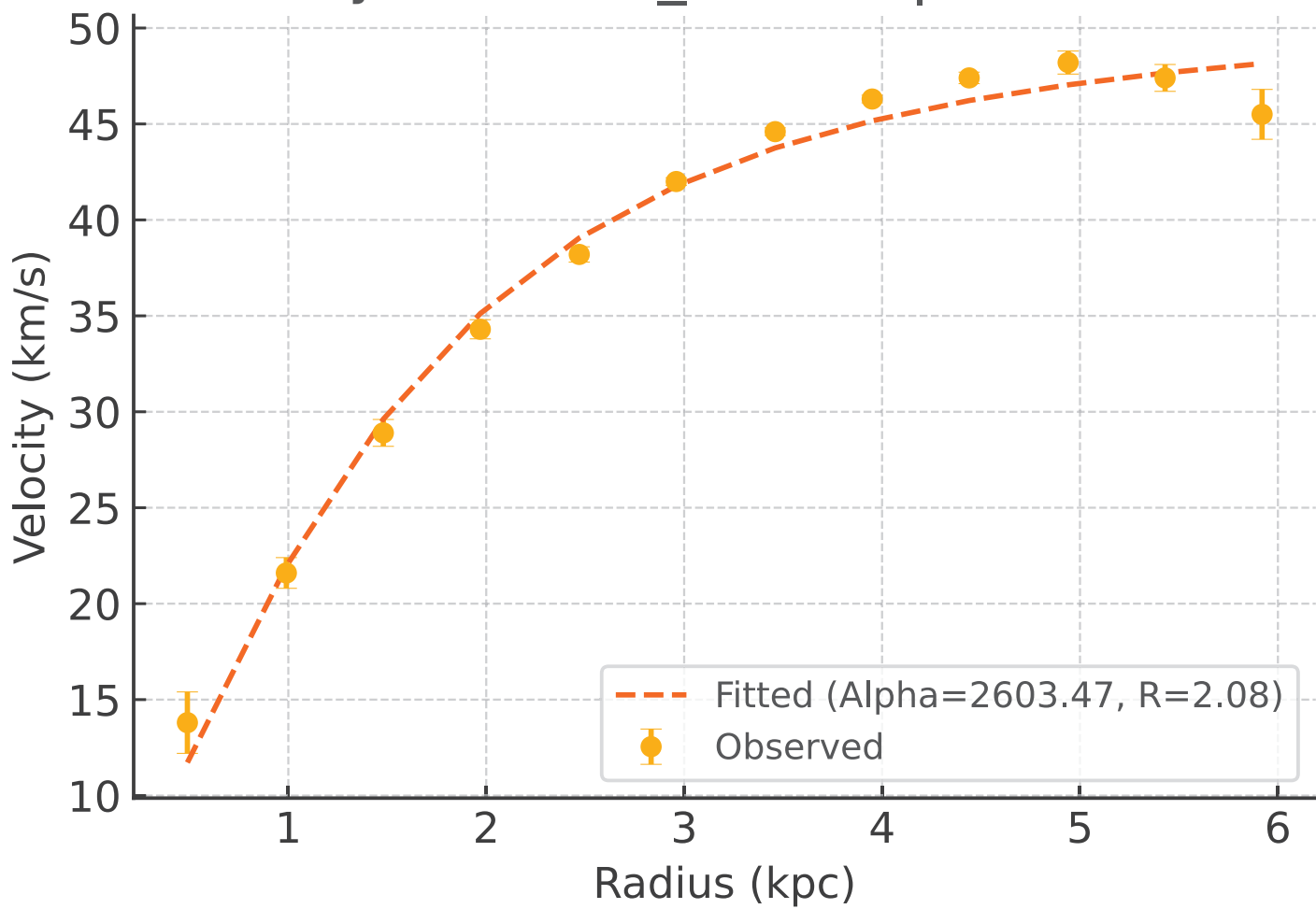
# Galaxy: D631-7\_rotmod | $R^2 = 0.992$



Galaxy: DDO064\_rotmod |  $R^2 = 0.958$

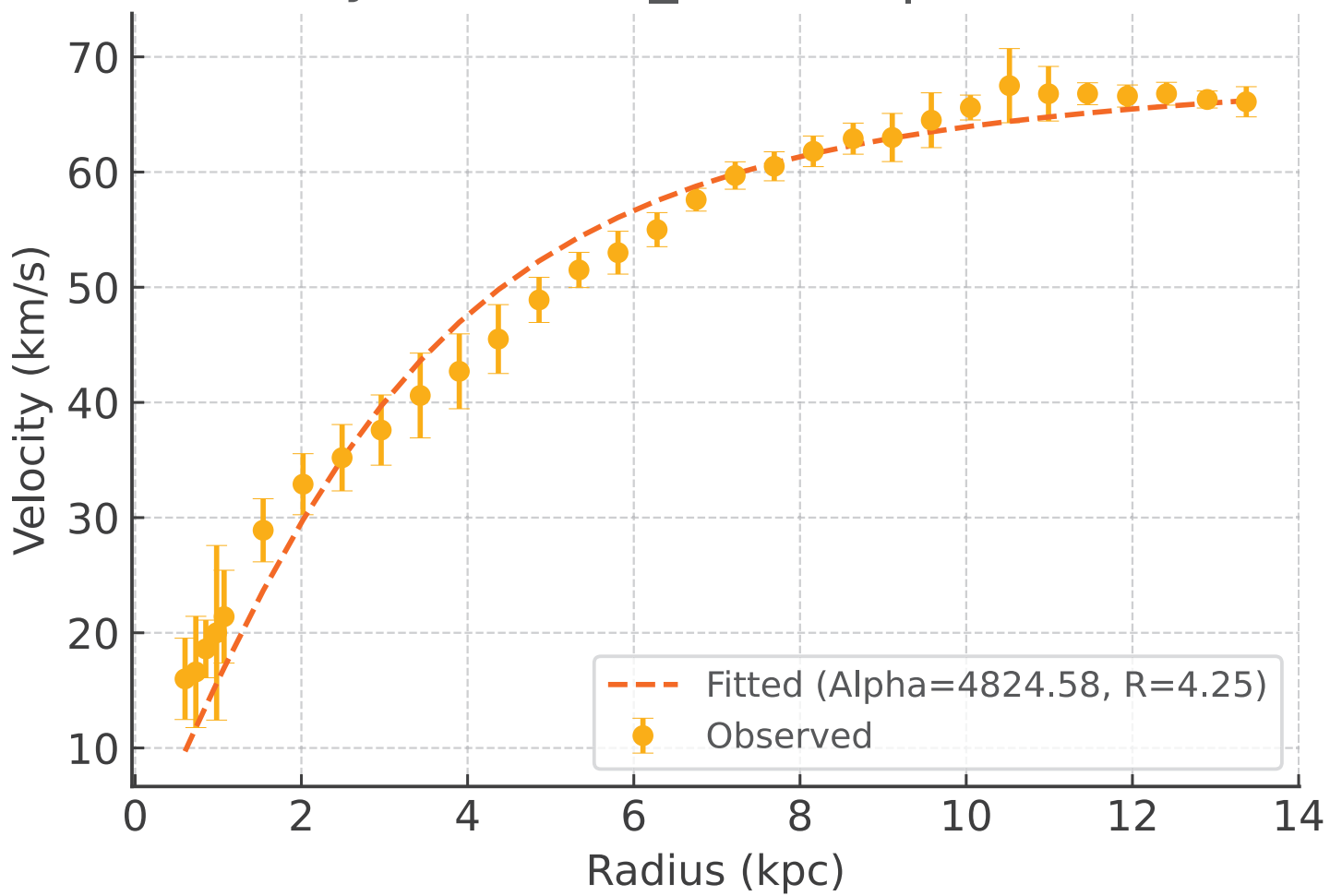


# Galaxy: DDO154\_rotmod | $R^2 = 0.987$

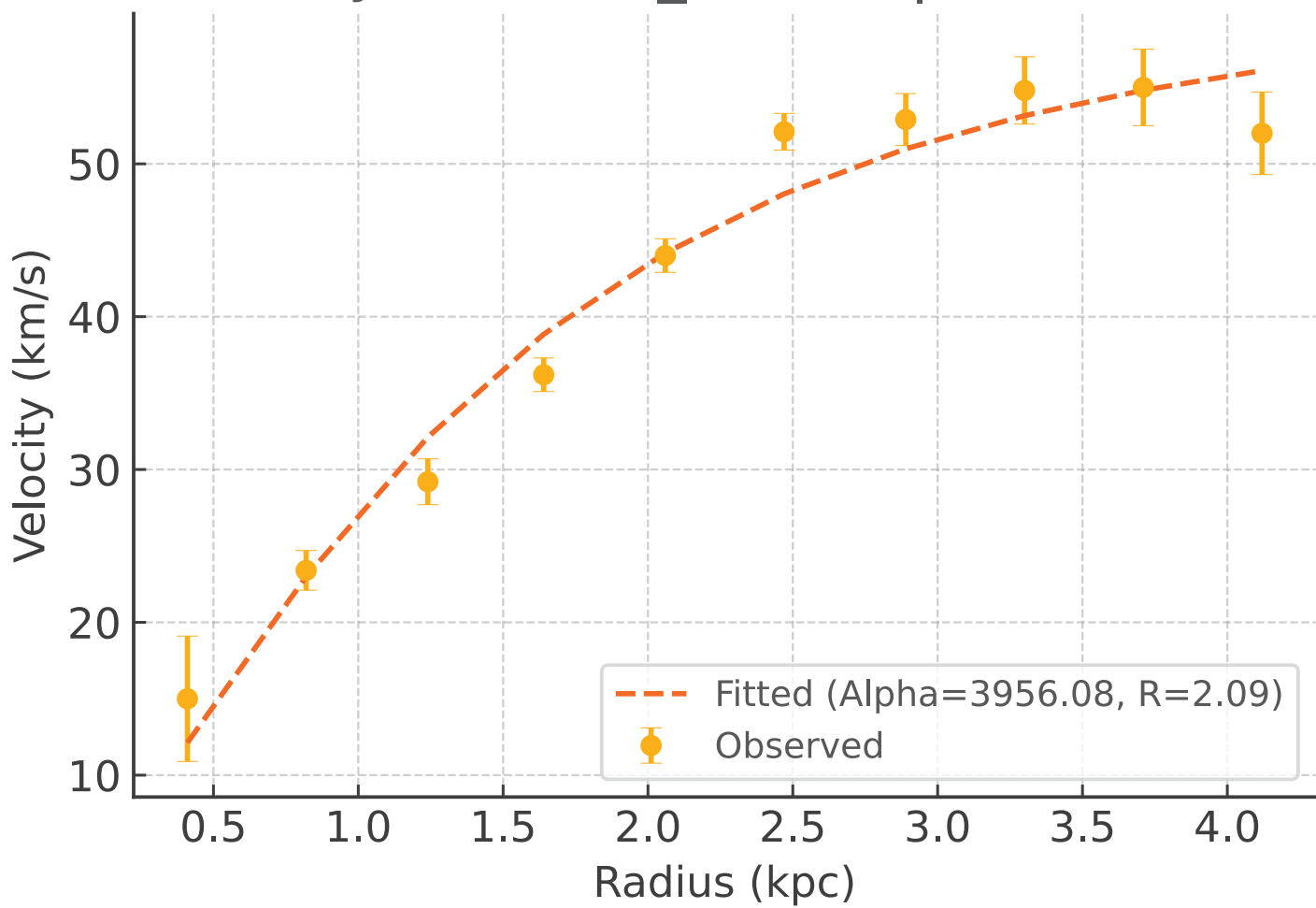




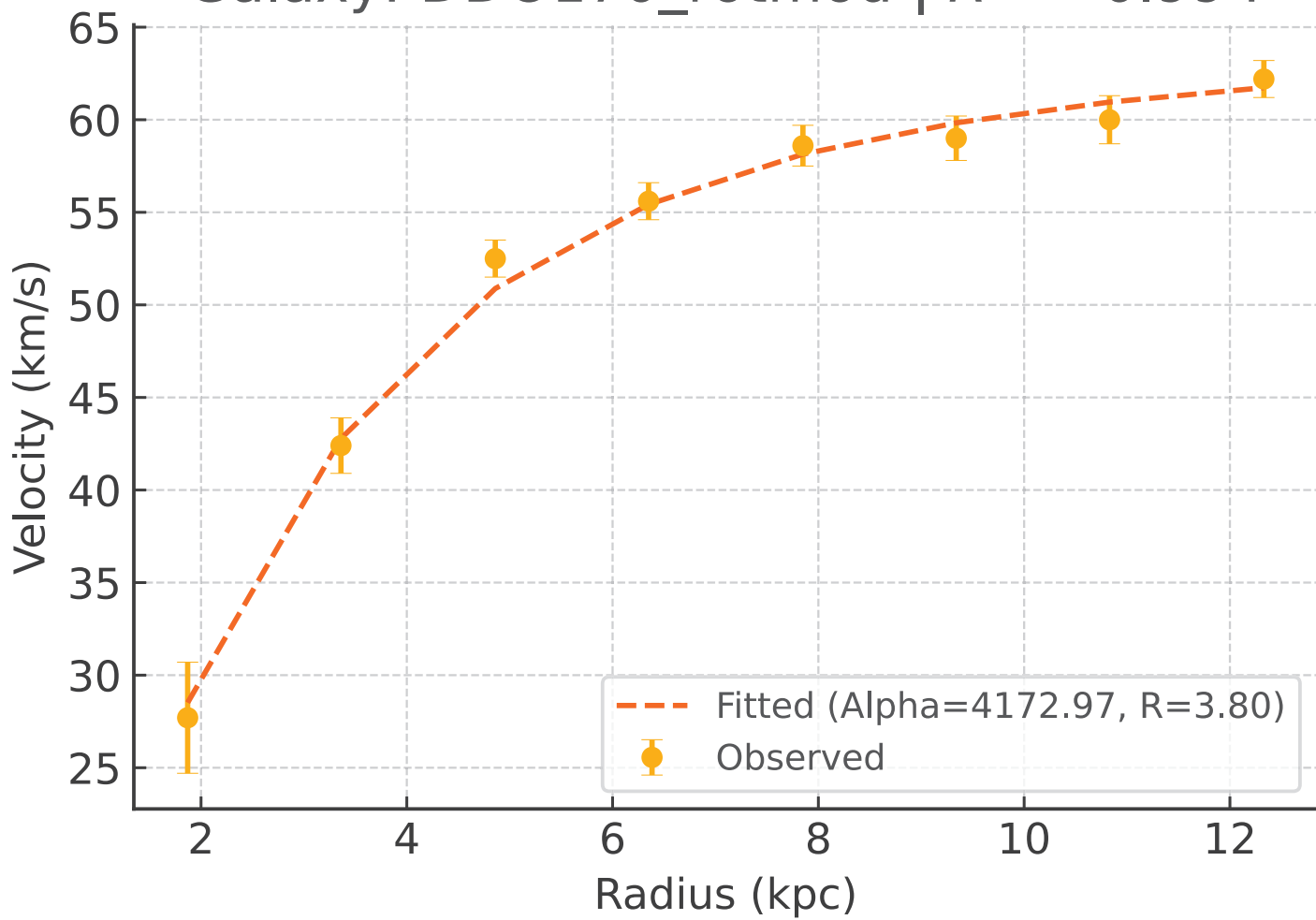
Galaxy: DDO161\_rotmod |  $R^2 = 0.971$



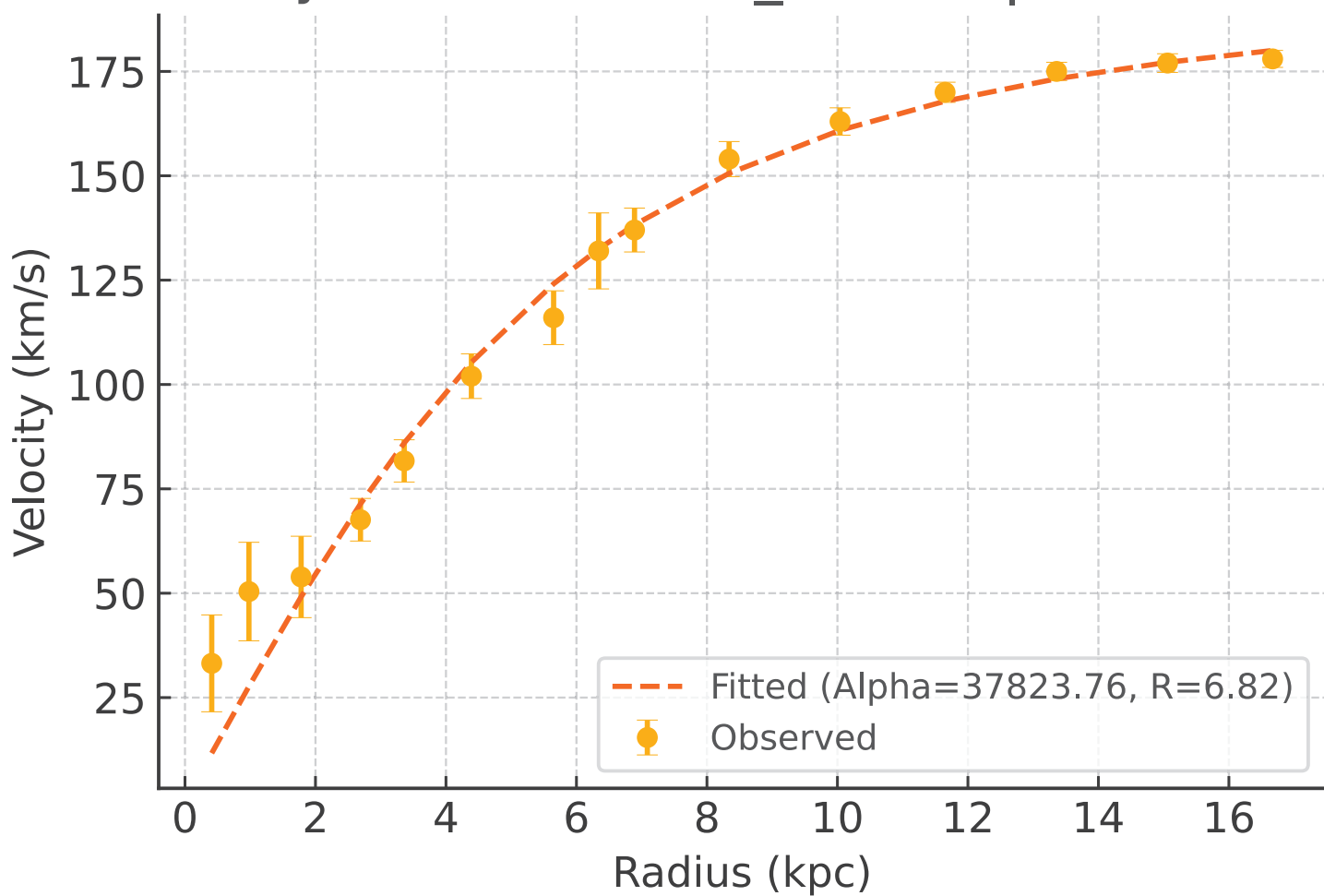
Galaxy: DDO168\_rotmod |  $R^2 = 0.967$



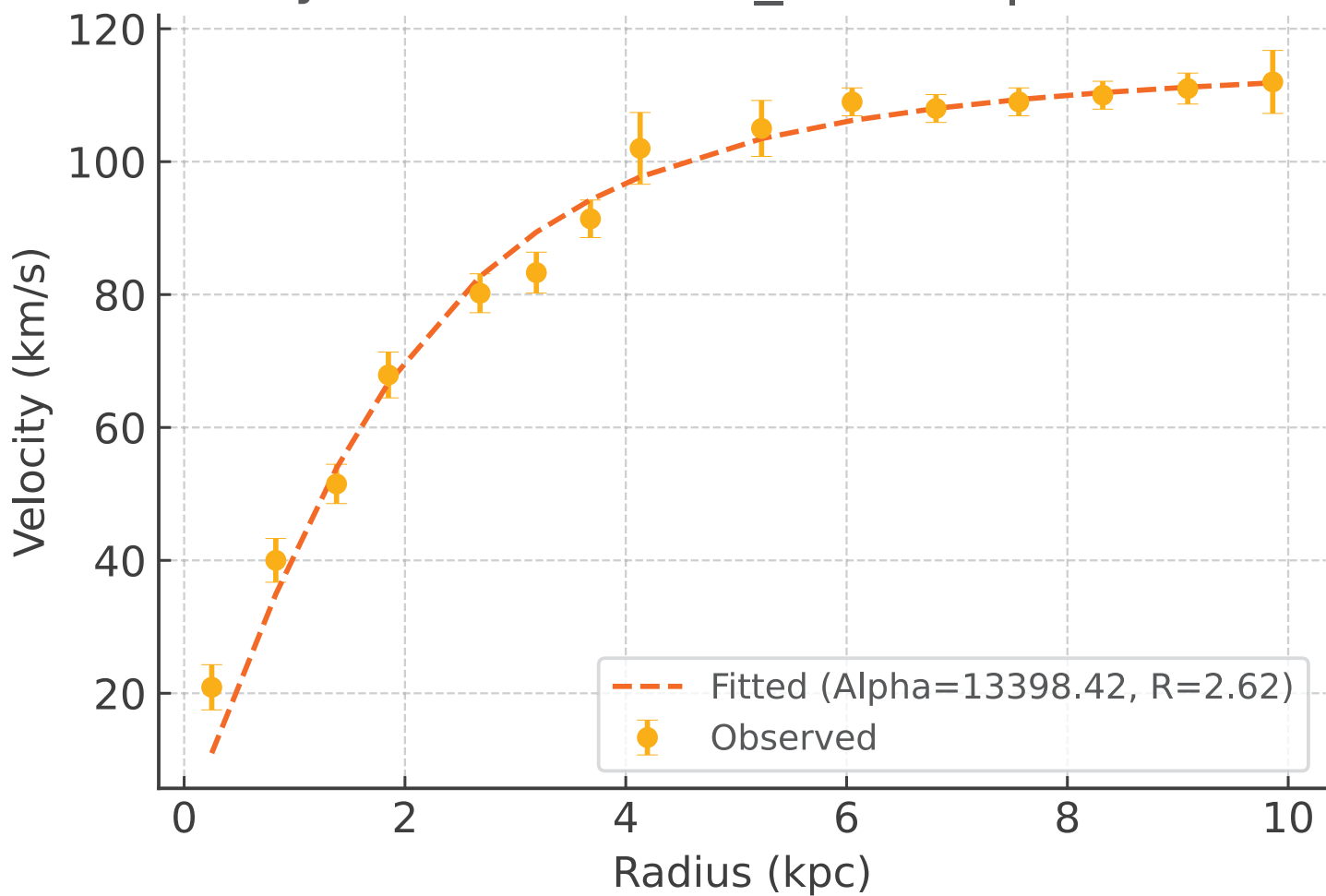
Galaxy: DDO170\_rotmod |  $R^2 = 0.994$



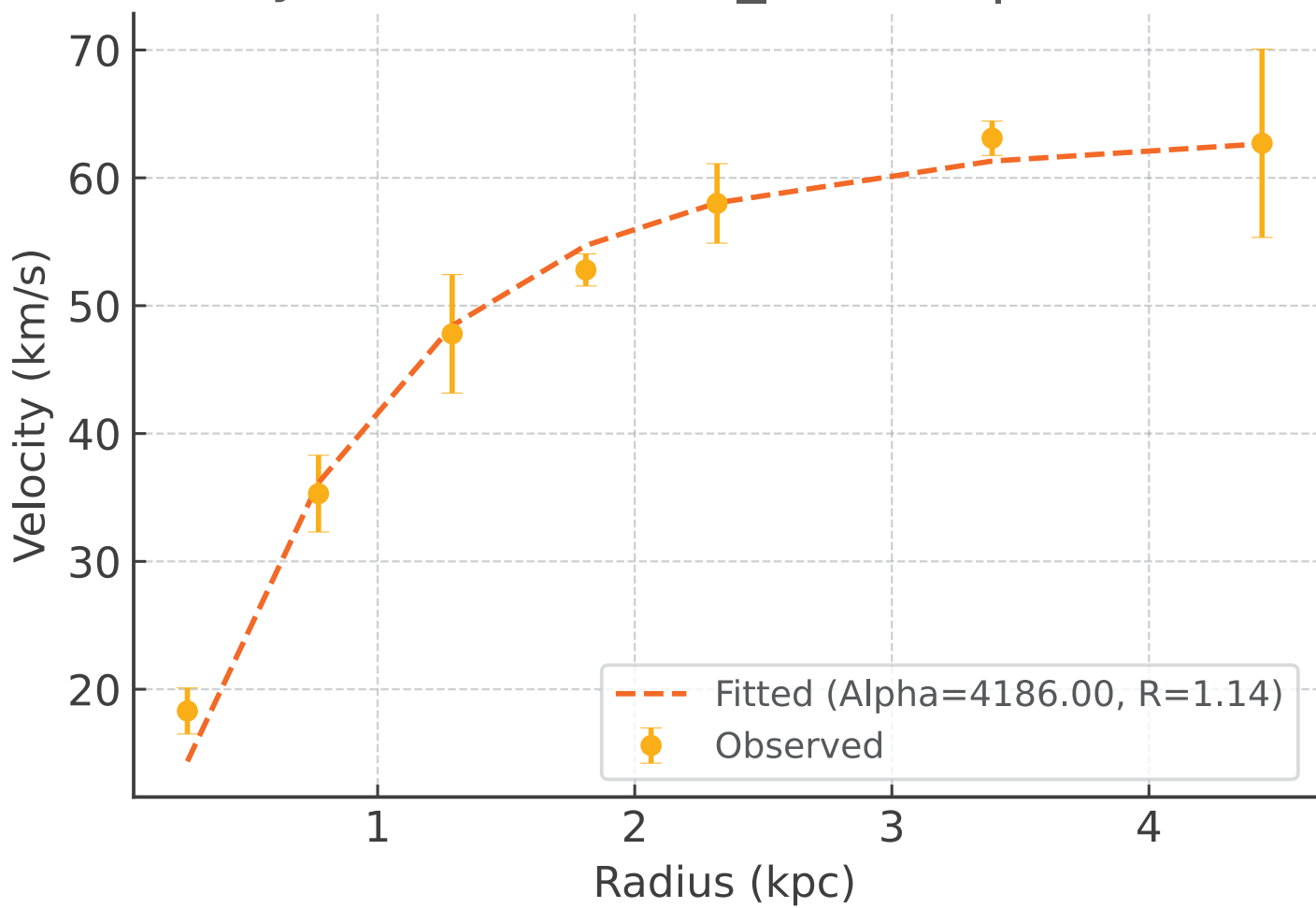
Galaxy: ESO079-G014\_rotmod |  $R^2 = 0.969$



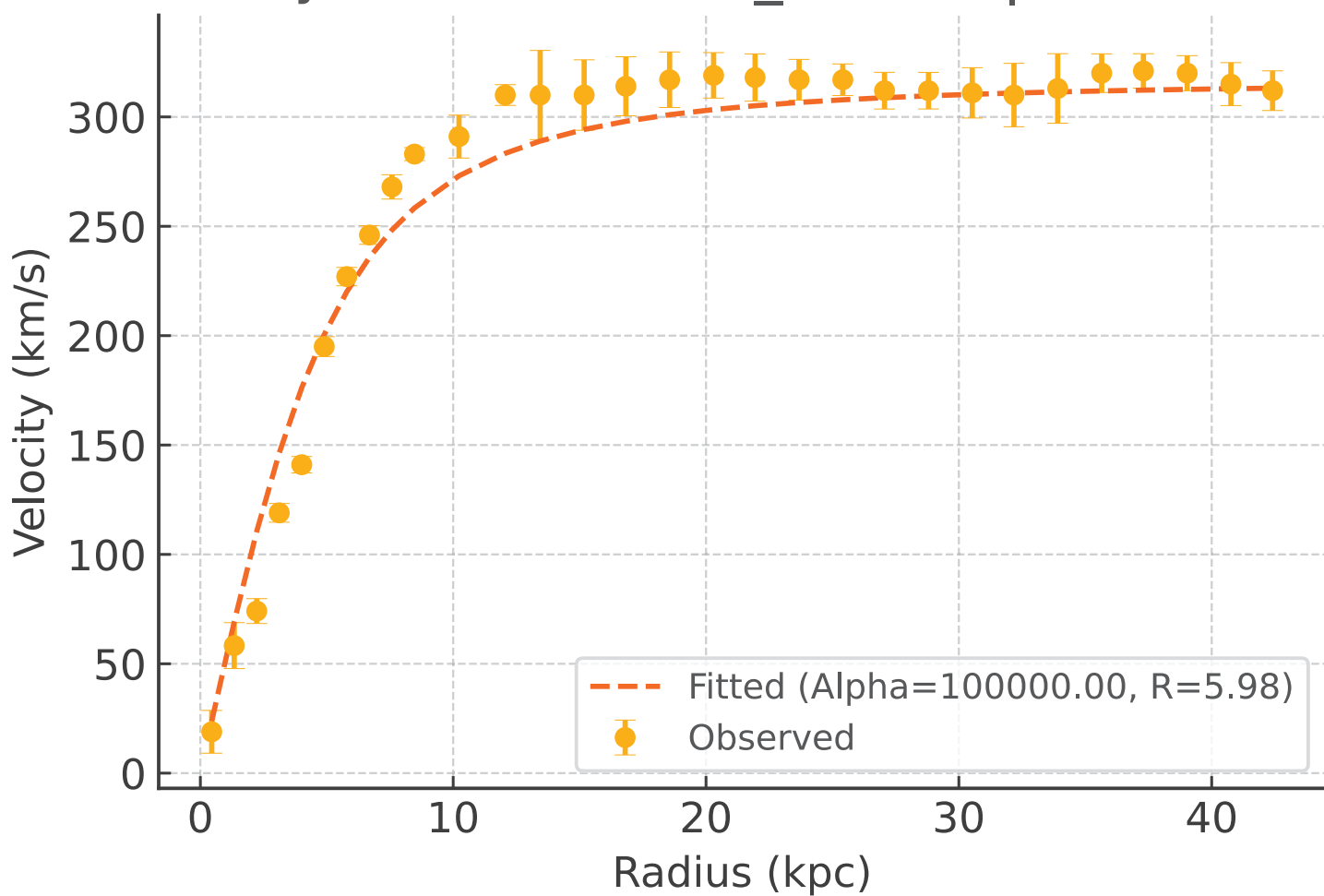
# Galaxy: ESO116-G012\_rotmod | $R^2 = 0.982$



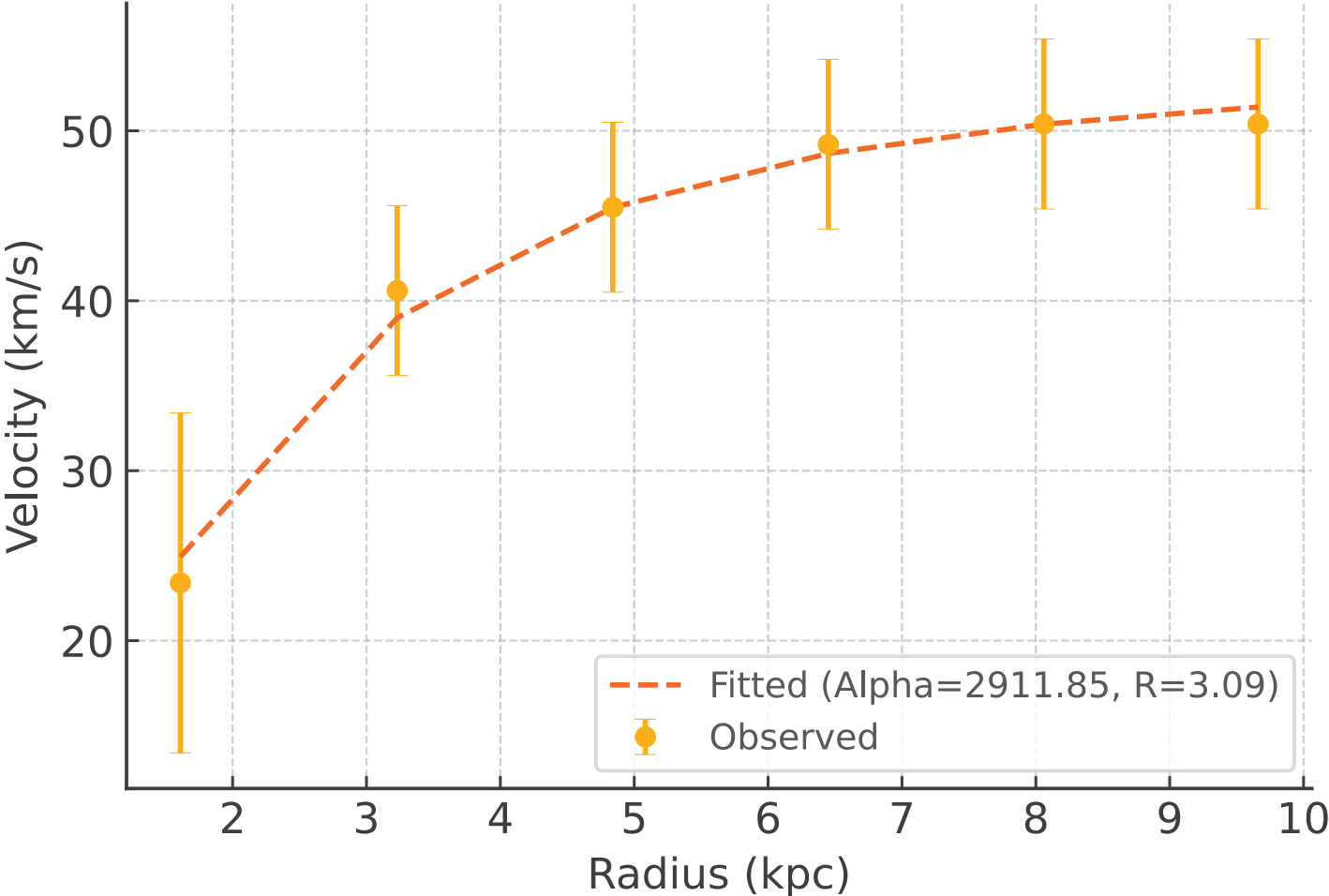
Galaxy: ESO444-G084\_rotmod |  $R^2 = 0.985$



# Galaxy: ESO563-G021\_rotmod | $R^2 = 0.967$

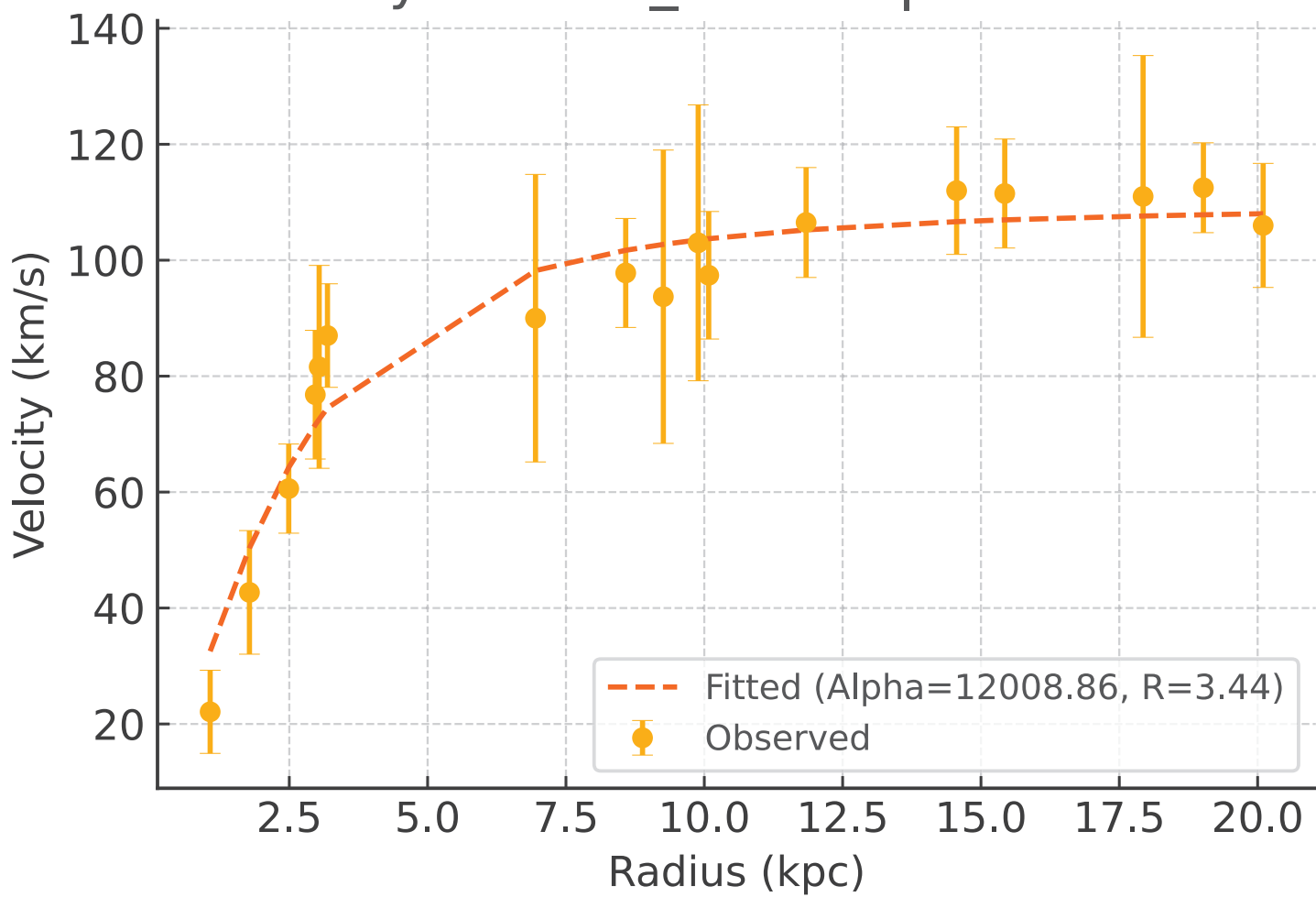


Galaxy: F561-1\_rotmod |  $R^2 = 0.989$

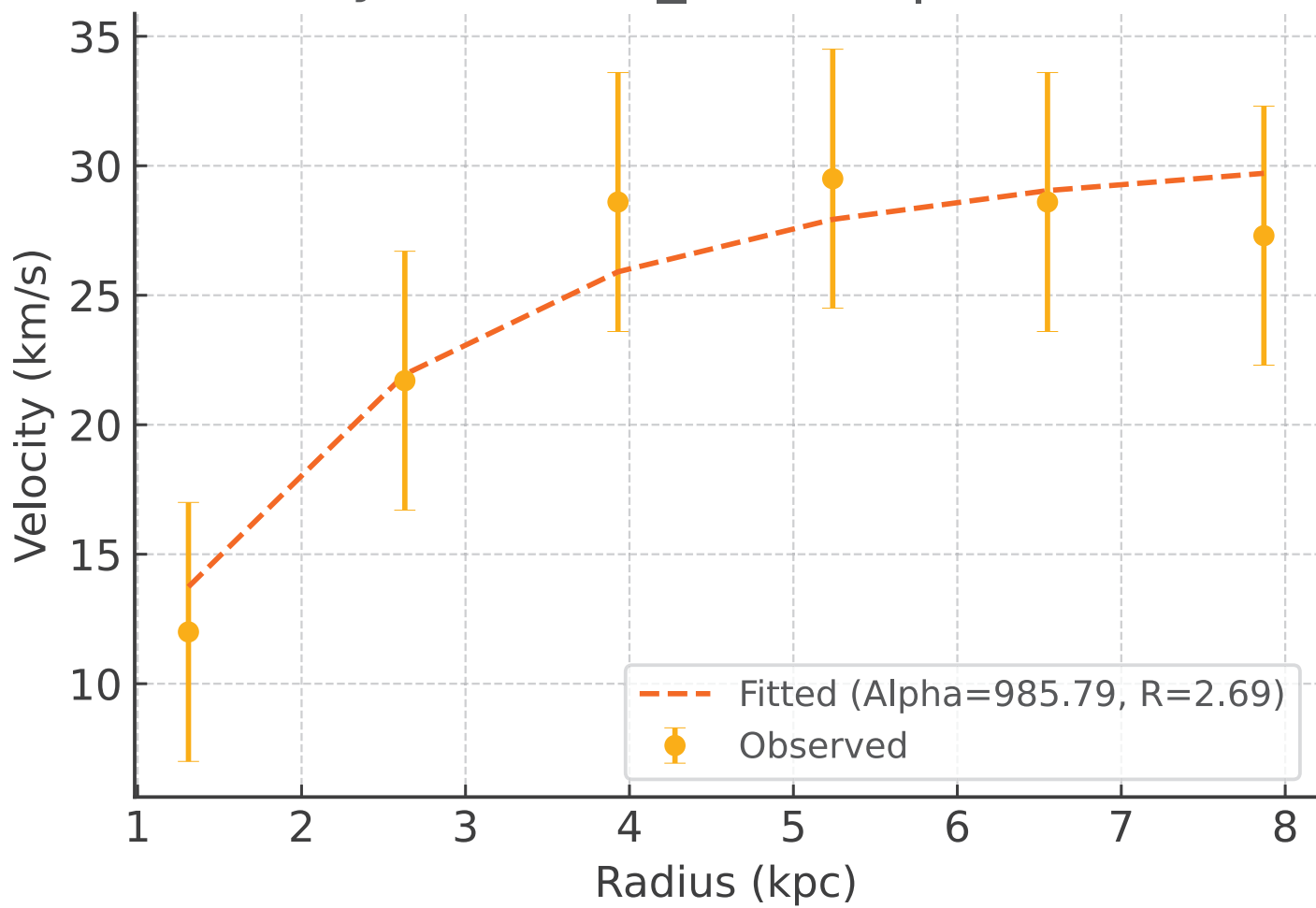




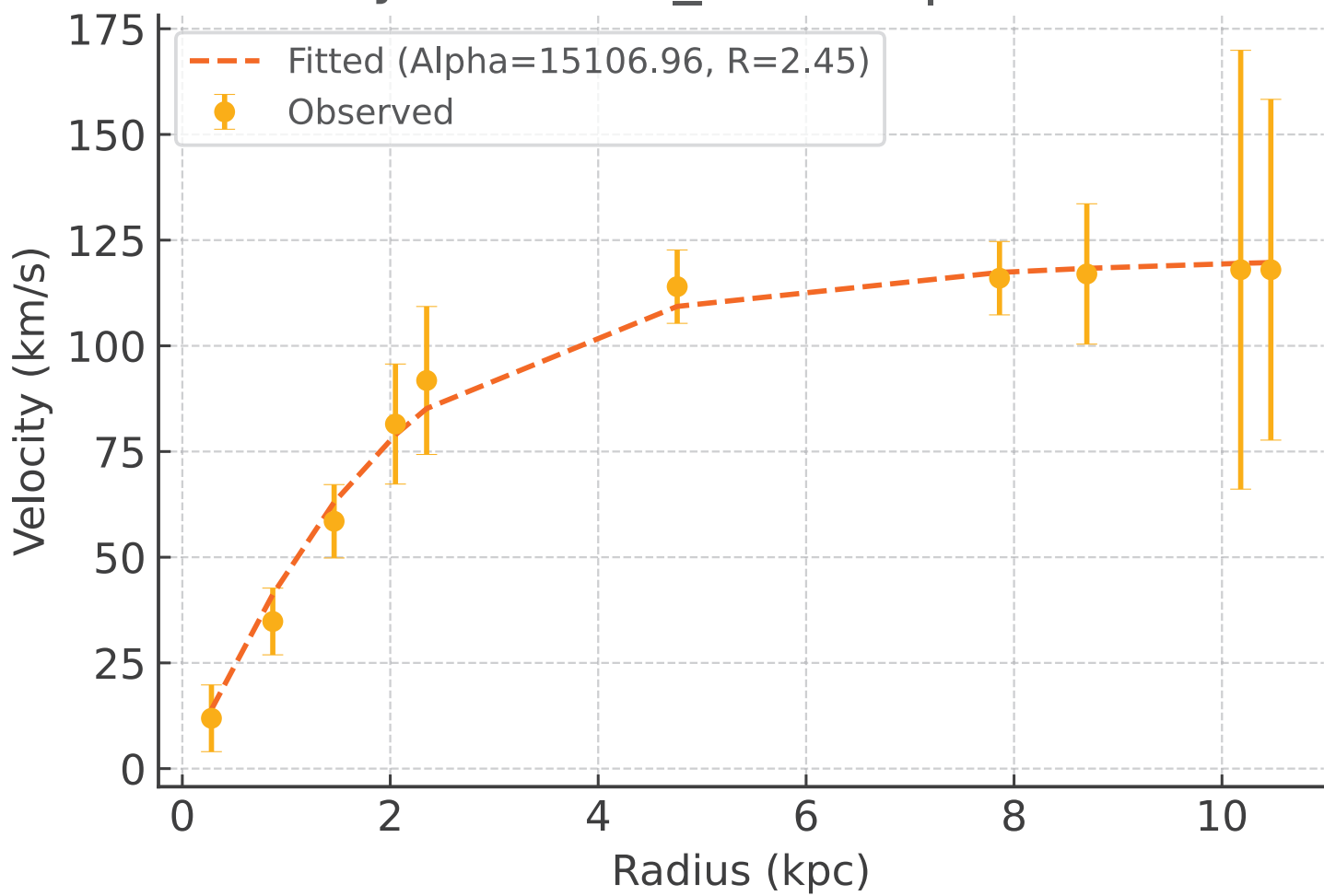
Galaxy: F563-1\_rotmod |  $R^2 = 0.931$



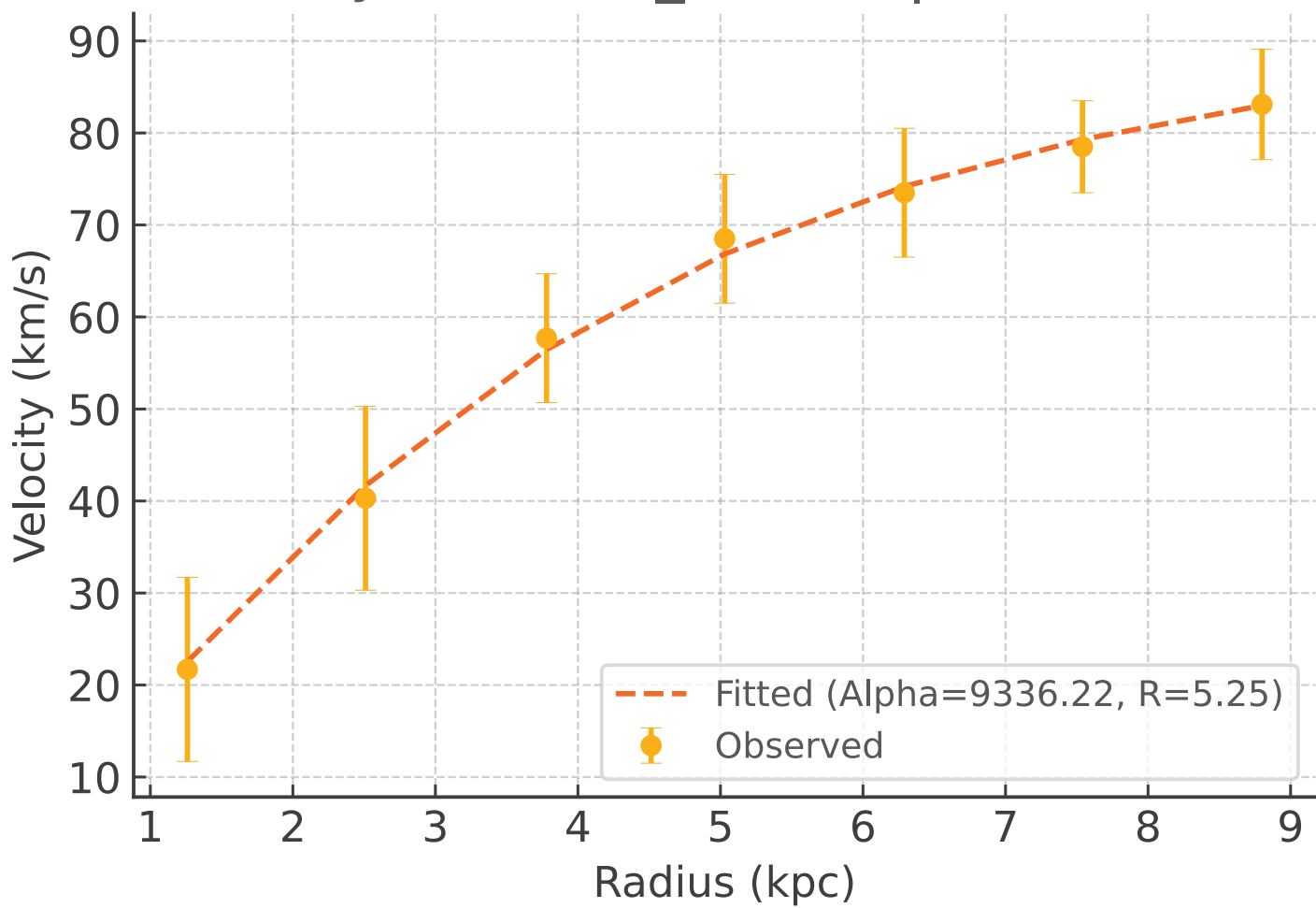
Galaxy: F563-V1\_rotmod |  $R^2 = 0.918$



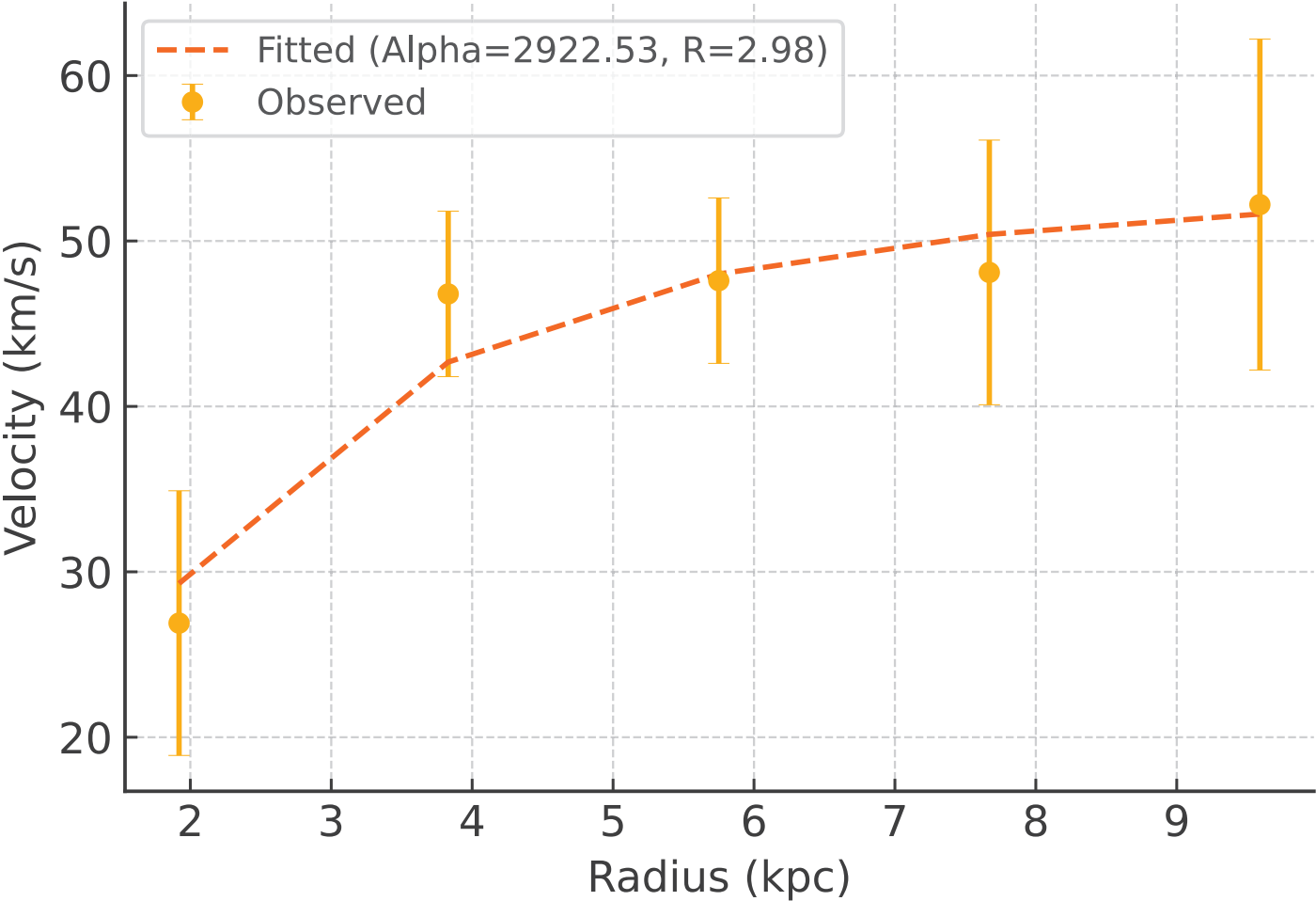
# Galaxy: F563-V2\_rotmod | $R^2 = 0.989$



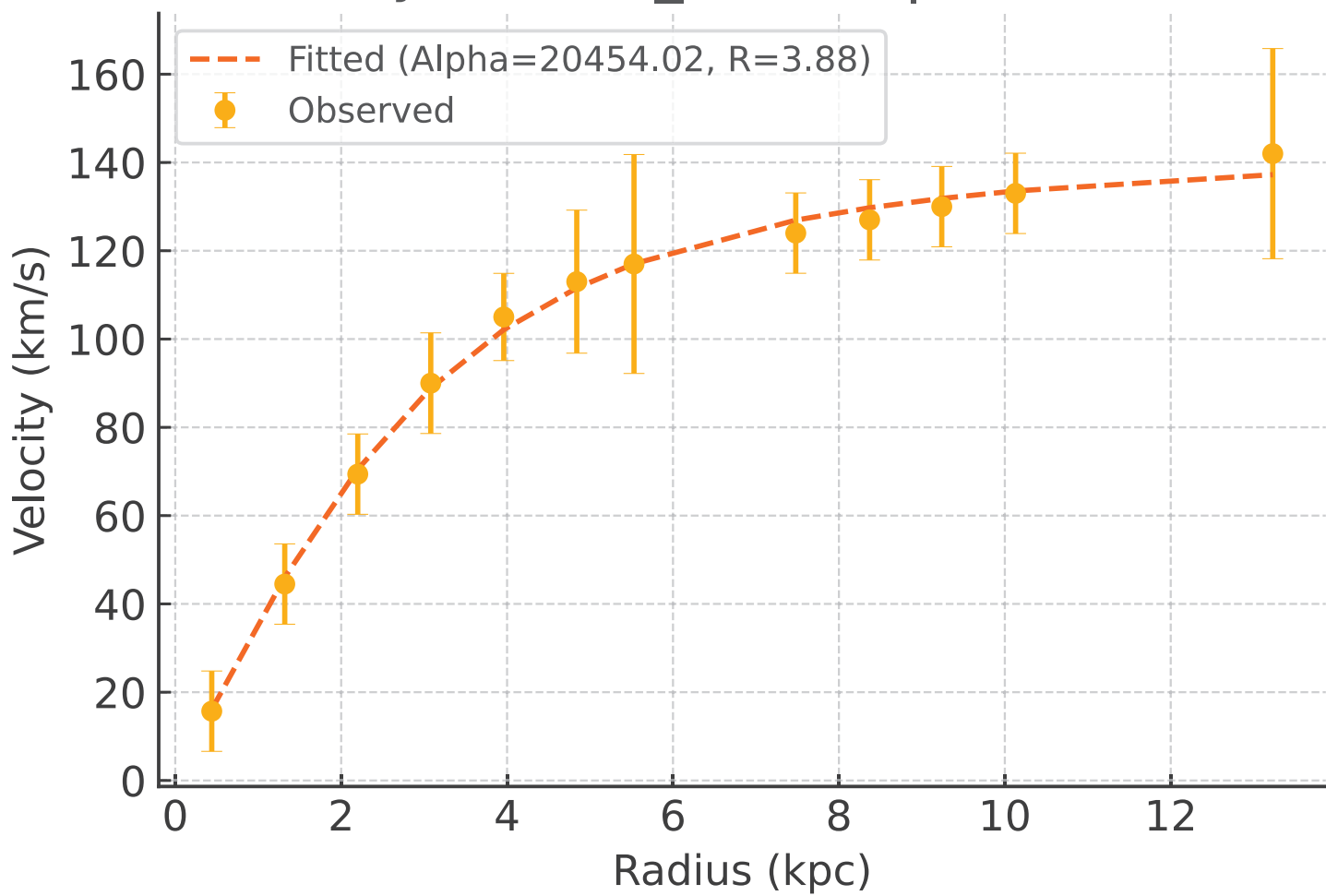
Galaxy: F565-V2\_rotmod |  $R^2 = 0.997$



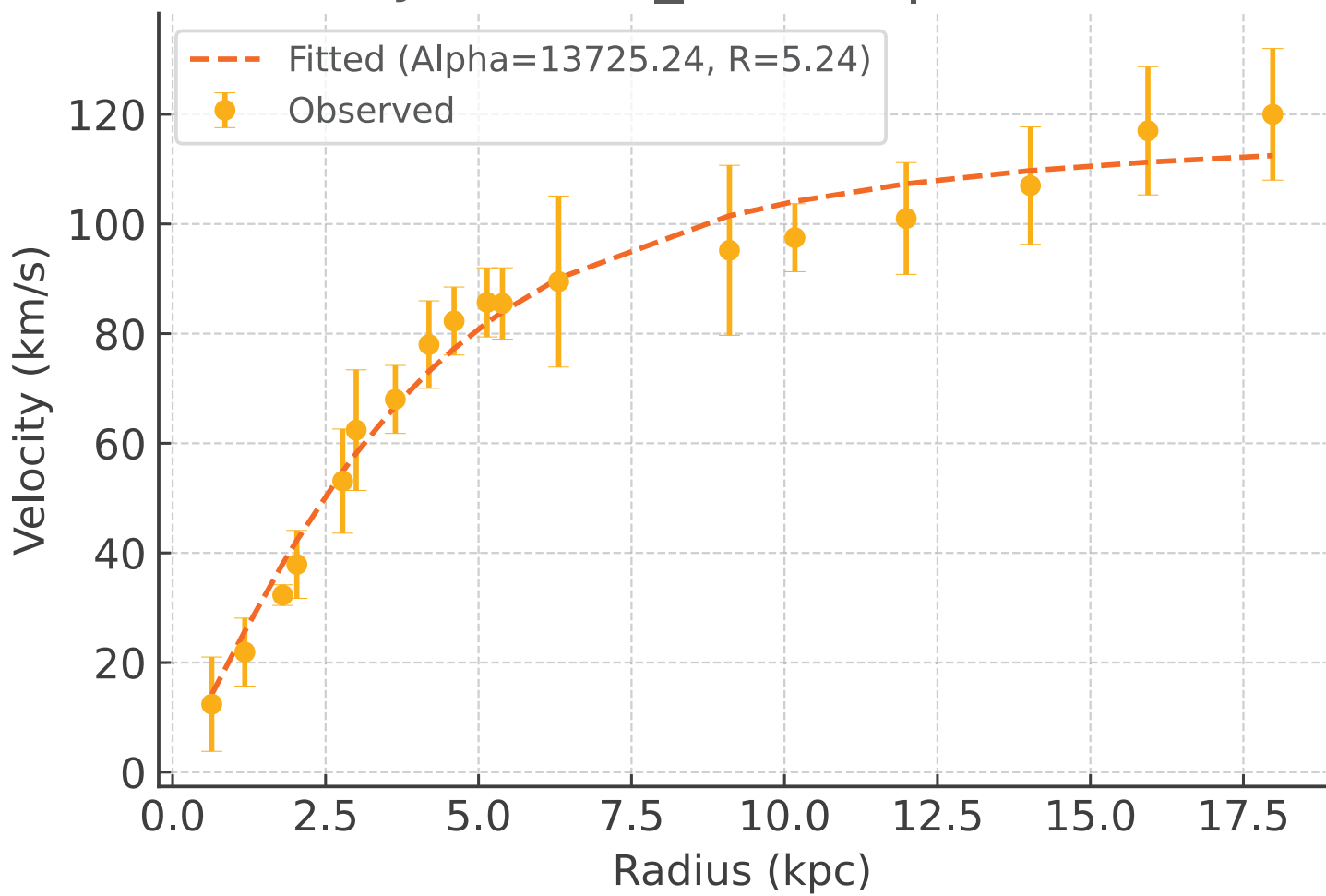
# Galaxy: F567-2\_rotmod | $R^2 = 0.928$



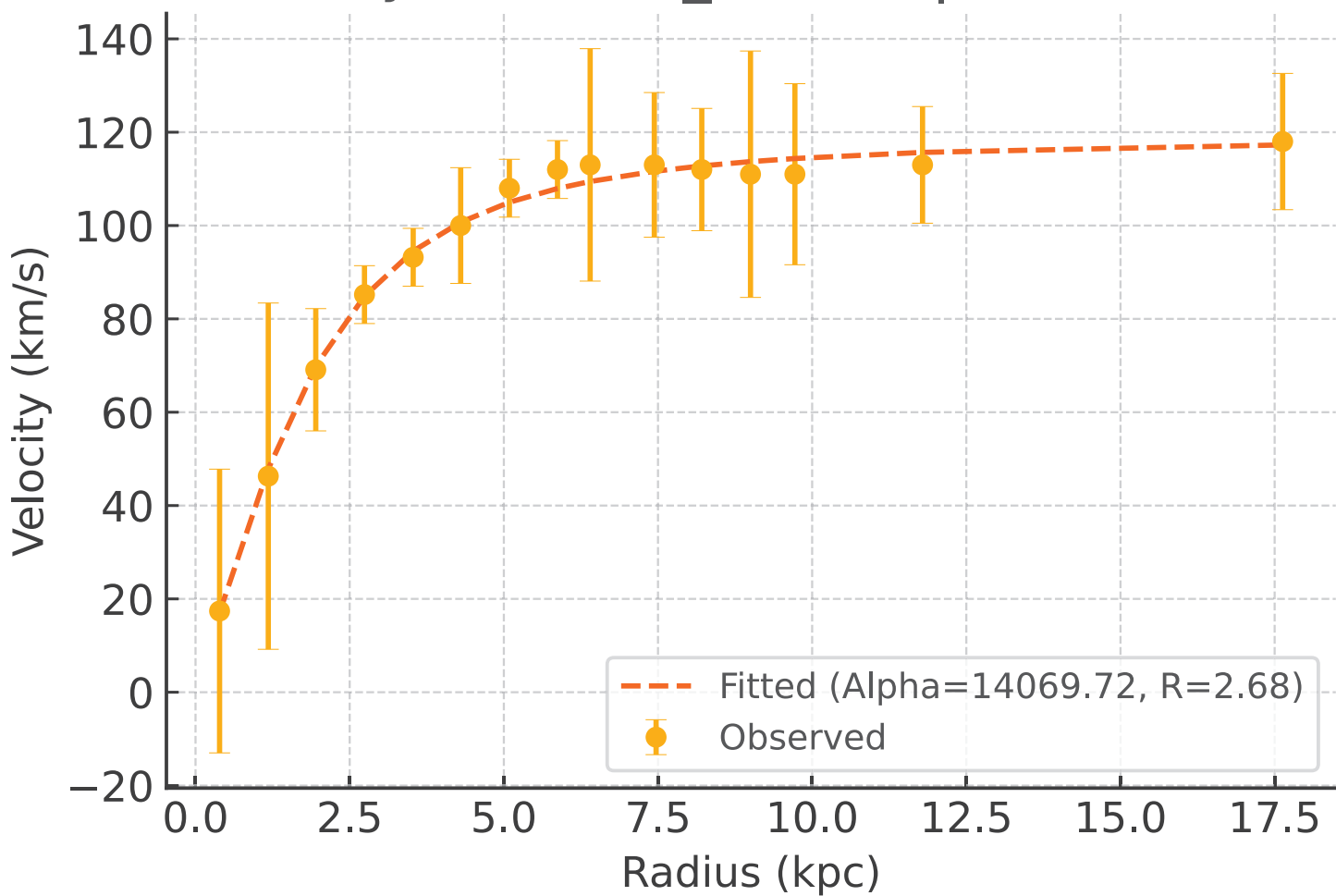
# Galaxy: F568-1\_rotmod | $R^2 = 0.997$



# Galaxy: F568-3\_rotmod | $R^2 = 0.978$

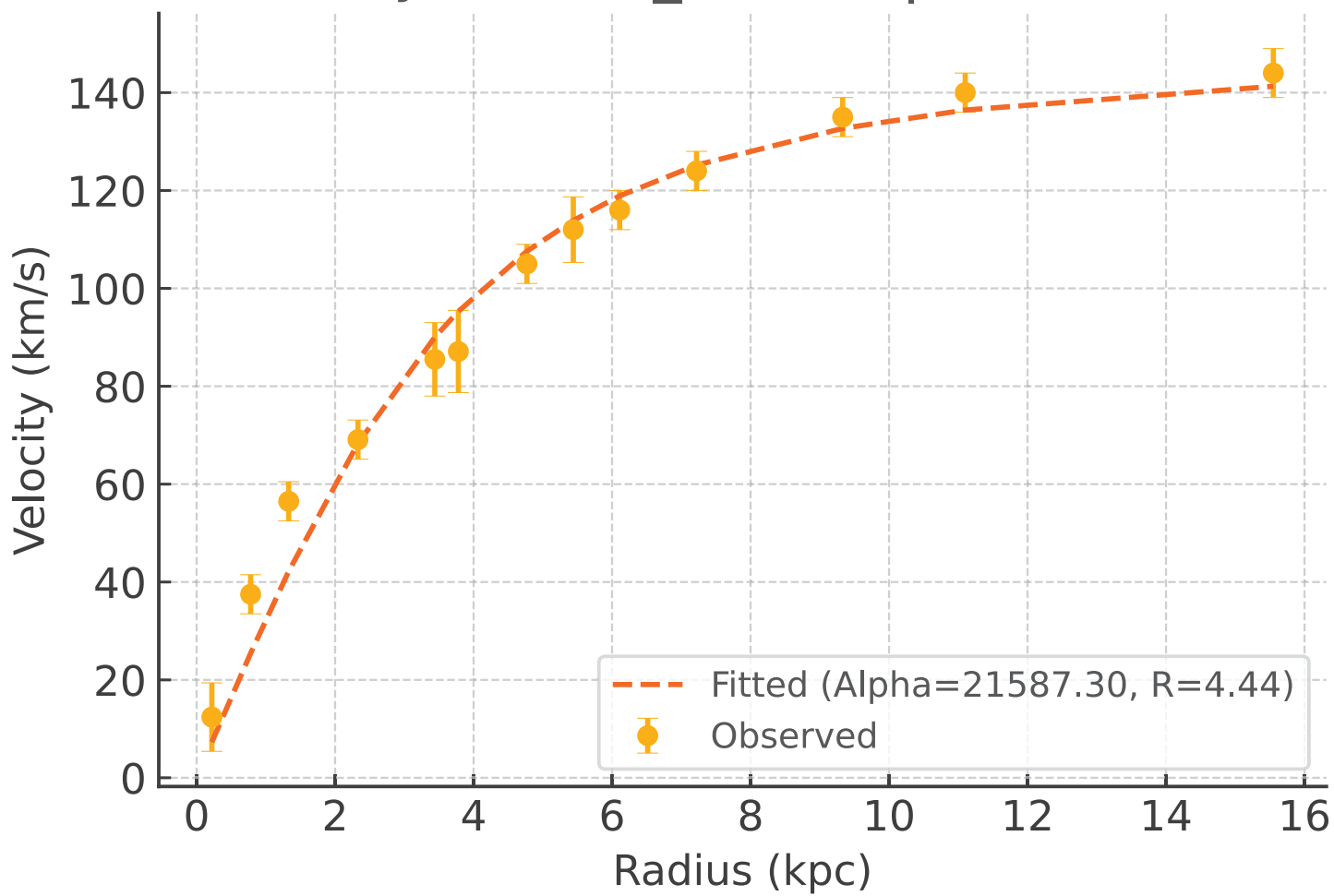


Galaxy: F568-V1\_rotmod |  $R^2 = 0.994$

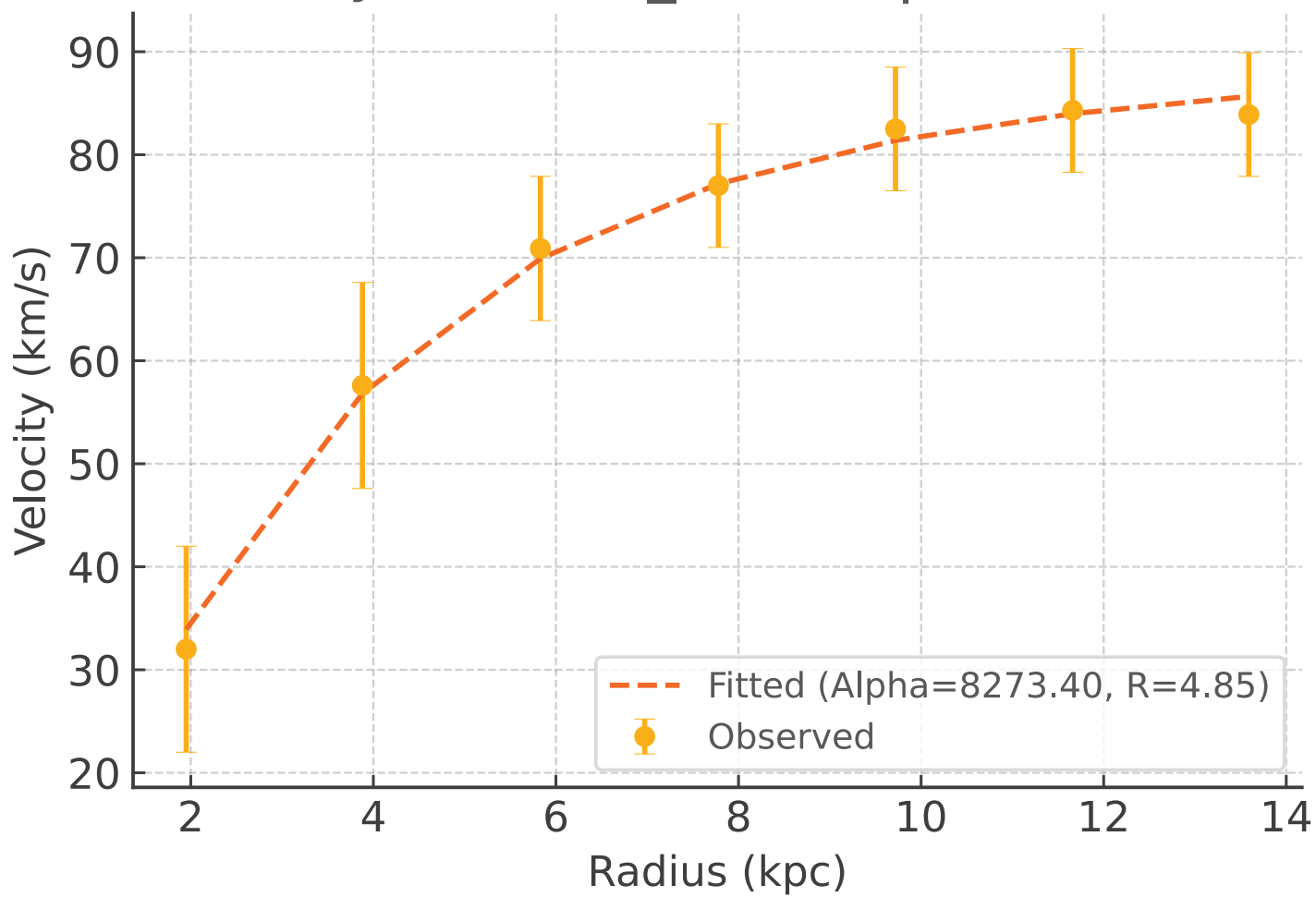




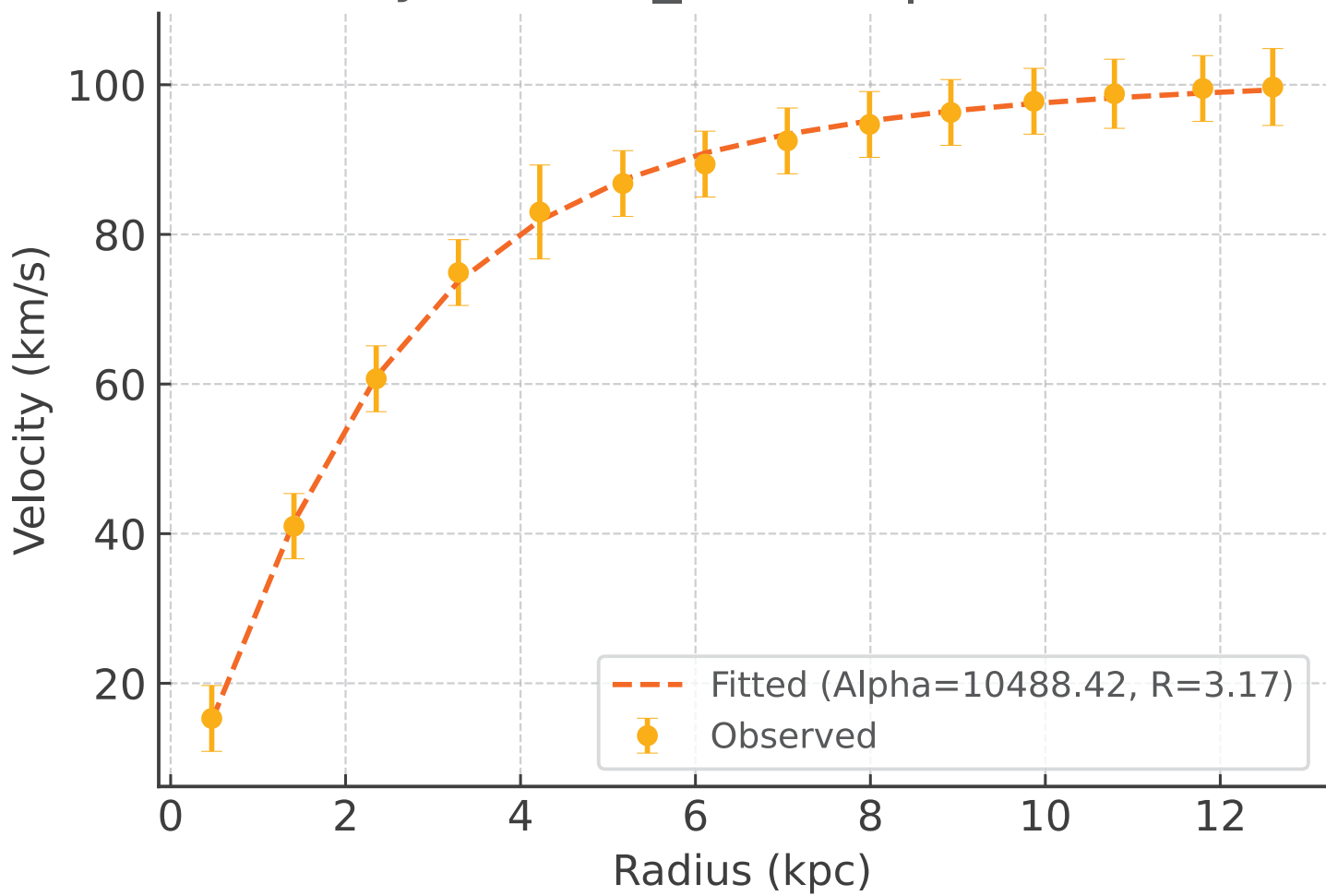
Galaxy: F571-8\_rotmod |  $R^2 = 0.975$



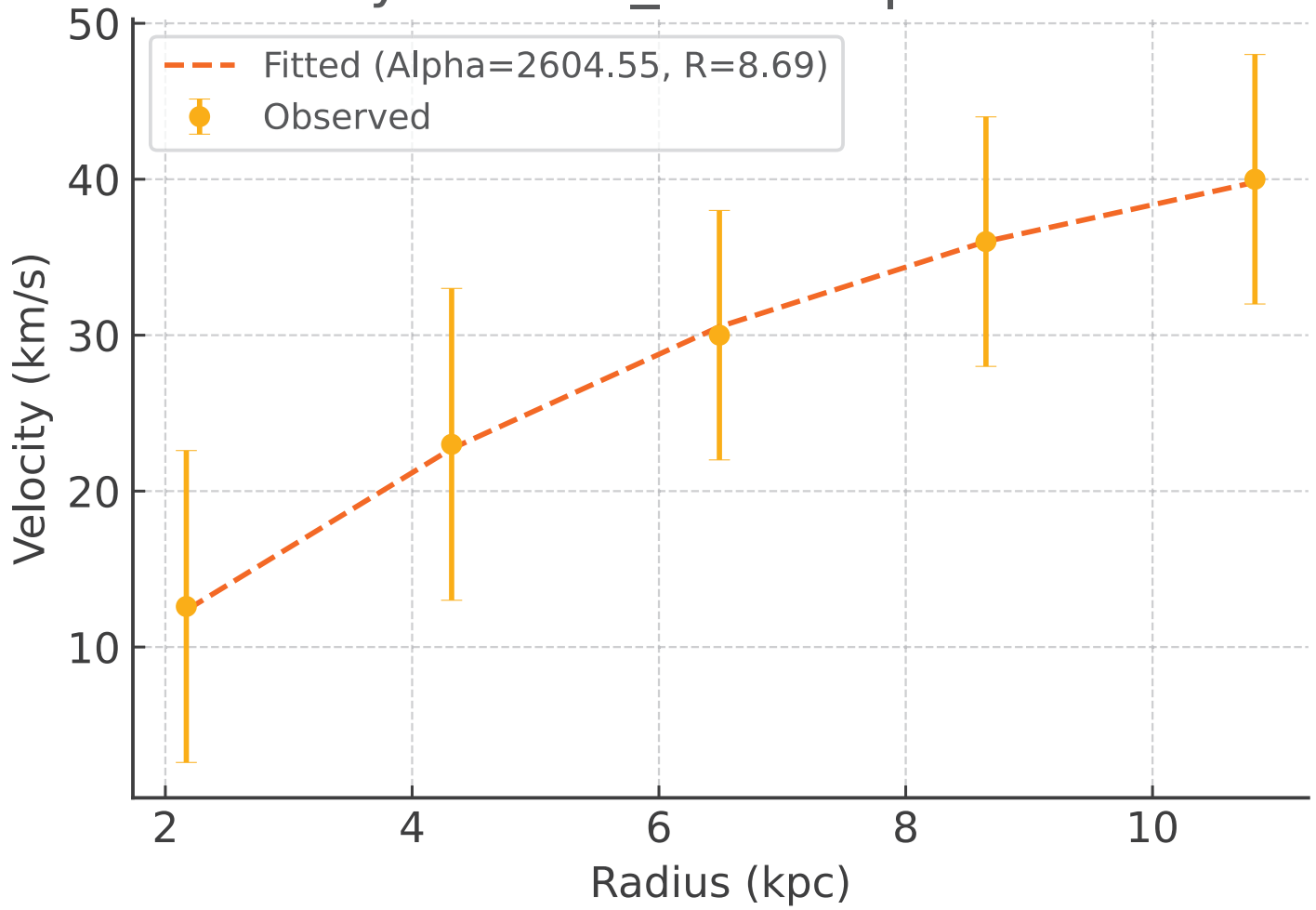
Galaxy: F571-V1\_rotmod |  $R^2 = 0.996$



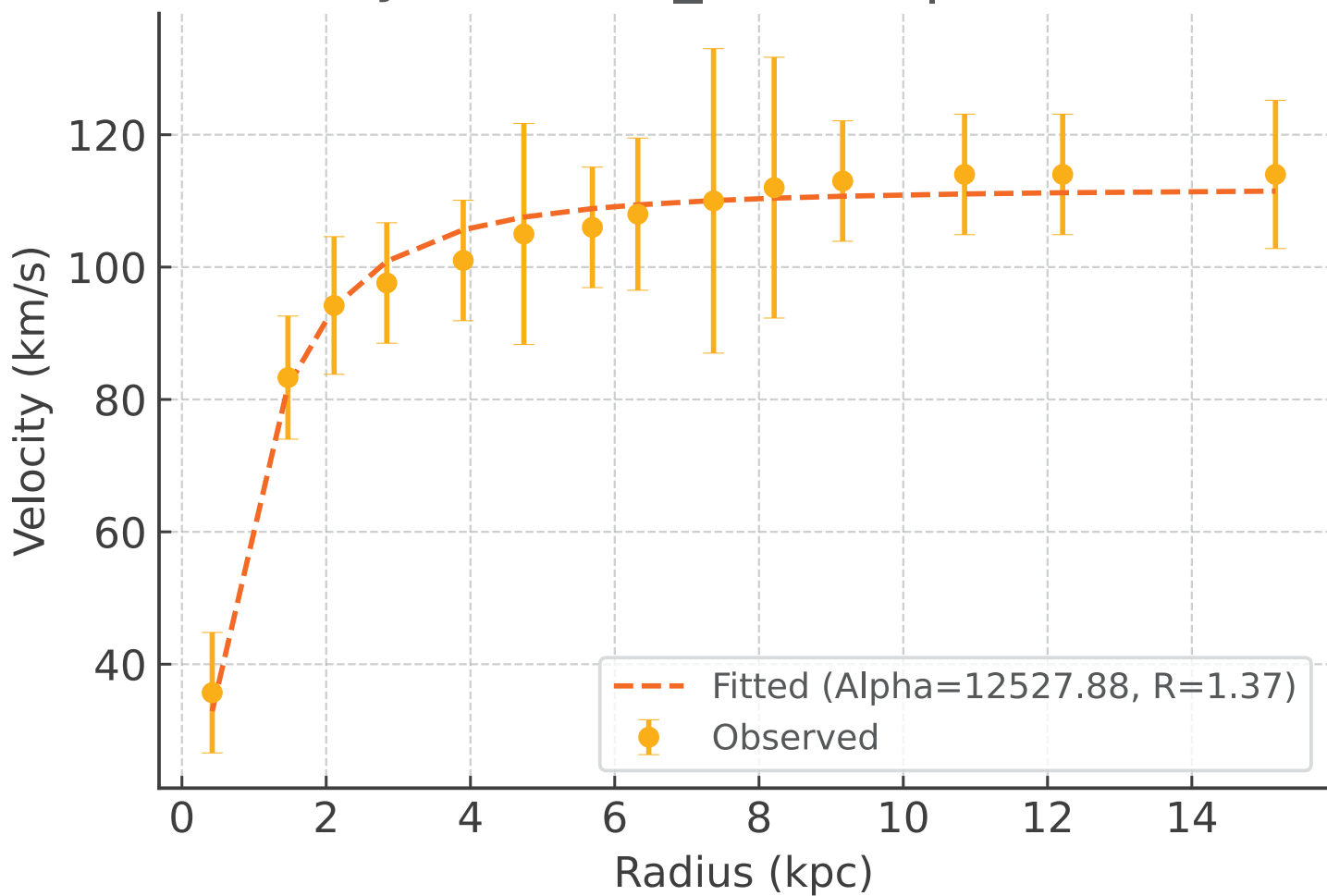
Galaxy: F574-1\_rotmod |  $R^2 = 0.999$



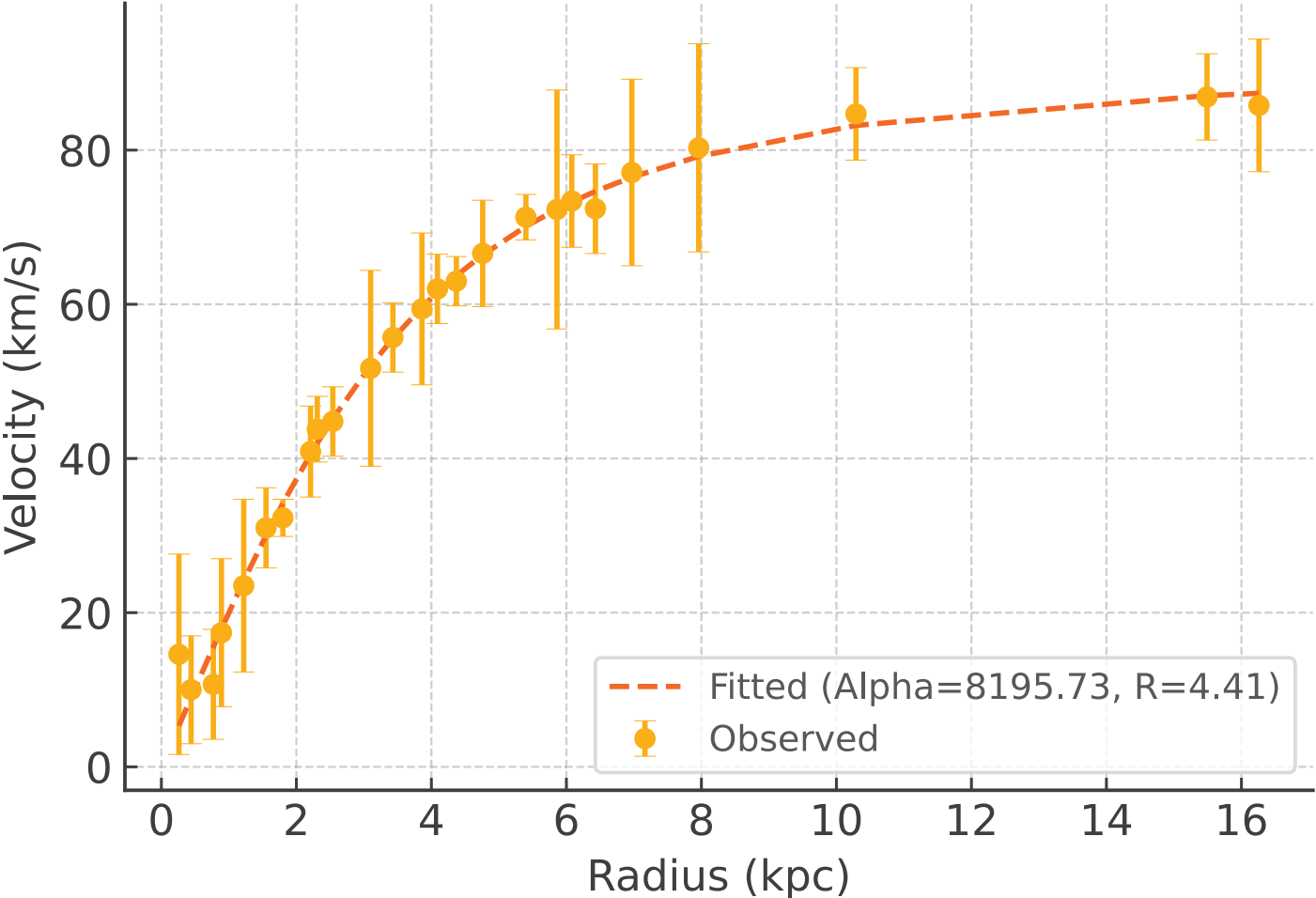
# Galaxy: F574-2\_rotmod | $R^2 = 0.999$



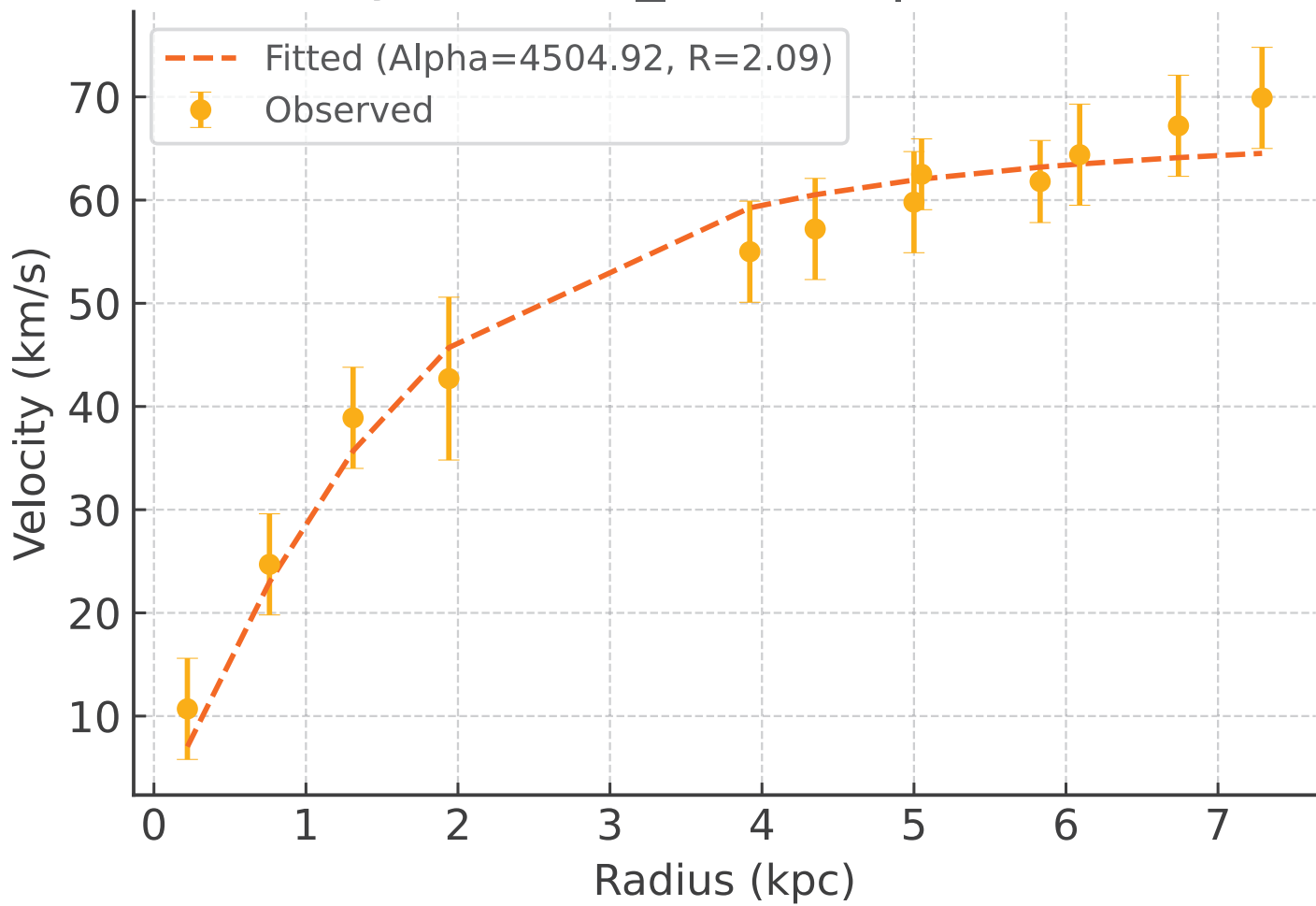
Galaxy: F579-V1\_rotmod |  $R^2 = 0.984$



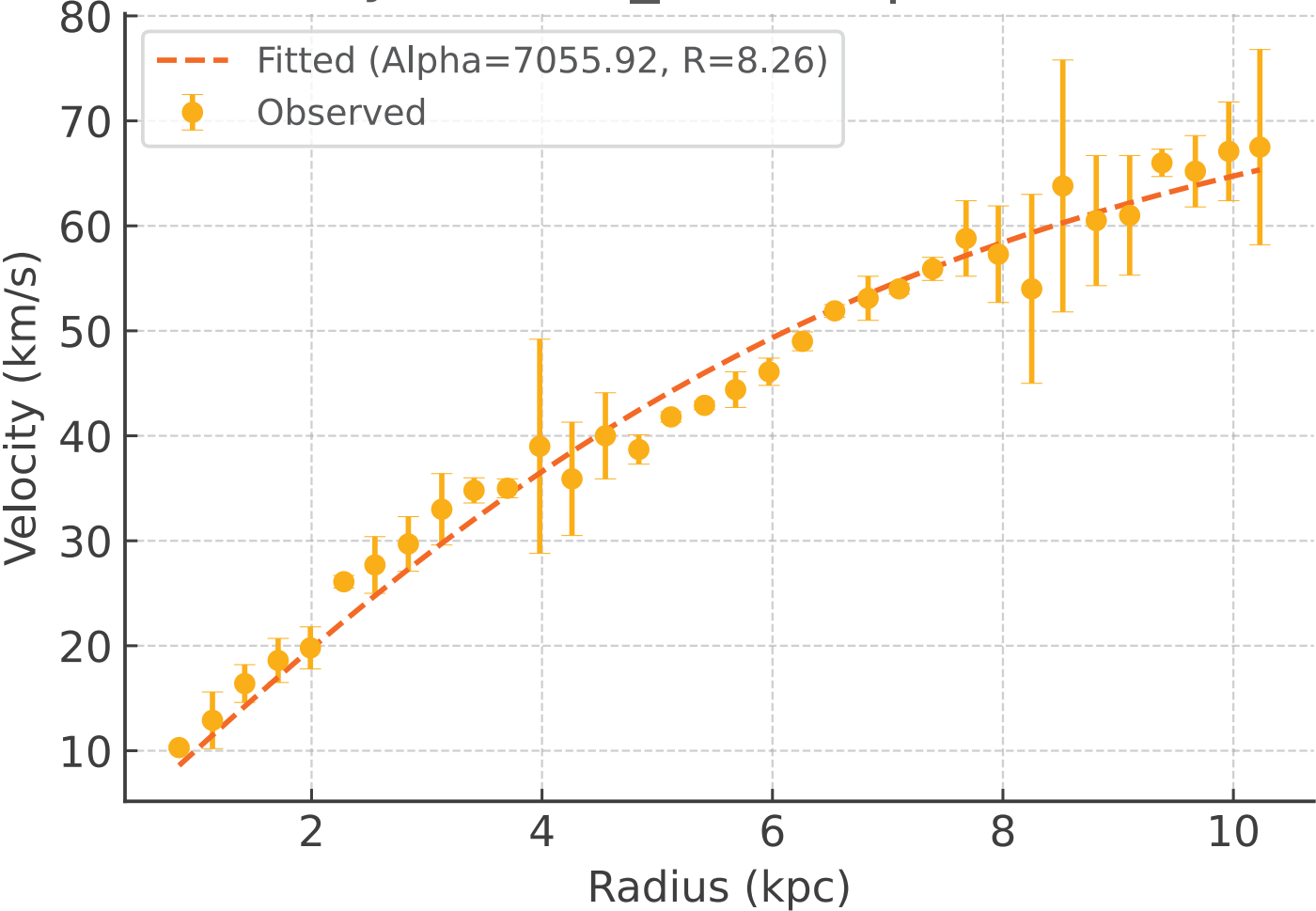
Galaxy: F583-1\_rotmod |  $R^2 = 0.991$



# Galaxy: F583-4\_rotmod | $R^2 = 0.970$

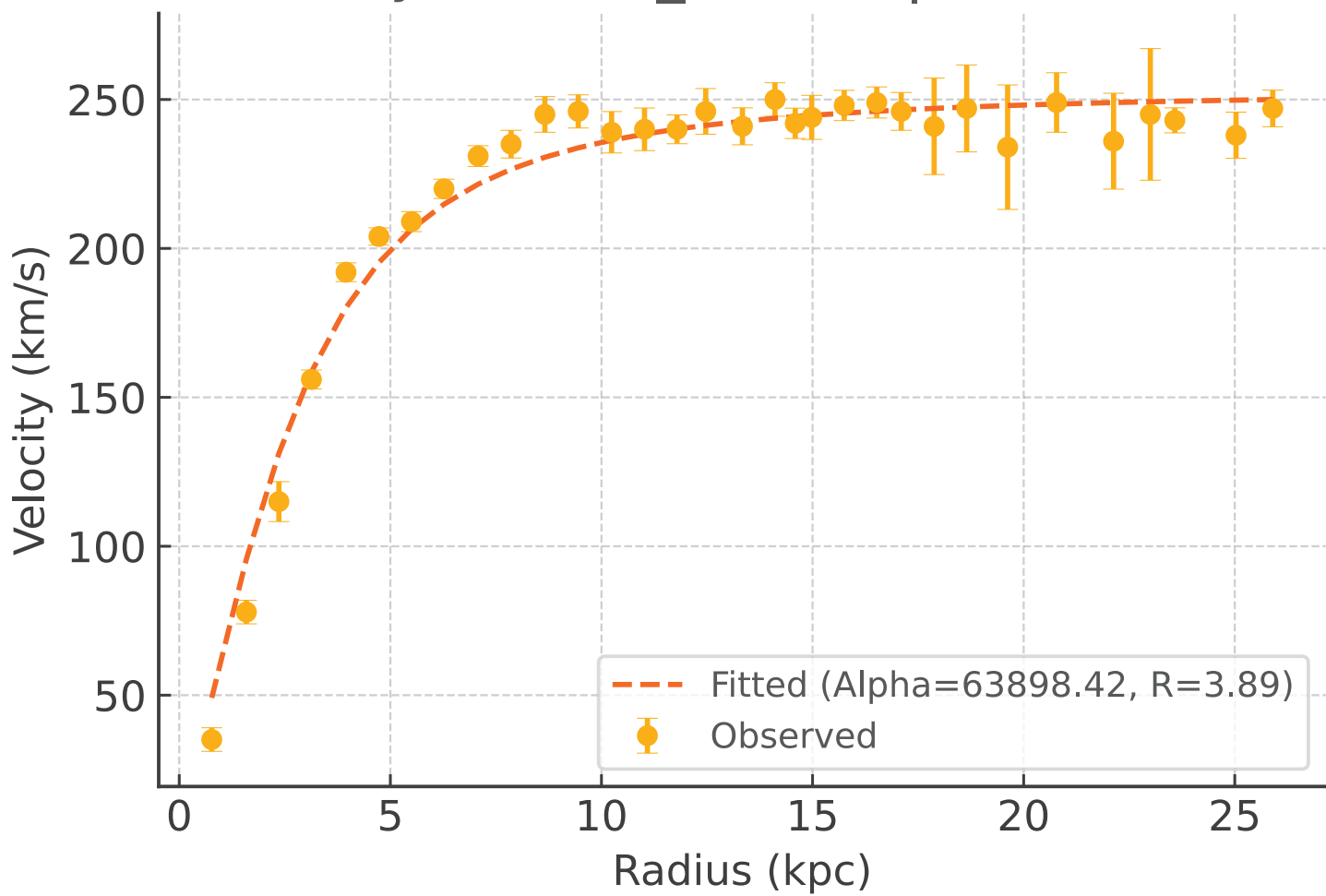


# Galaxy: IC2574\_rotmod | $R^2 = 0.979$

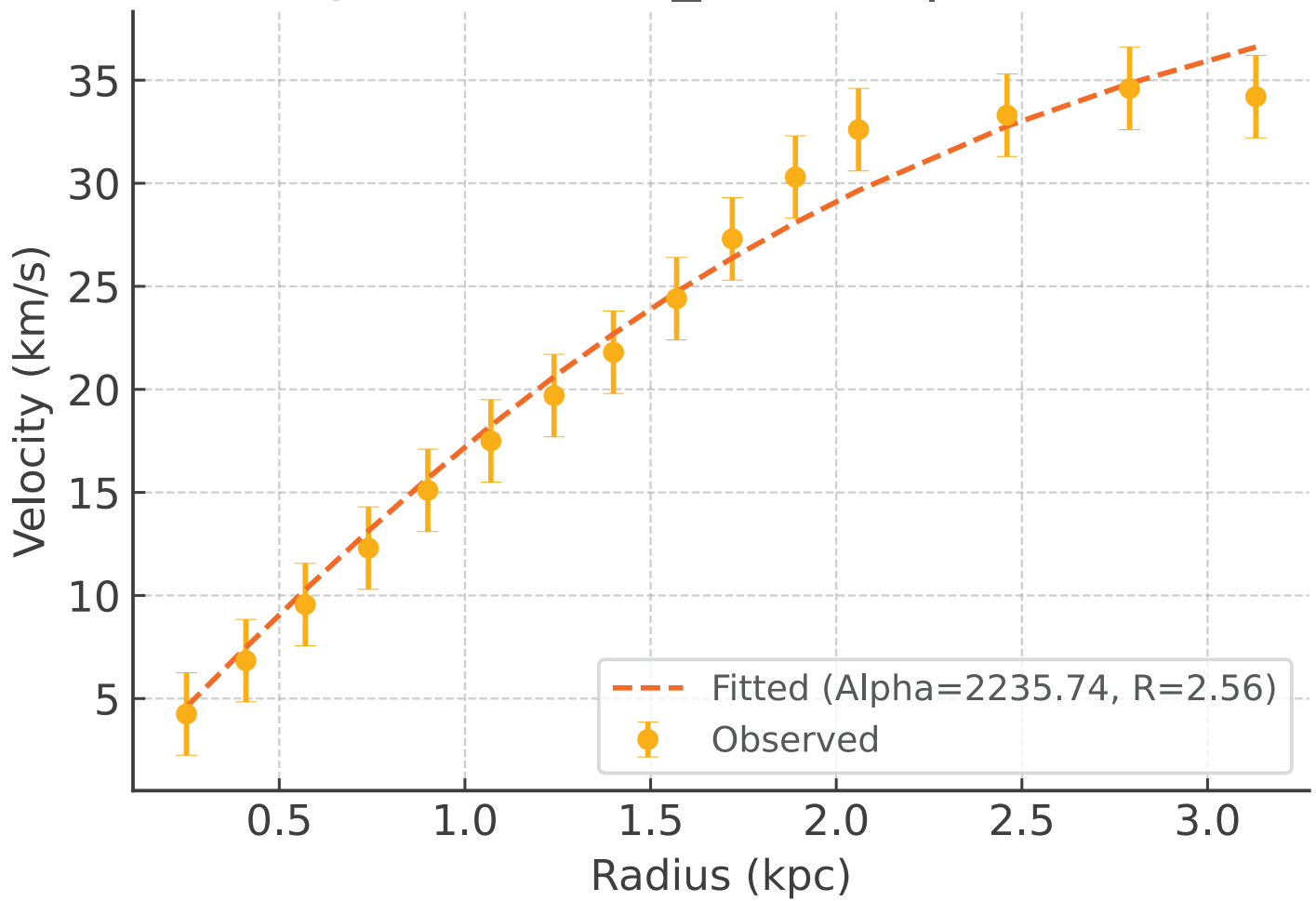




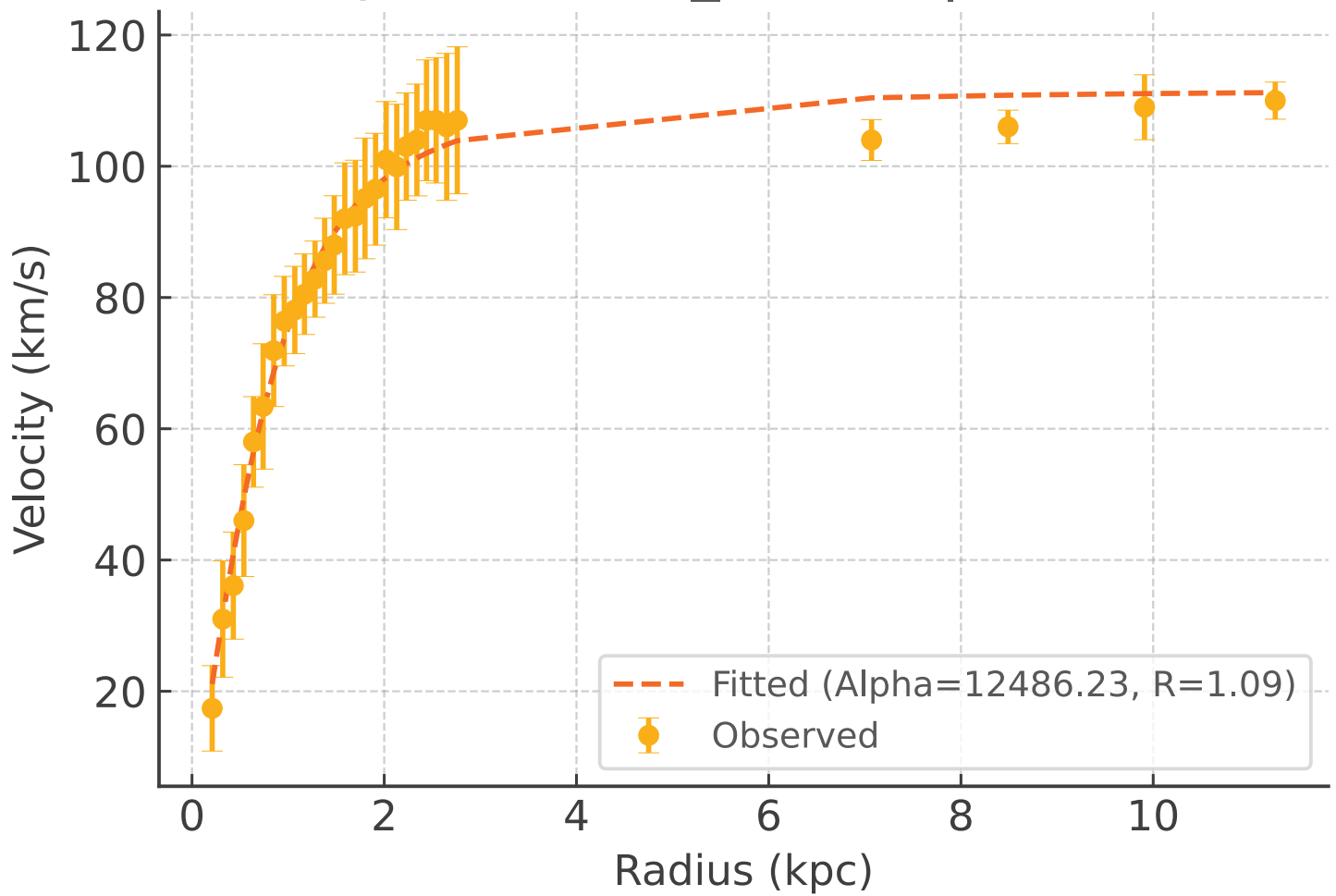
Galaxy: IC4202\_rotmod |  $R^2 = 0.973$



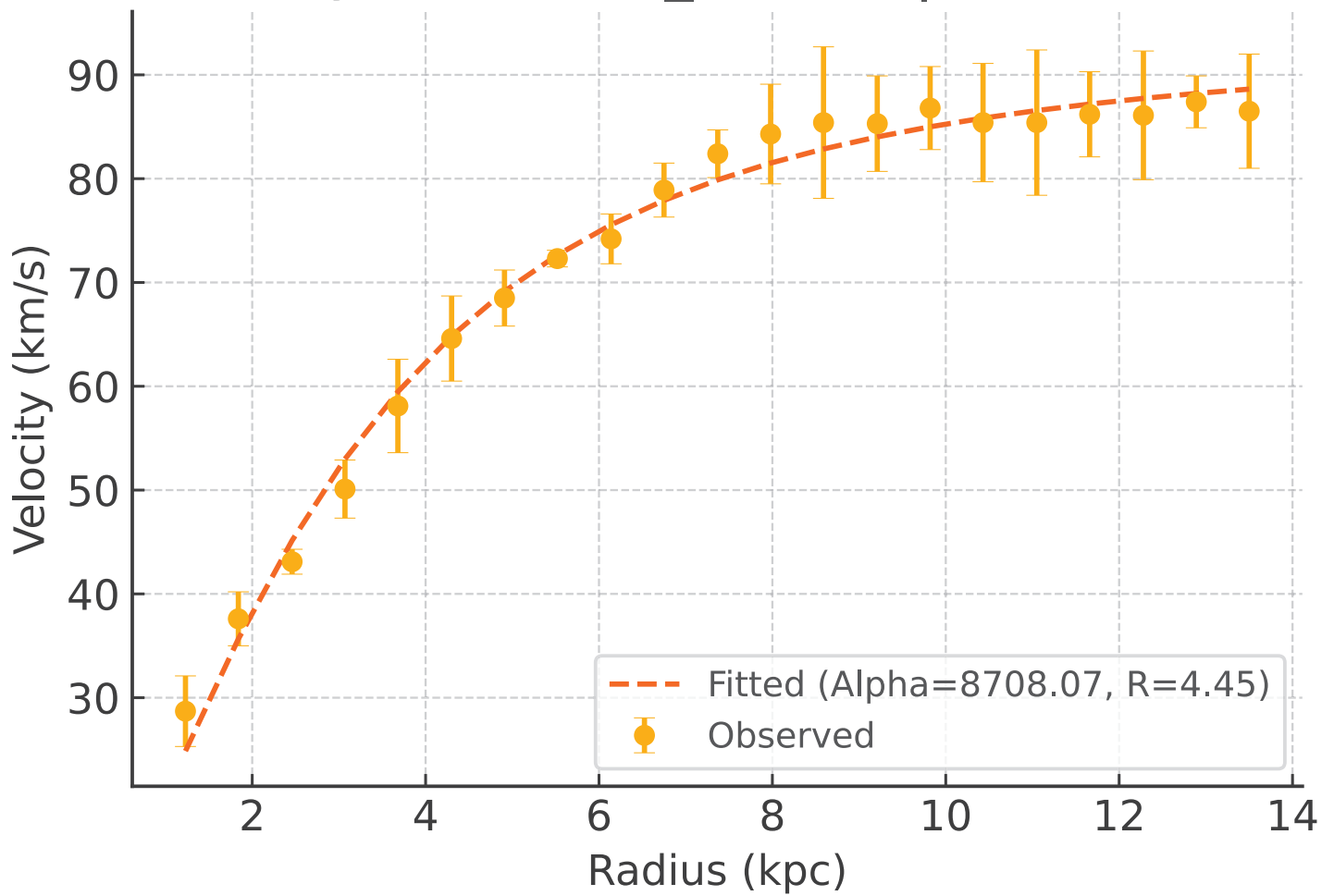
Galaxy: KK98-251\_rotmod |  $R^2 = 0.984$



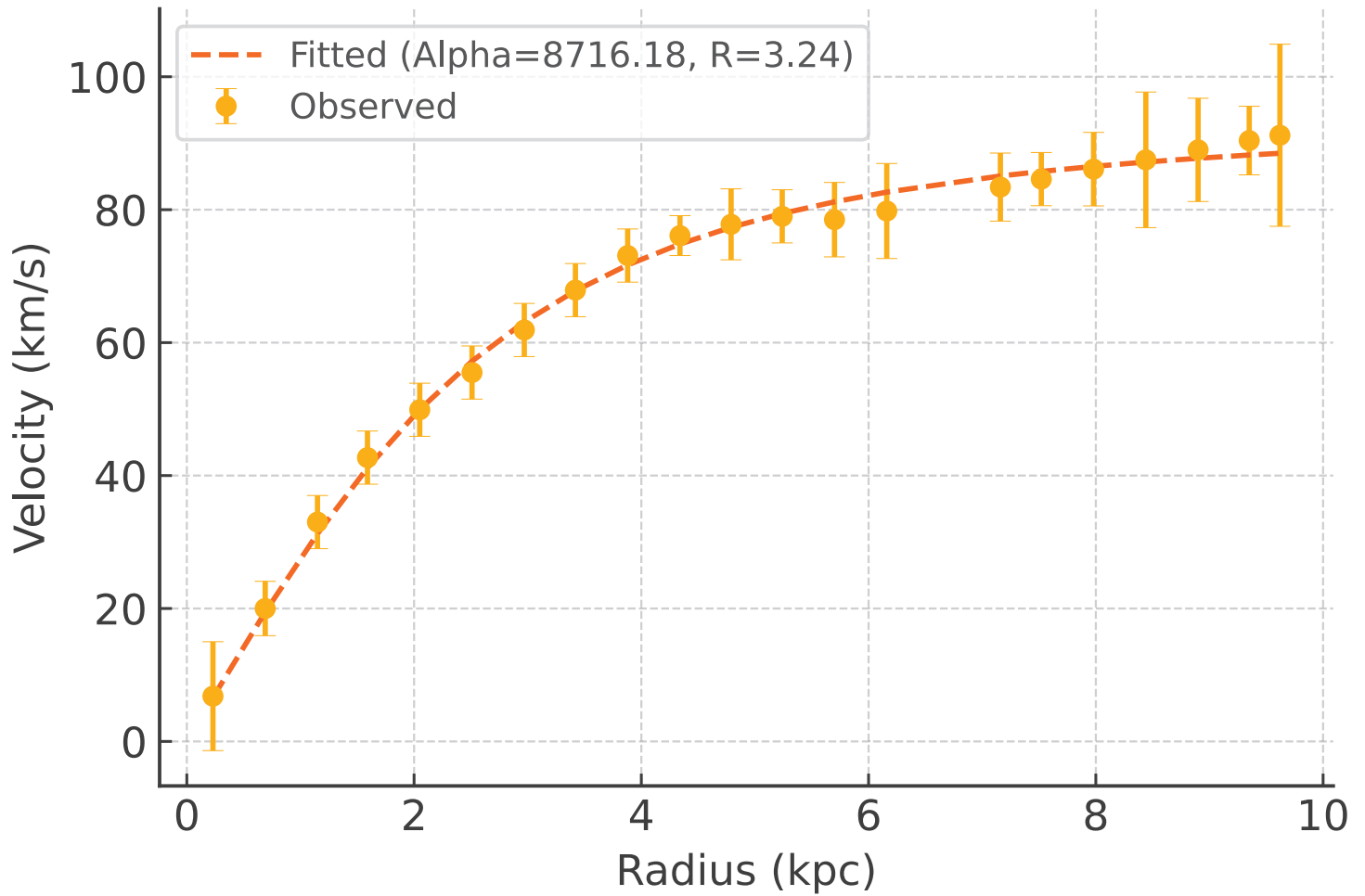
Galaxy: NGC0024\_rotmod |  $R^2 = 0.987$



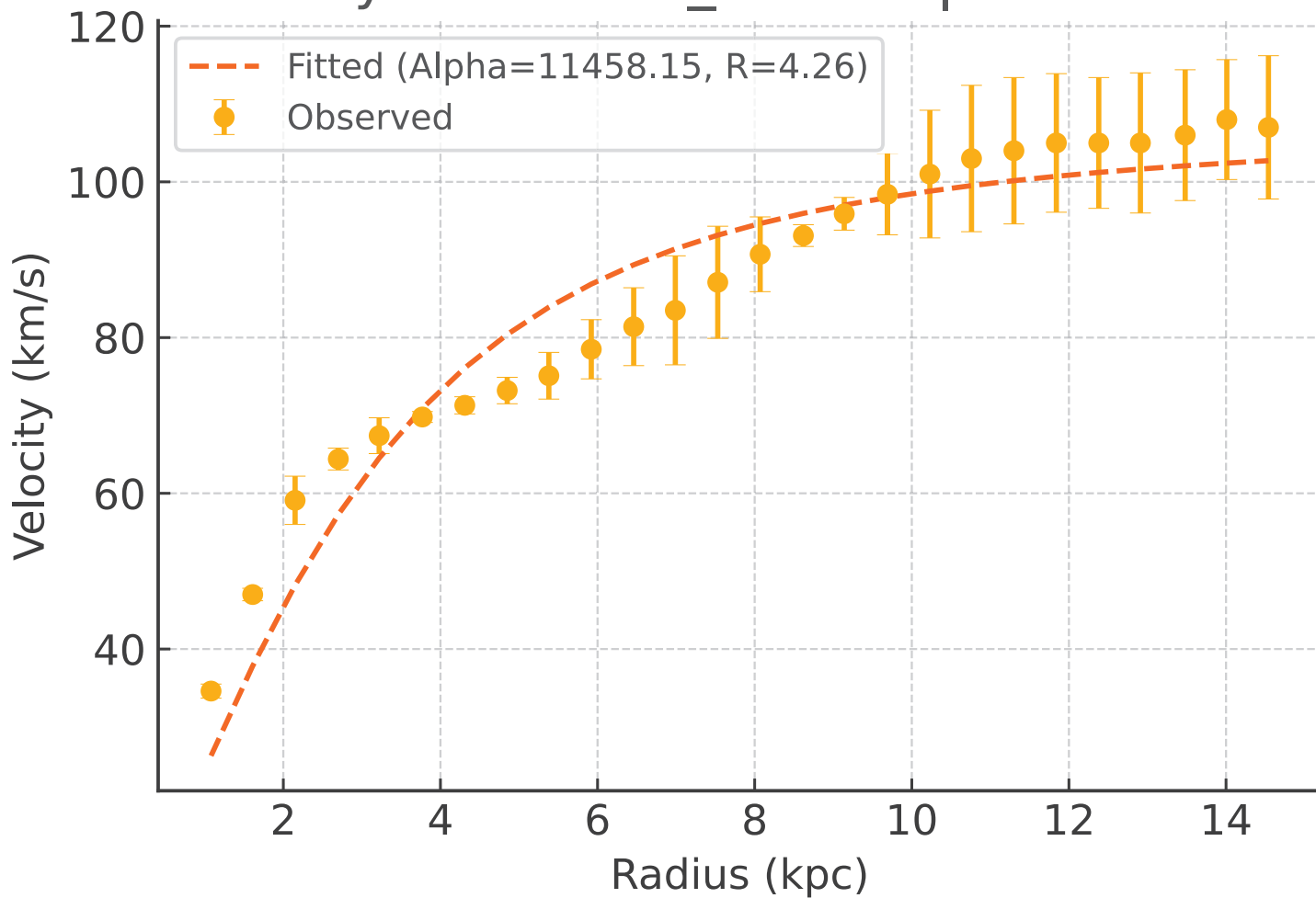
Galaxy: NGC0055\_rotmod |  $R^2 = 0.989$



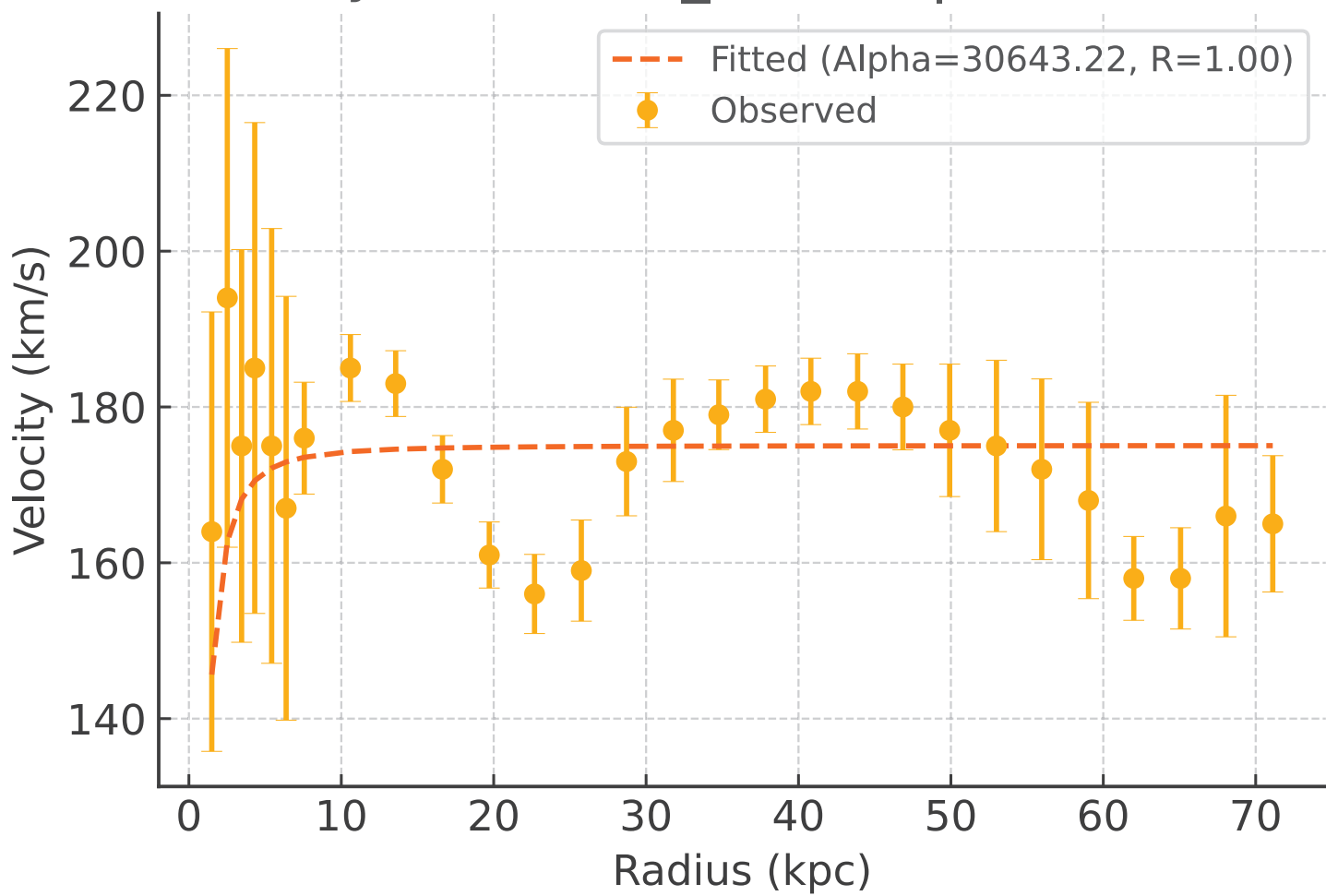
# Galaxy: NGC0100\_rotmod | $R^2 = 0.996$



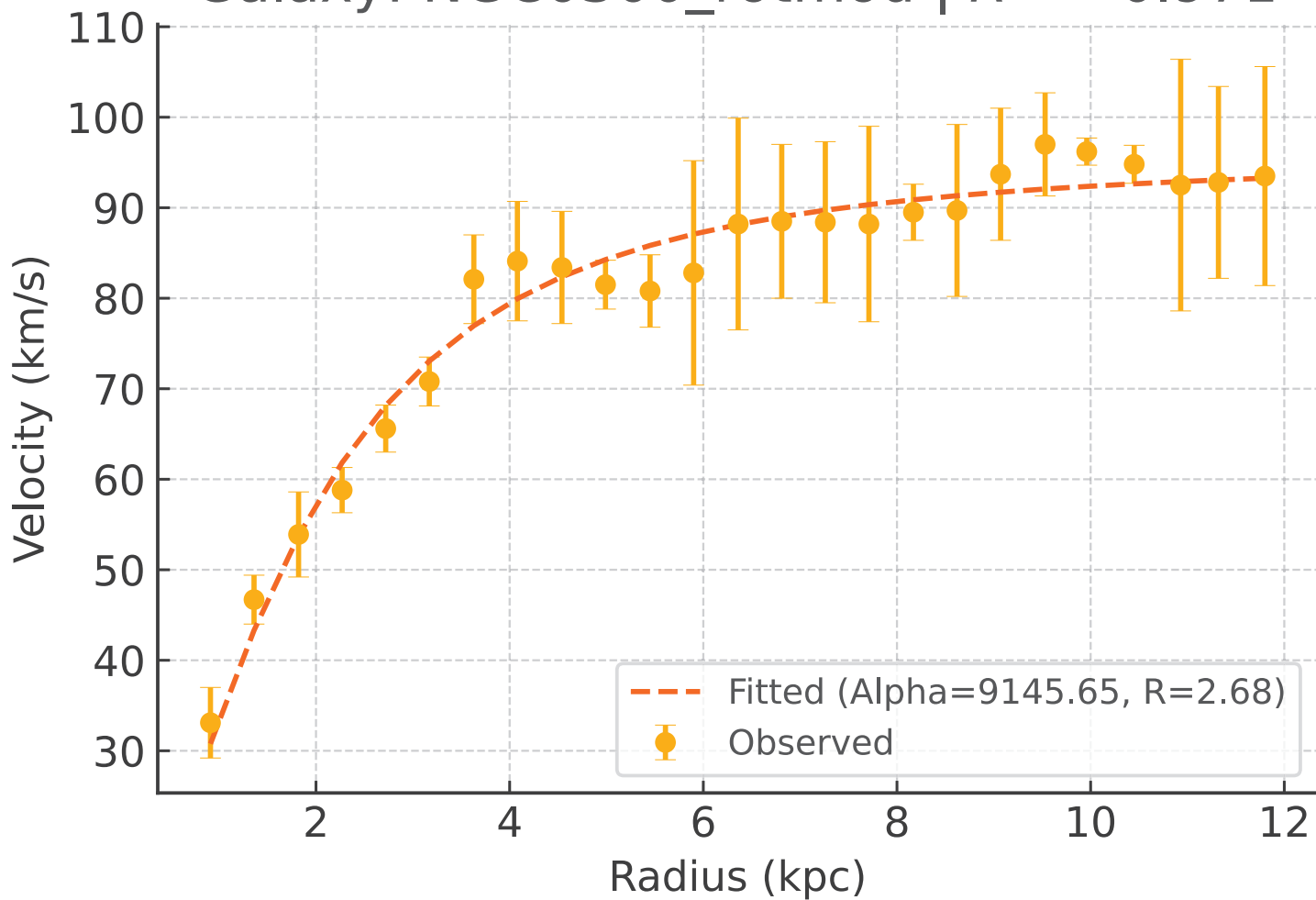
# Galaxy: NGC0247\_rotmod | $R^2 = 0.913$



# Galaxy: NGC0289\_rotmod | $R^2 = -0.433$

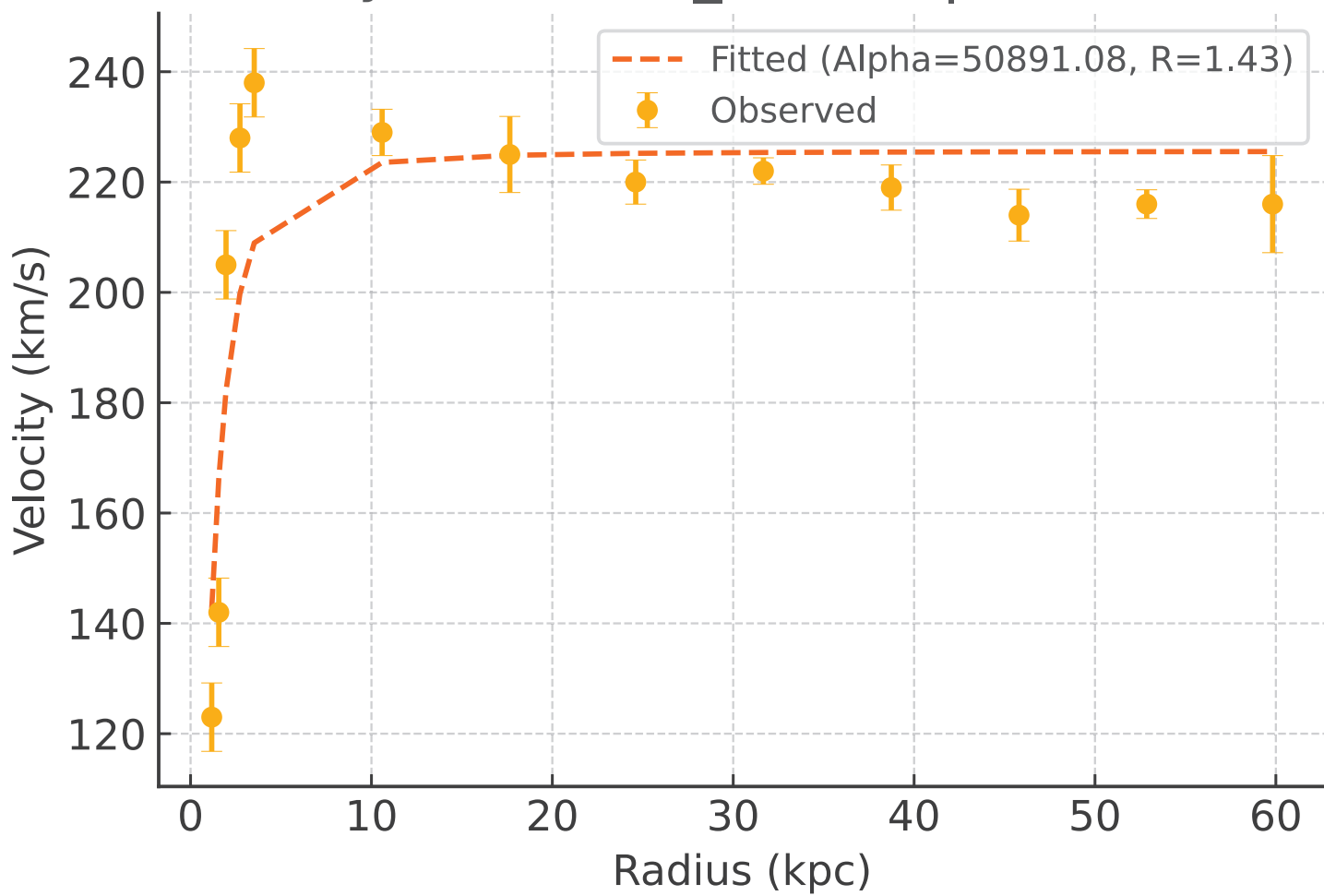


Galaxy: NGC0300\_rotmod |  $R^2 = 0.971$

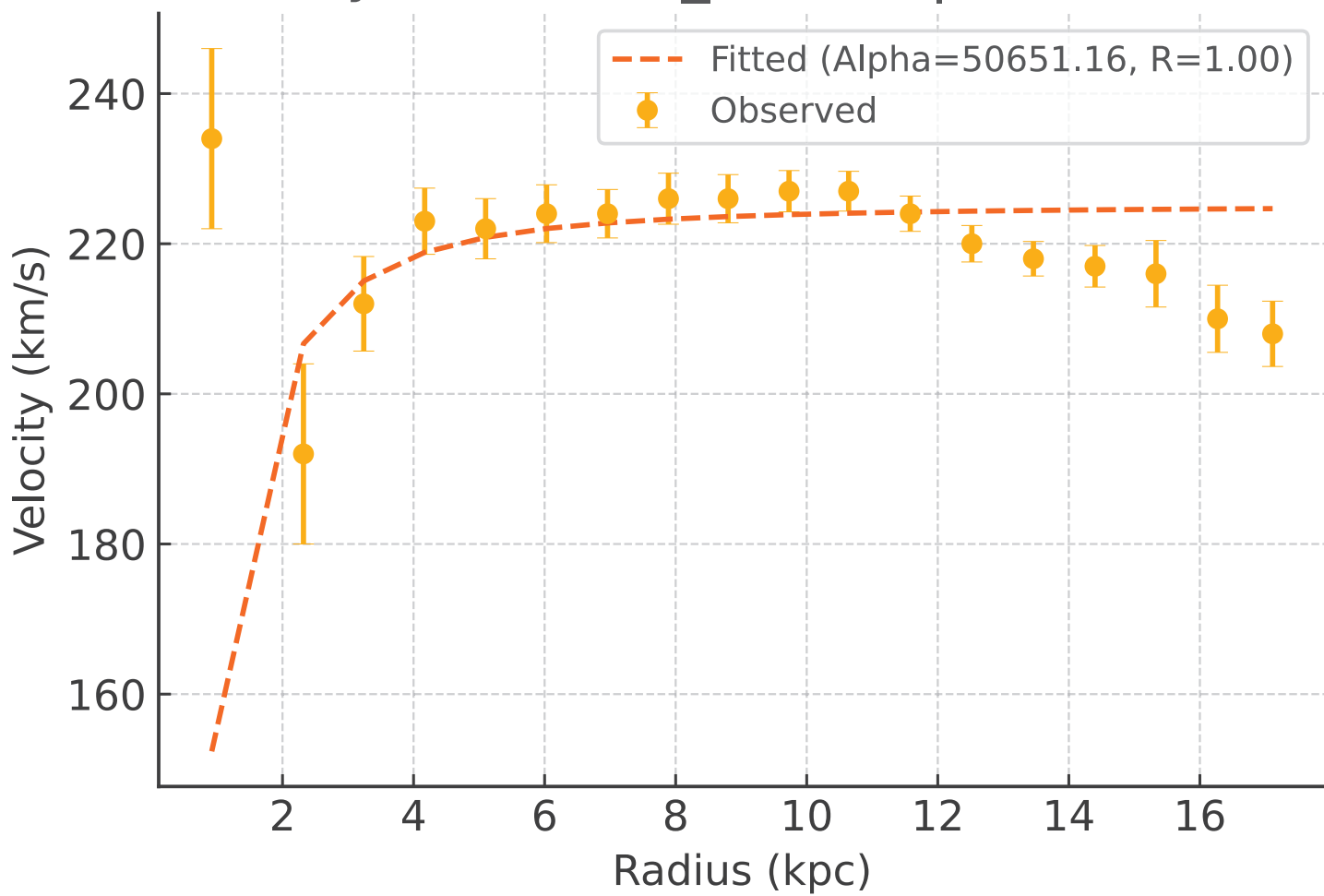




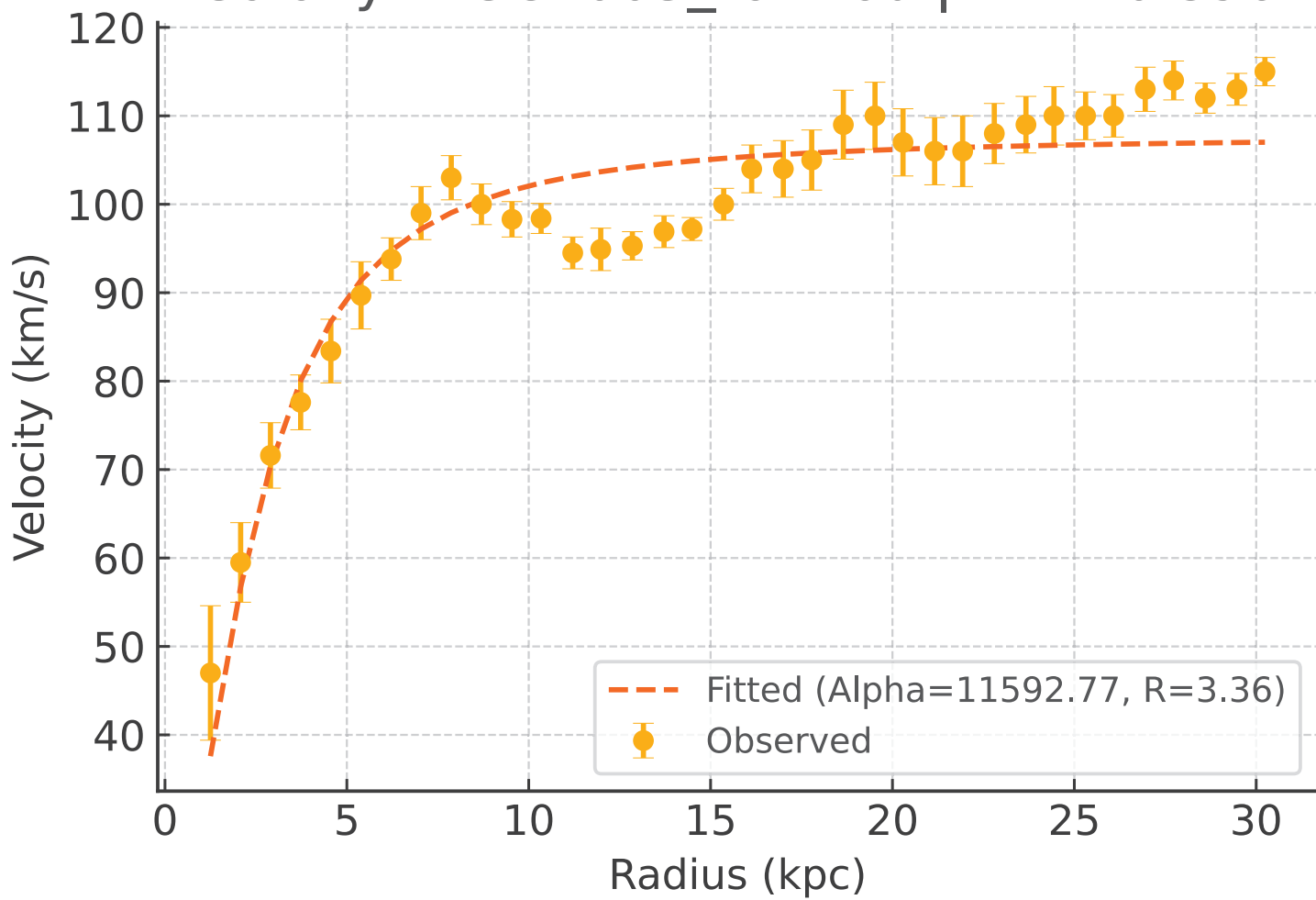
# Galaxy: NGC0801\_rotmod | $R^2 = 0.750$



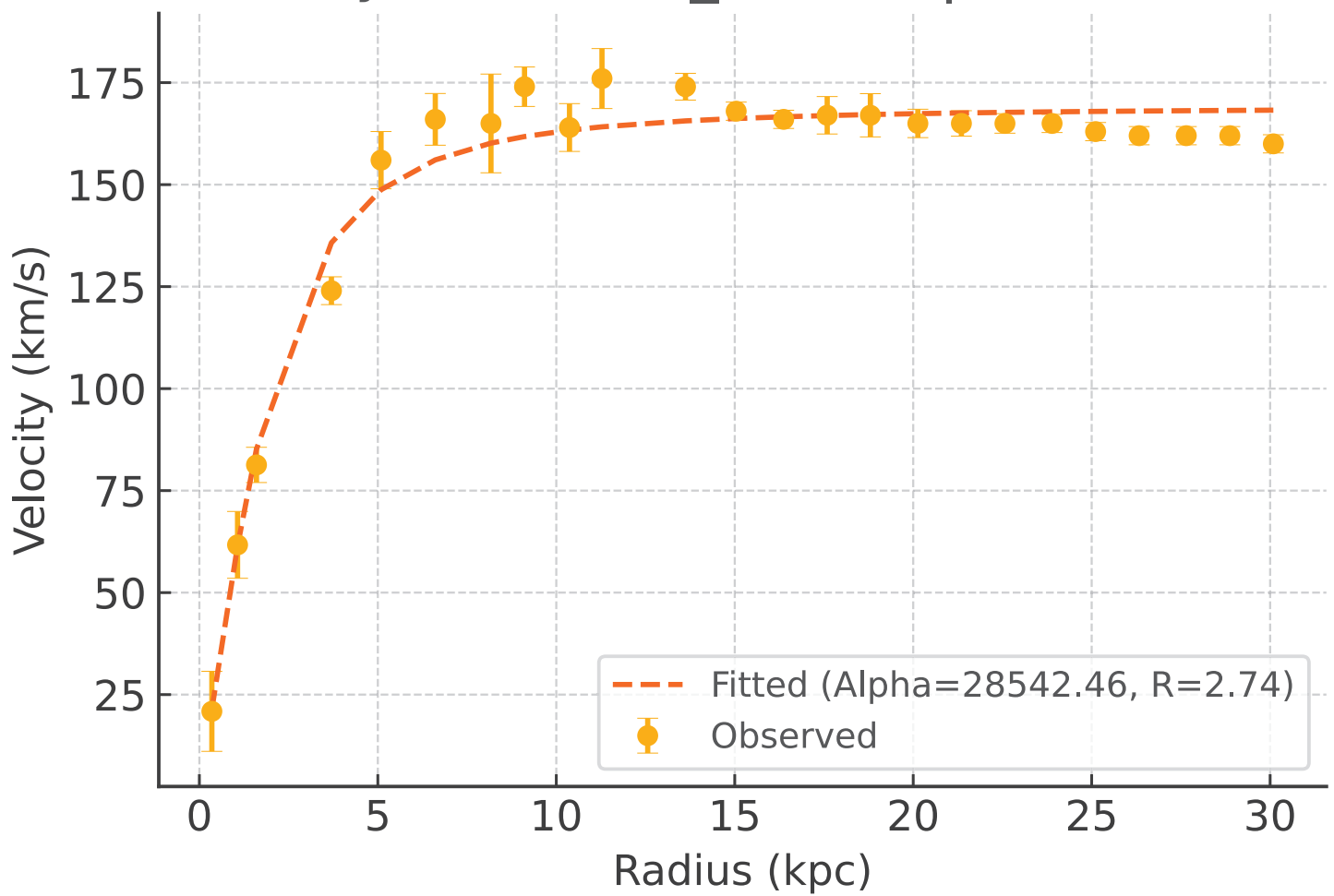
# Galaxy: NGC0891\_rotmod | $R^2 = -3.944$



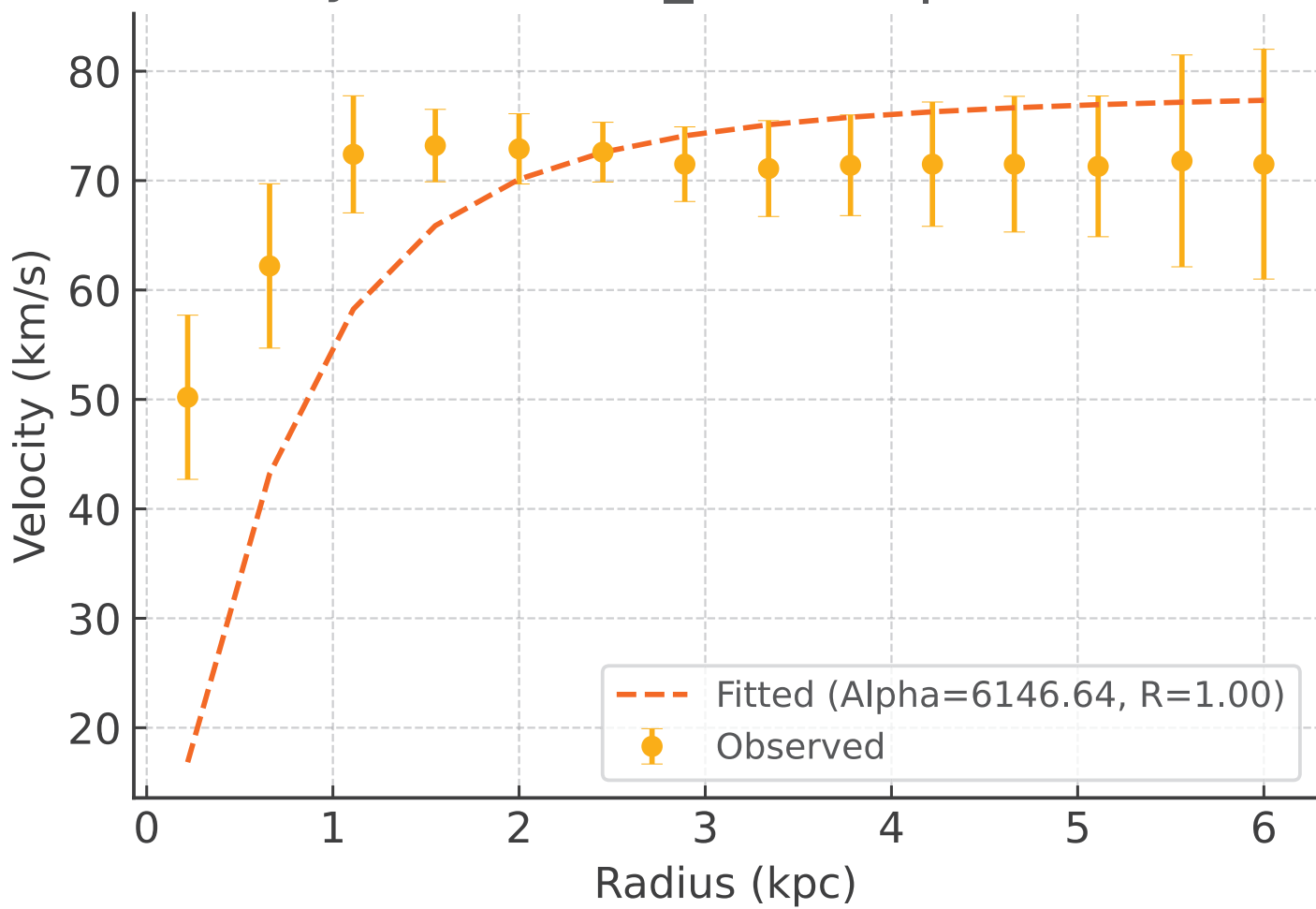
Galaxy: NGC1003\_rotmod |  $R^2 = 0.896$



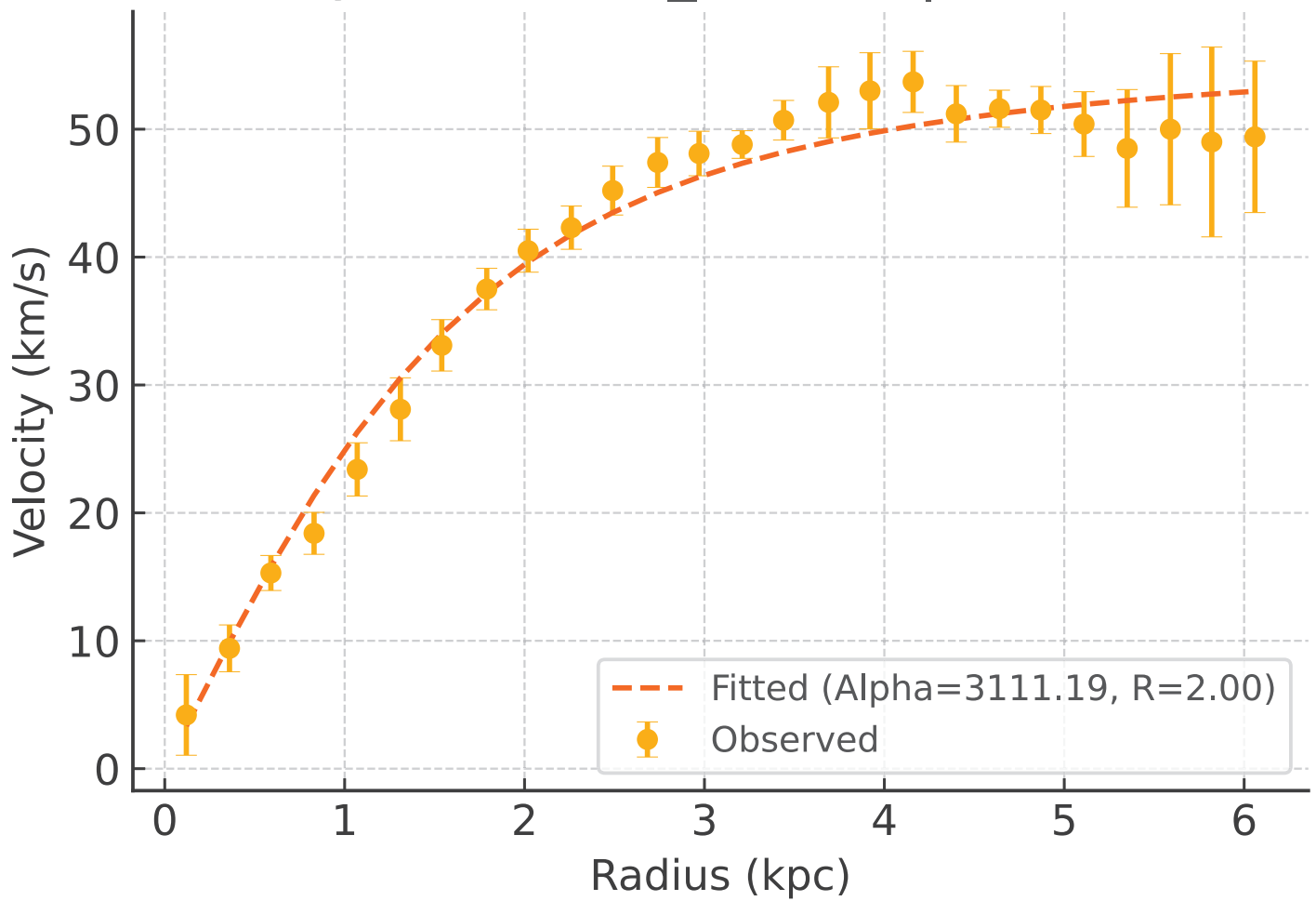
Galaxy: NGC1090\_rotmod |  $R^2 = 0.974$



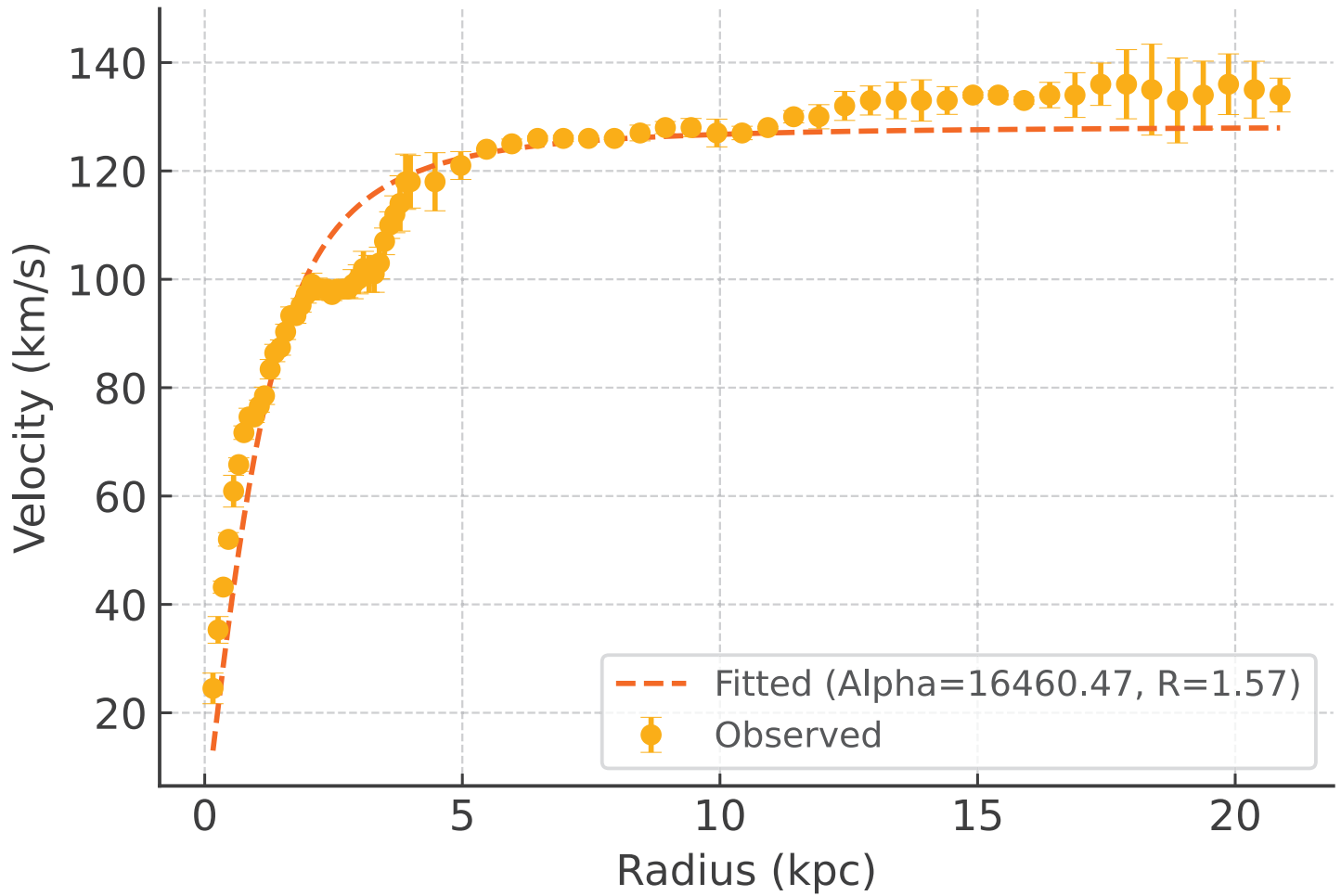
Galaxy: NGC1705\_rotmod |  $R^2 = -2.848$



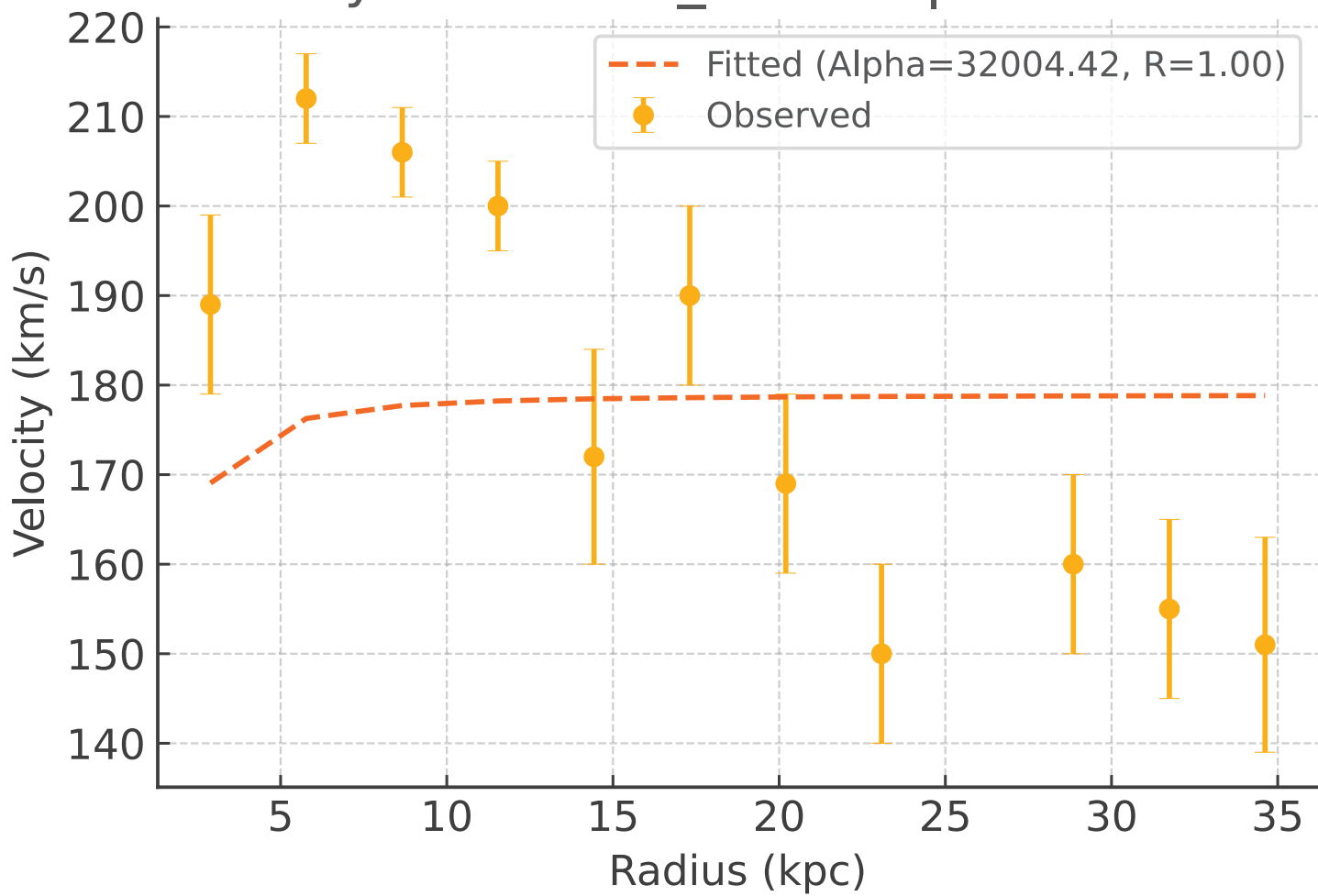
Galaxy: NGC2366\_rotmod |  $R^2 = 0.976$



# Galaxy: NGC2403\_rotmod | $R^2 = 0.905$

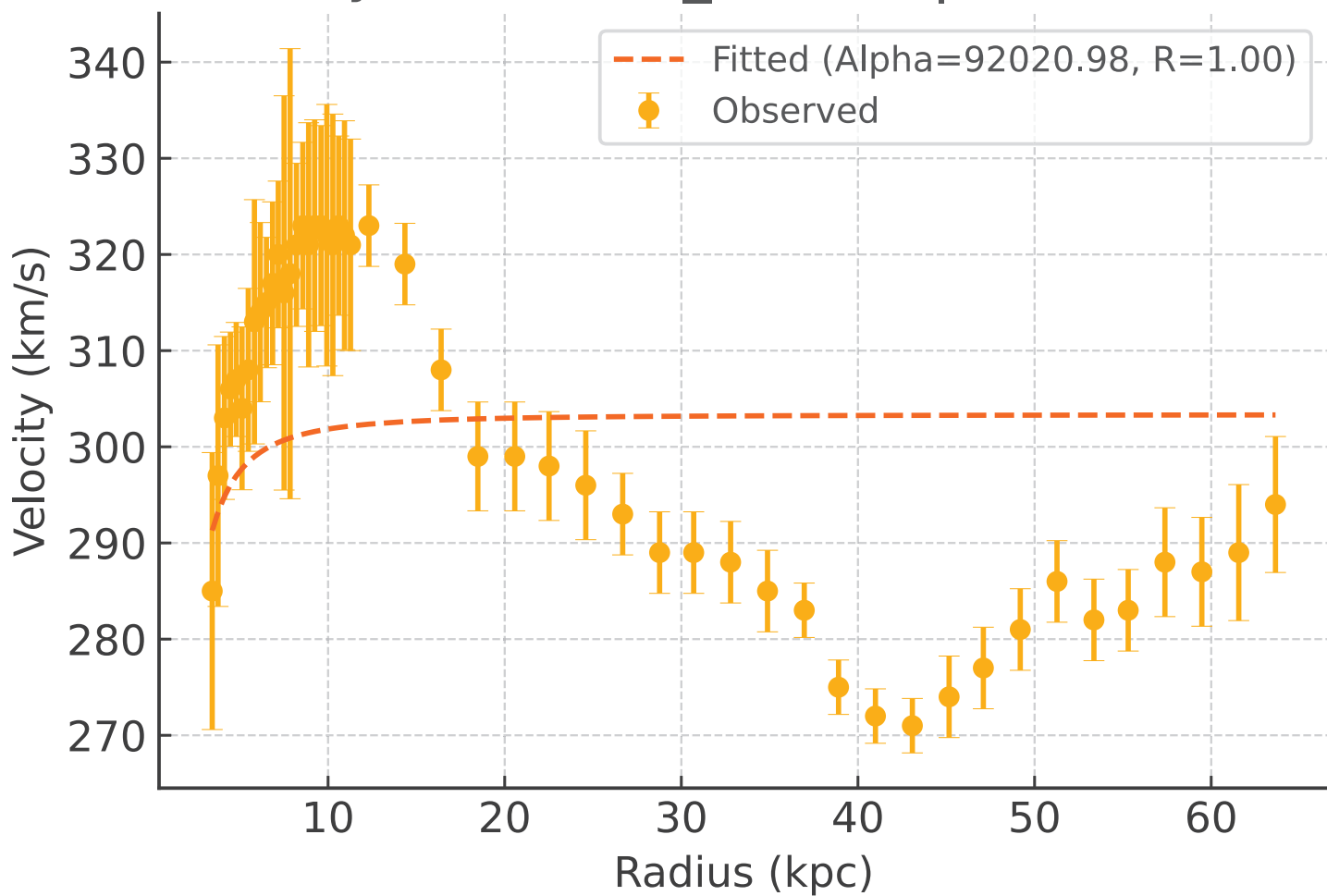


# Galaxy: NGC2683\_rotmod | $R^2 = -0.109$

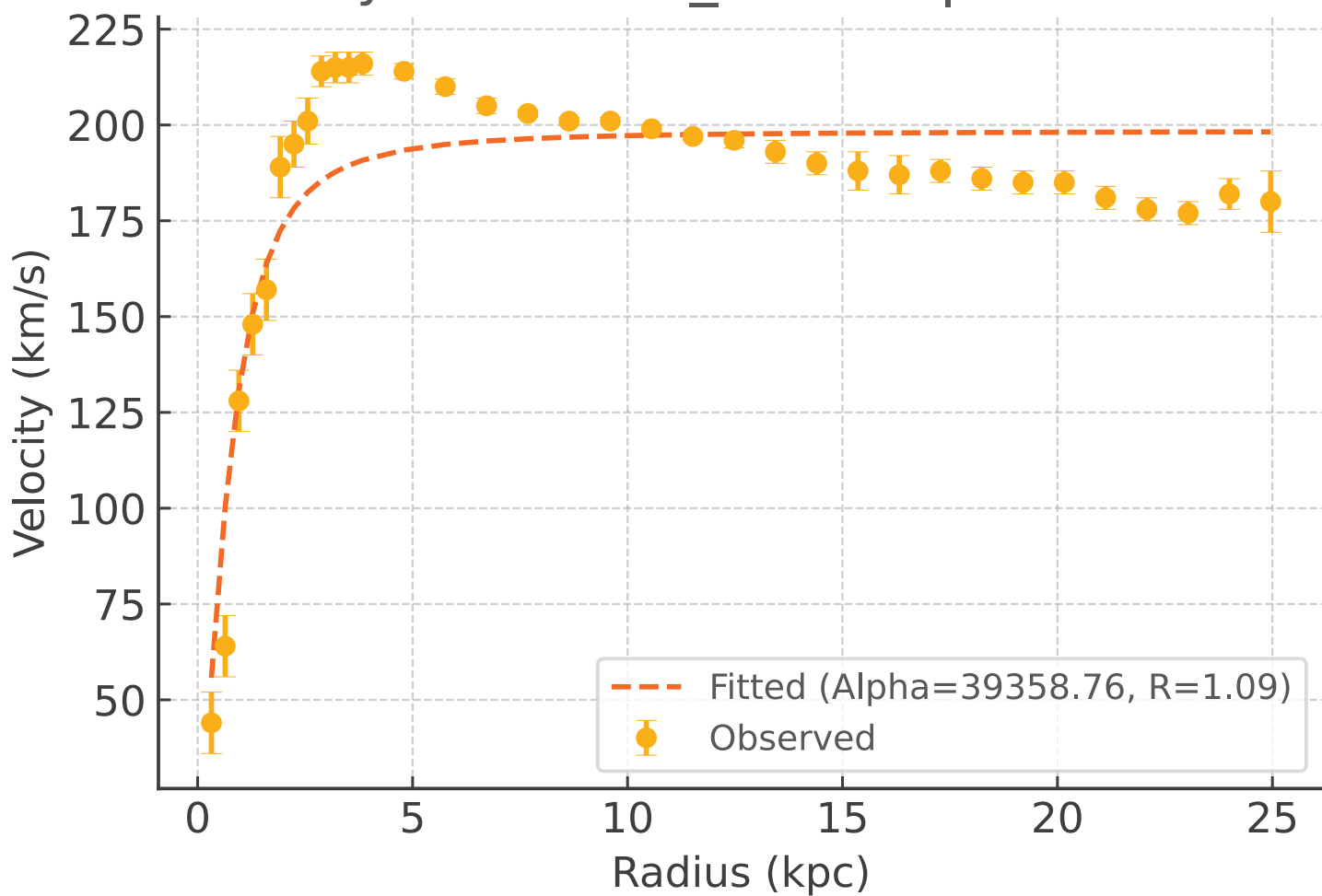




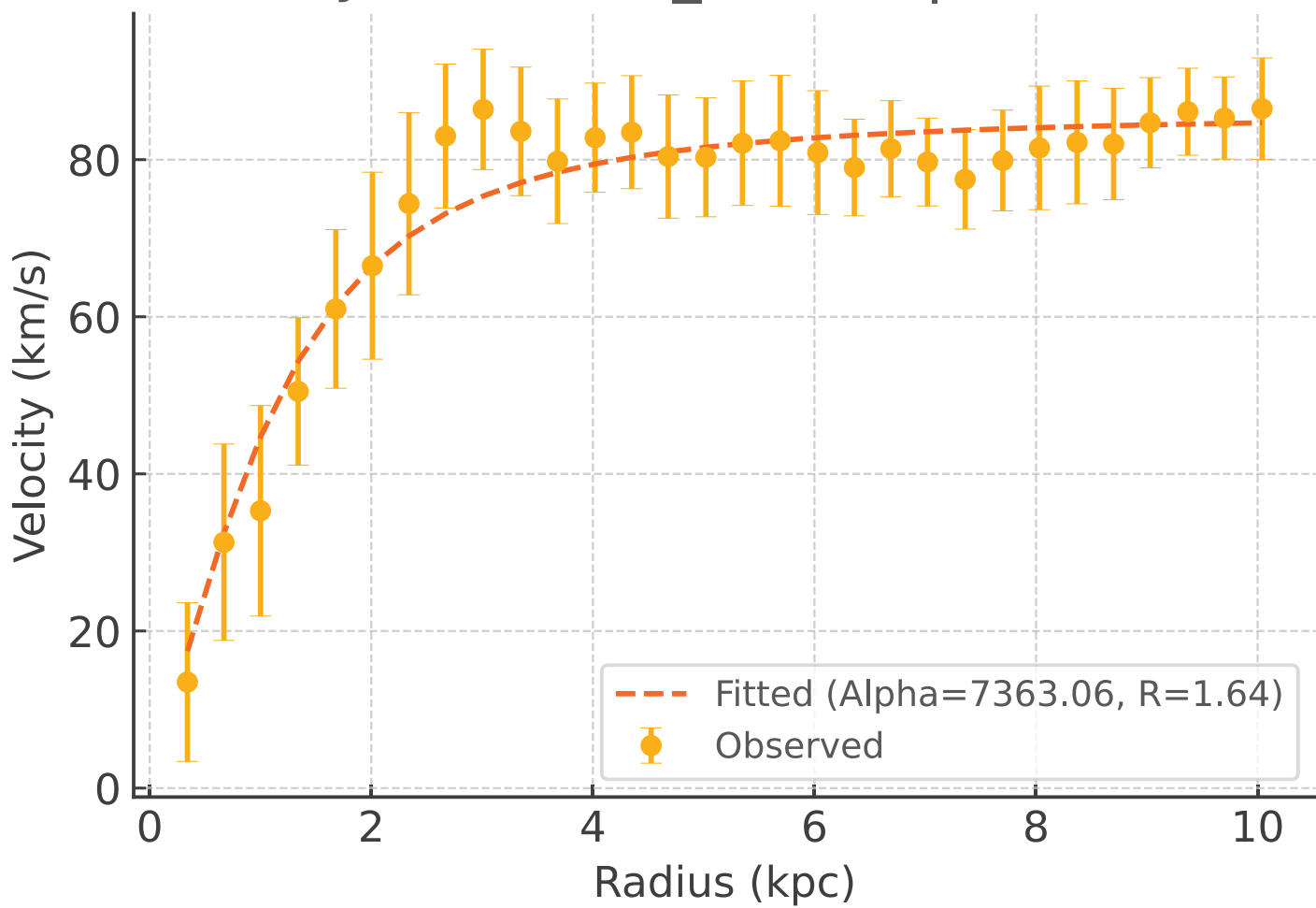
# Galaxy: NGC2841\_rotmod | $R^2 = -0.106$



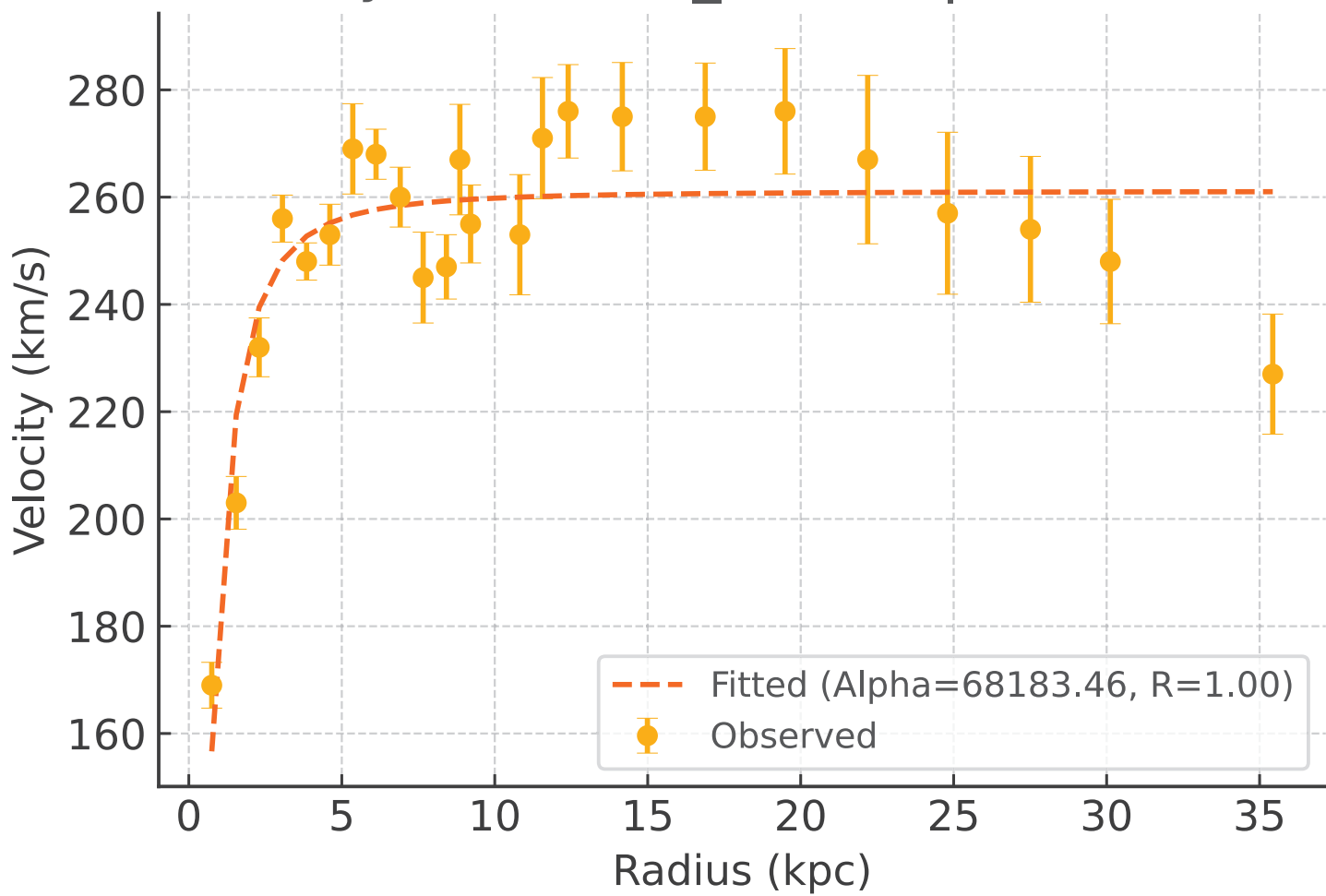
# Galaxy: NGC2903\_rotmod | $R^2 = 0.815$



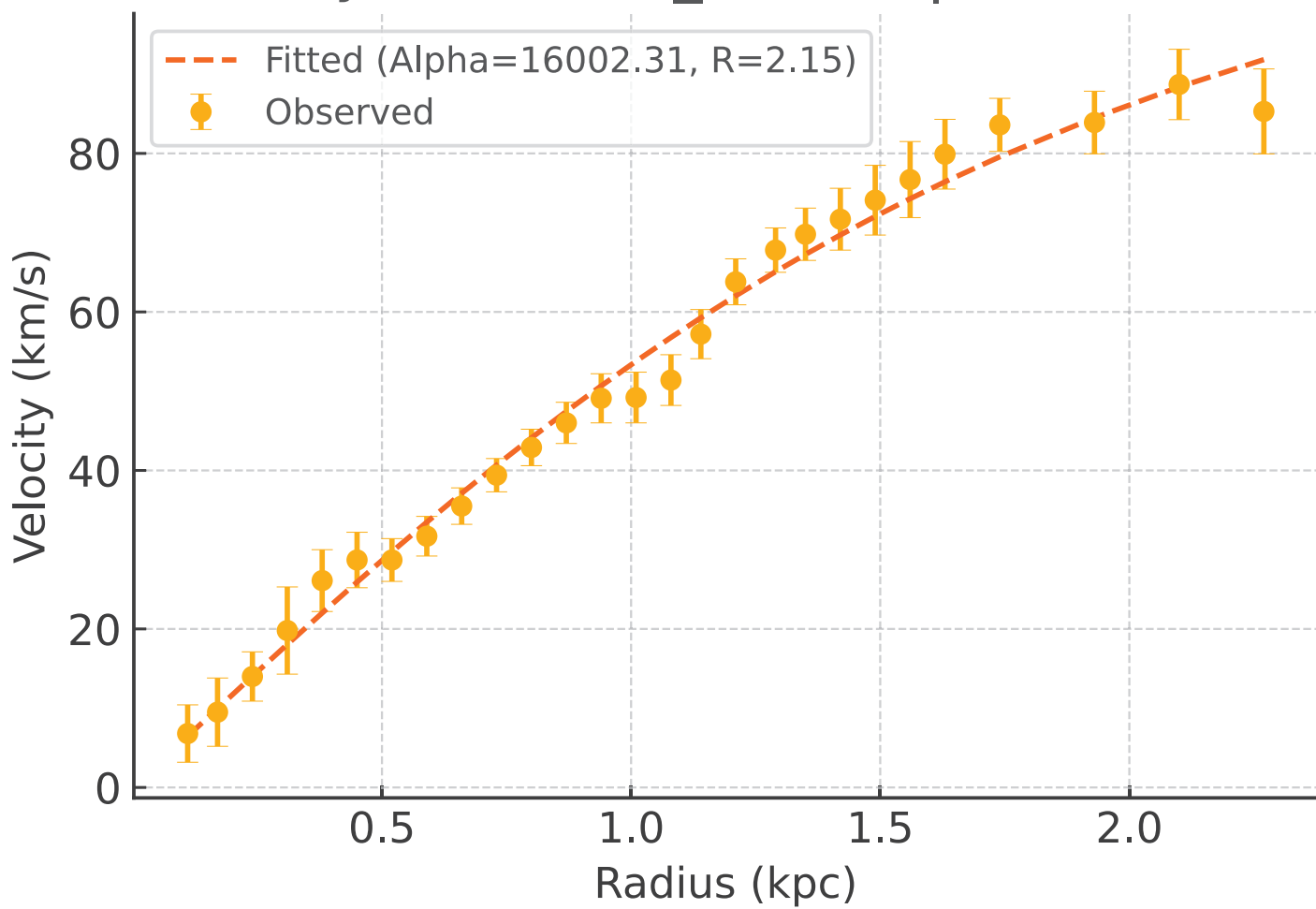
Galaxy: NGC2915\_rotmod |  $R^2 = 0.943$



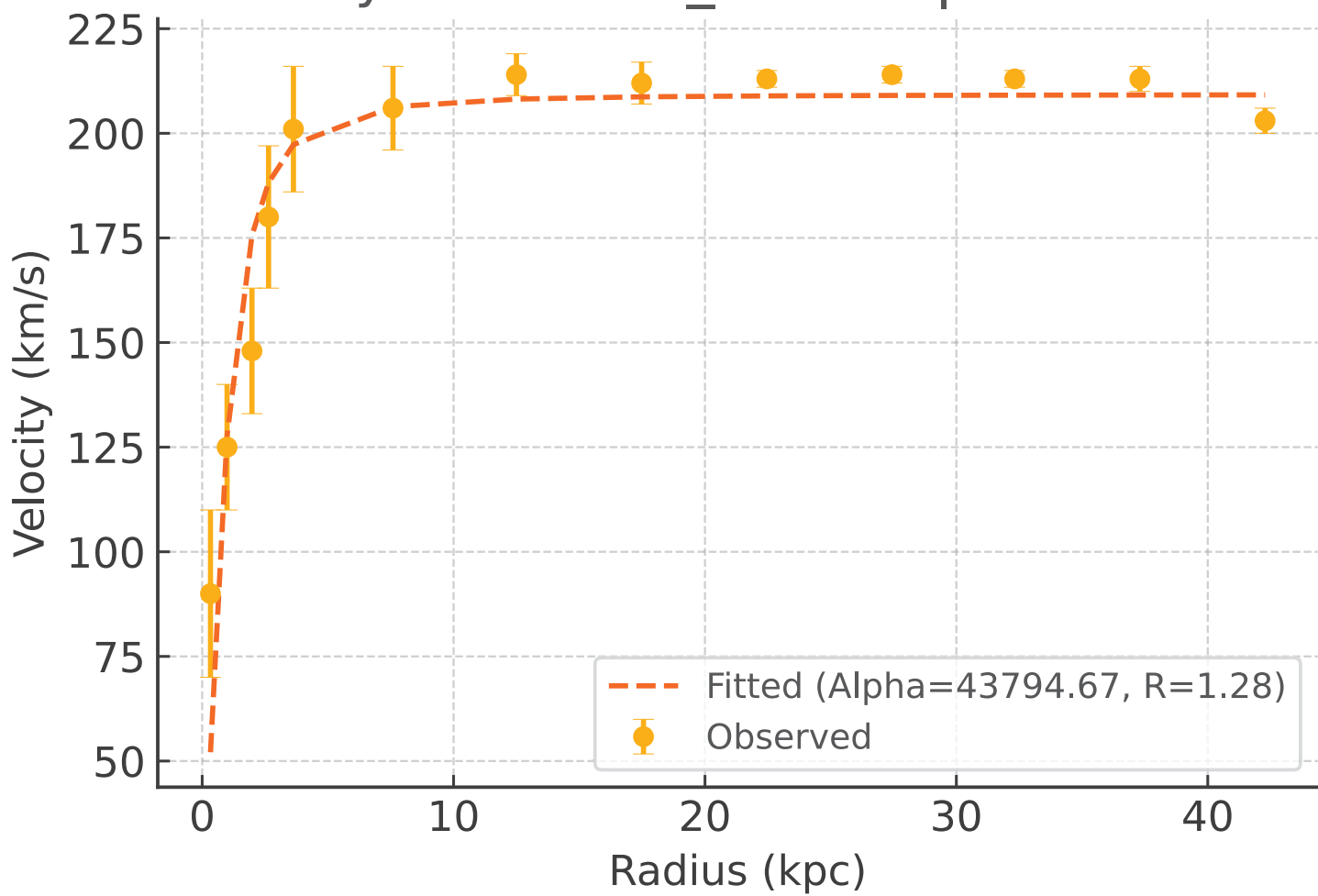
# Galaxy: NGC2955\_rotmod | $R^2 = 0.735$



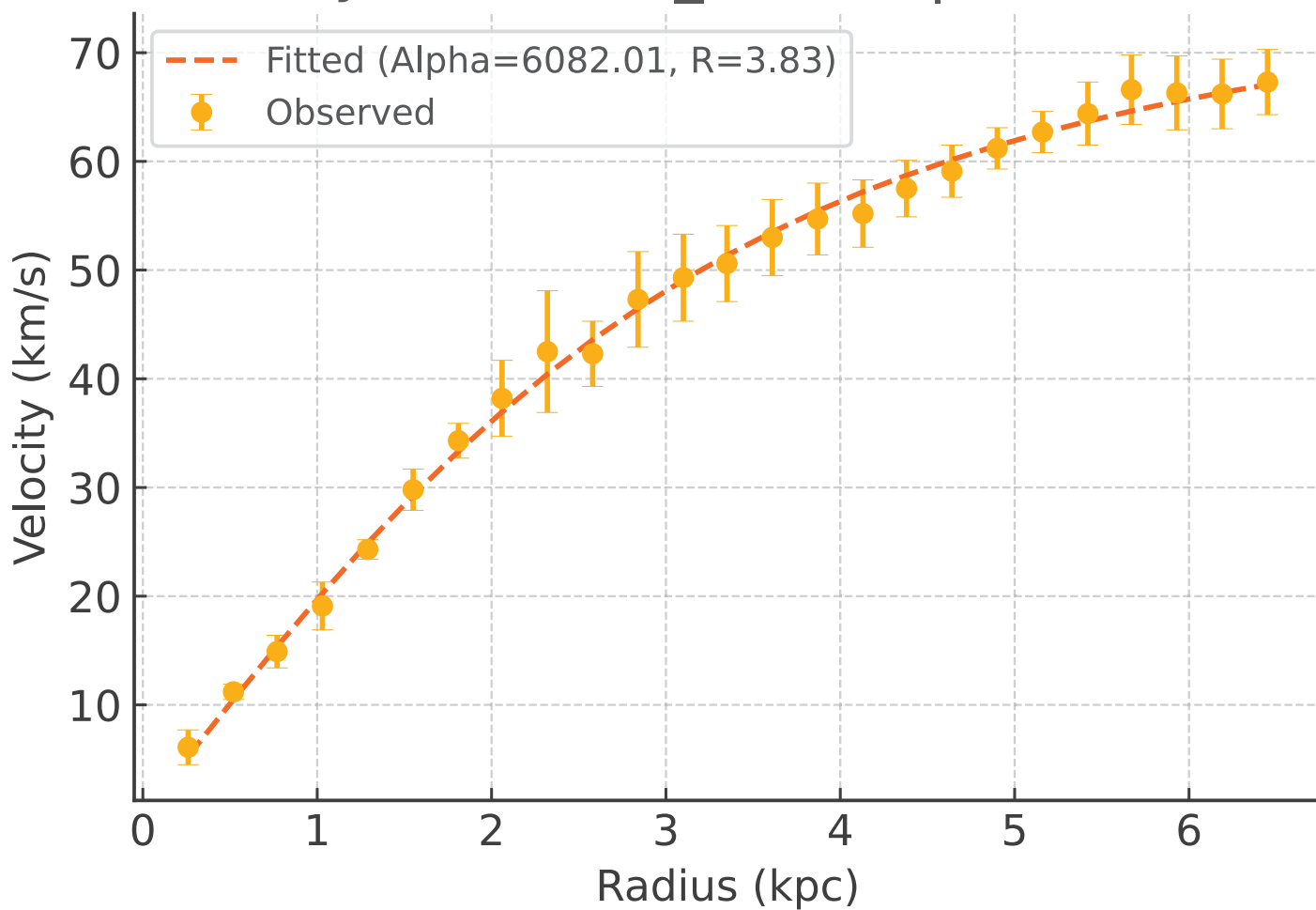
# Galaxy: NGC2976\_rotmod | $R^2 = 0.988$



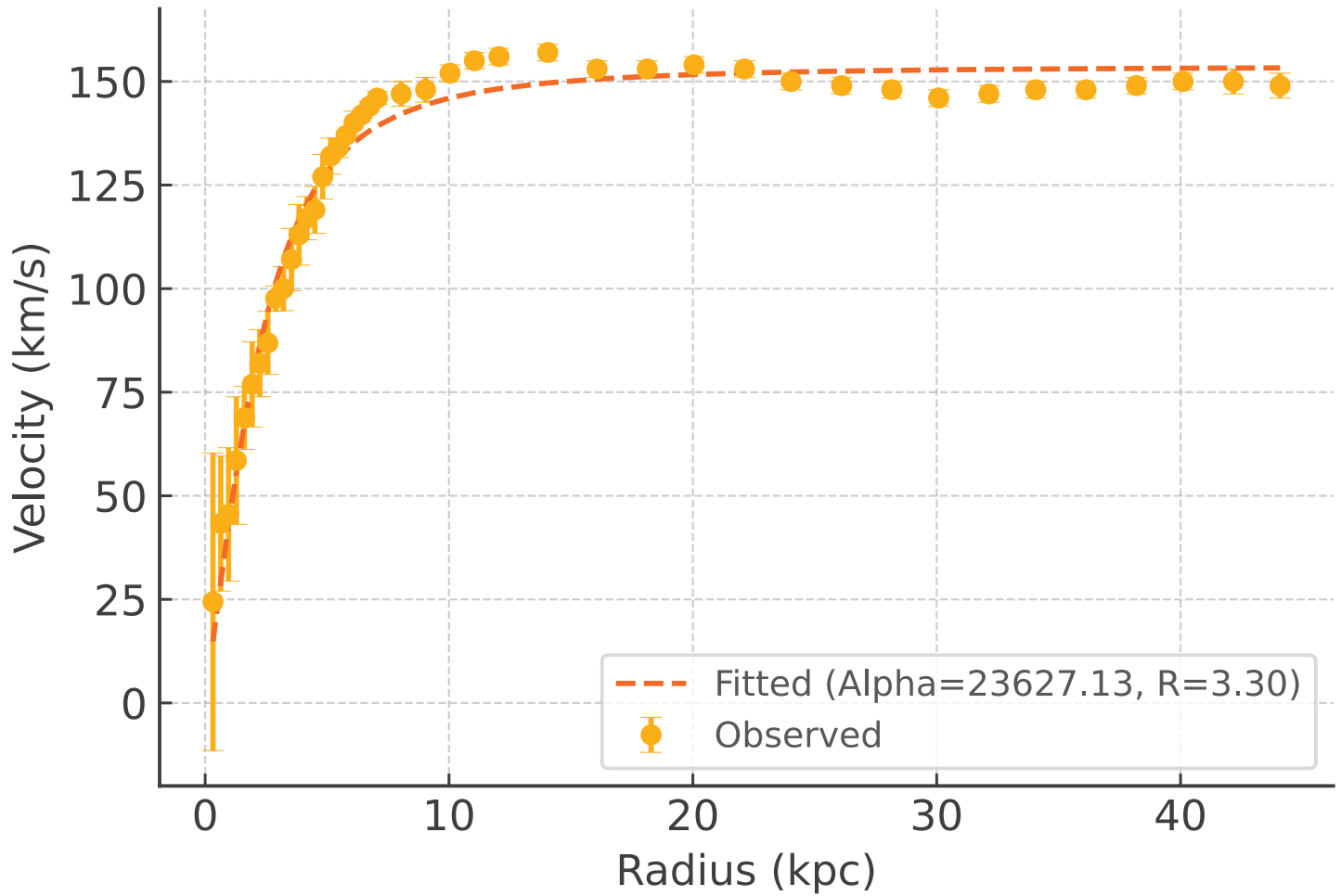
# Galaxy: NGC2998\_rotmod | $R^2 = 0.876$



# Galaxy: NGC3109\_rotmod | $R^2 = 0.997$

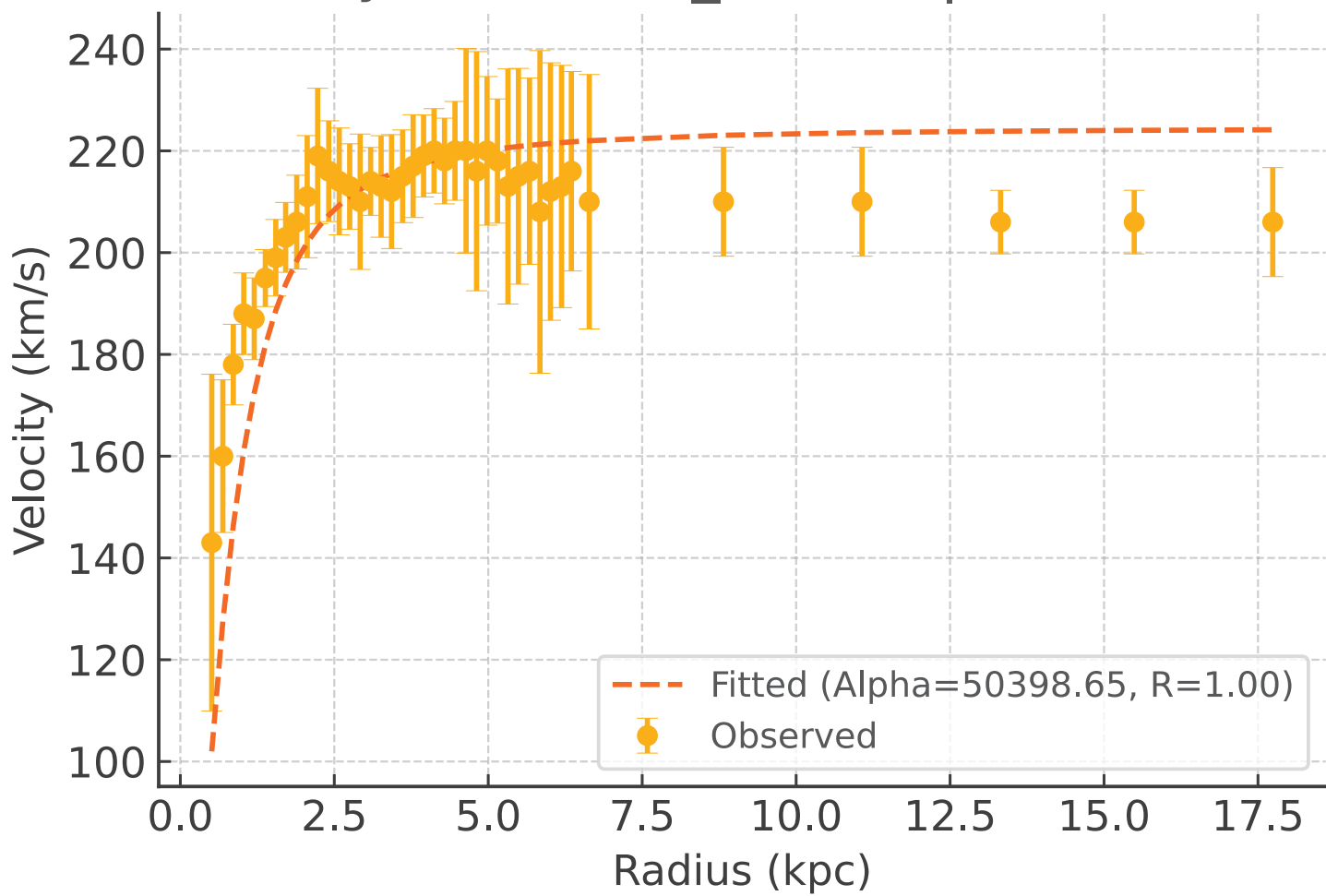


Galaxy: NGC3198\_rotmod |  $R^2 = 0.978$

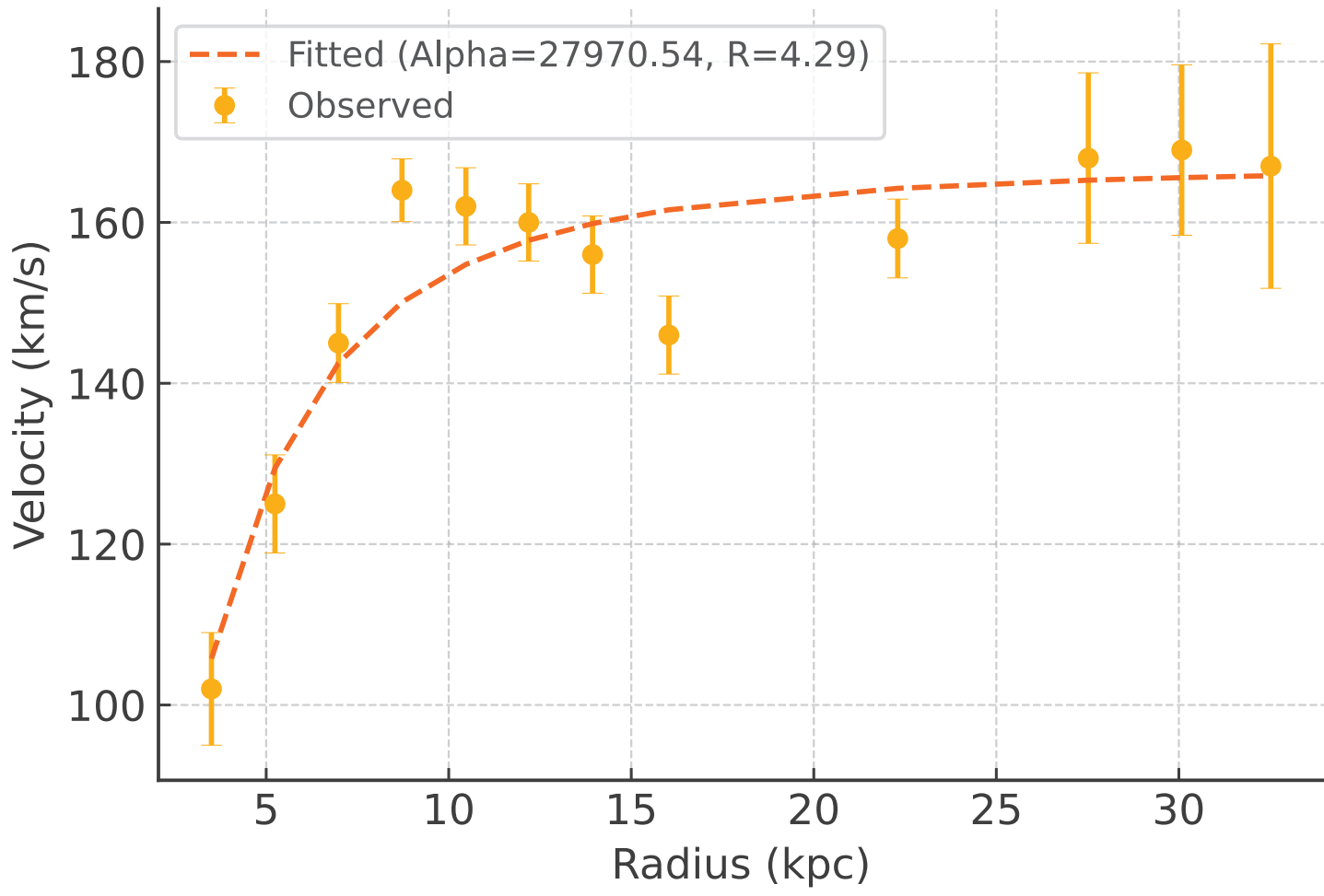




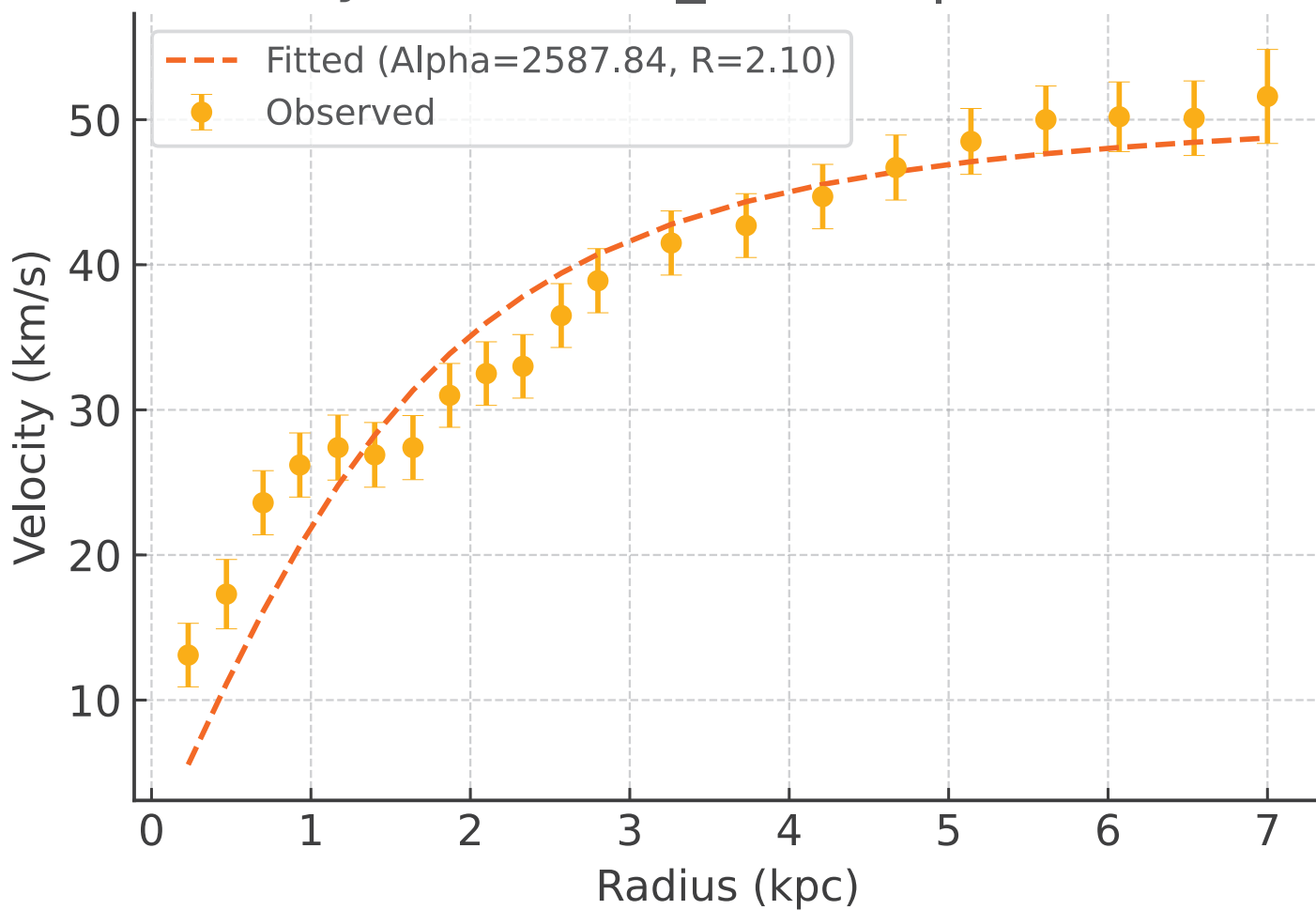
# Galaxy: NGC3521\_rotmod | $R^2 = 0.267$



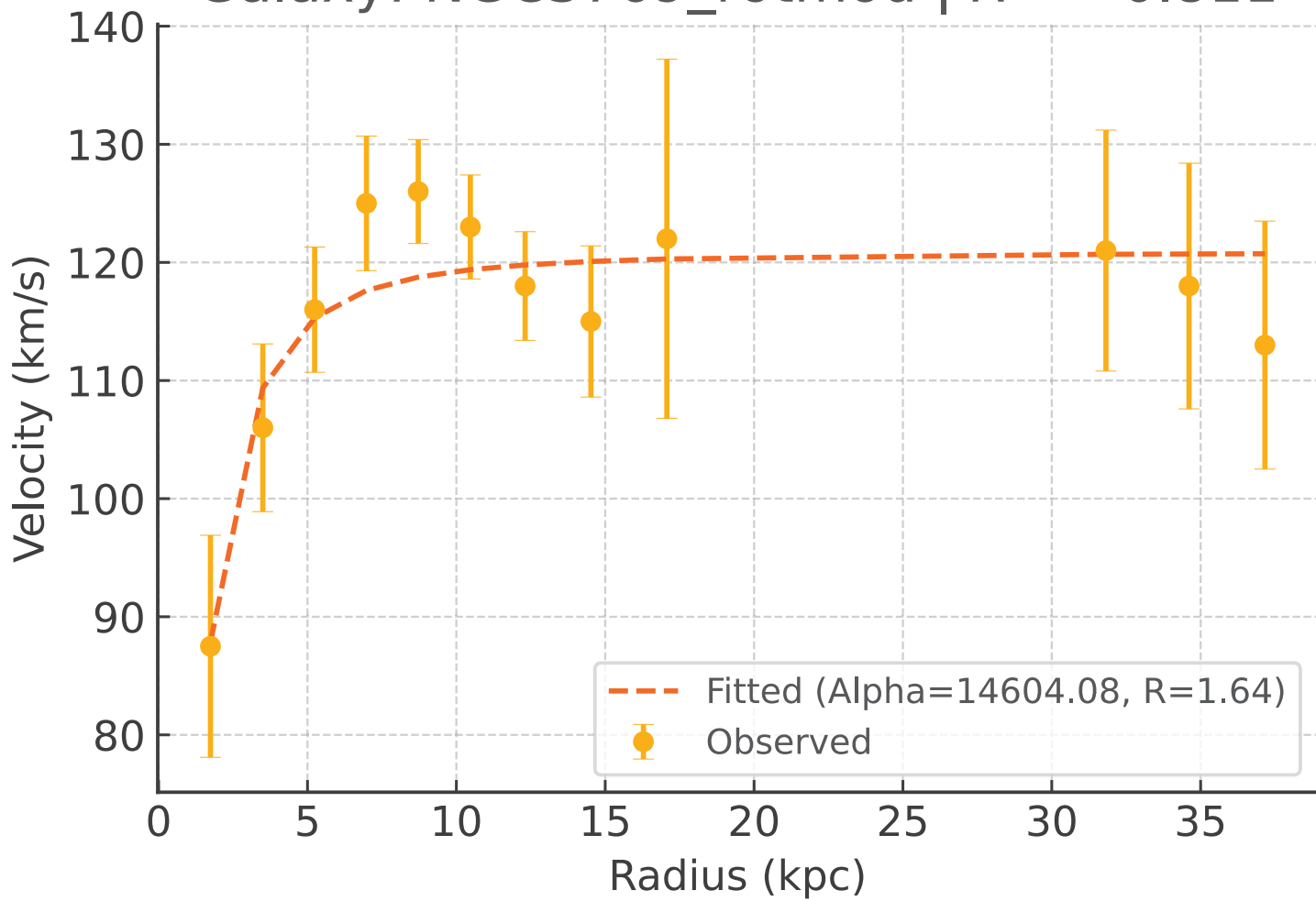
# Galaxy: NGC3726\_rotmod | $R^2 = 0.863$



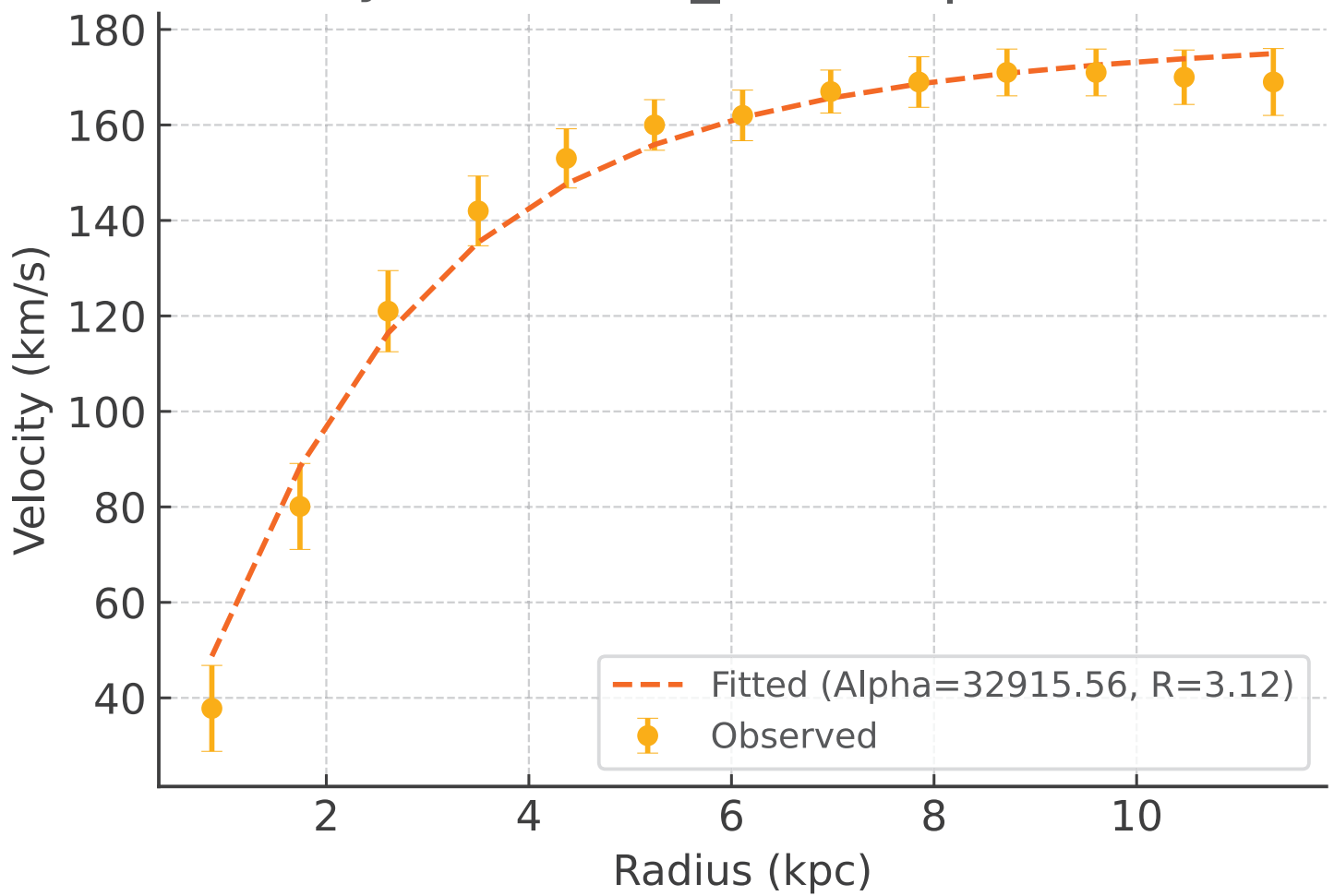
# Galaxy: NGC3741\_rotmod | $R^2 = 0.891$



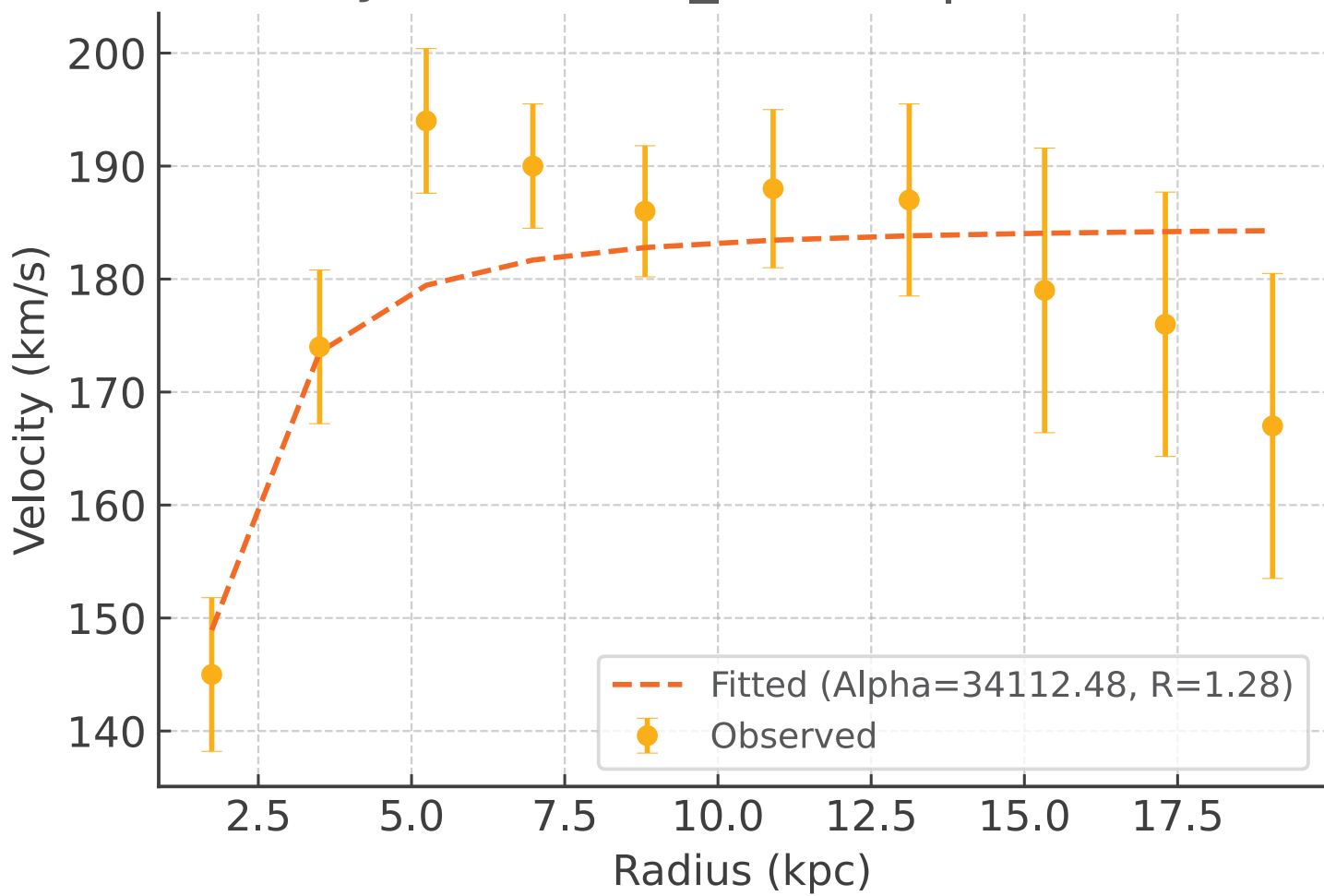
Galaxy: NGC3769\_rotmod |  $R^2 = 0.811$



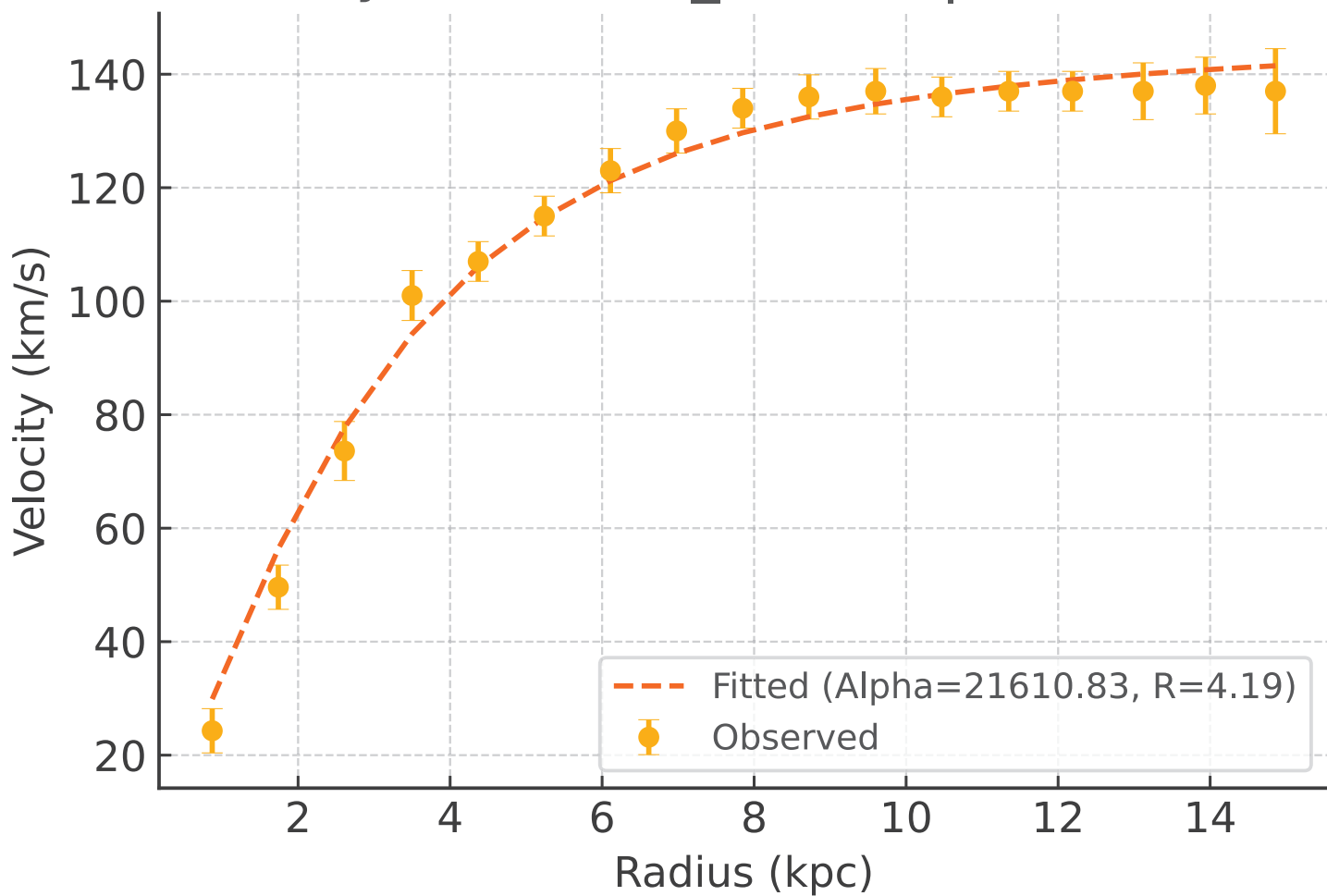
Galaxy: NGC3877\_rotmod |  $R^2 = 0.983$



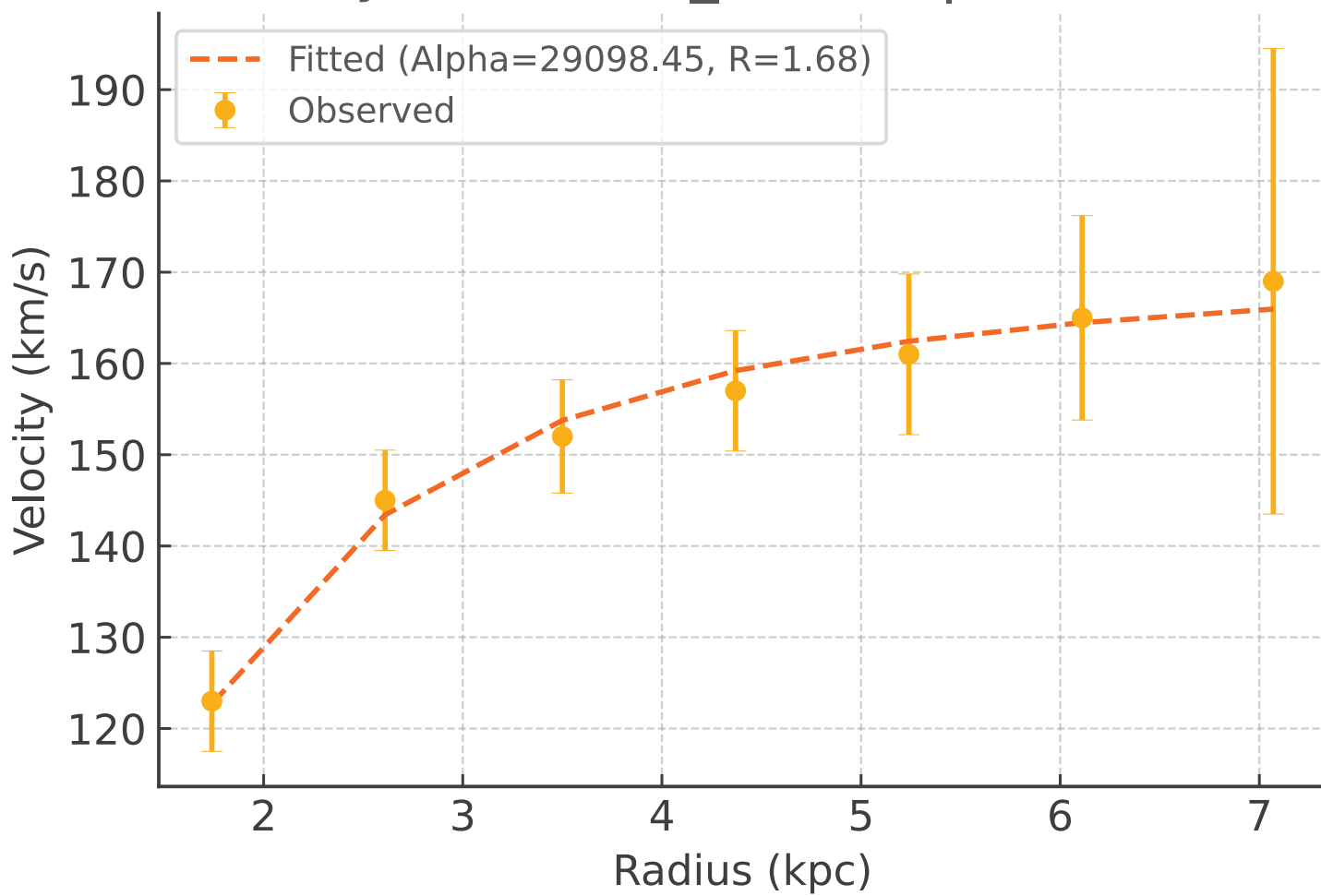
# Galaxy: NGC3893\_rotmod | $R^2 = 0.611$



Galaxy: NGC3917\_rotmod |  $R^2 = 0.987$

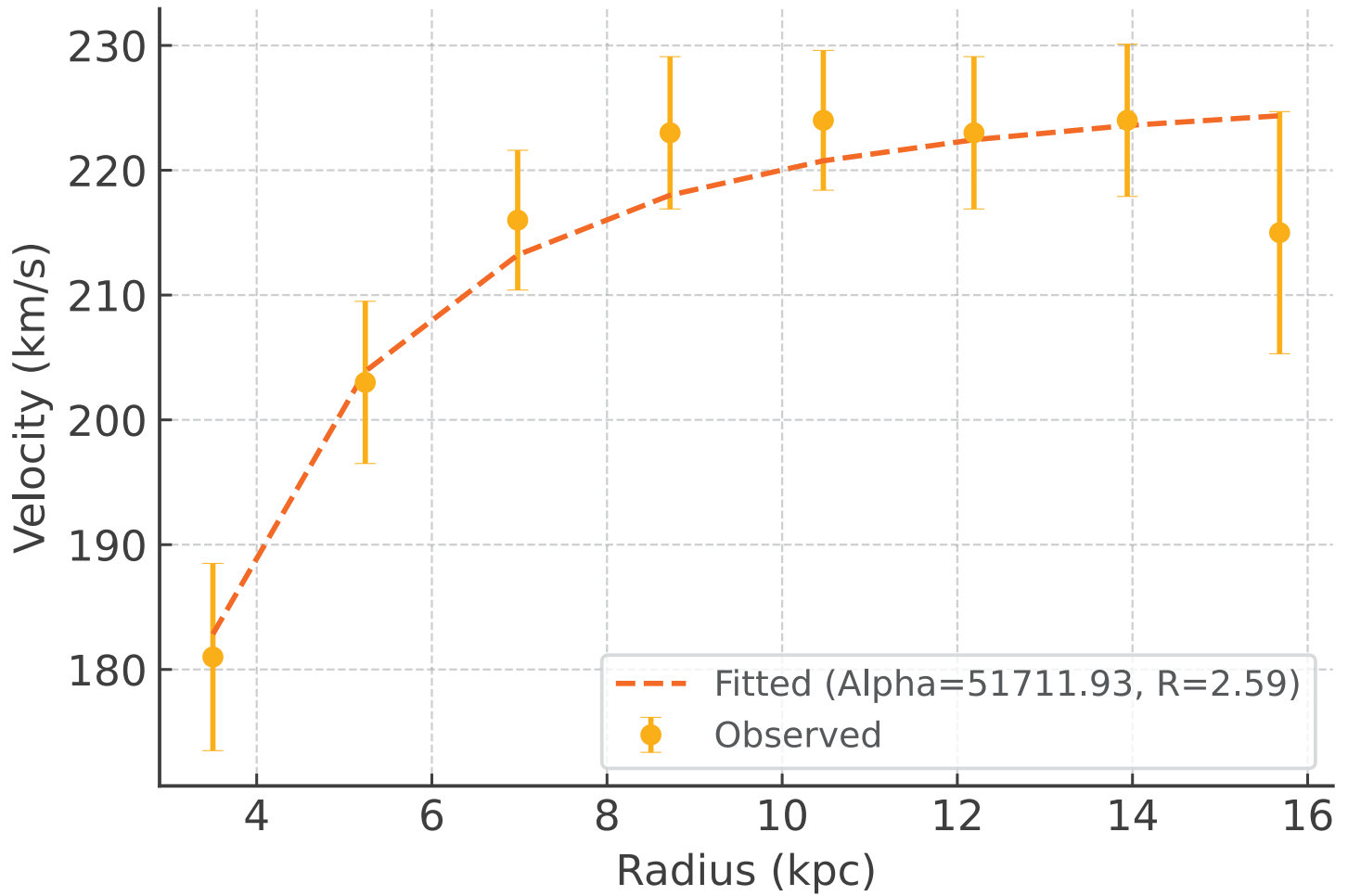


# Galaxy: NGC3949\_rotmod | $R^2 = 0.985$

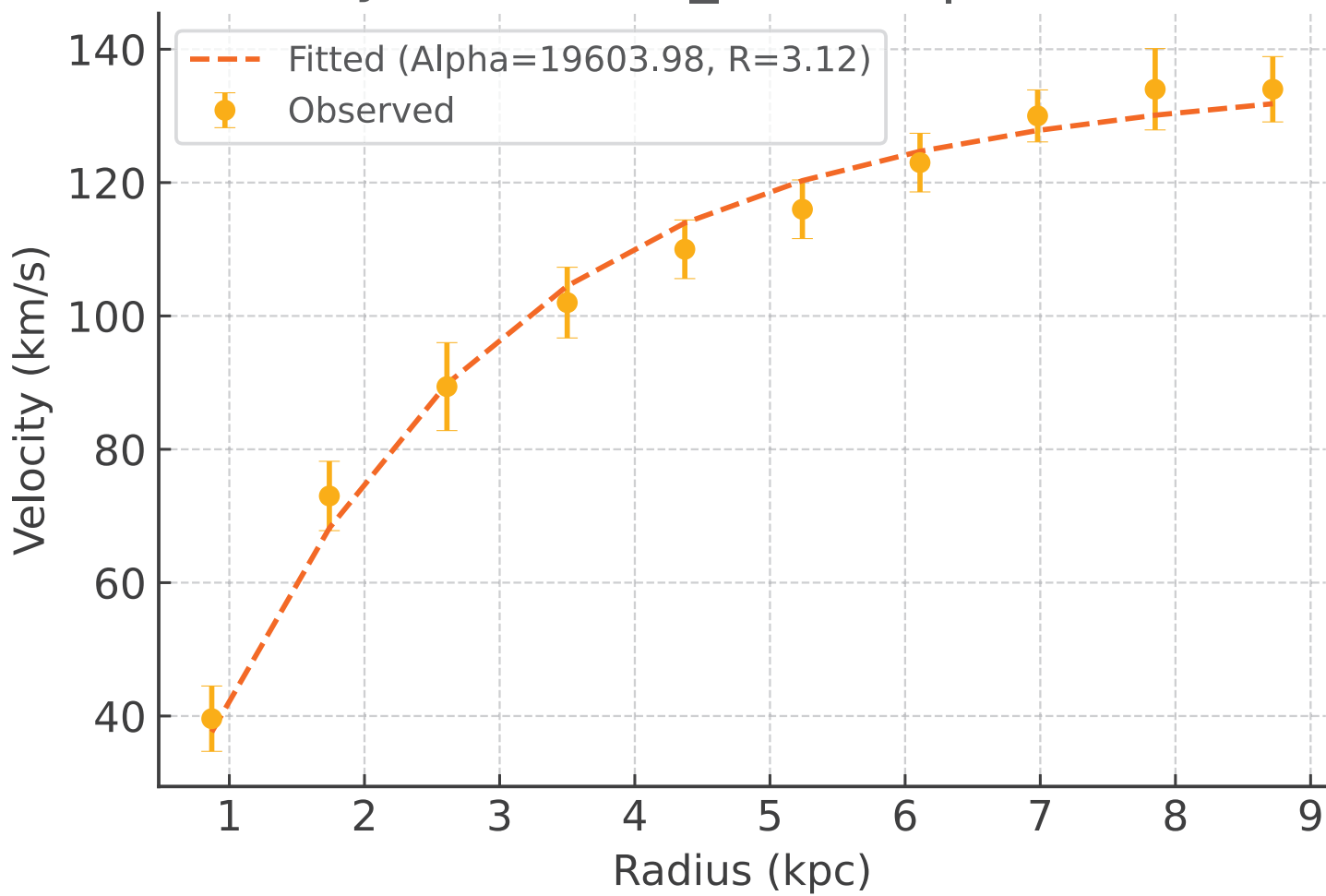




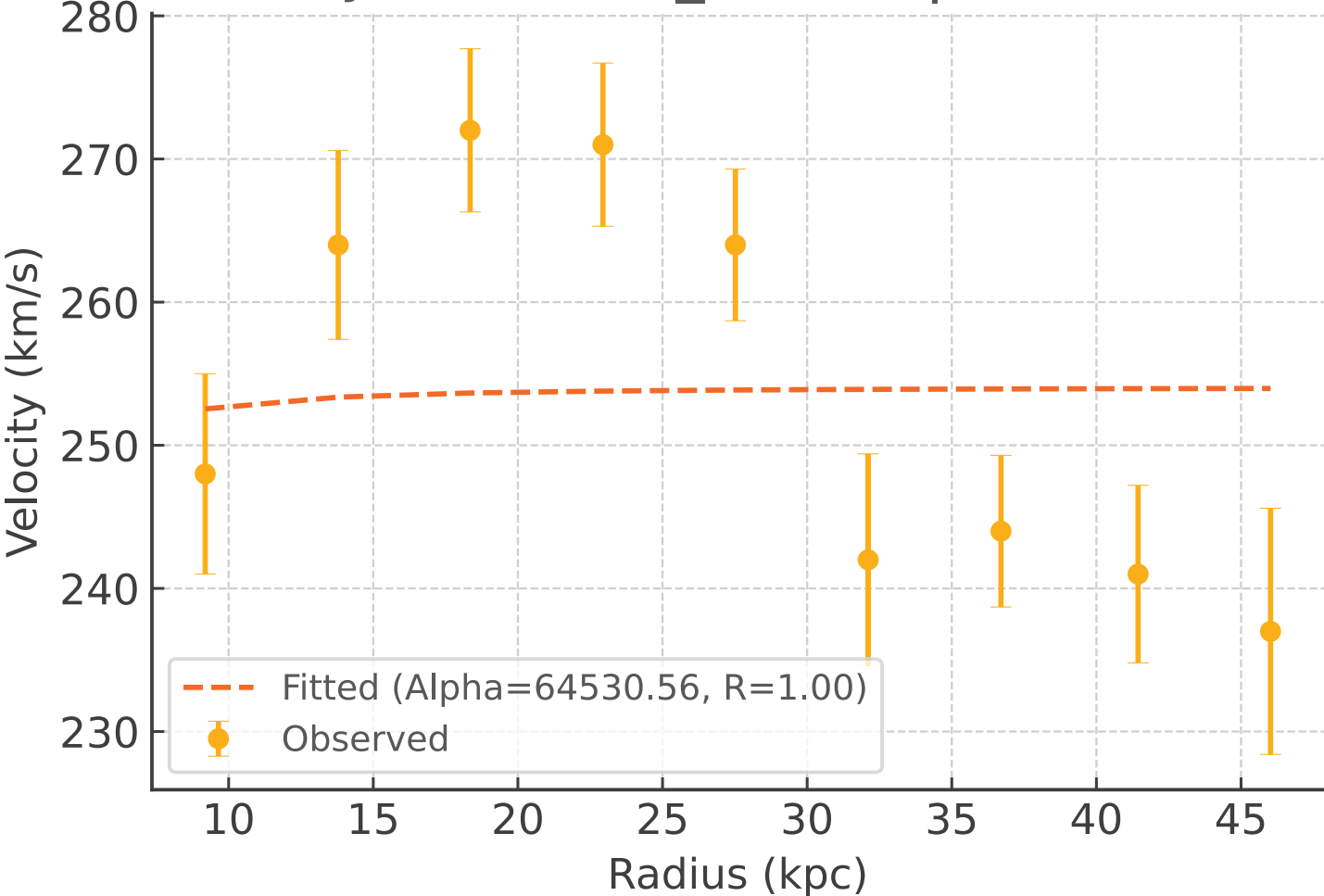
Galaxy: NGC3953\_rotmod |  $R^2 = 0.914$



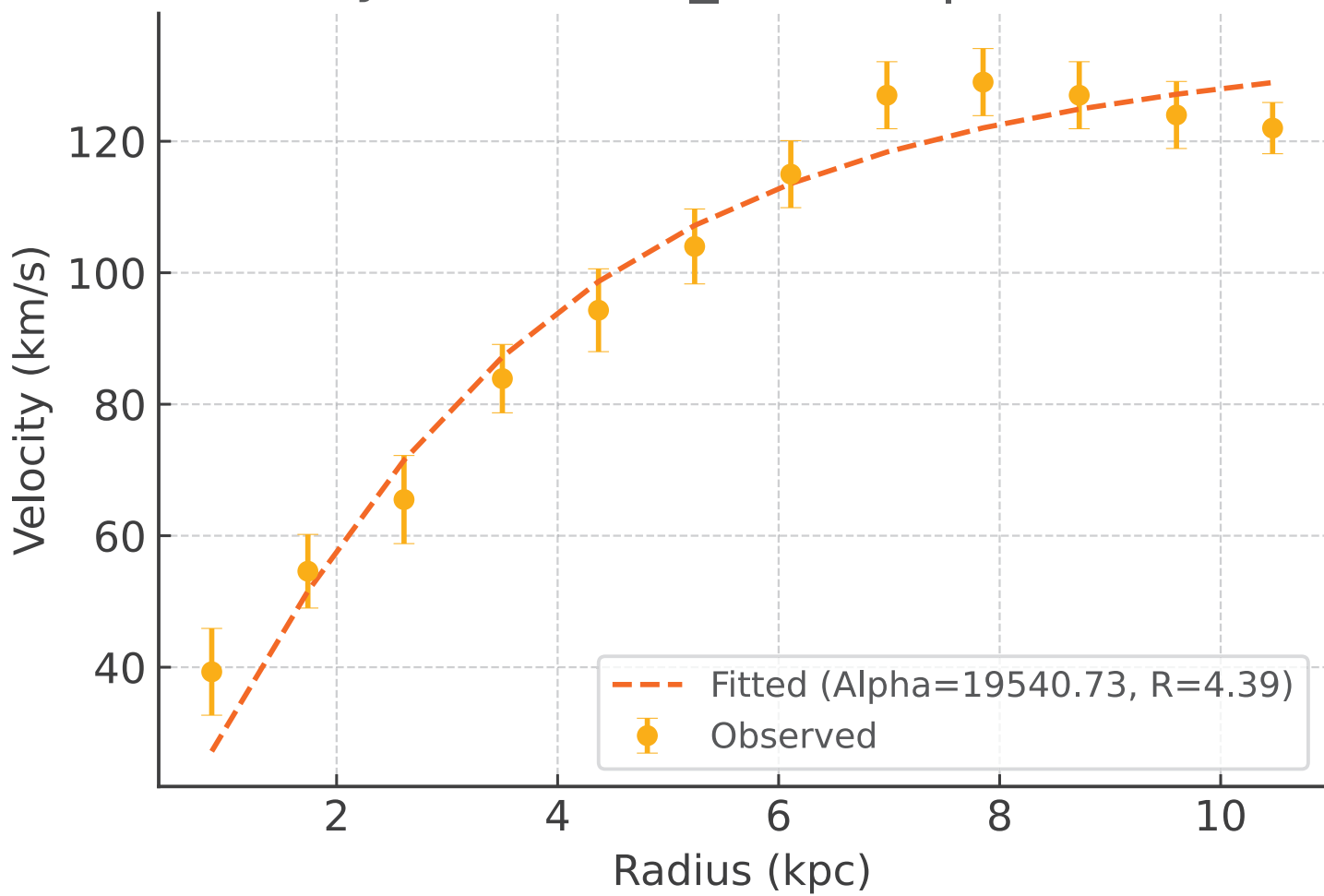
# Galaxy: NGC3972\_rotmod | $R^2 = 0.989$



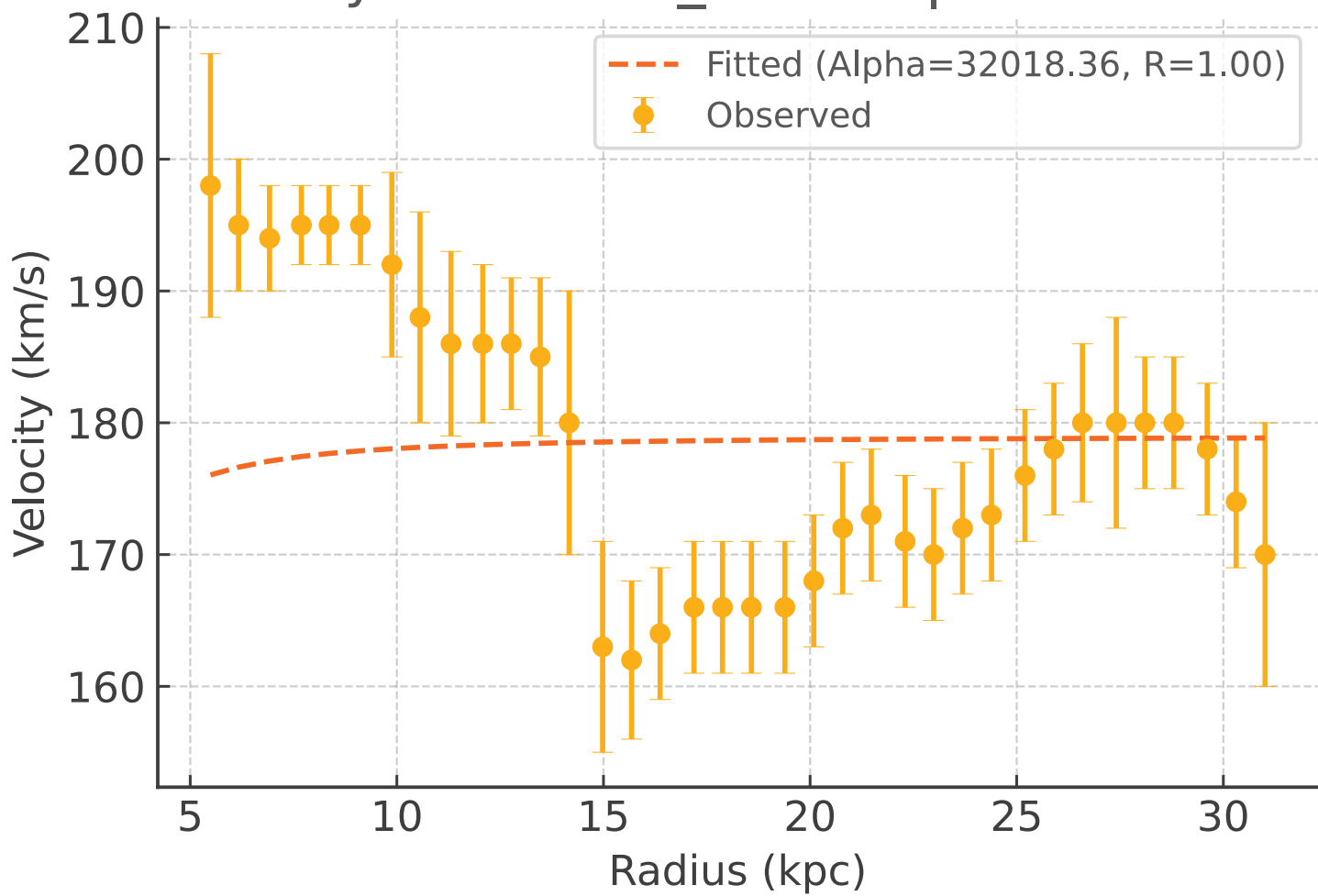
Galaxy: NGC3992\_rotmod |  $R^2 = -0.010$



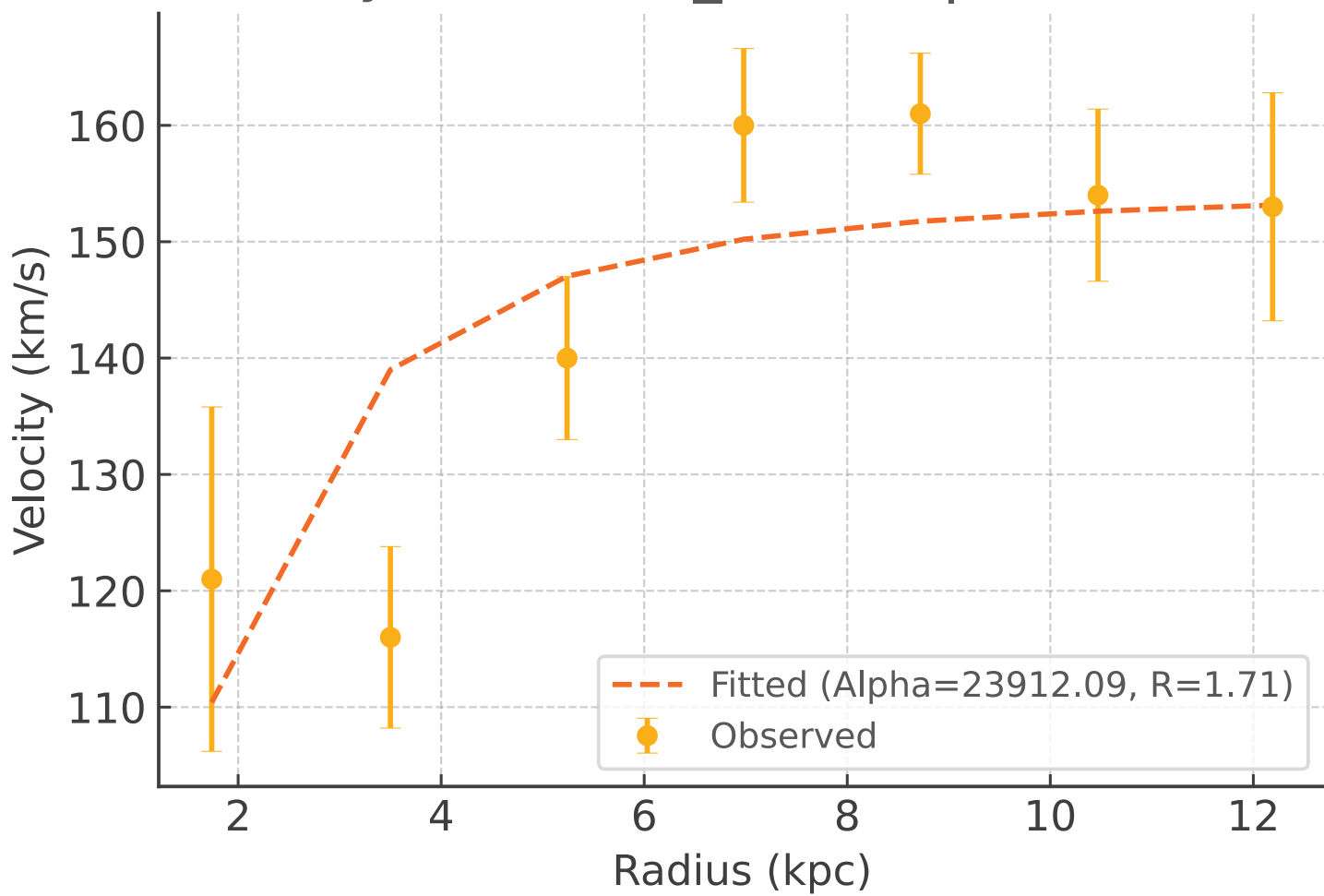
Galaxy: NGC4010\_rotmod |  $R^2 = 0.961$



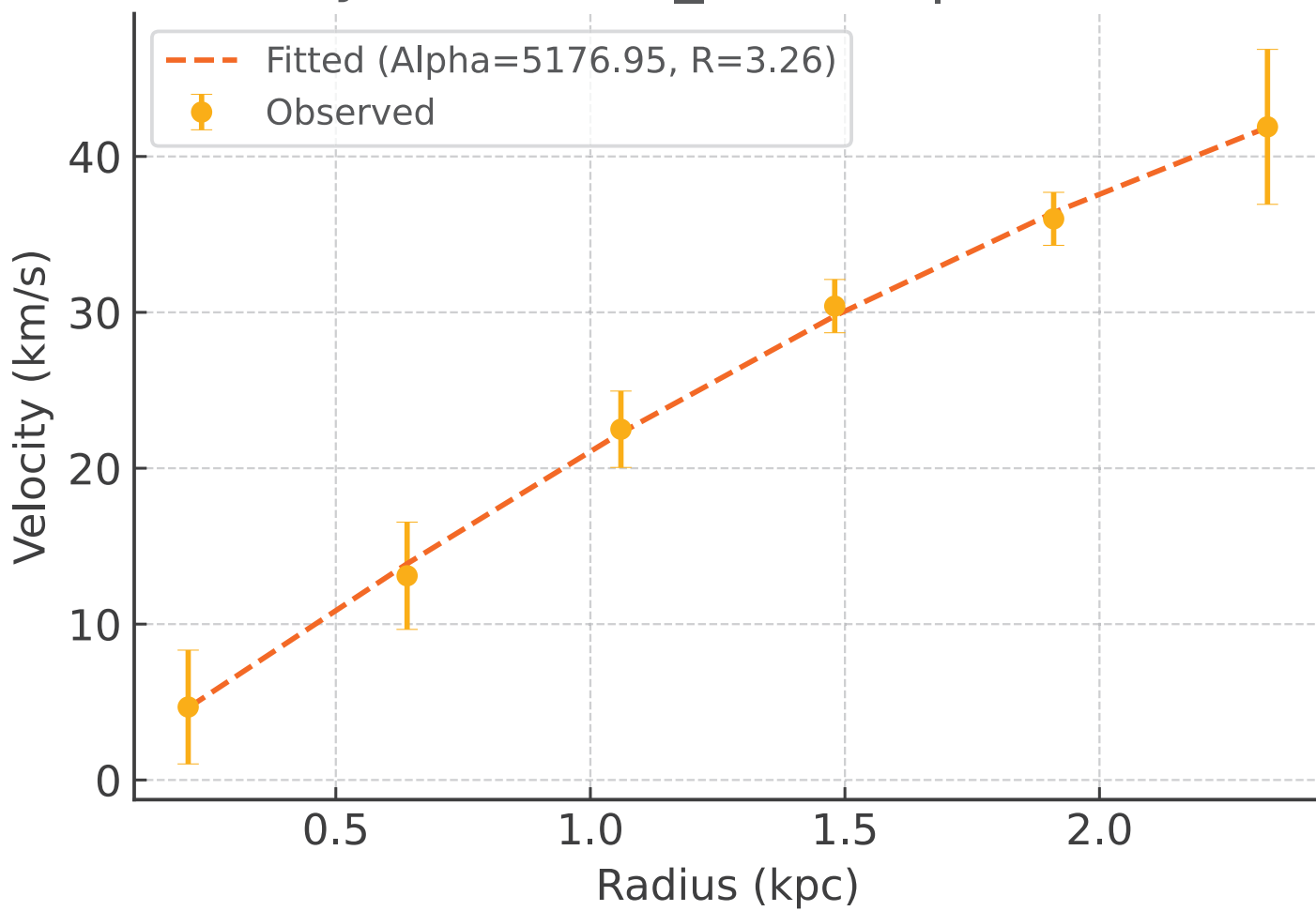
# Galaxy: NGC4013\_rotmod | $R^2 = -0.095$



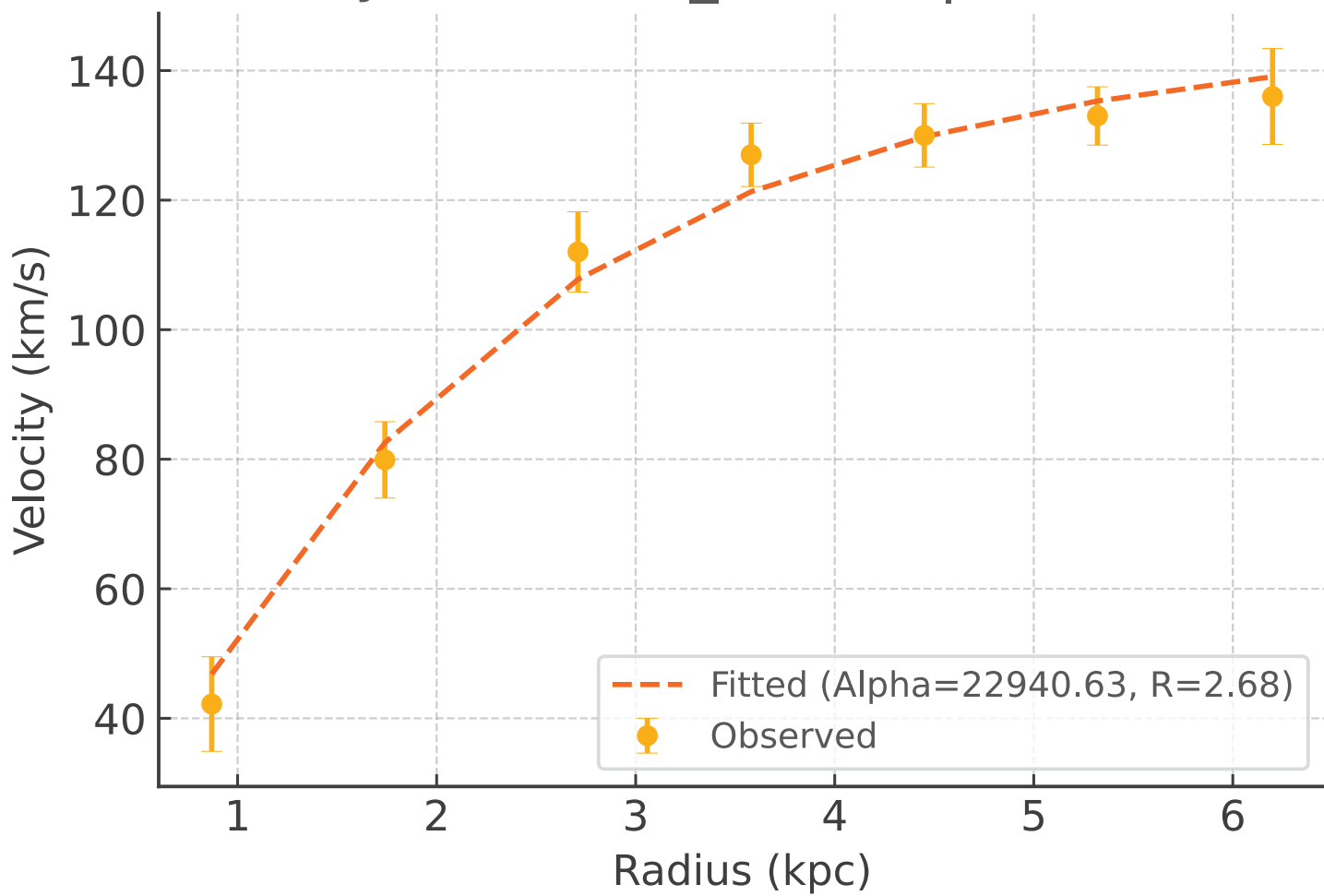
# Galaxy: NGC4051\_rotmod | $R^2 = 0.575$



# Galaxy: NGC4068\_rotmod | $R^2 = 0.999$

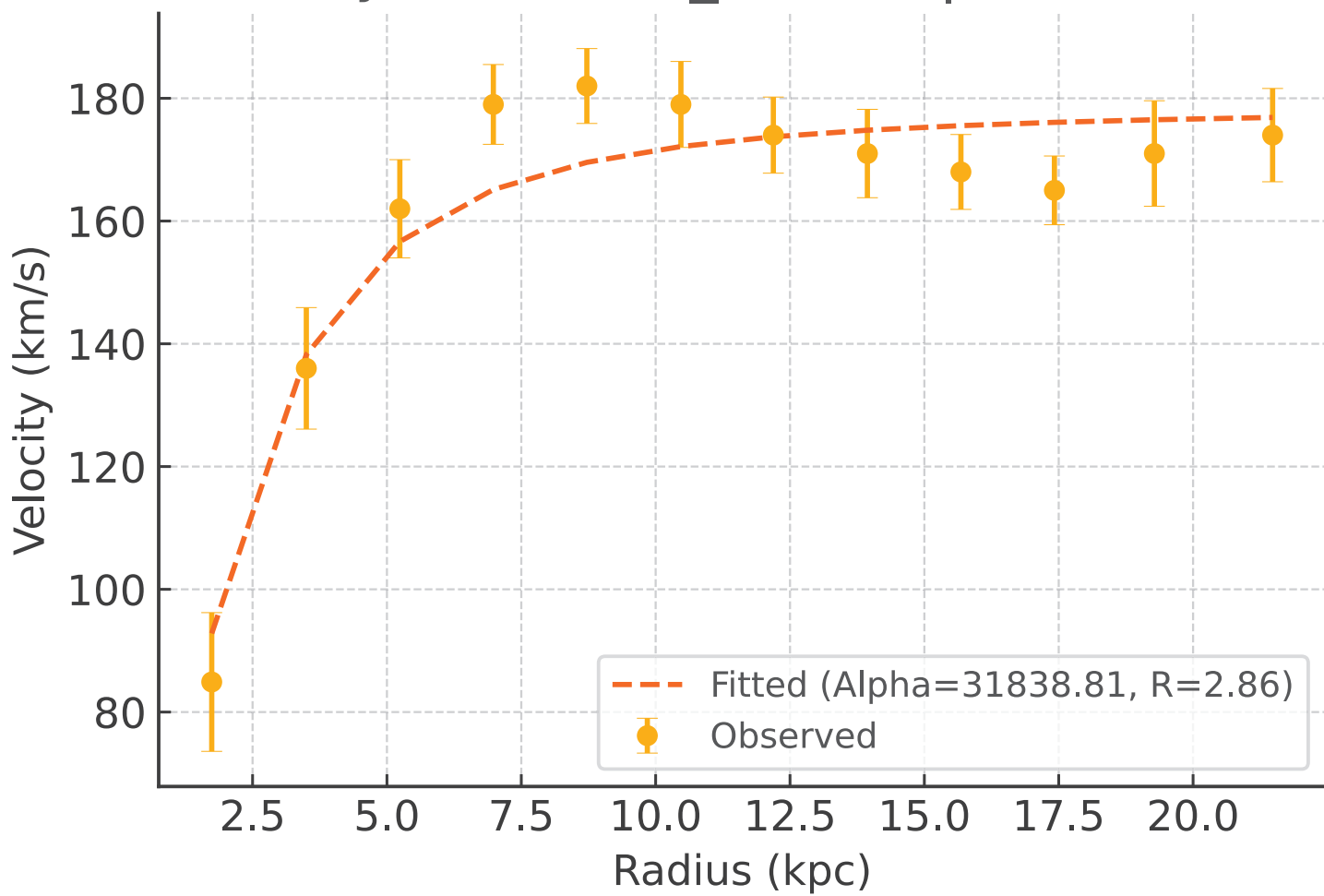


Galaxy: NGC4085\_rotmod |  $R^2 = 0.987$

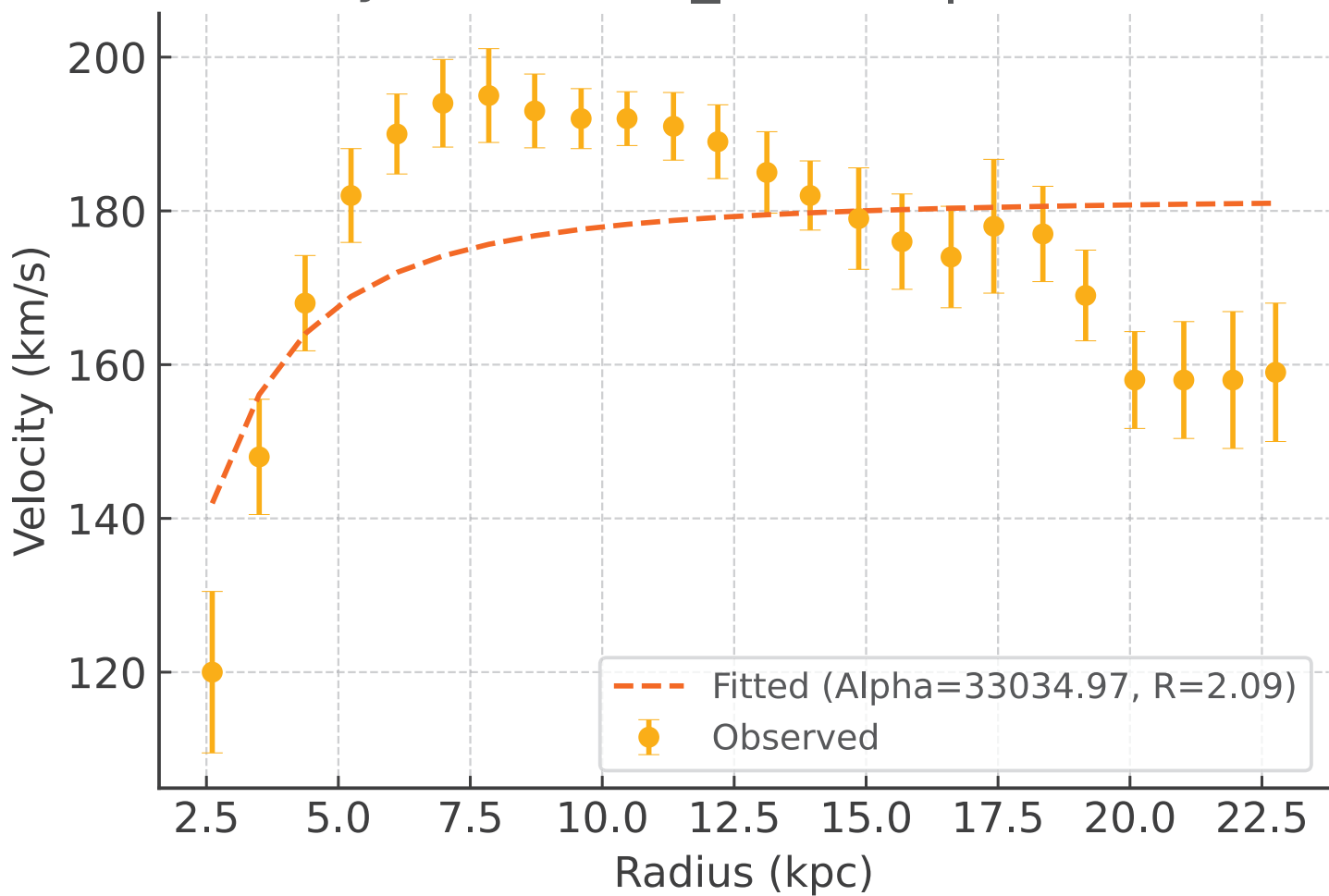




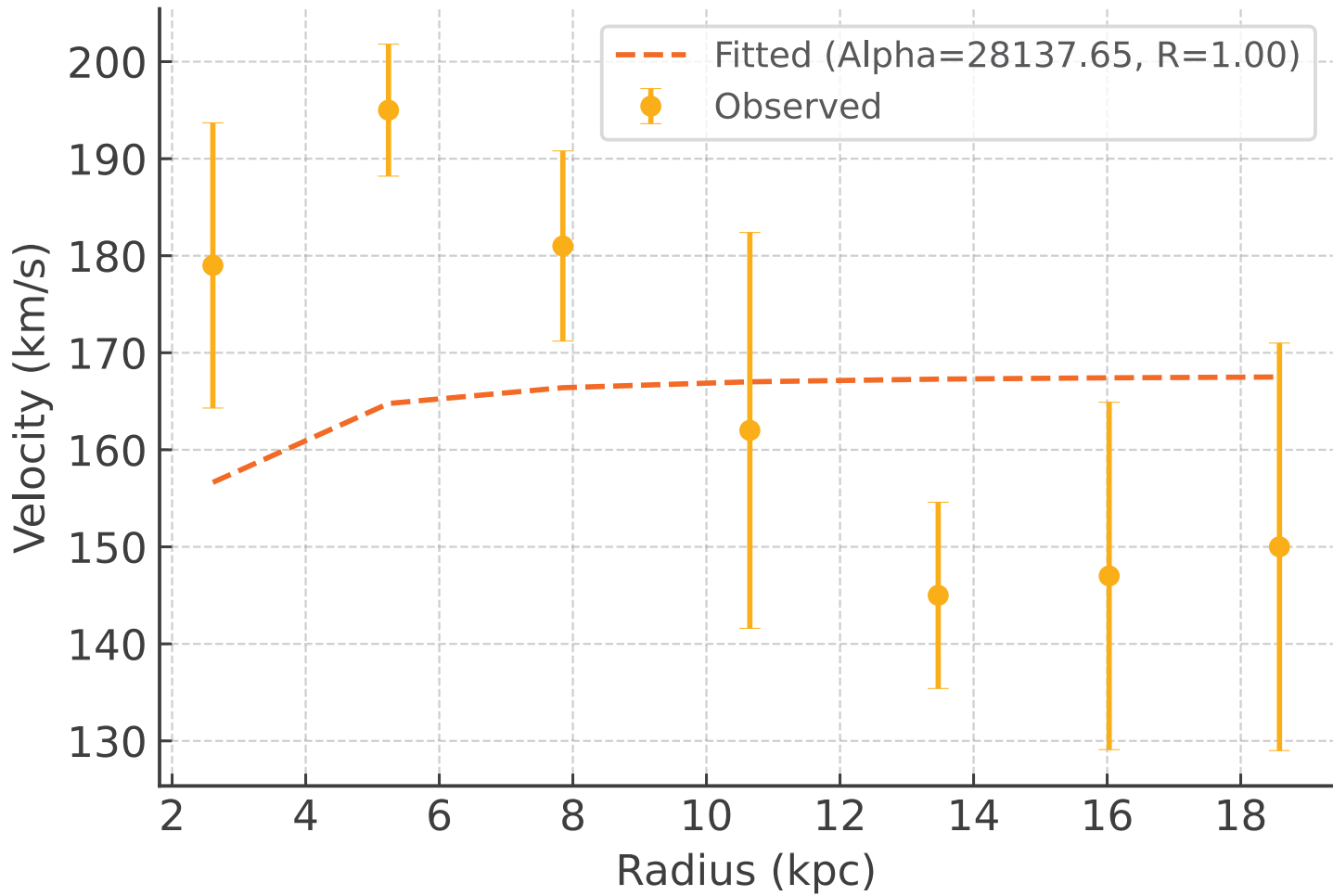
# Galaxy: NGC4088\_rotmod | $R^2 = 0.911$



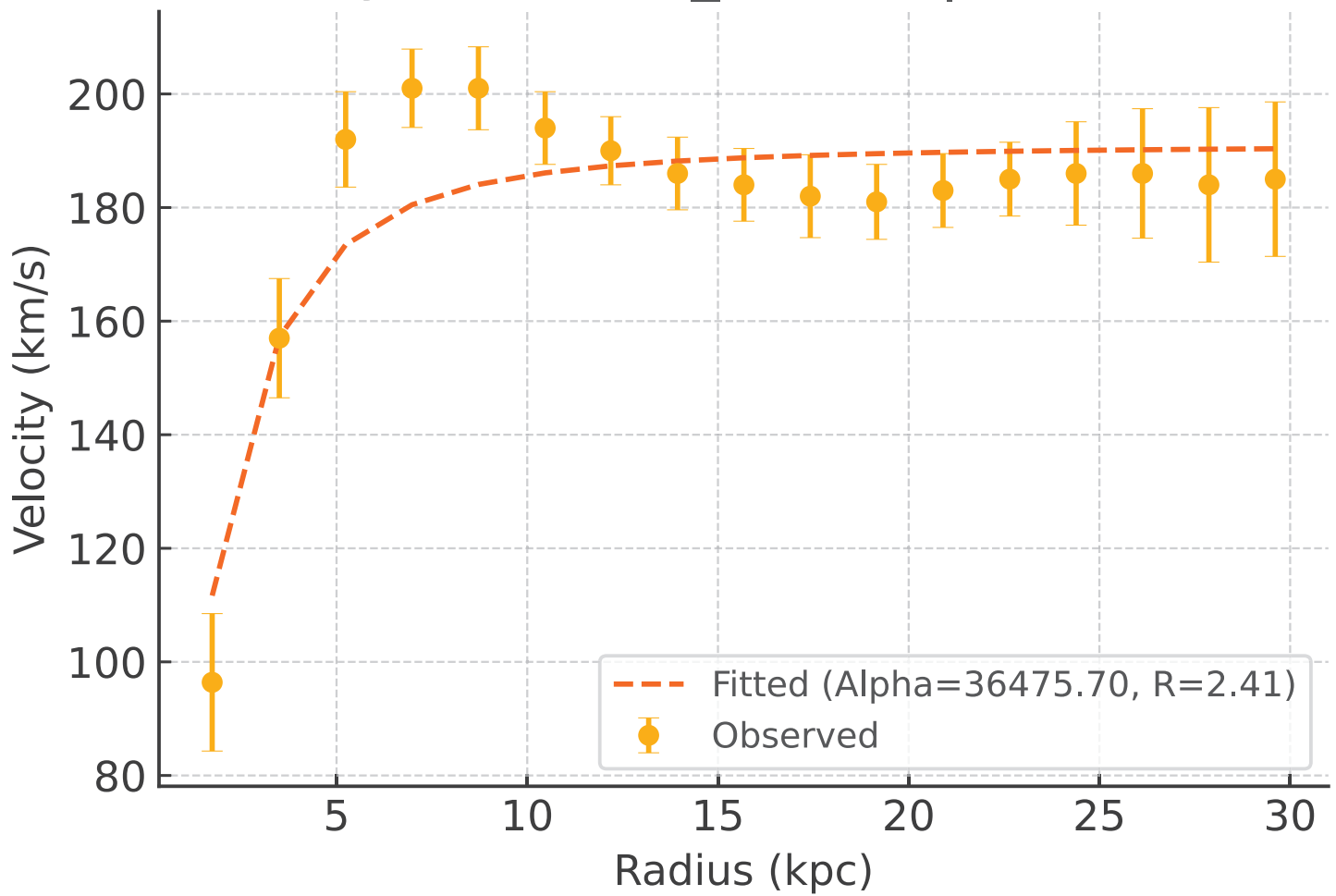
# Galaxy: NGC4100\_rotmod | $R^2 = 0.331$



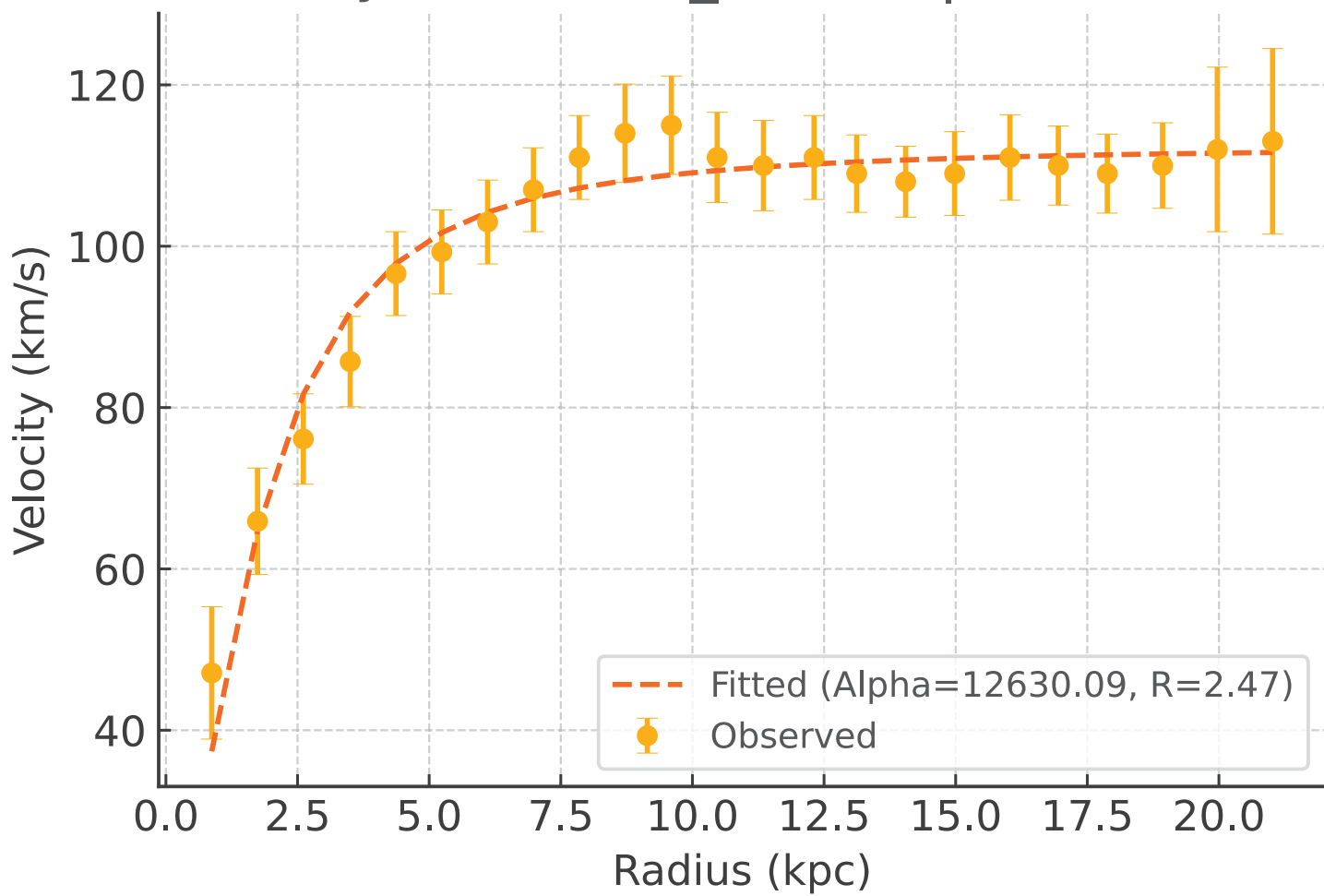
# Galaxy: NGC4138\_rotmod | $R^2 = -0.244$



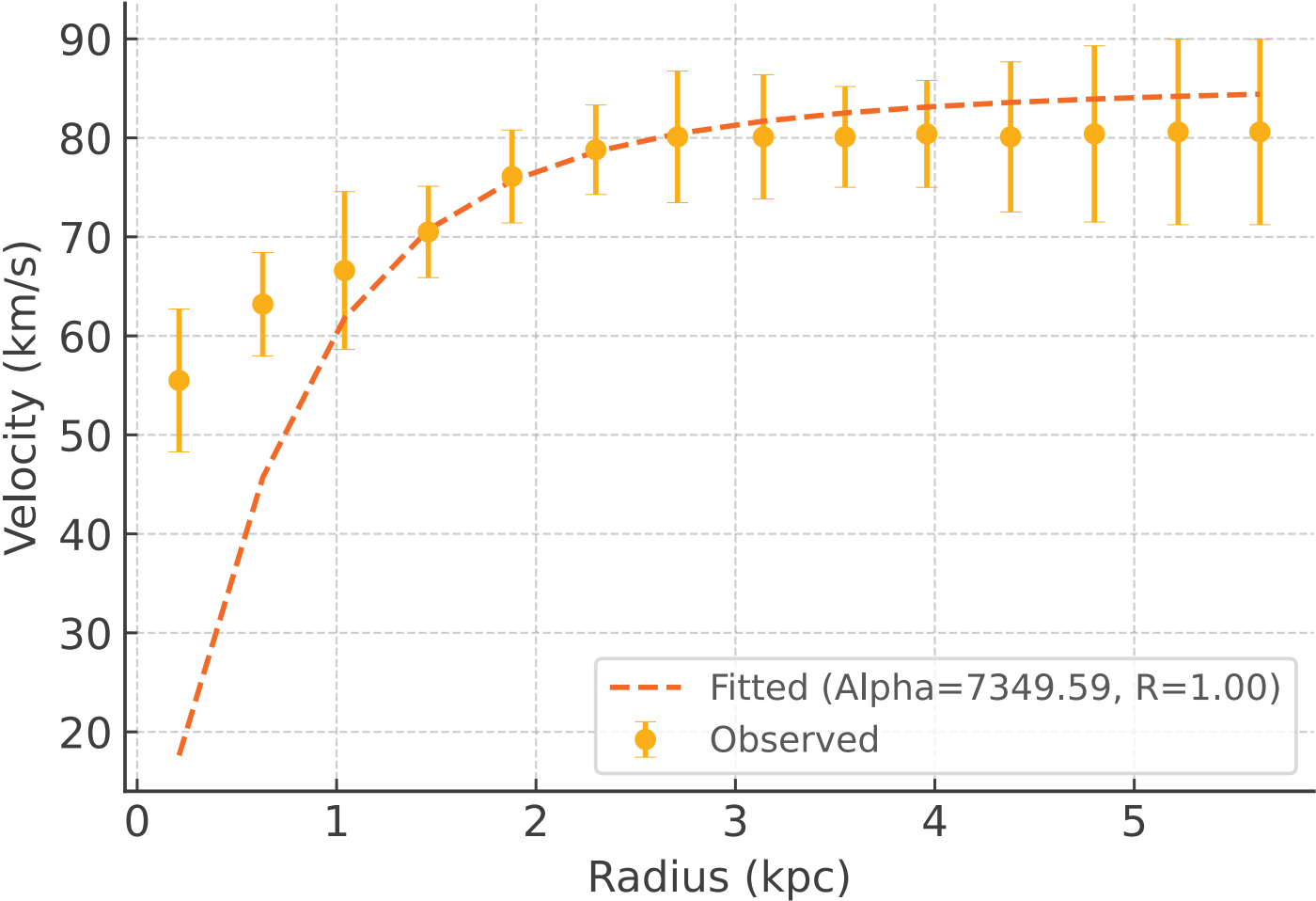
Galaxy: NGC4157\_rotmod |  $R^2 = 0.814$



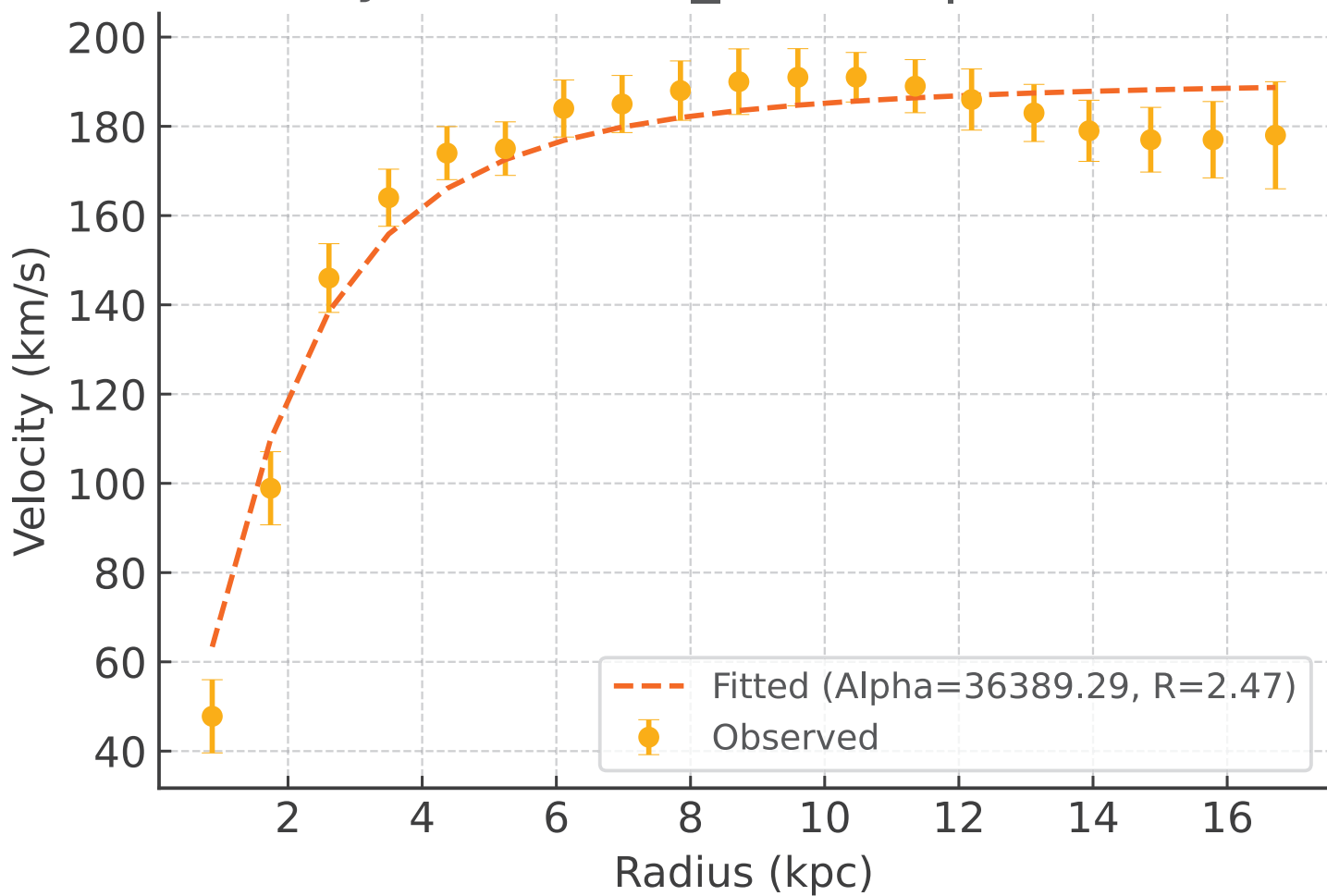
Galaxy: NGC4183\_rotmod |  $R^2 = 0.956$



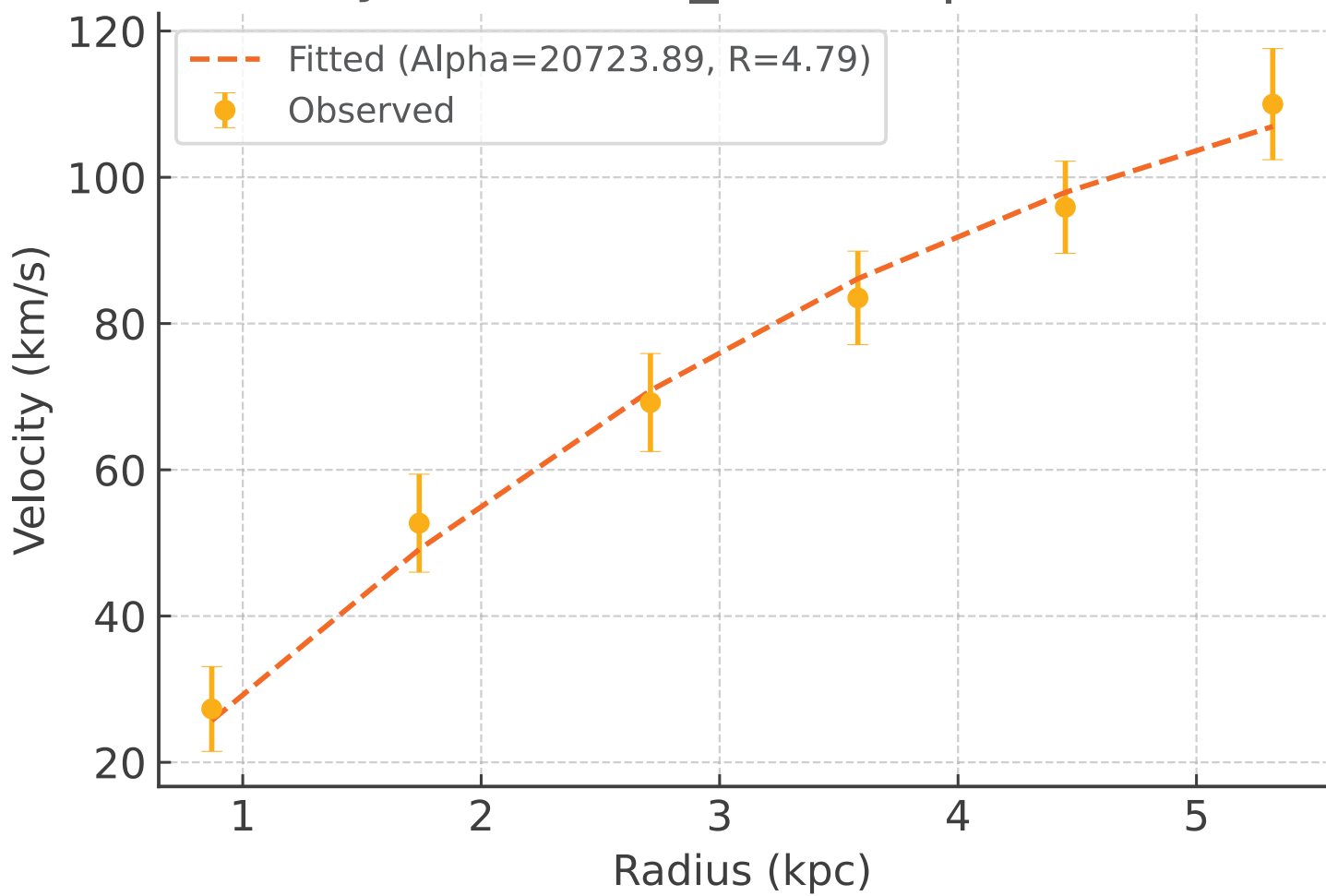
Galaxy: NGC4214\_rotmod |  $R^2 = -1.155$



# Galaxy: NGC4217\_rotmod | $R^2 = 0.947$

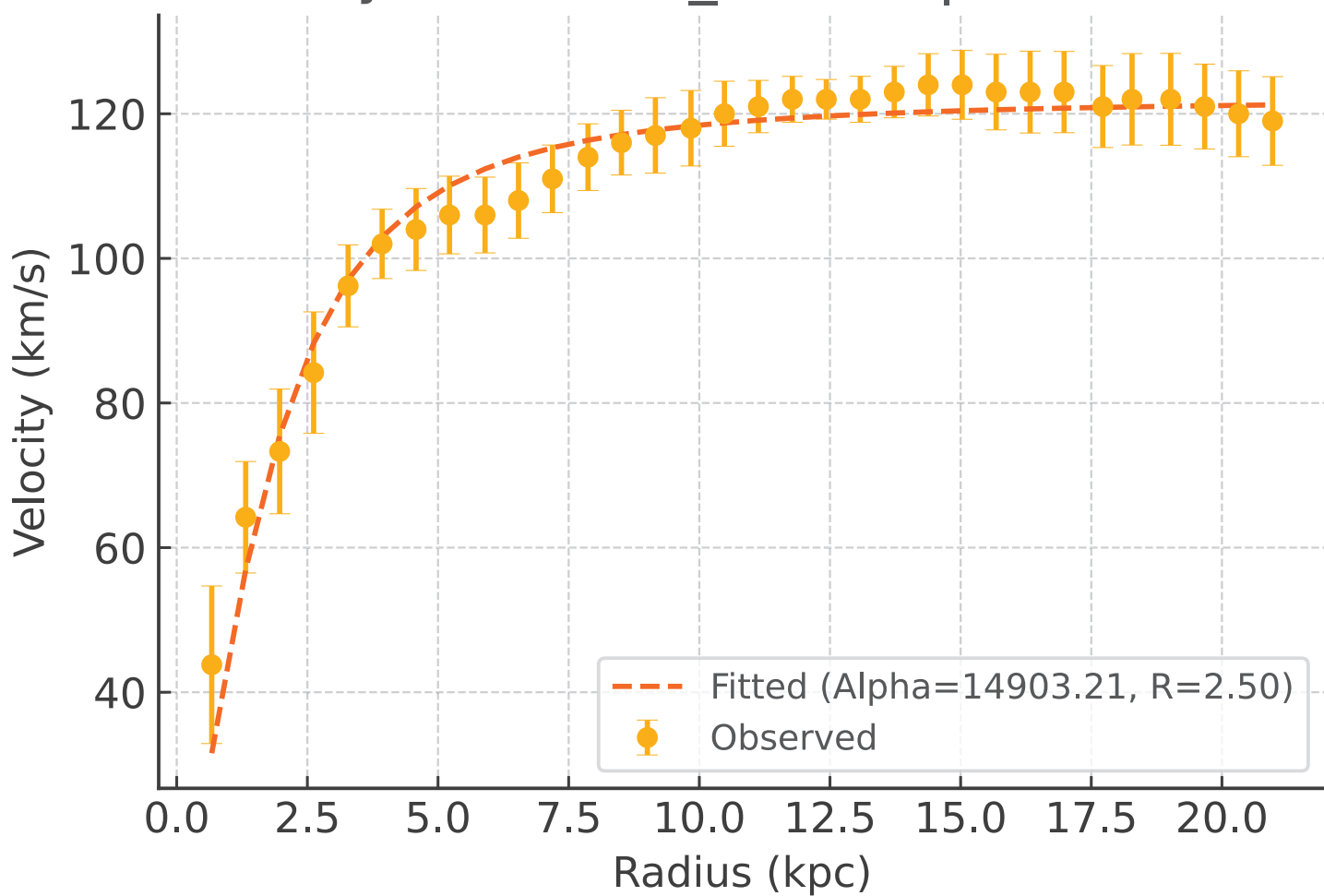


# Galaxy: NGC4389\_rotmod | $R^2 = 0.991$

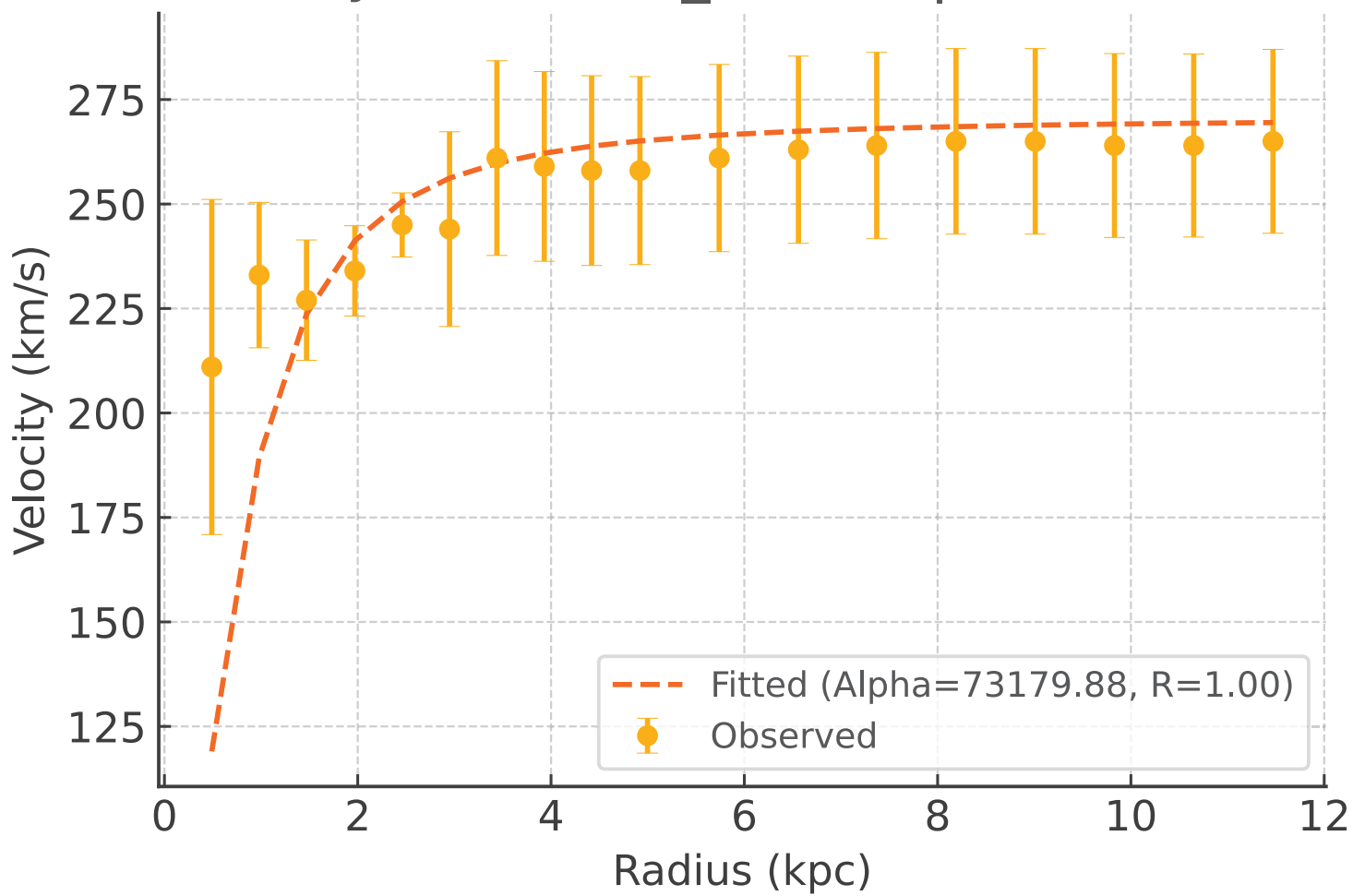




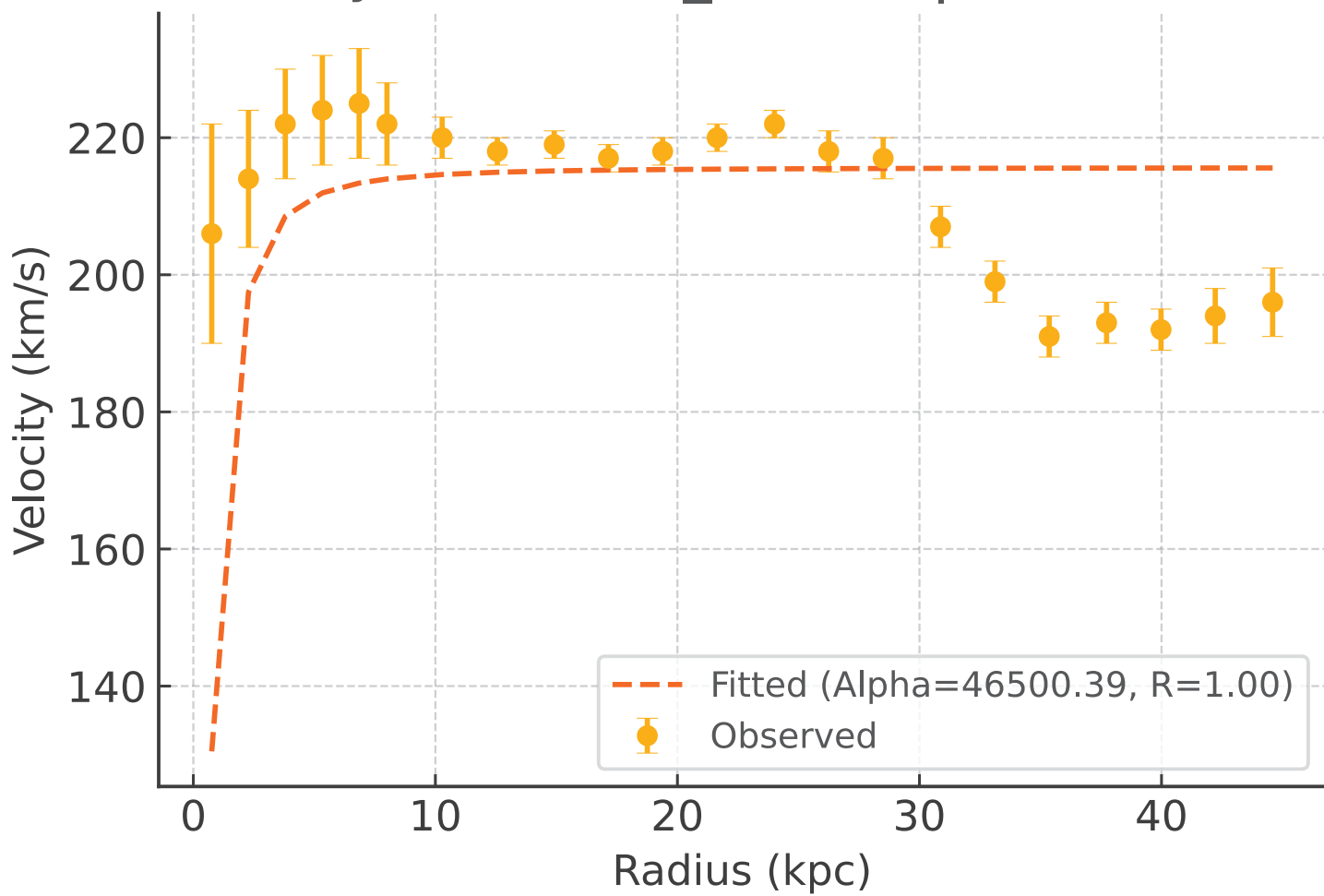
# Galaxy: NGC4559\_rotmod | $R^2 = 0.962$



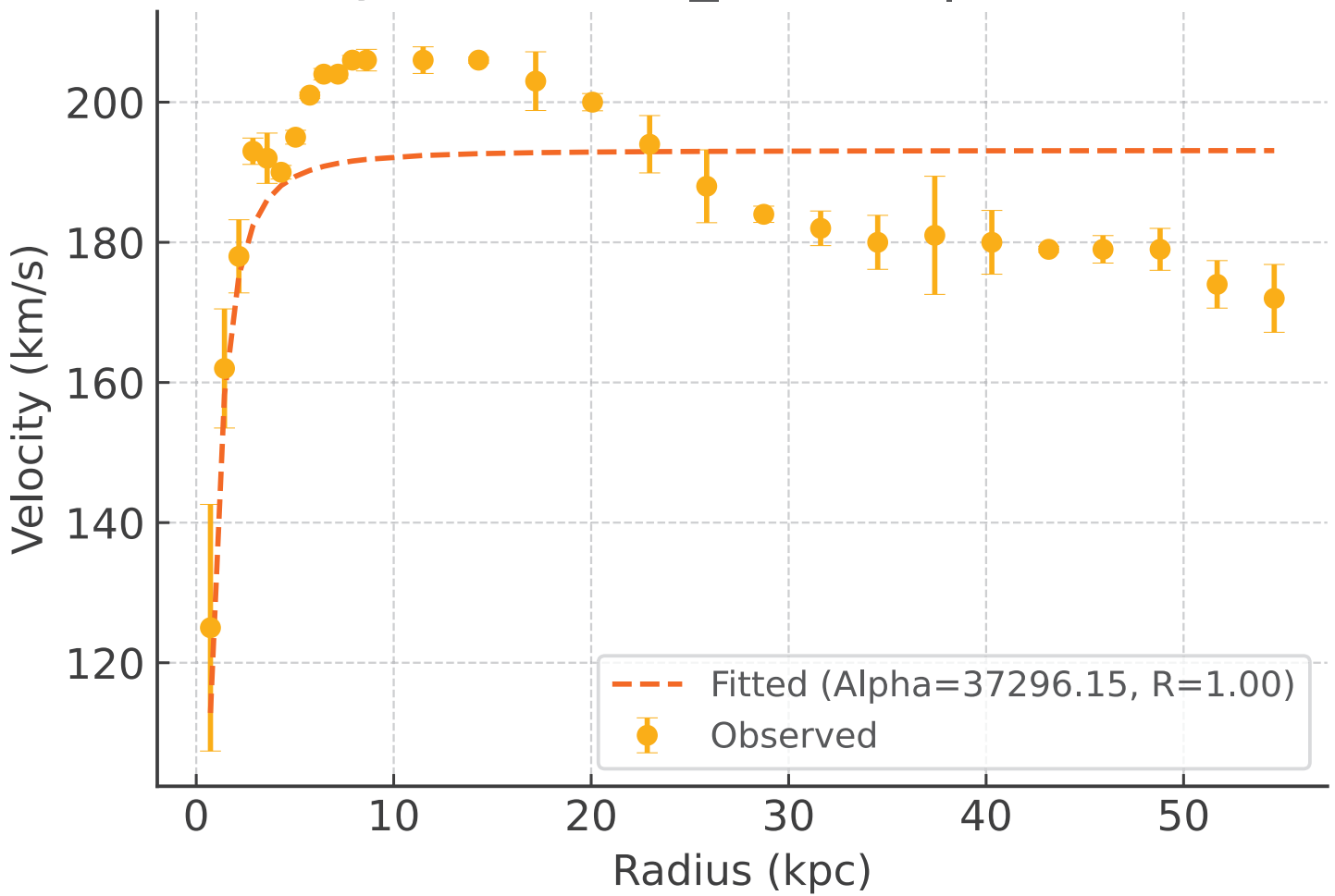
Galaxy: NGC5005\_rotmod |  $R^2 = -1.446$



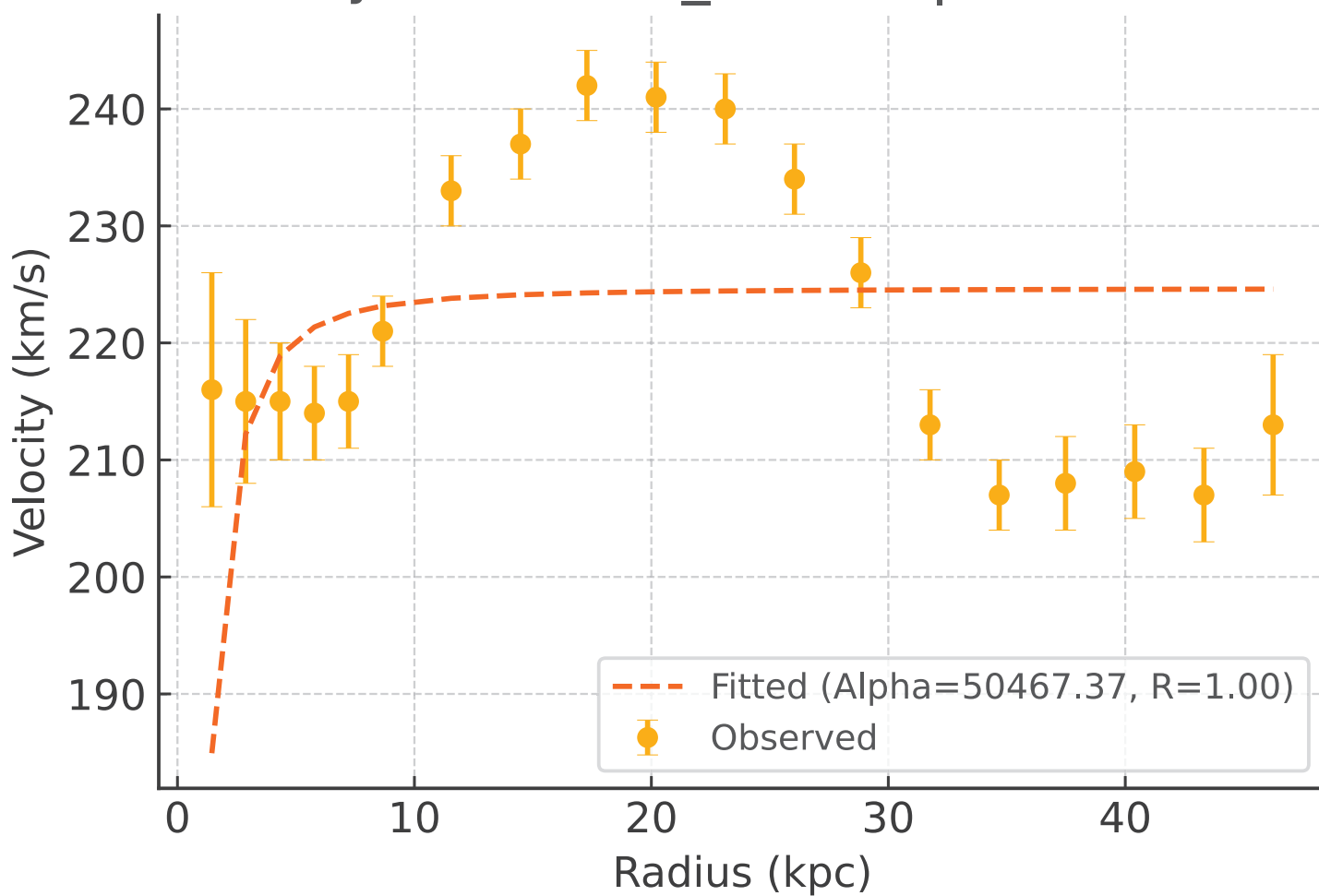
Galaxy: NGC5033\_rotmod |  $R^2 = -2.210$



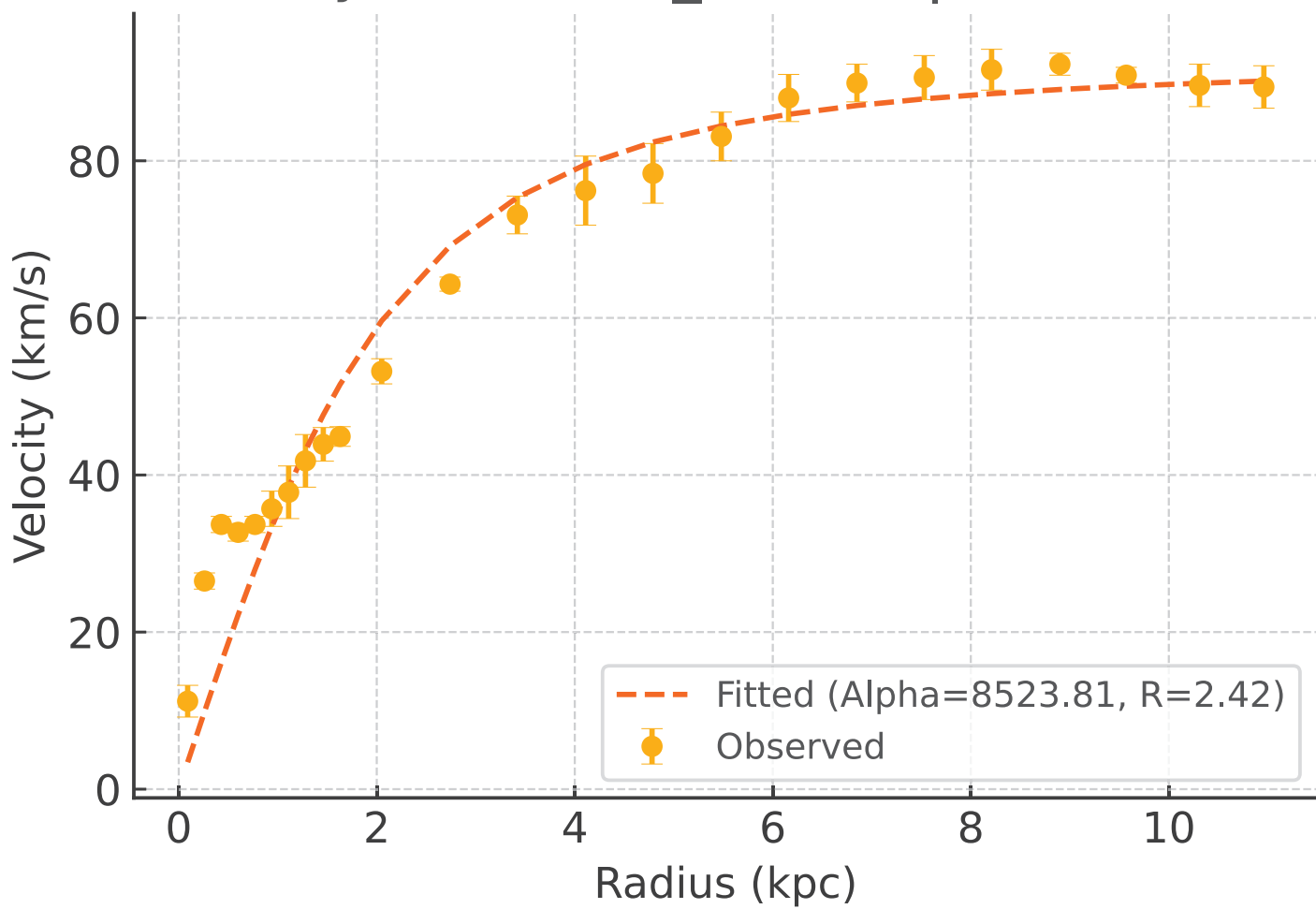
# Galaxy: NGC5055\_rotmod | $R^2 = 0.527$



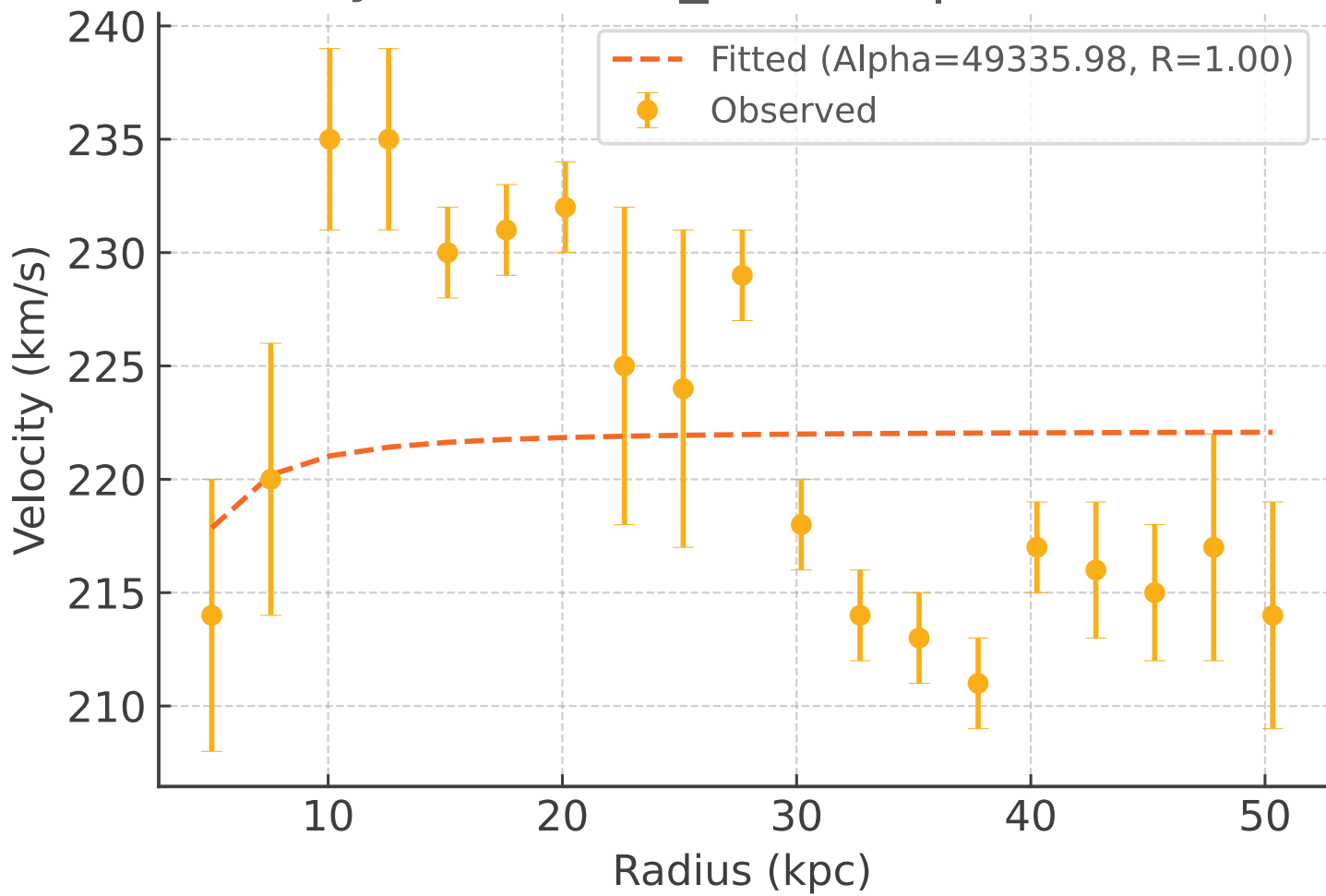
Galaxy: NGC5371\_rotmod |  $R^2 = -0.311$



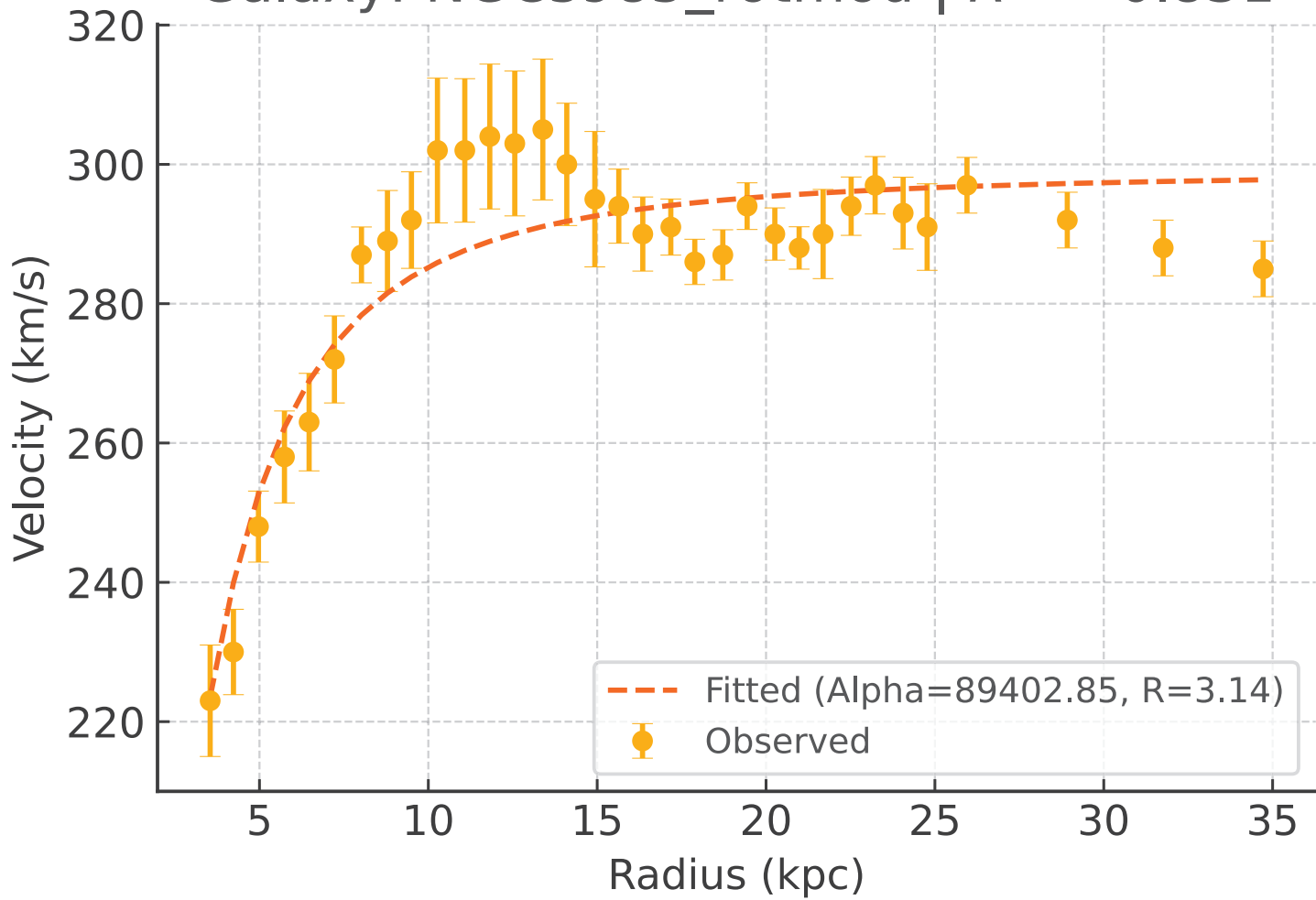
Galaxy: NGC5585\_rotmod |  $R^2 = 0.938$



# Galaxy: NGC5907\_rotmod | $R^2 = -0.010$

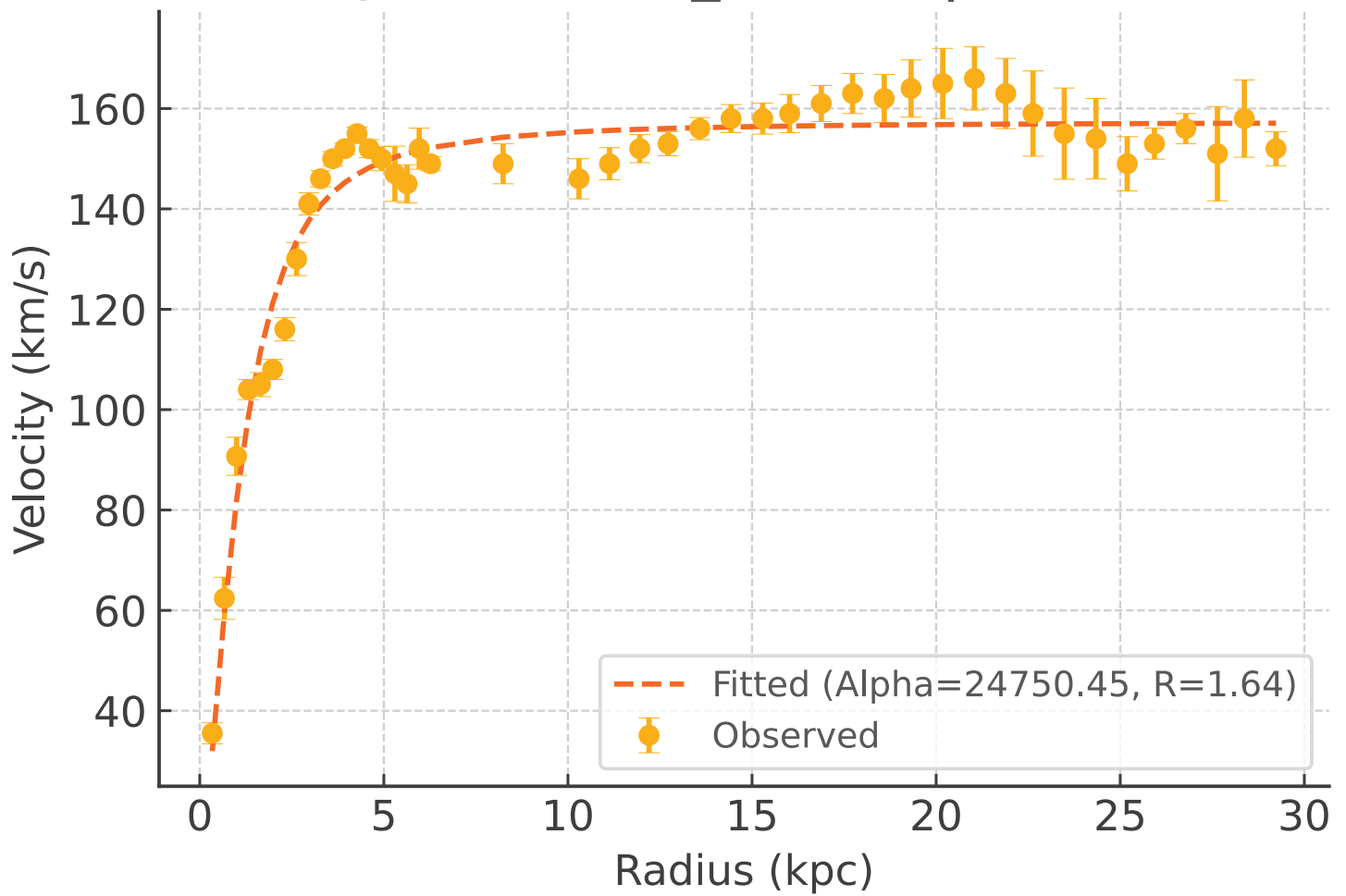


Galaxy: NGC5985\_rotmod |  $R^2 = 0.831$

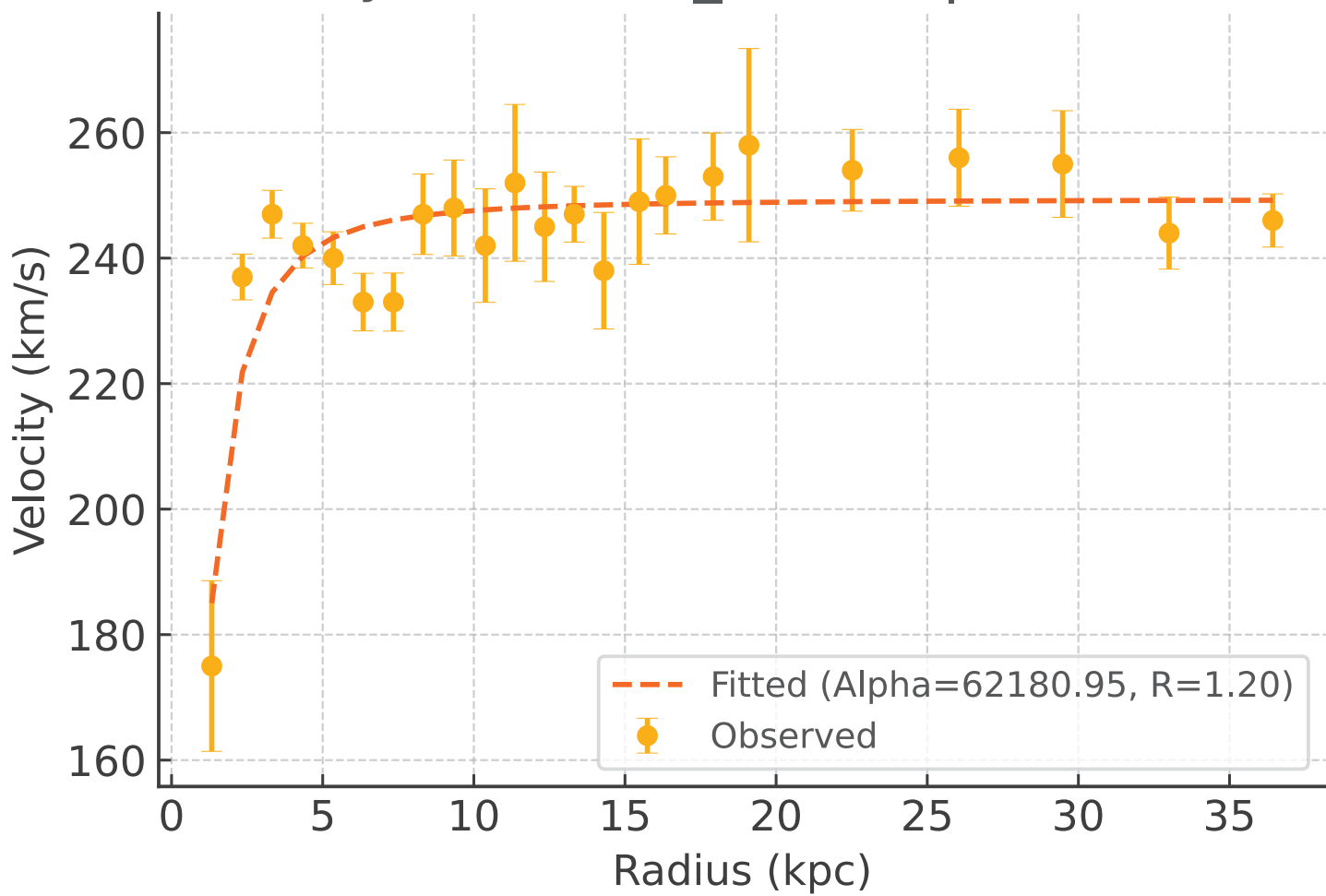




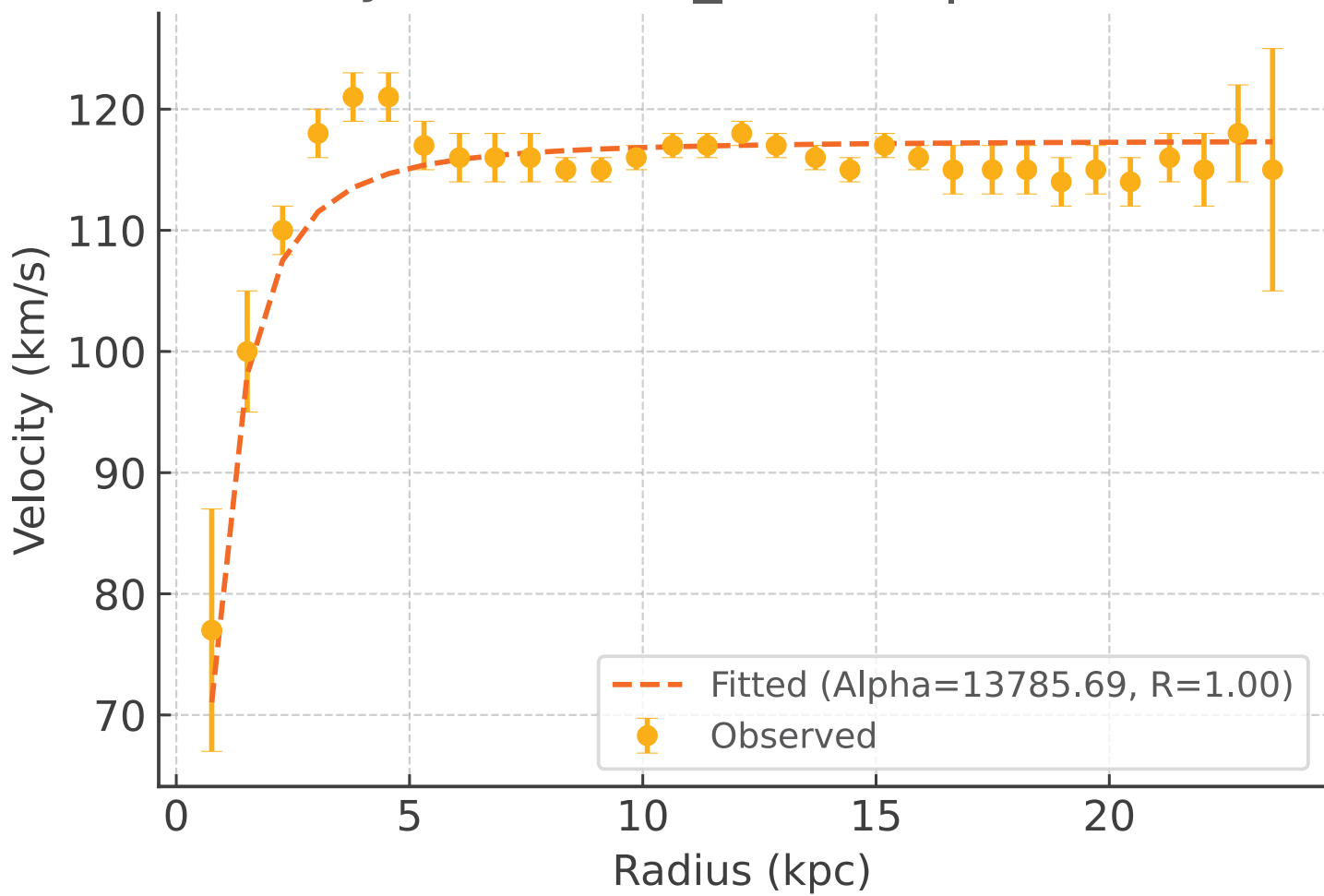
Galaxy: NGC6015\_rotmod |  $R^2 = 0.954$



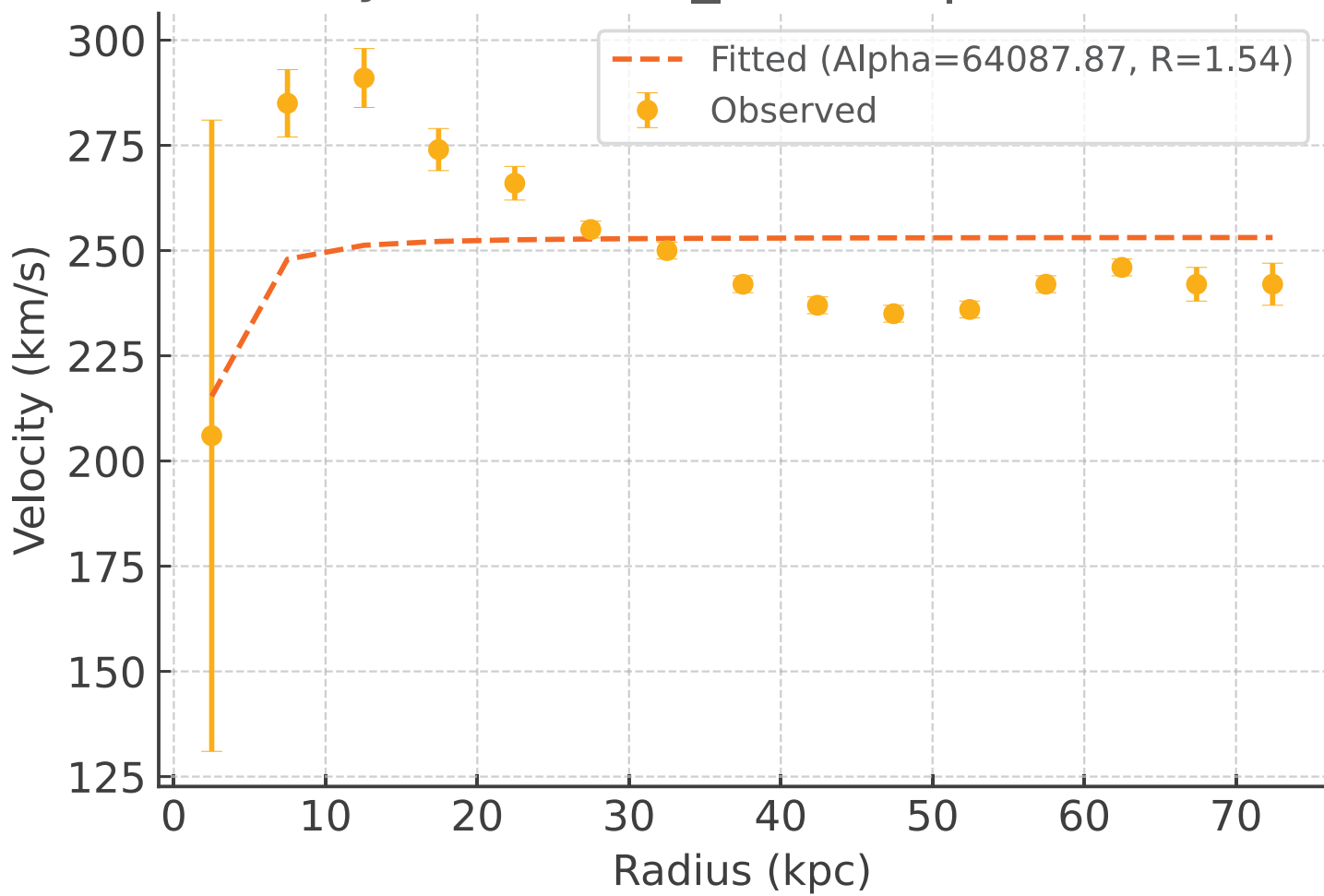
# Galaxy: NGC6195\_rotmod | $R^2 = 0.791$



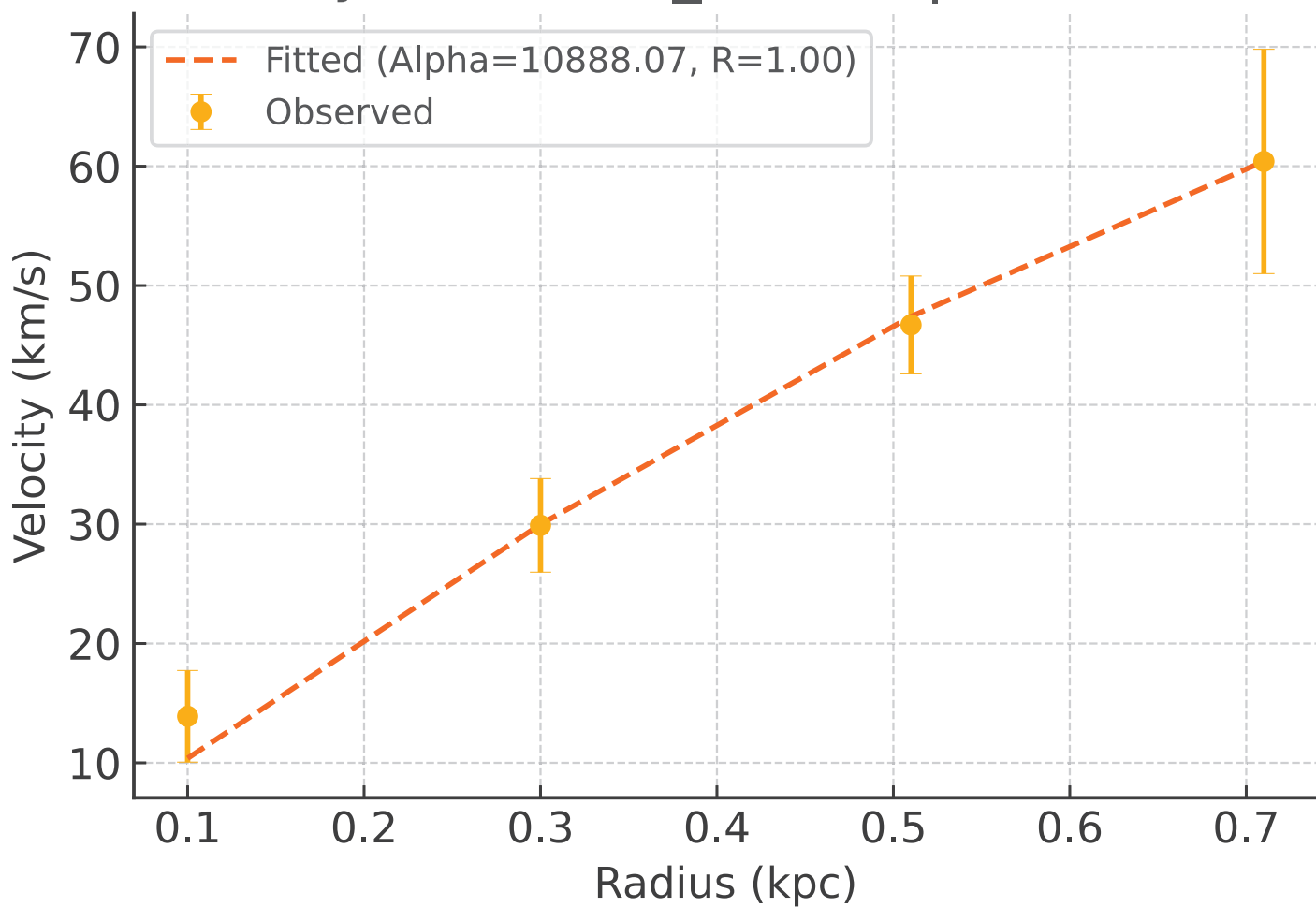
Galaxy: NGC6503\_rotmod |  $R^2 = 0.859$



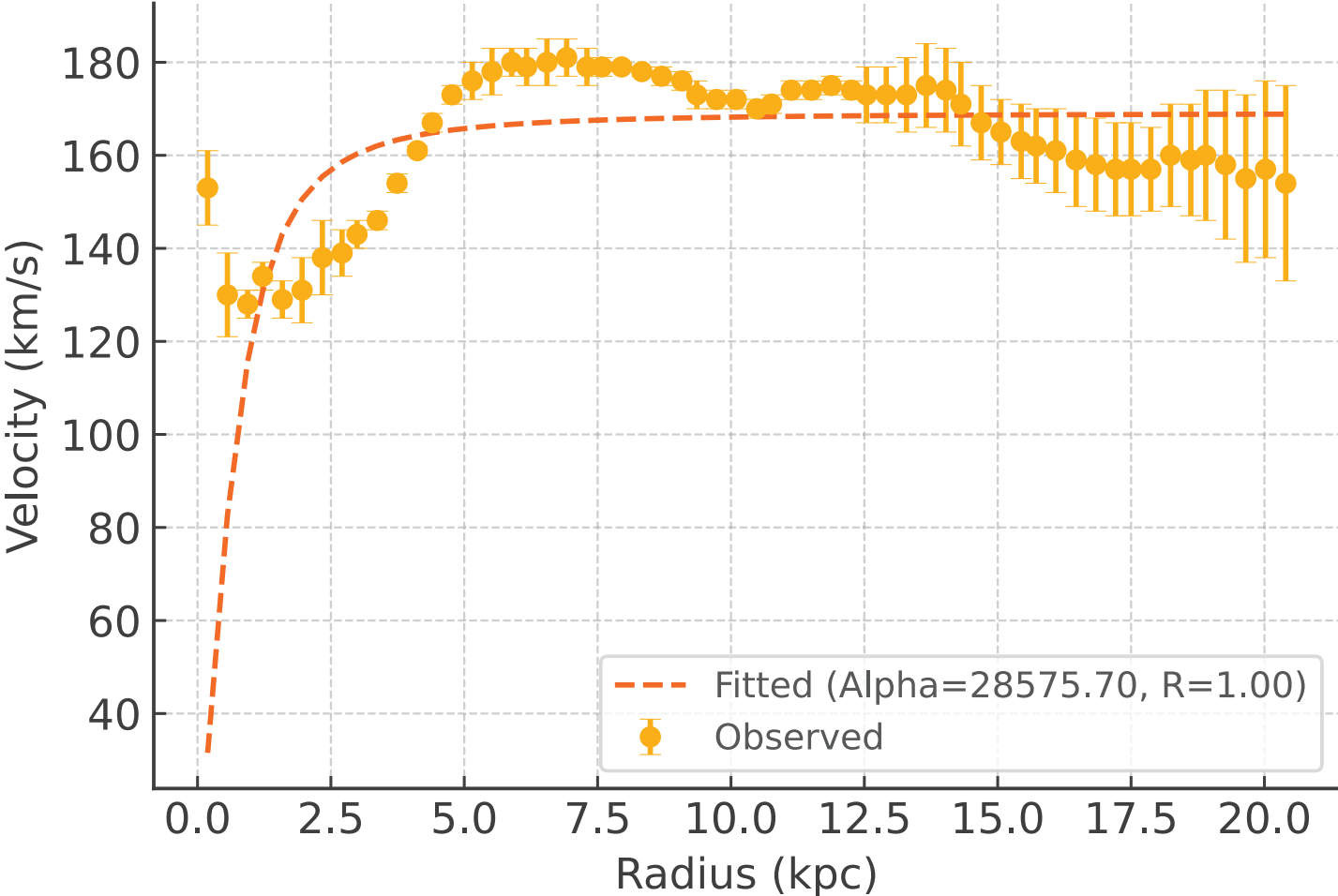
# Galaxy: NGC6674\_rotmod | $R^2 = 0.220$



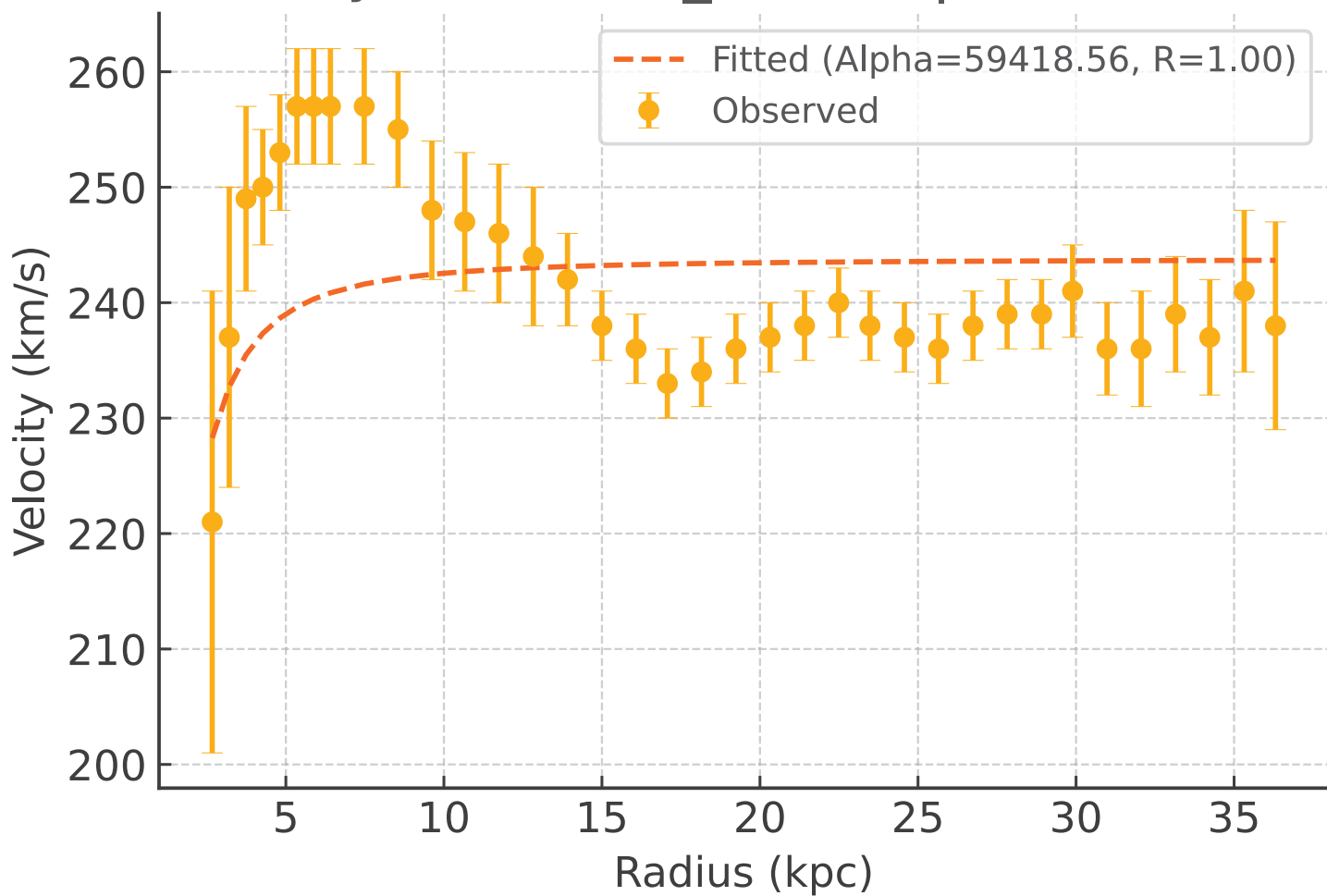
# Galaxy: NGC6789\_rotmod | $R^2 = 0.989$



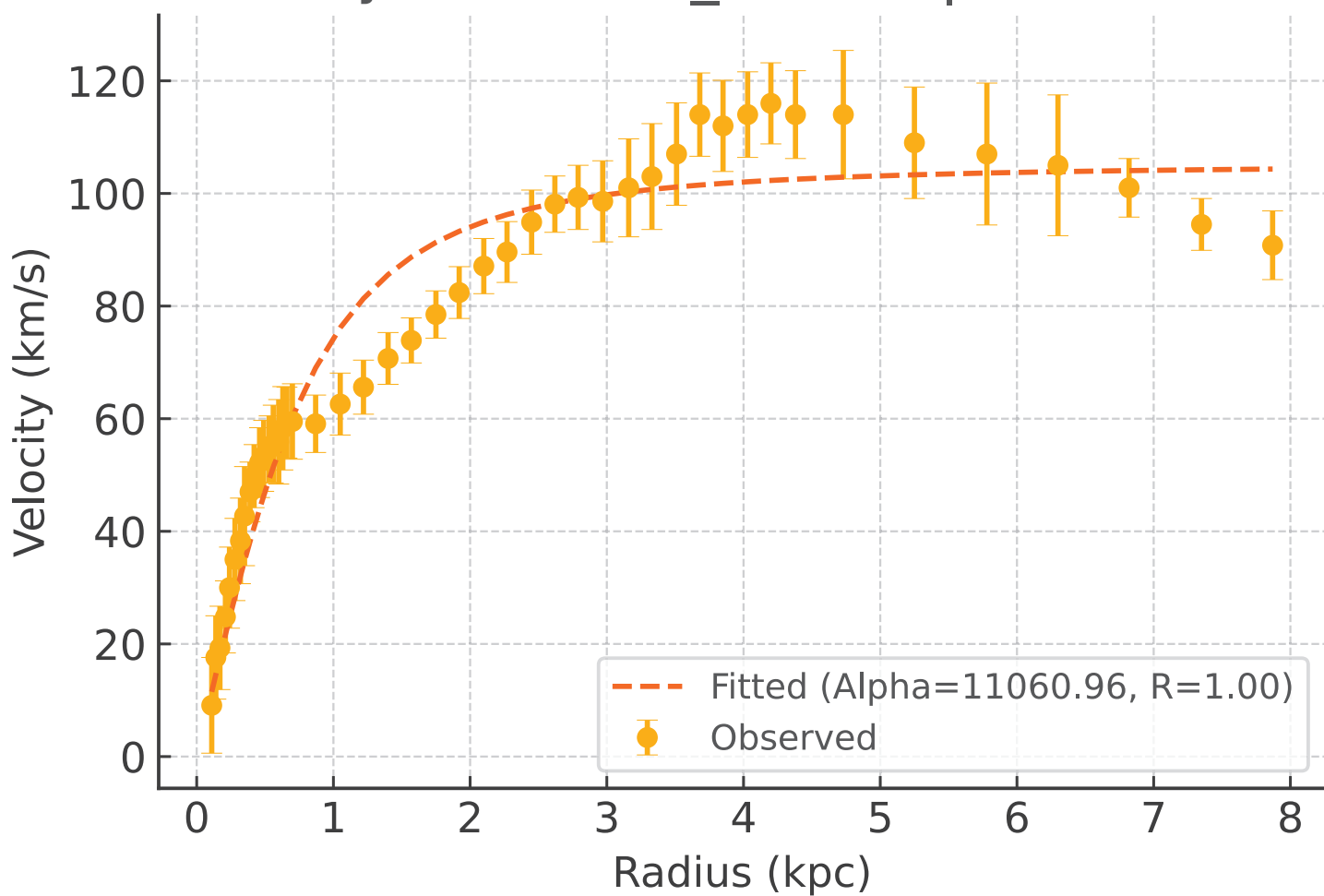
Galaxy: NGC6946\_rotmod |  $R^2 = -0.840$



# Galaxy: NGC7331\_rotmod | $R^2 = -0.175$

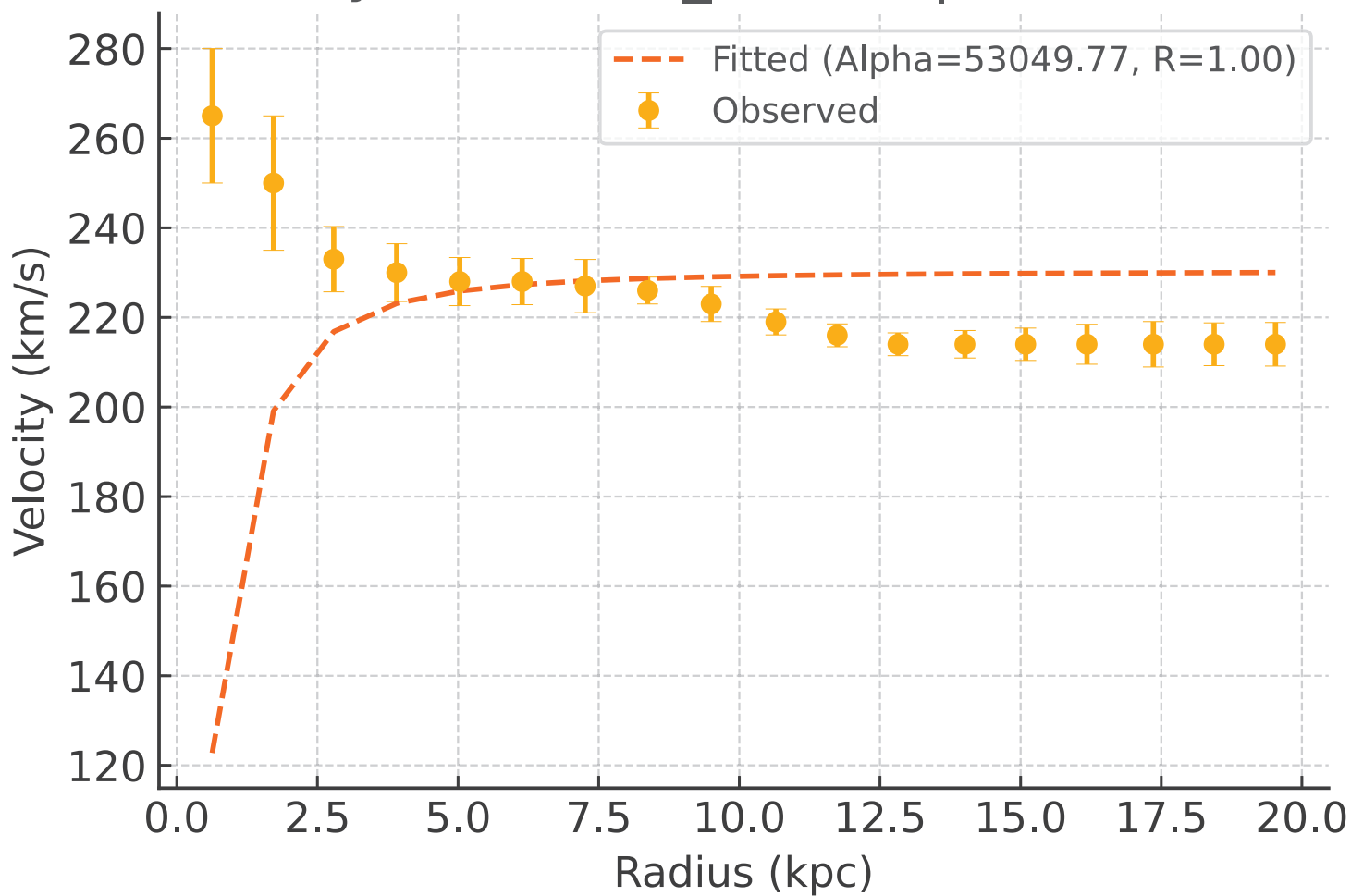


# Galaxy: NGC7793\_rotmod | $R^2 = 0.927$

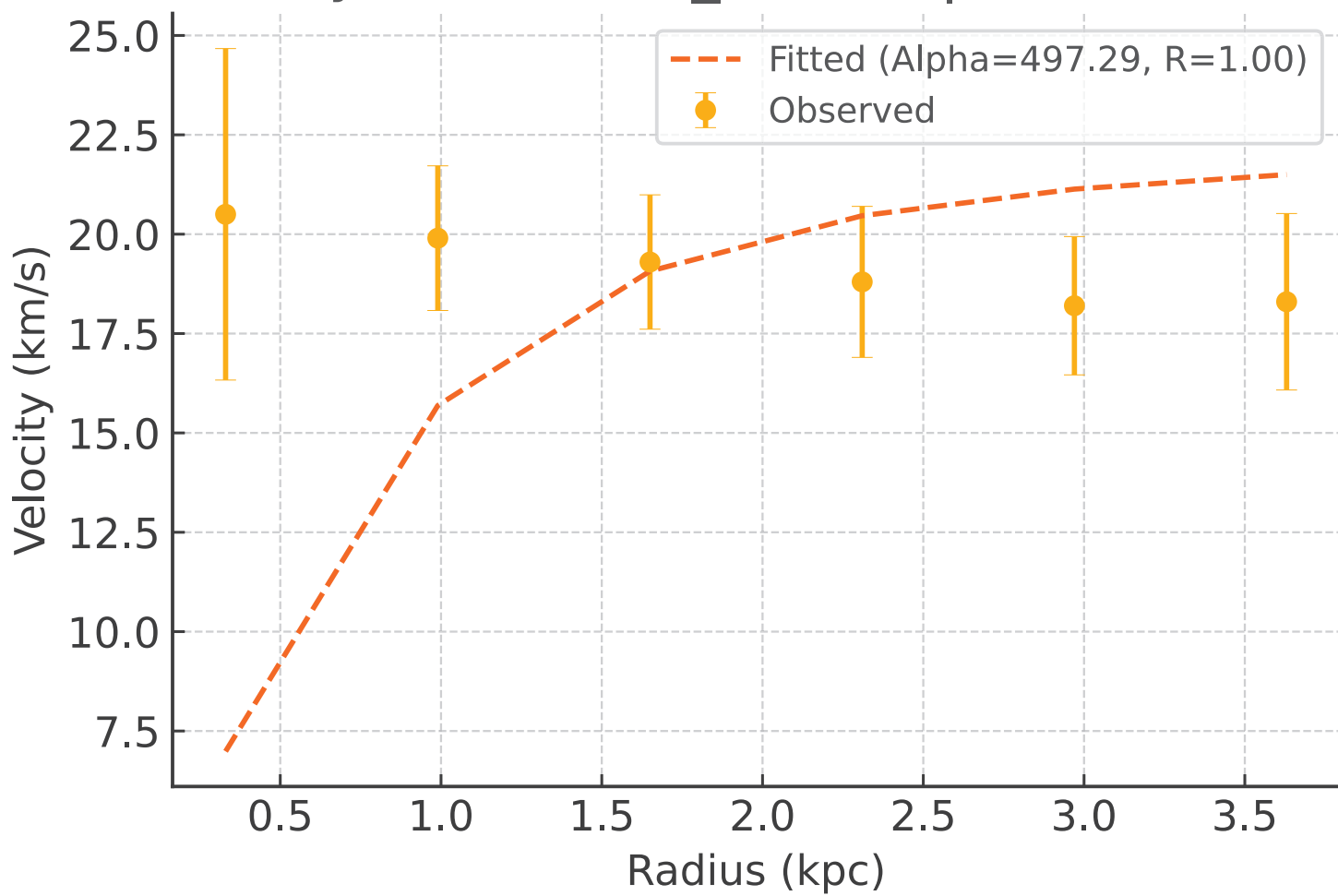




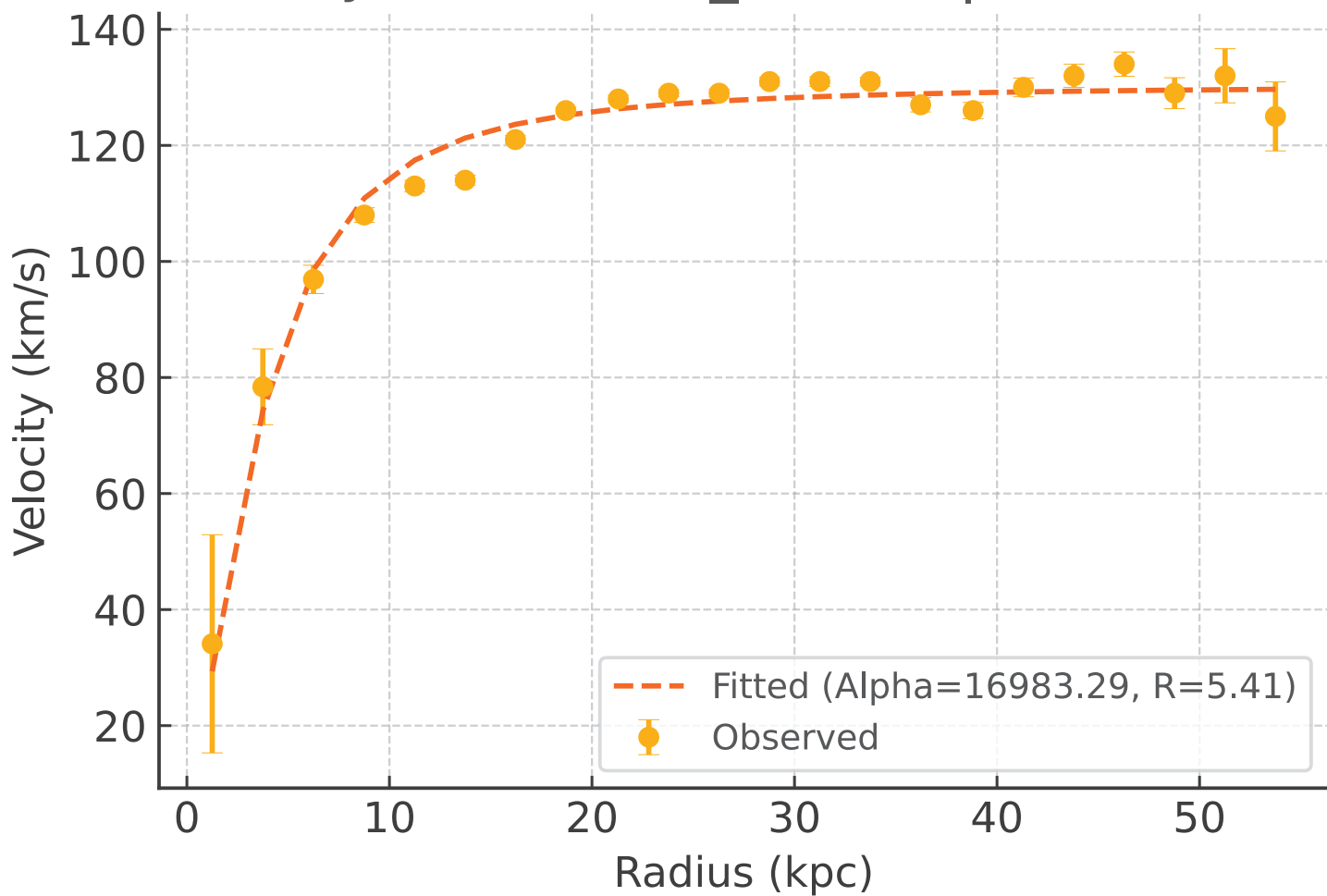
# Galaxy: NGC7814\_rotmod | $R^2 = -6.639$



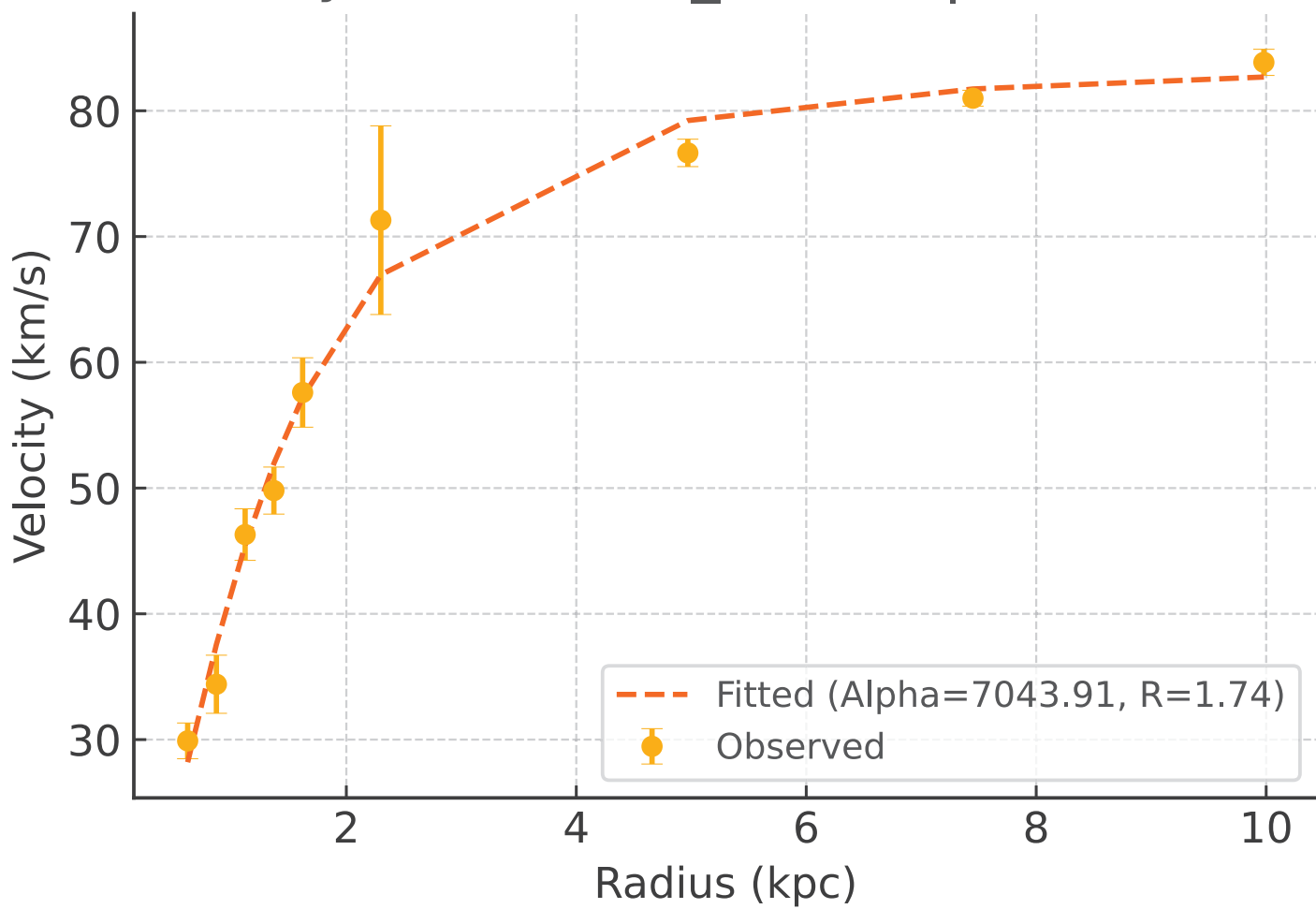
# Galaxy: PGC51017\_rotmod | $R^2 = -52.443$



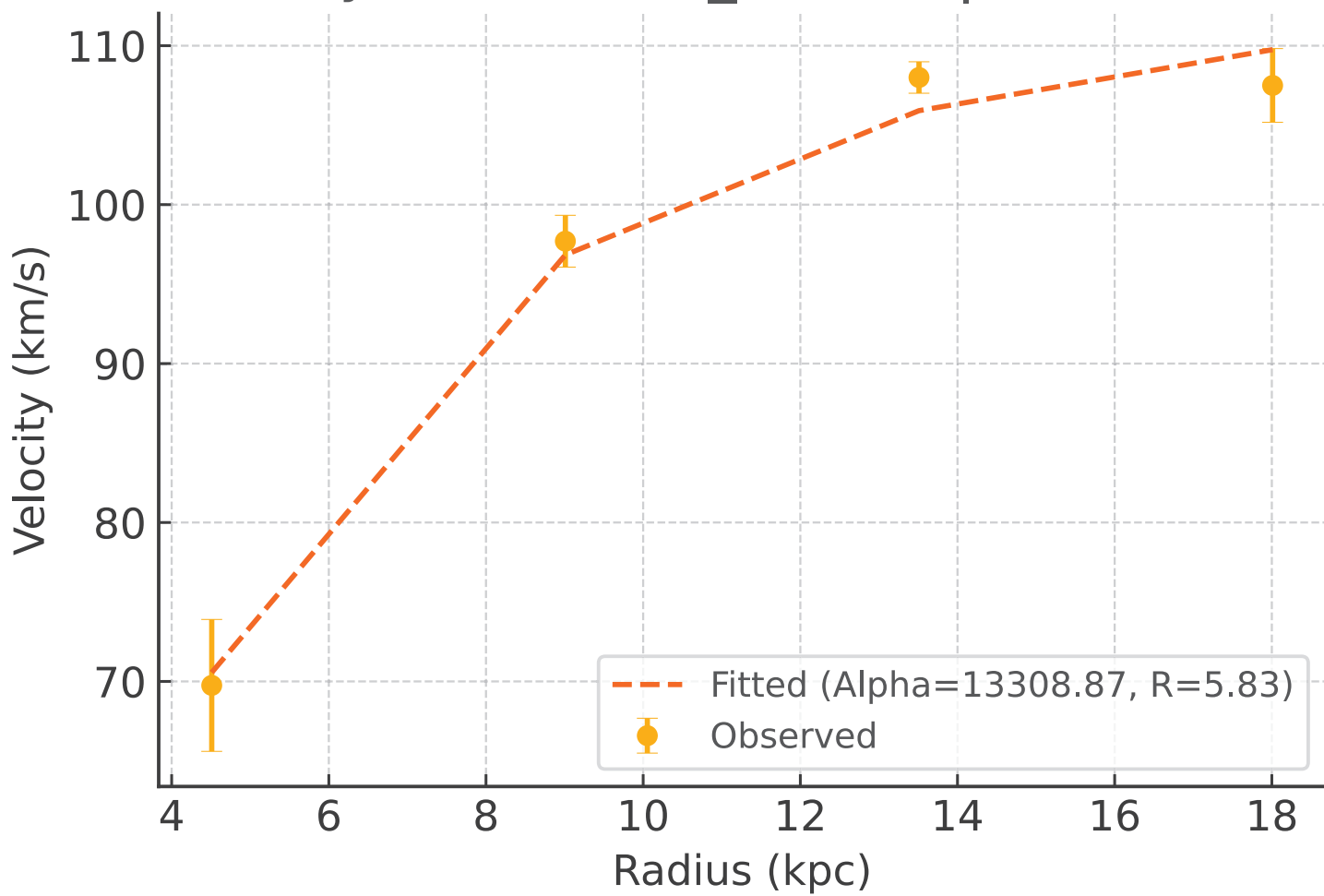
Galaxy: UGC00128\_rotmod |  $R^2 = 0.980$



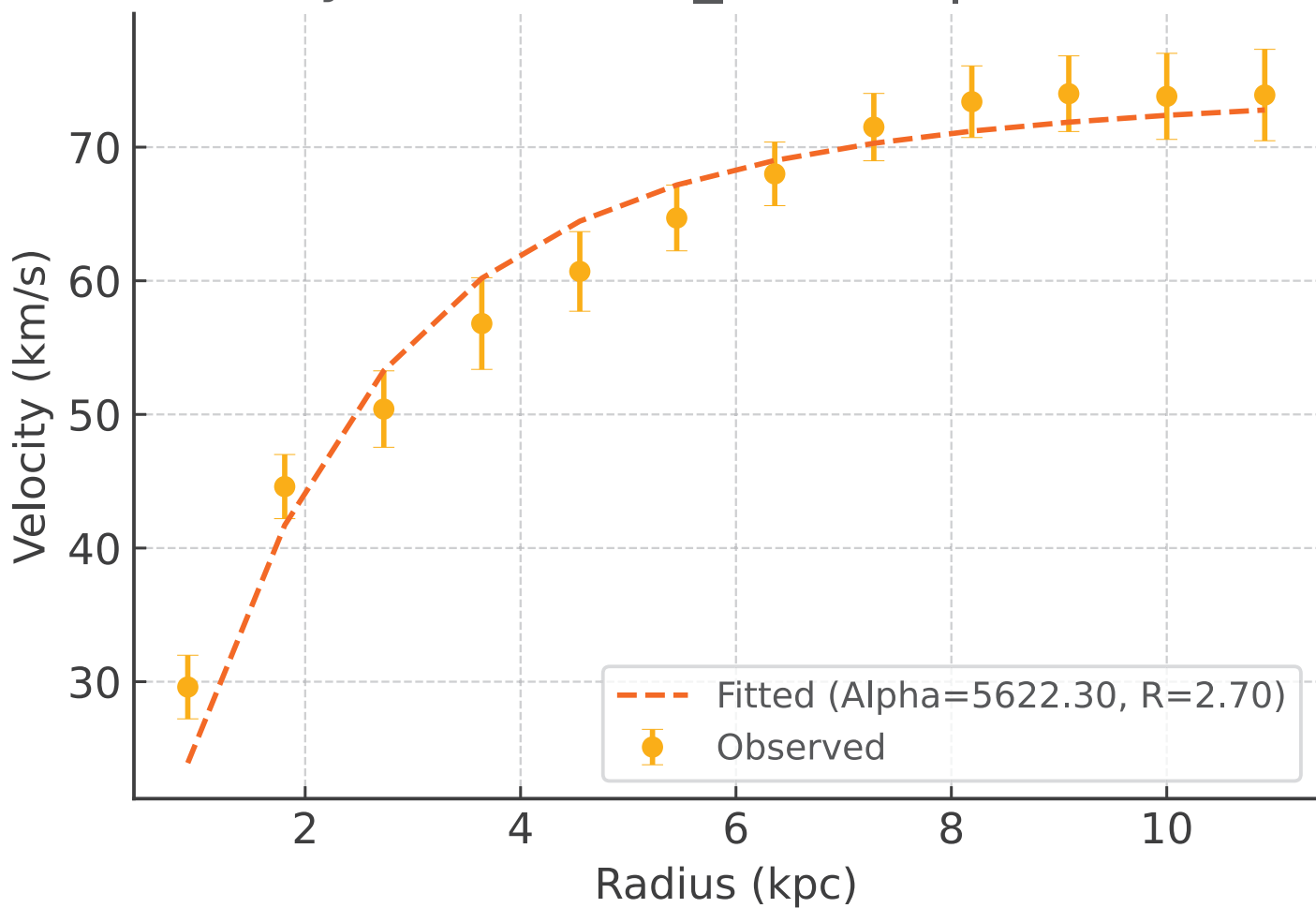
Galaxy: UGC00191\_rotmod |  $R^2 = 0.986$



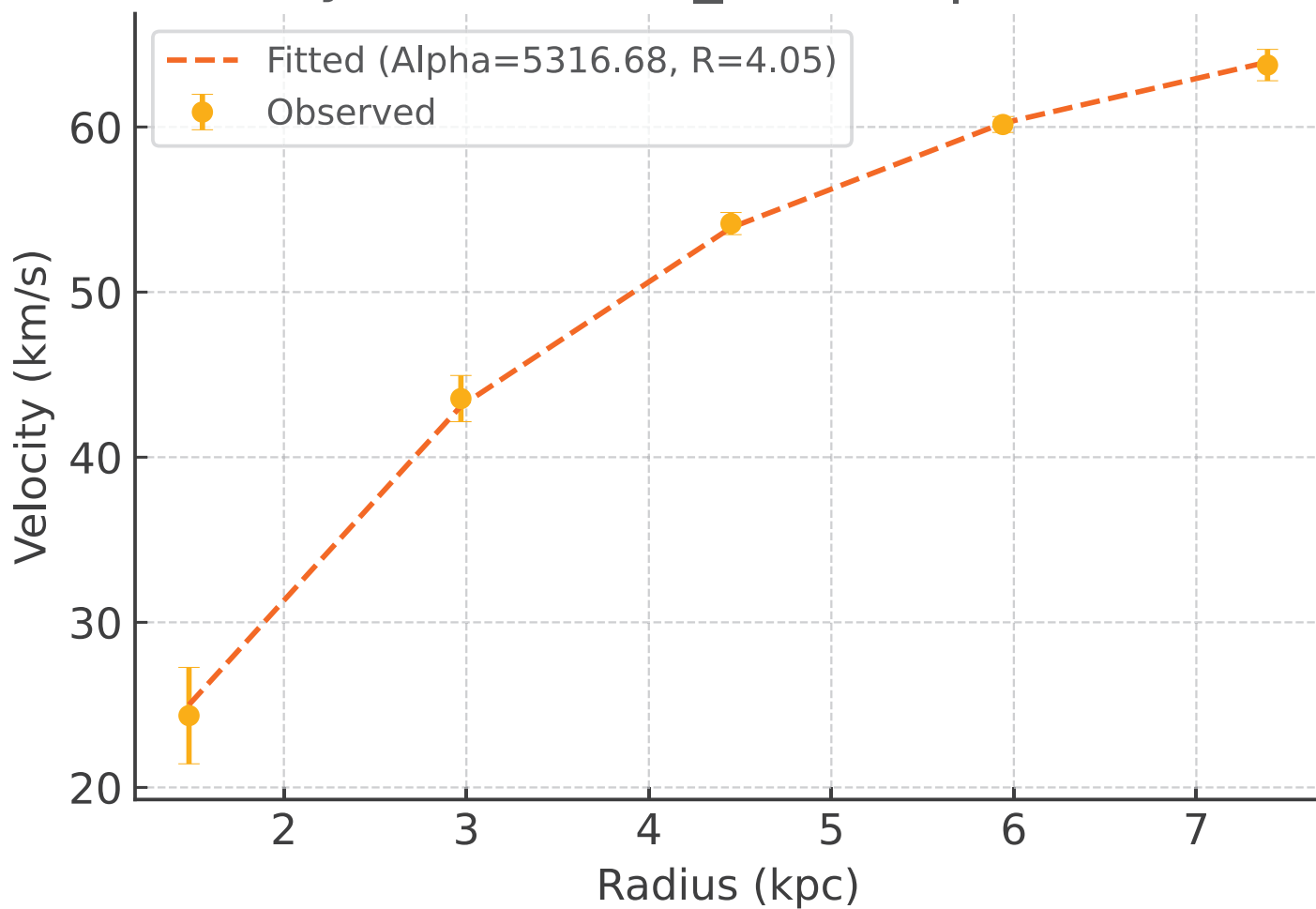
Galaxy: UGC00634\_rotmod |  $R^2 = 0.989$



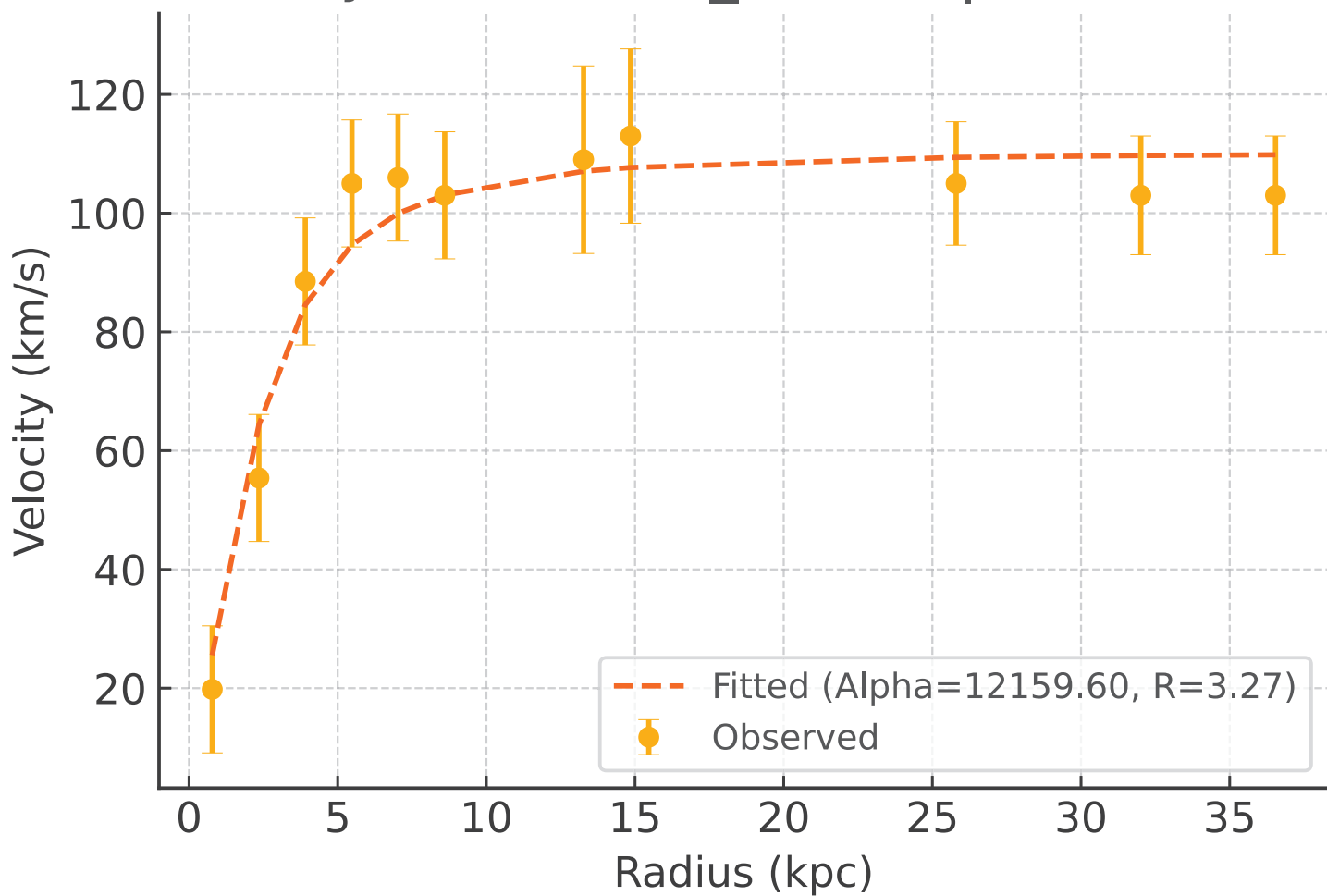
Galaxy: UGC00731\_rotmod |  $R^2 = 0.957$



# Galaxy: UGC00891\_rotmod | $R^2 = 0.999$

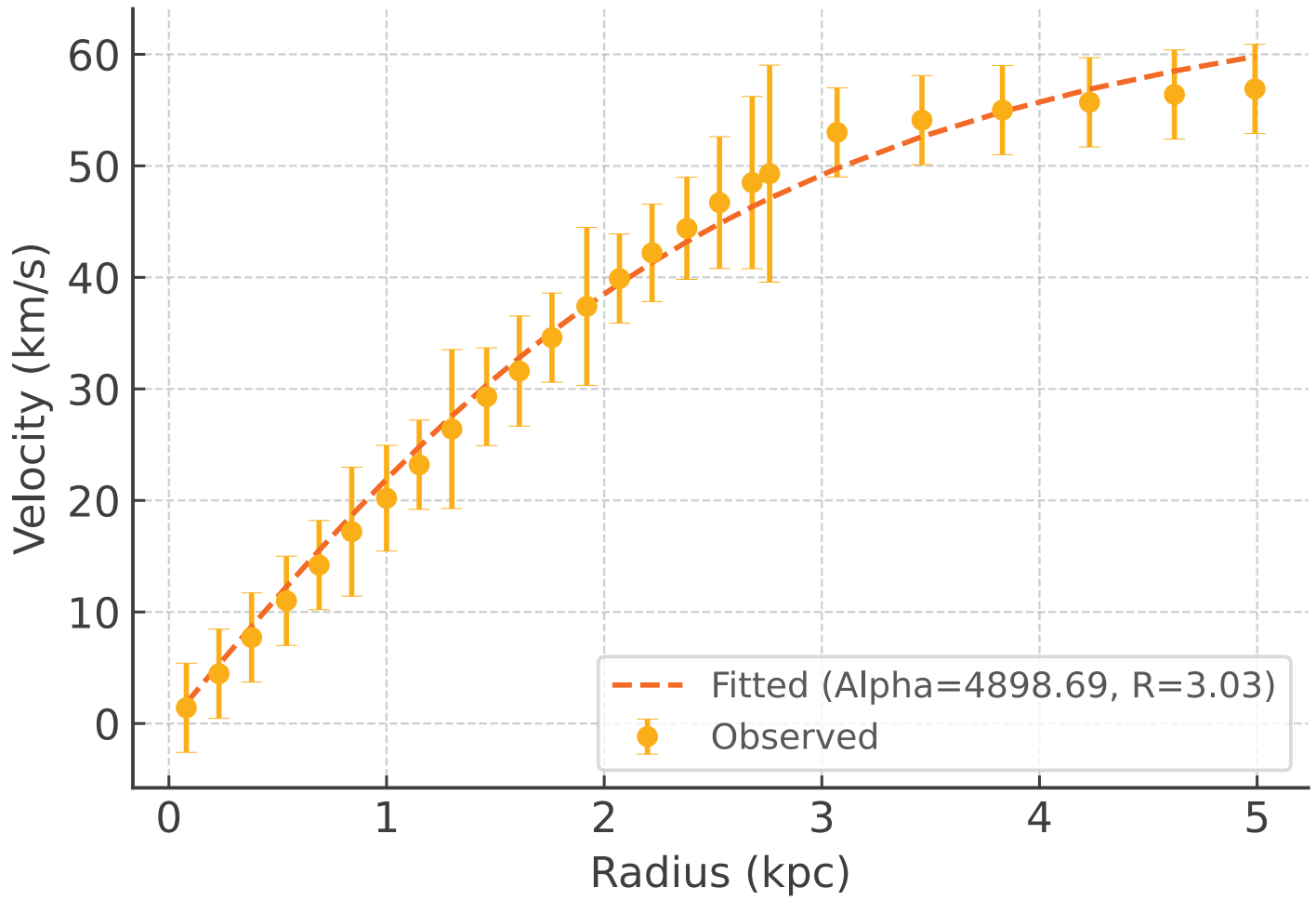


Galaxy: UGC01230\_rotmod |  $R^2 = 0.949$

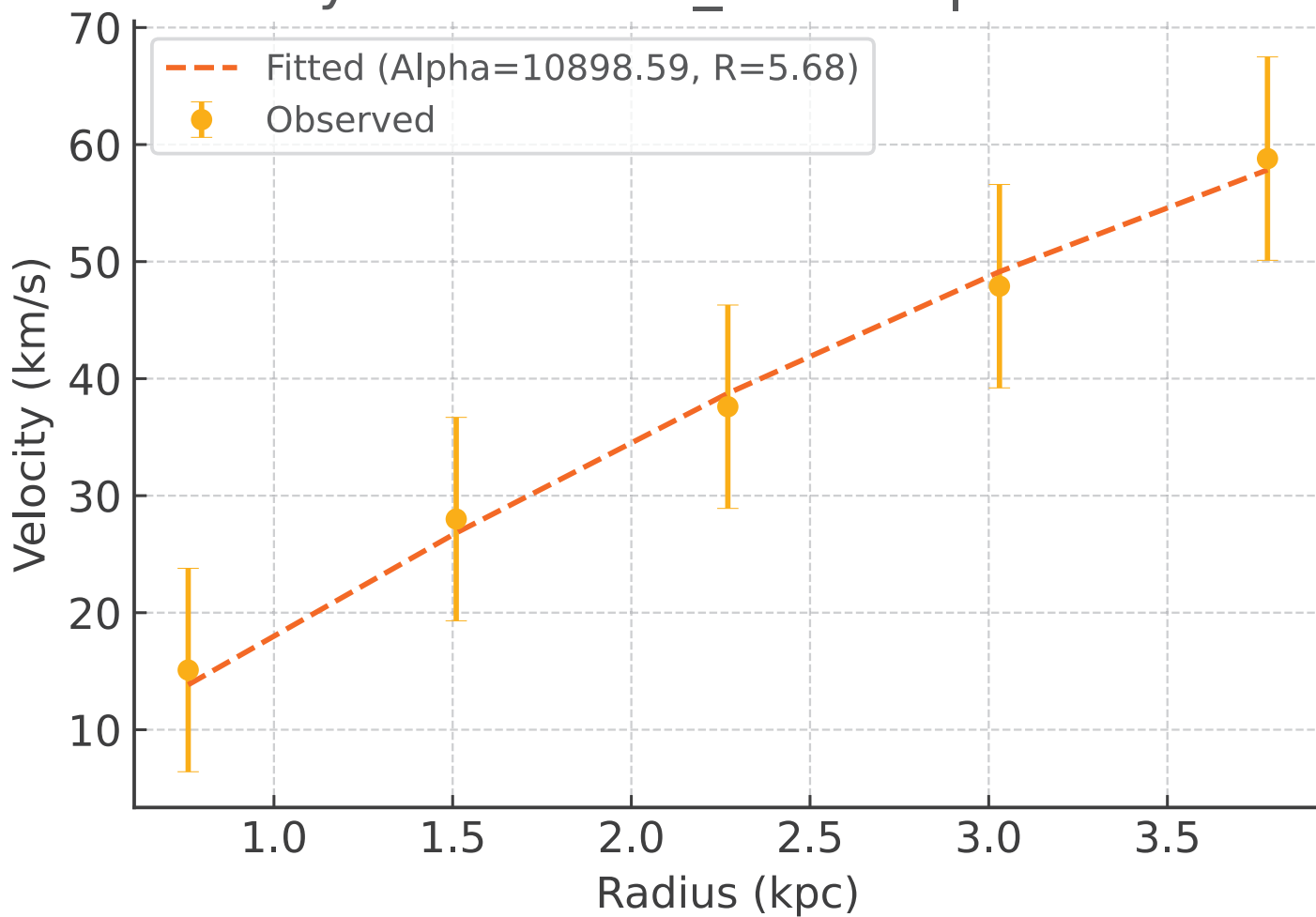




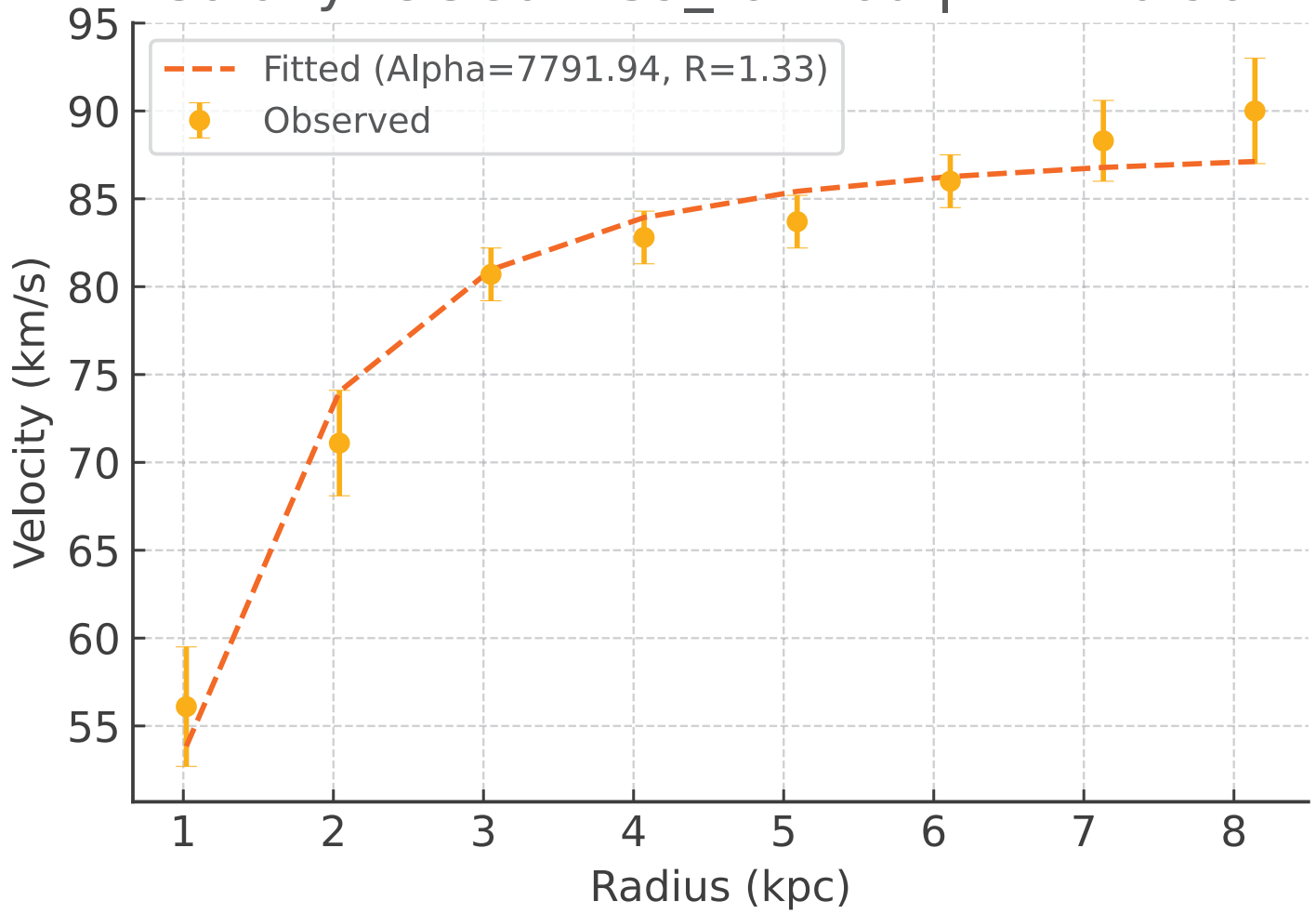
Galaxy: UGC01281\_rotmod |  $R^2 = 0.992$



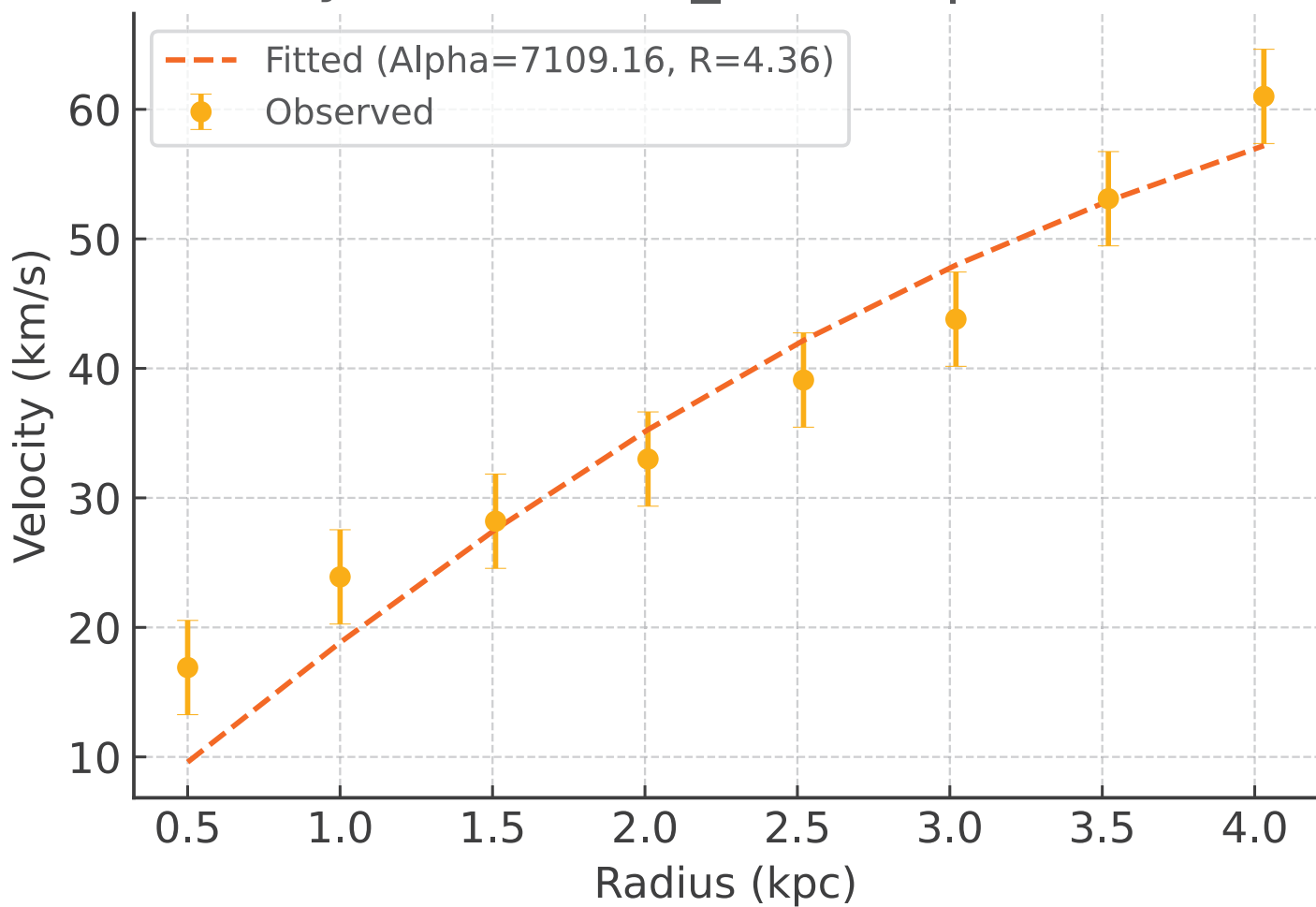
# Galaxy: UGC02023\_rotmod | $R^2 = 0.994$



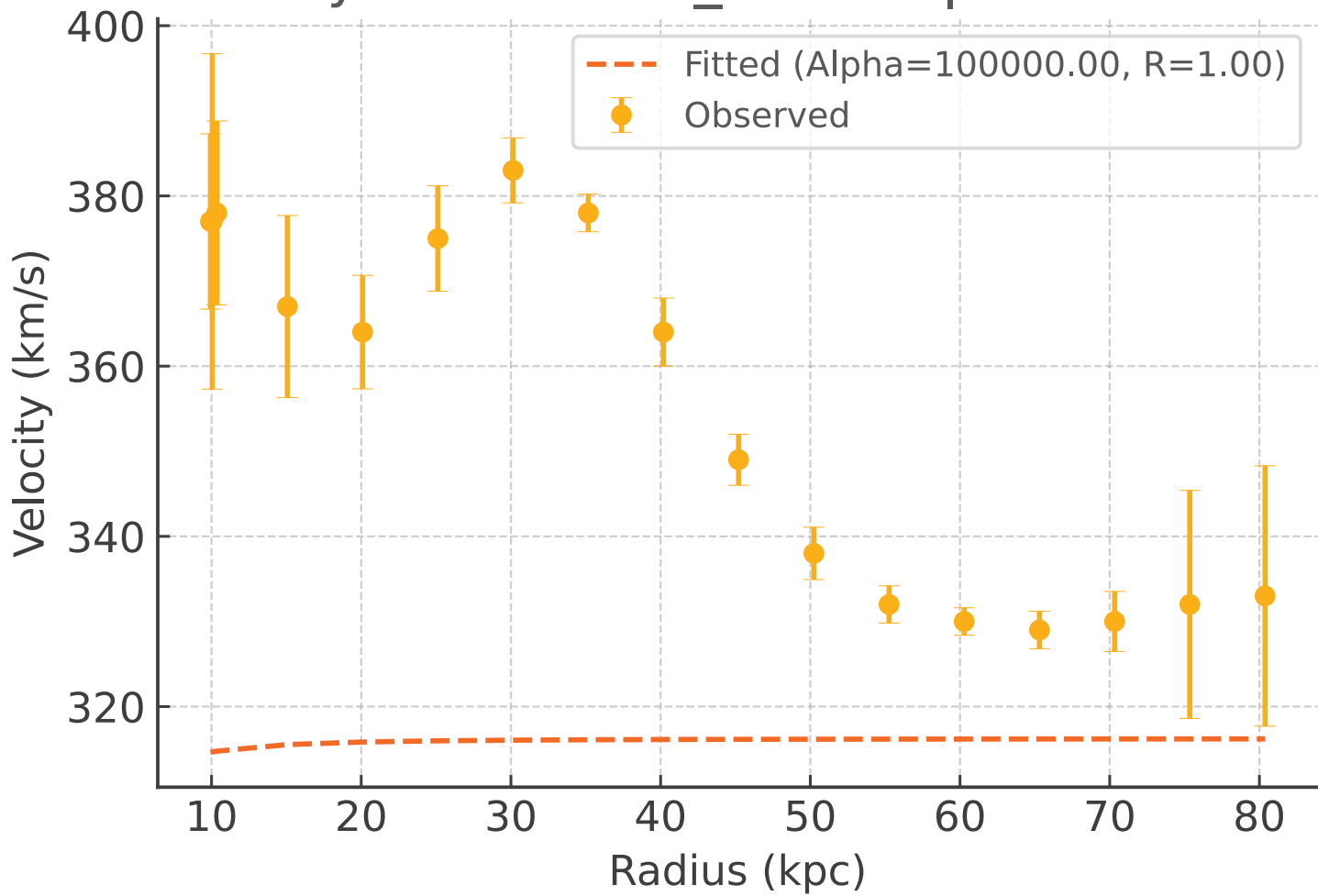
# Galaxy: UGC02259\_rotmod | $R^2 = 0.967$



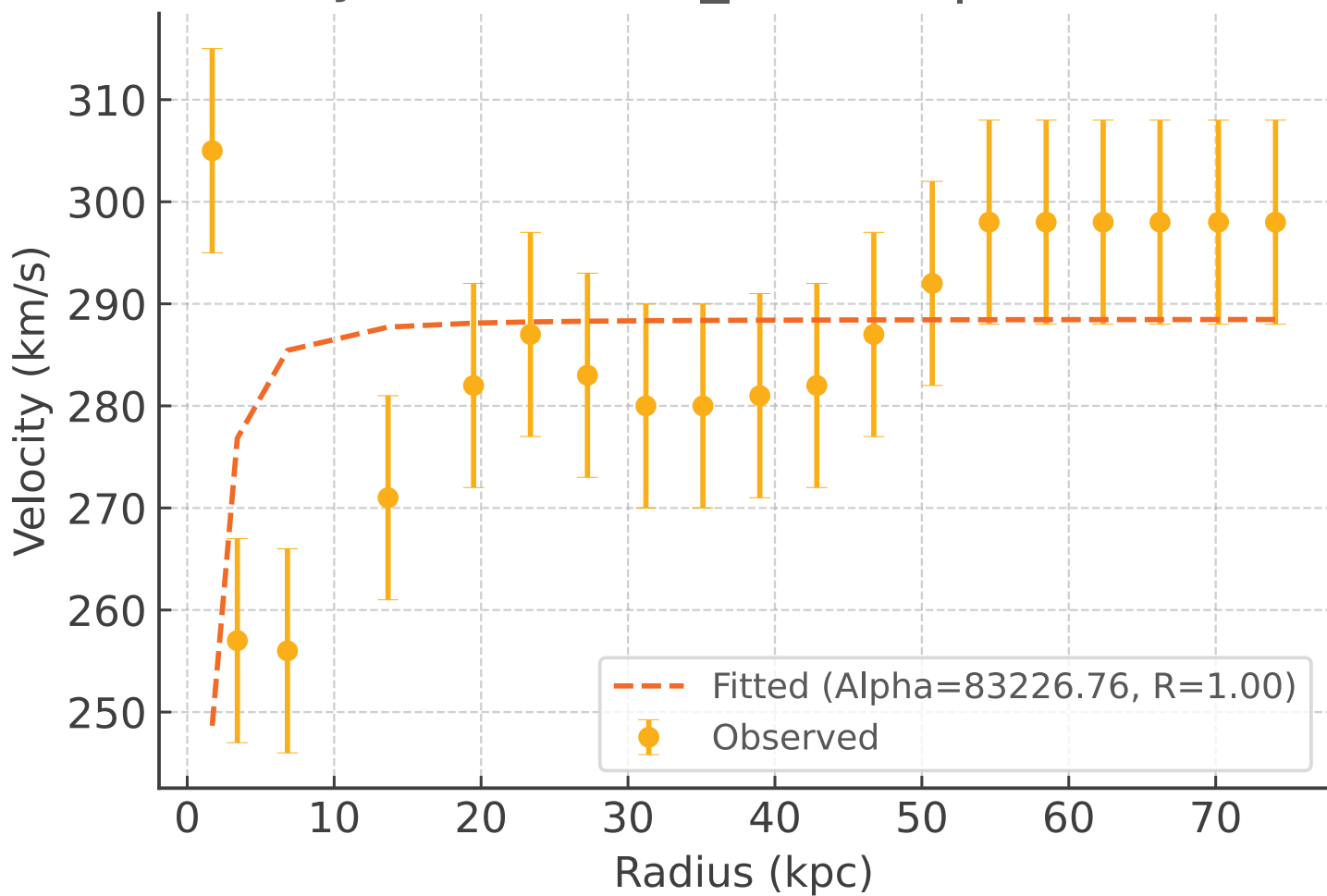
# Galaxy: UGC02455\_rotmod | $R^2 = 0.919$



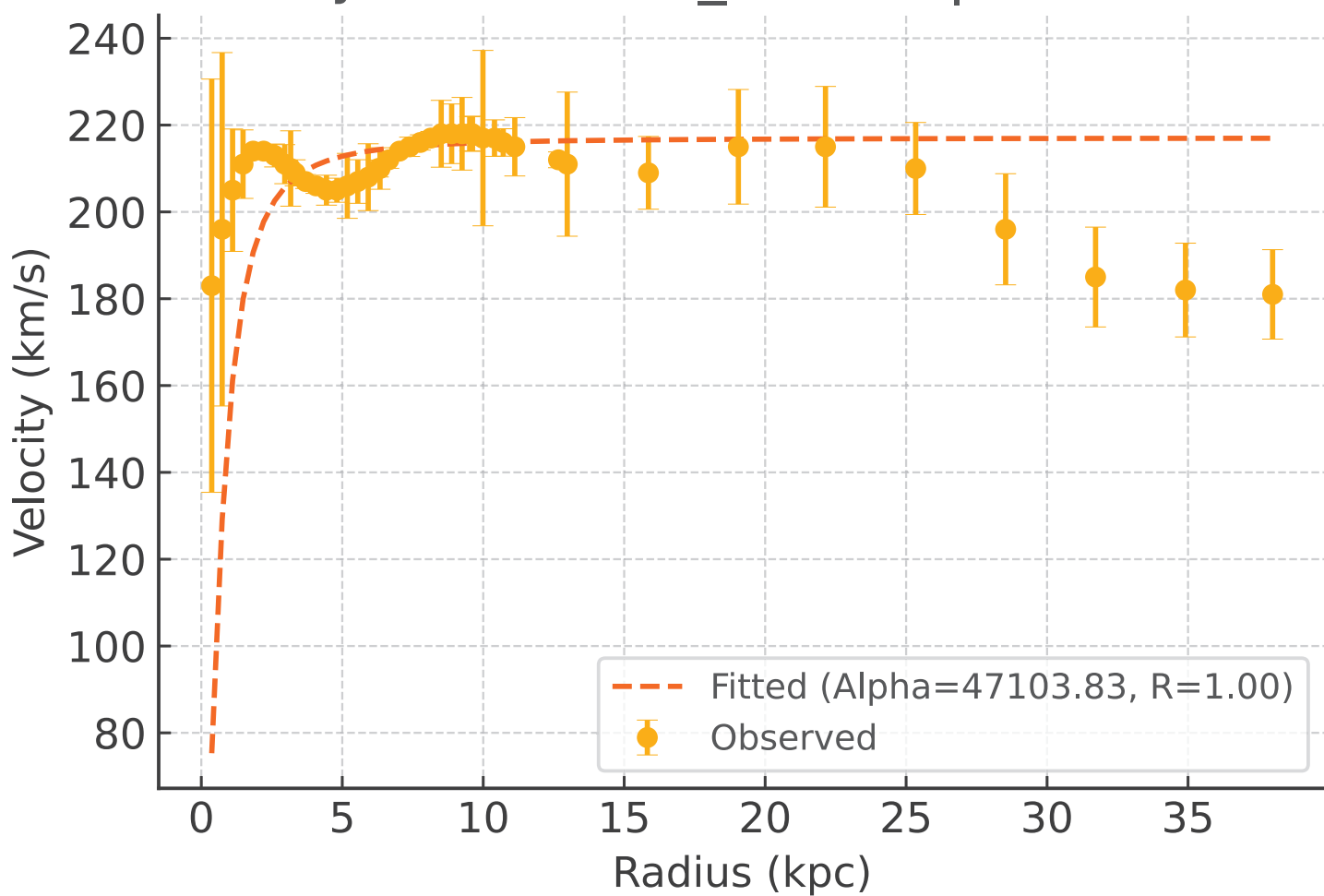
# Galaxy: UGC02487\_rotmod | $R^2 = -3.613$



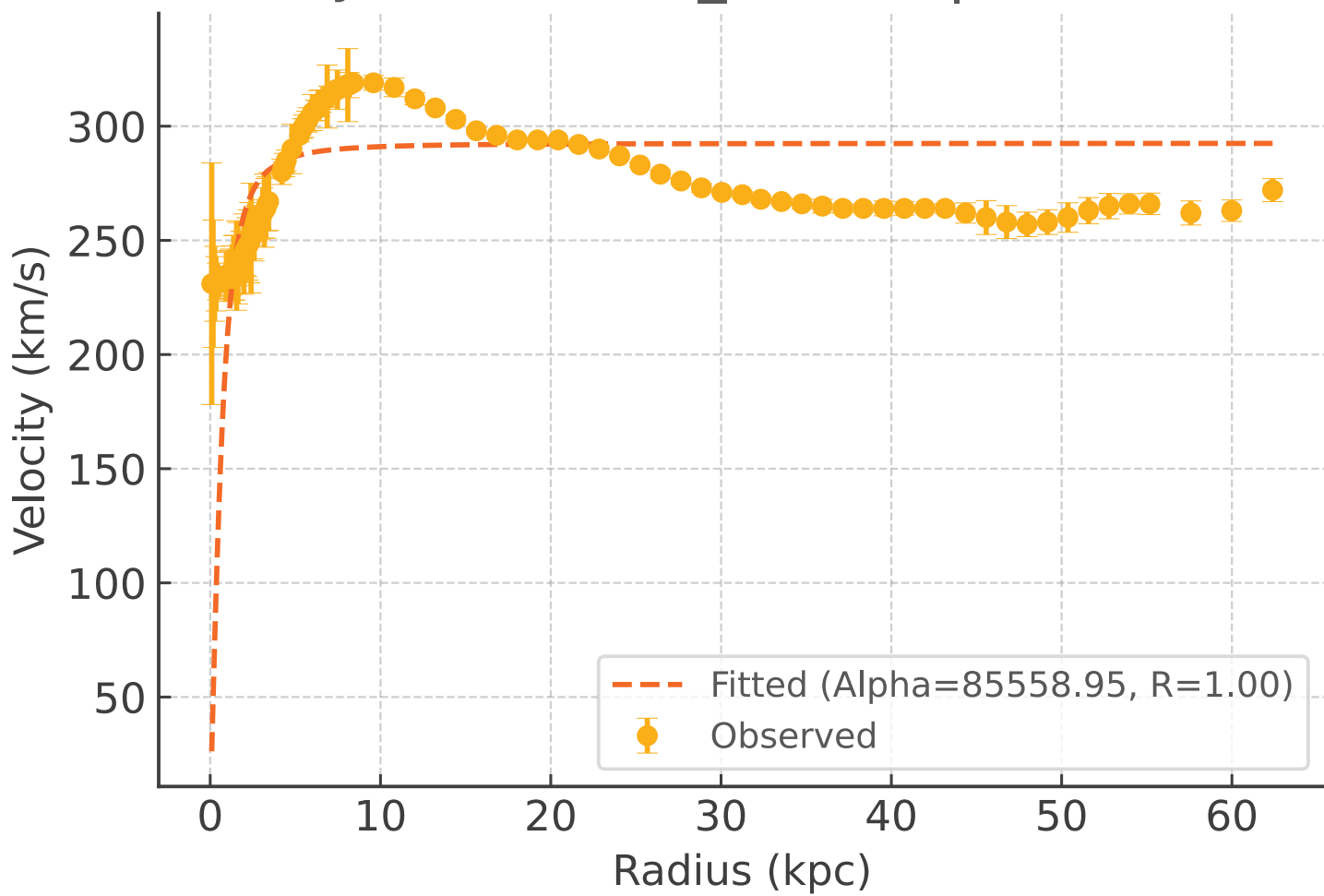
Galaxy: UGC02885\_rotmod |  $R^2 = -0.657$



Galaxy: UGC02916\_rotmod |  $R^2 = -4.891$

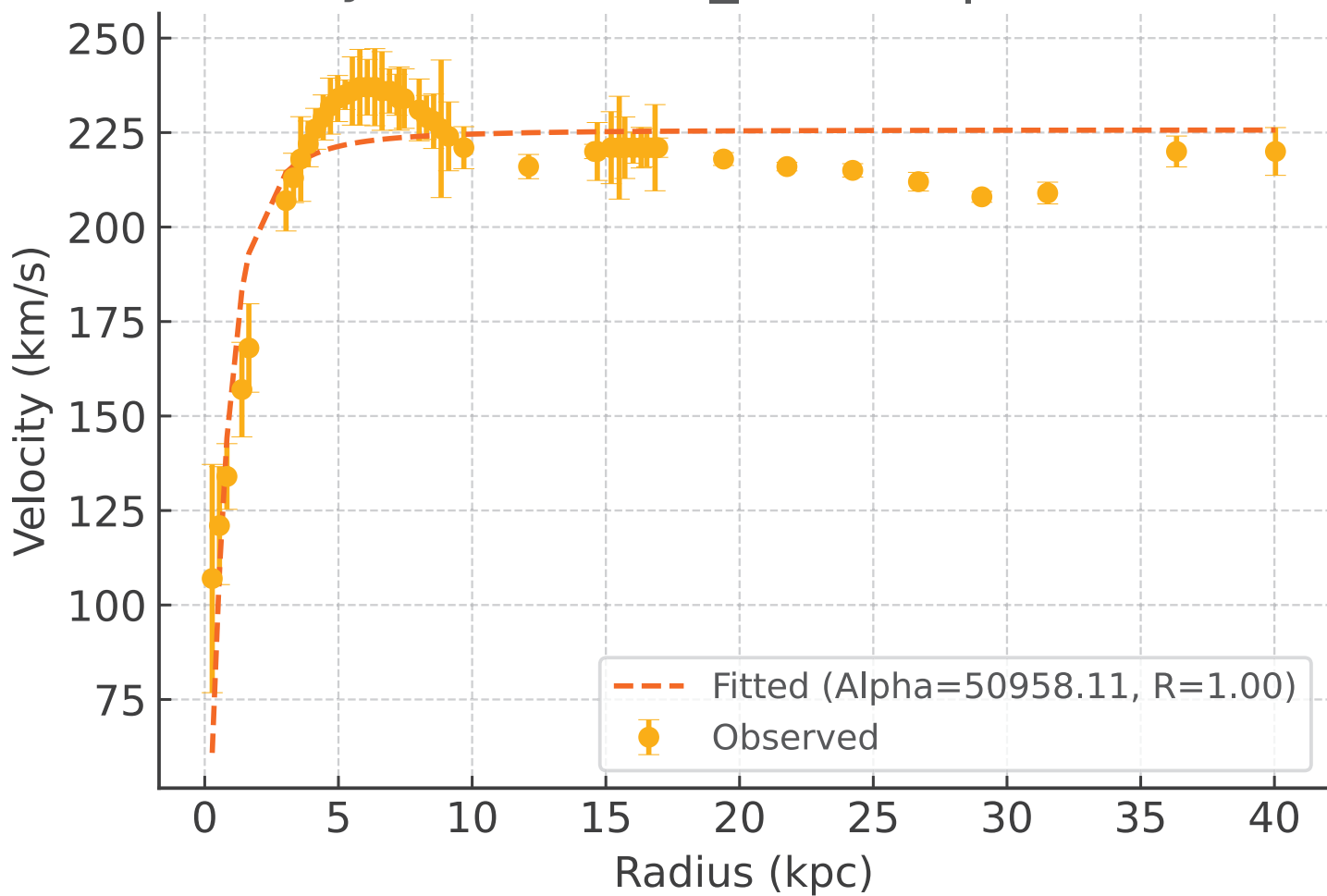


Galaxy: UGC02953\_rotmod |  $R^2 = -1.210$

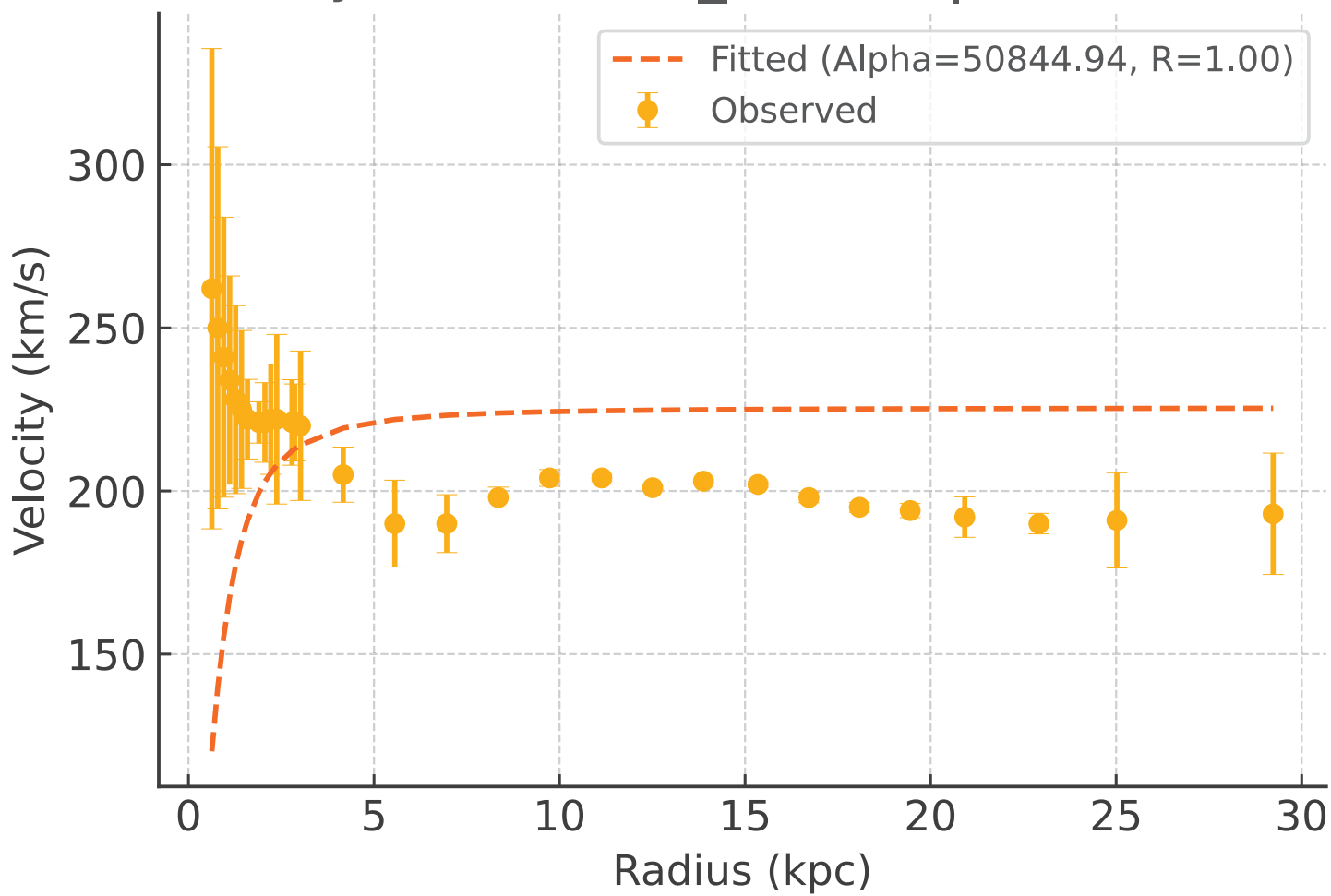




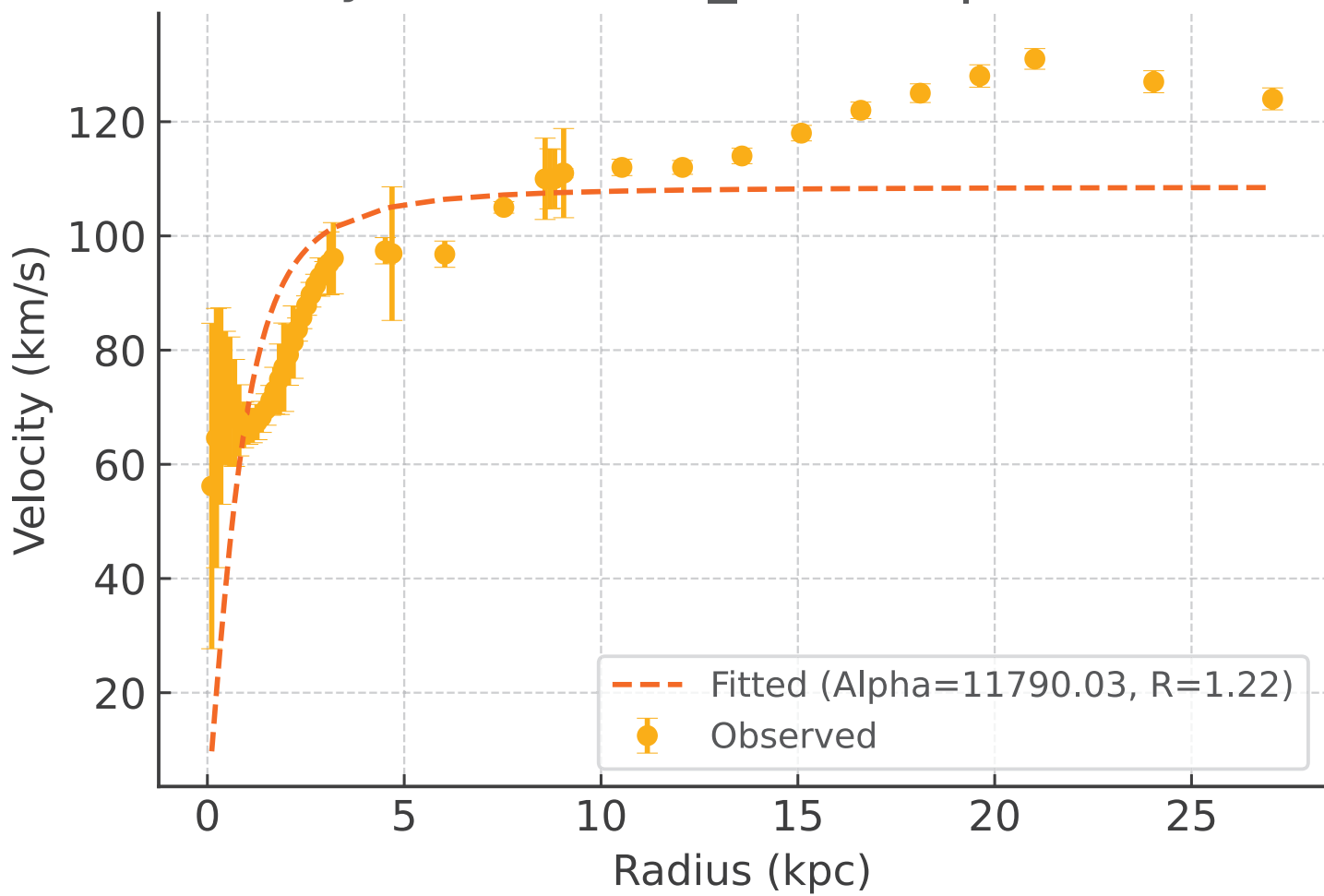
# Galaxy: UGC03205\_rotmod | $R^2 = 0.815$



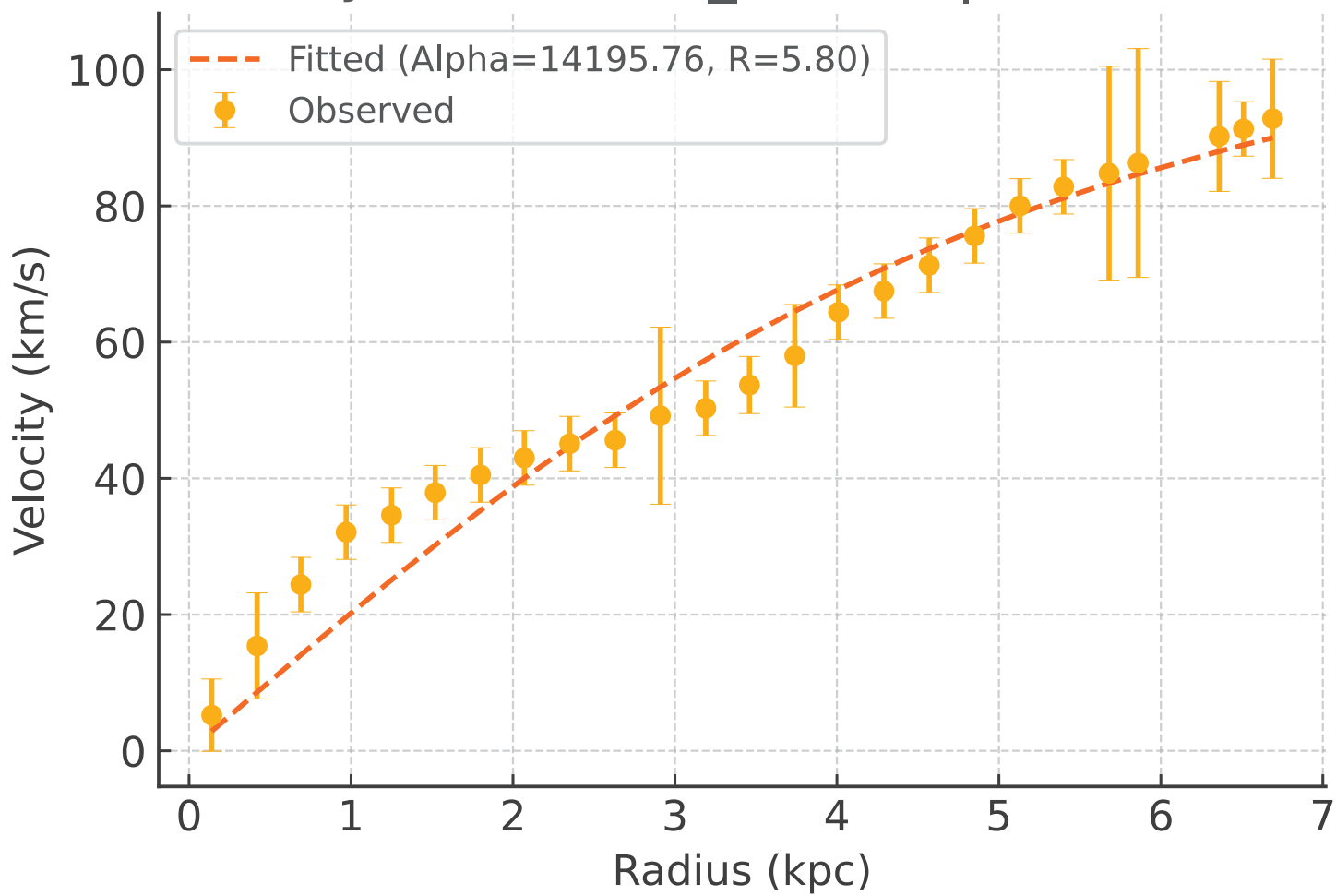
# Galaxy: UGC03546\_rotmod | $R^2 = -5.009$



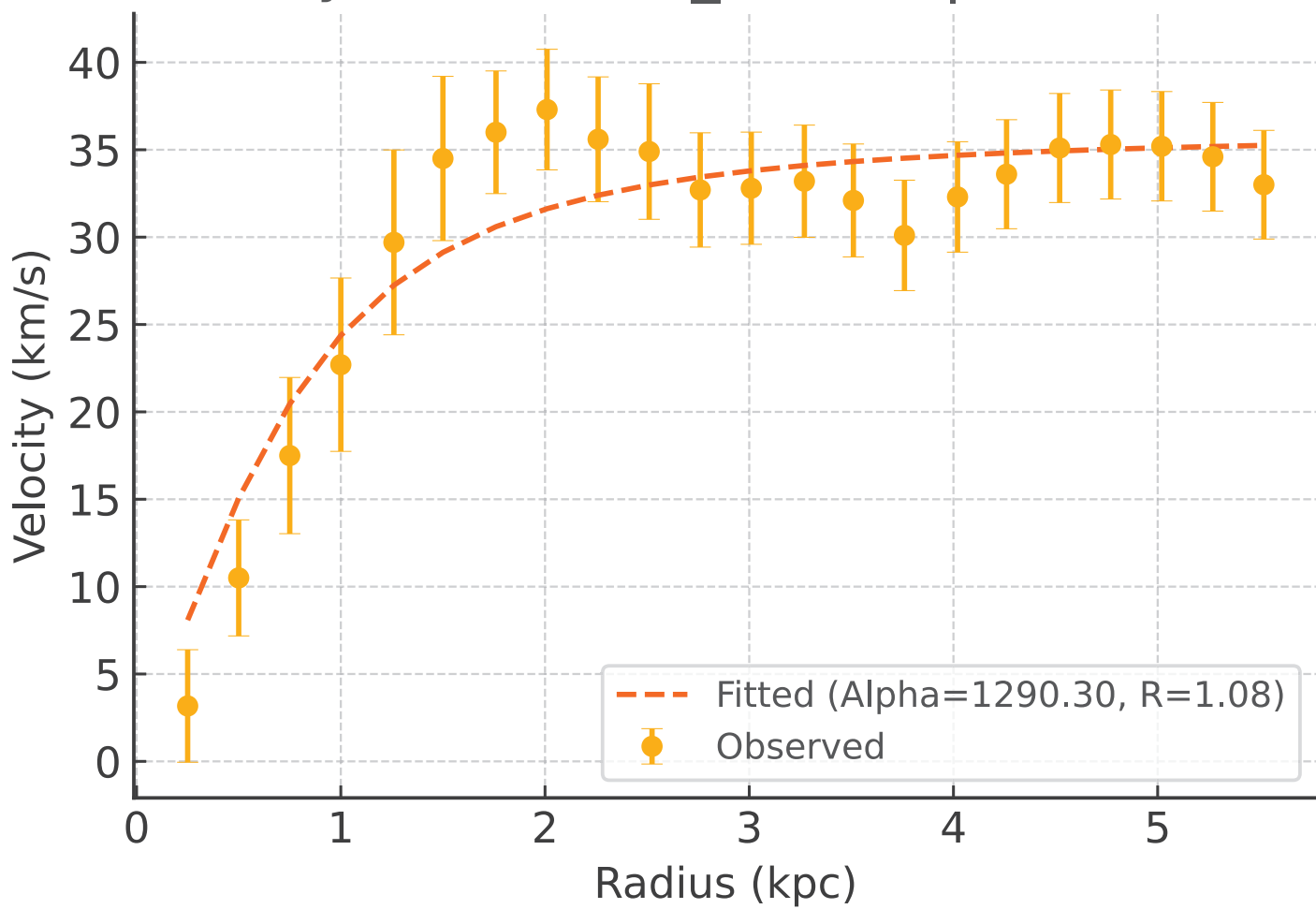
Galaxy: UGC03580\_rotmod |  $R^2 = 0.385$



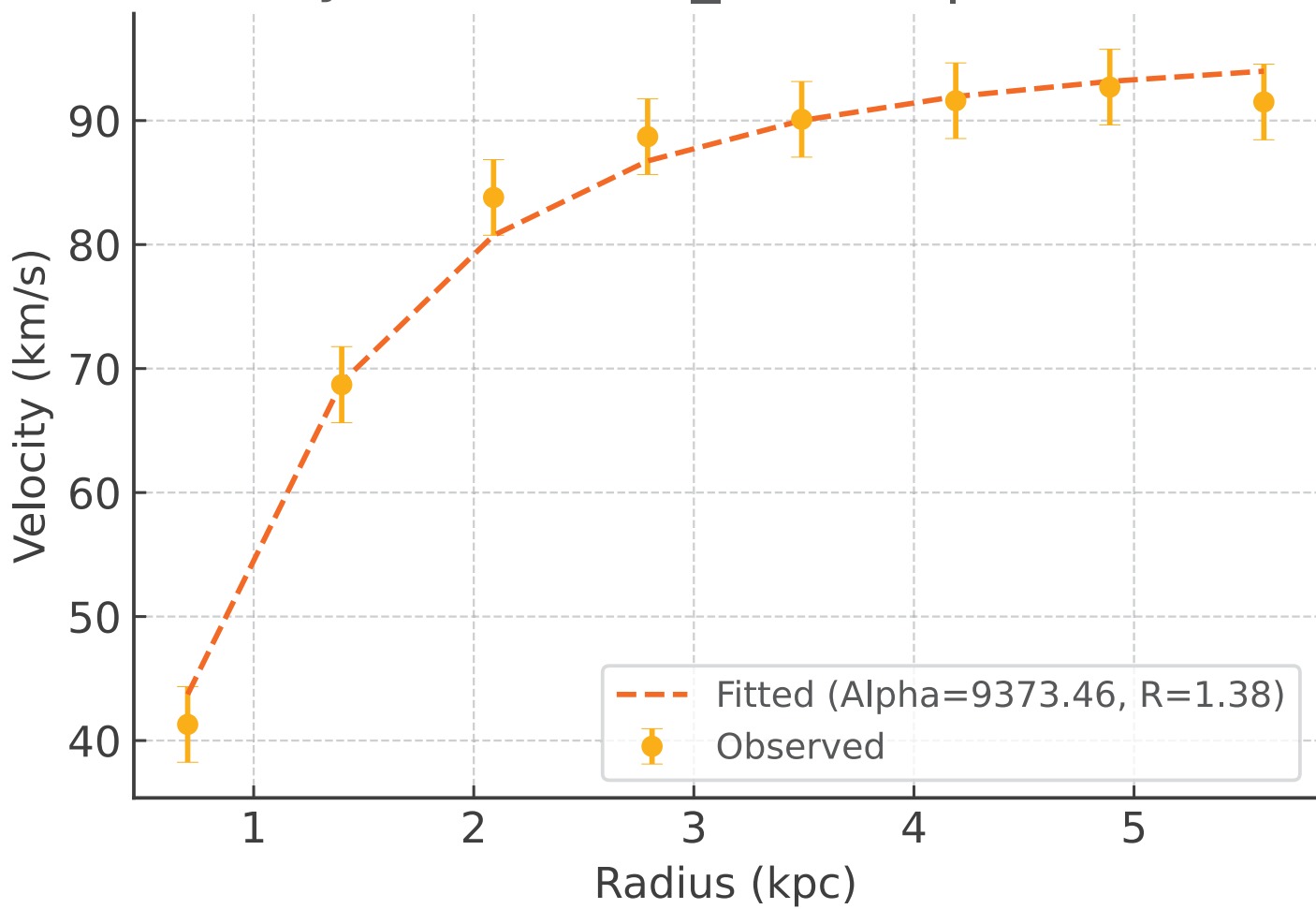
# Galaxy: UGC04278\_rotmod | $R^2 = 0.951$



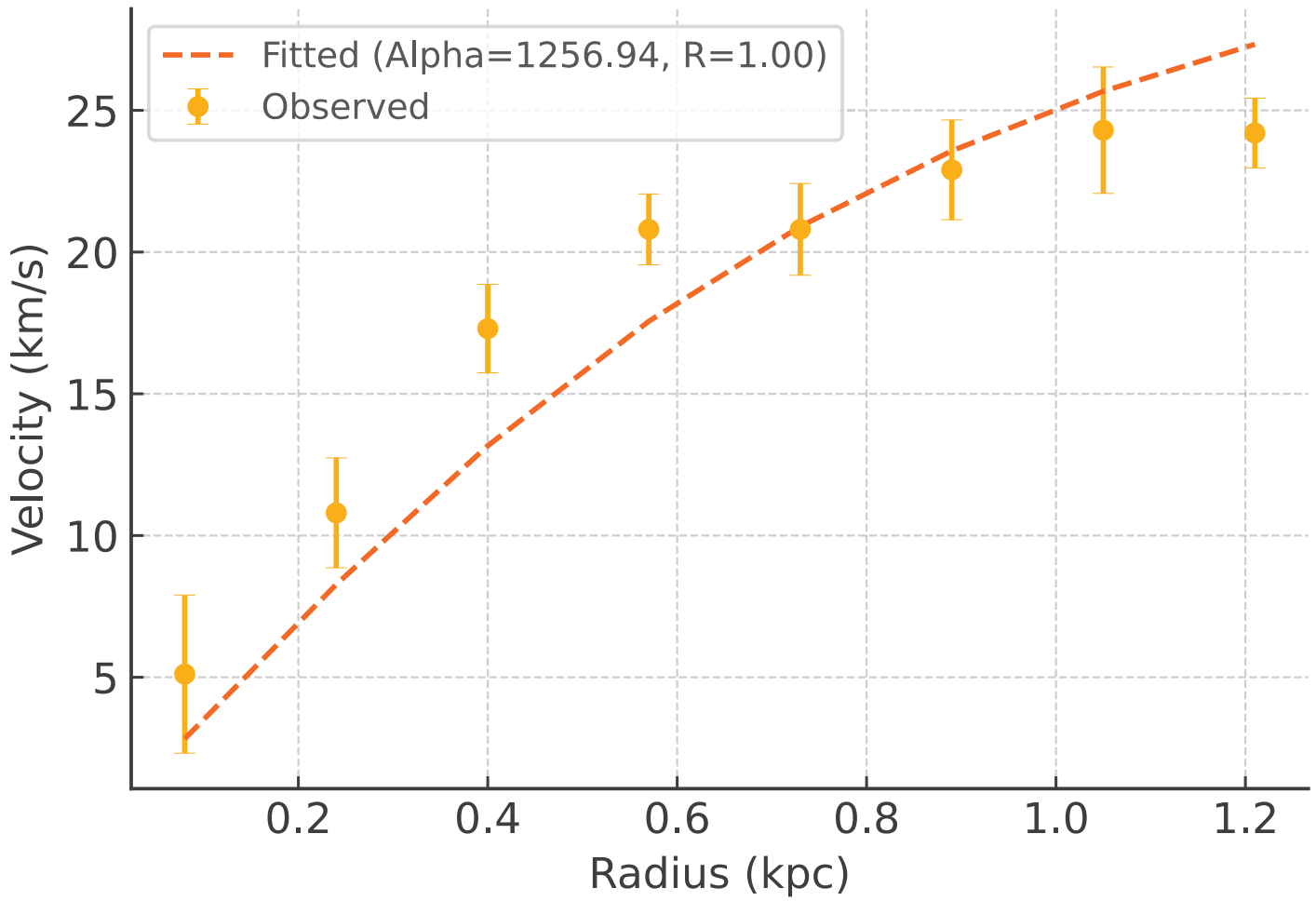
Galaxy: UGC04305\_rotmod |  $R^2 = 0.874$



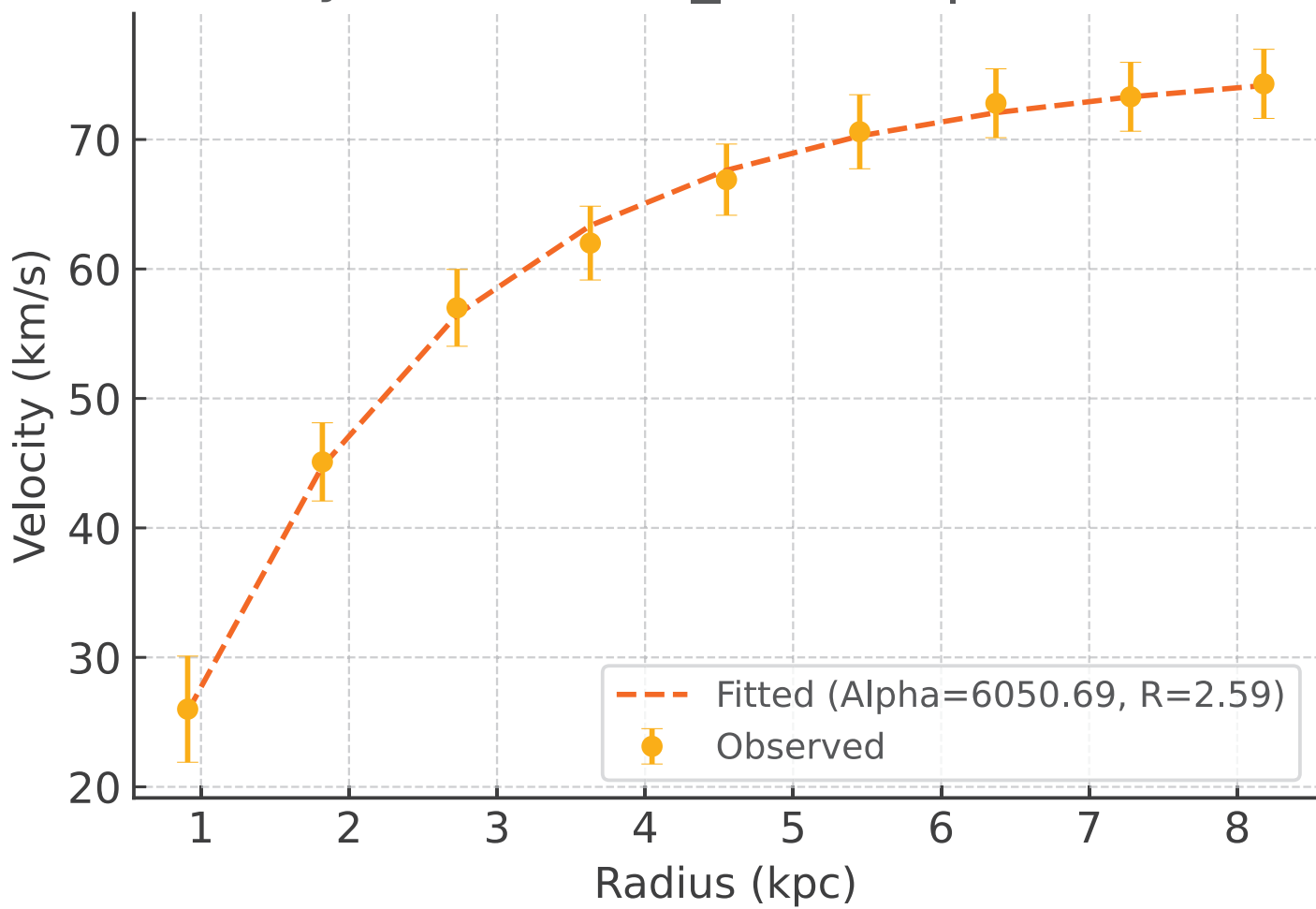
Galaxy: UGC04325\_rotmod |  $R^2 = 0.989$



# Galaxy: UGC04483\_rotmod | $R^2 = 0.847$

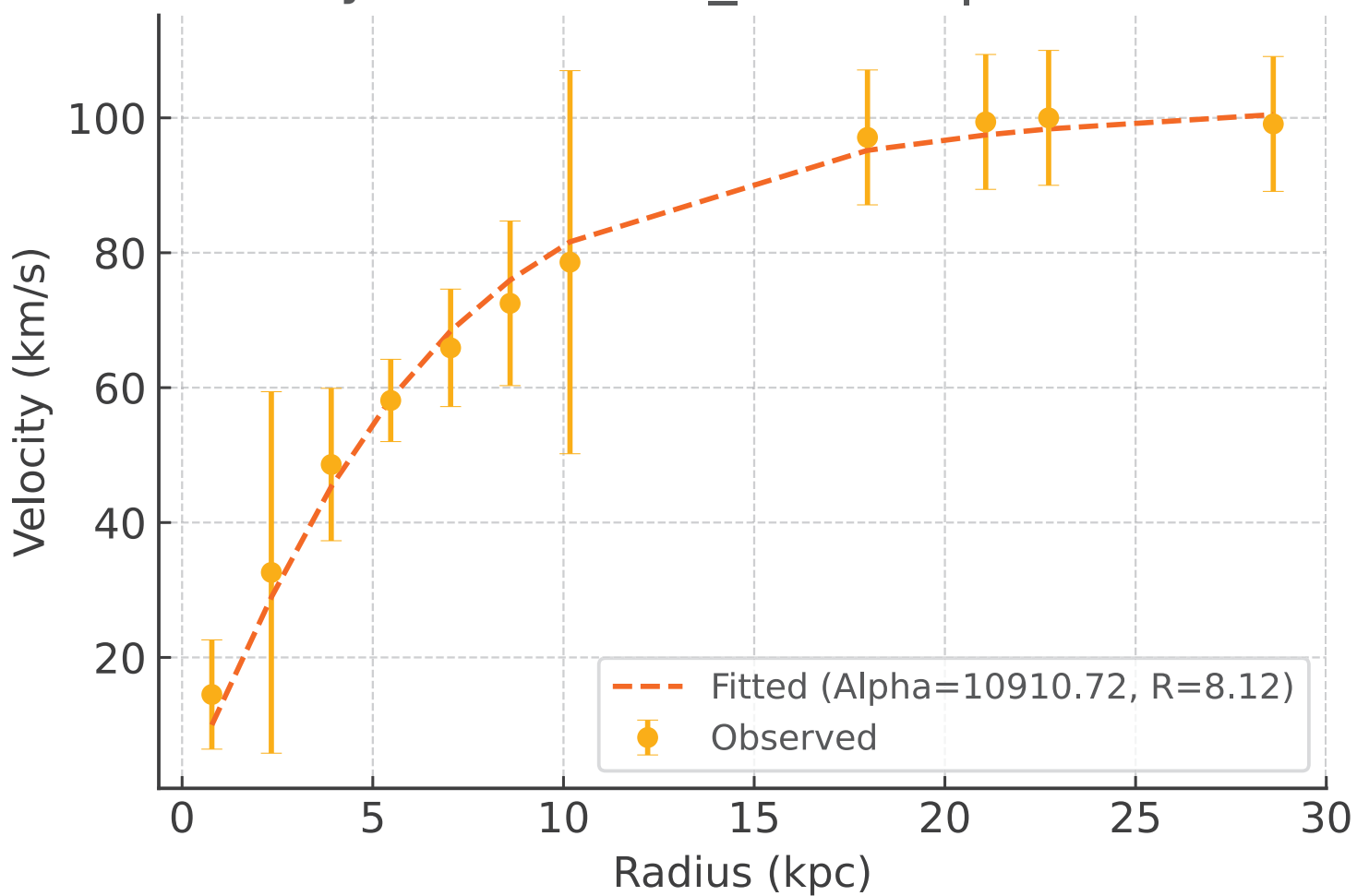


Galaxy: UGC04499\_rotmod |  $R^2 = 0.998$

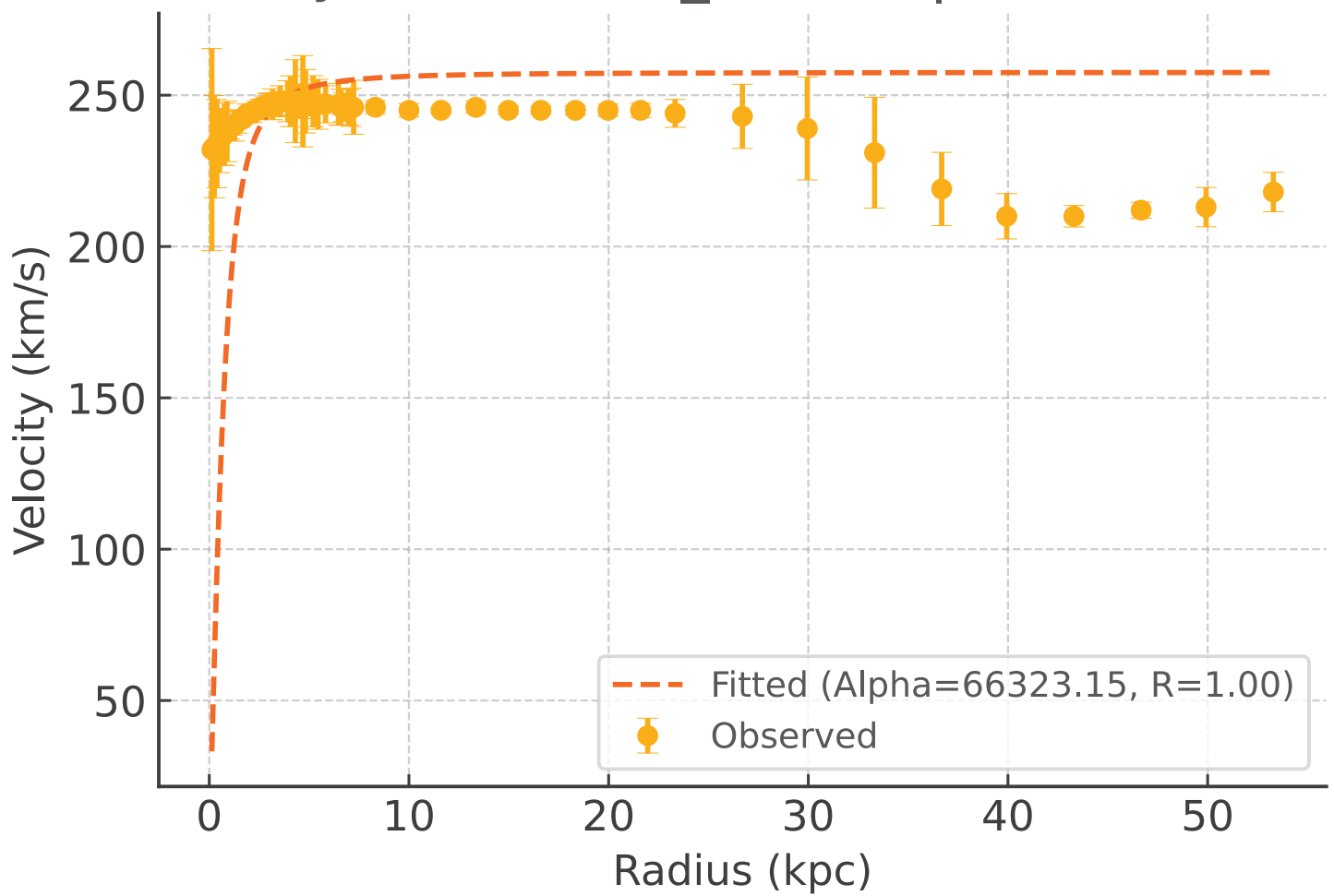




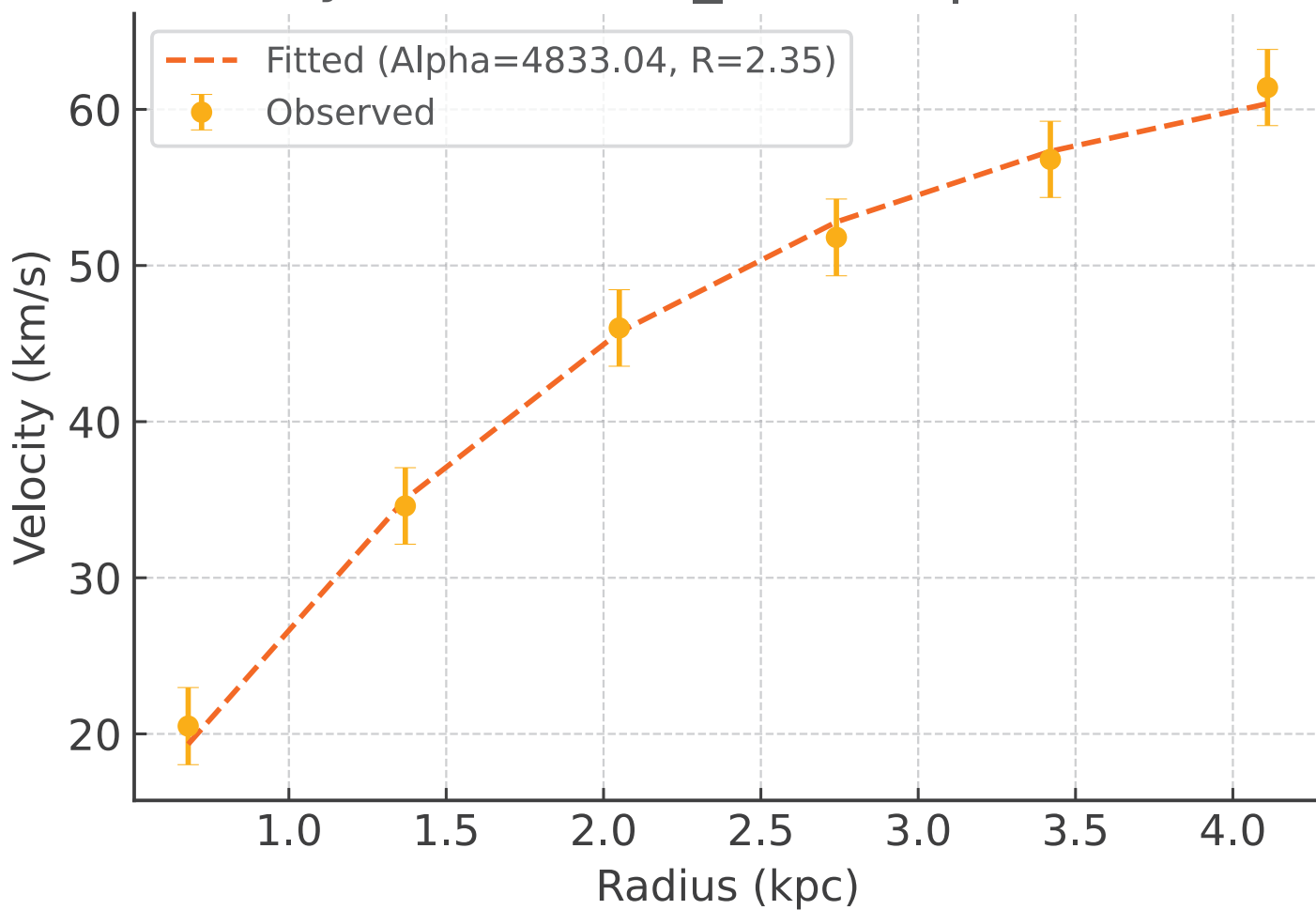
Galaxy: UGC05005\_rotmod |  $R^2 = 0.990$



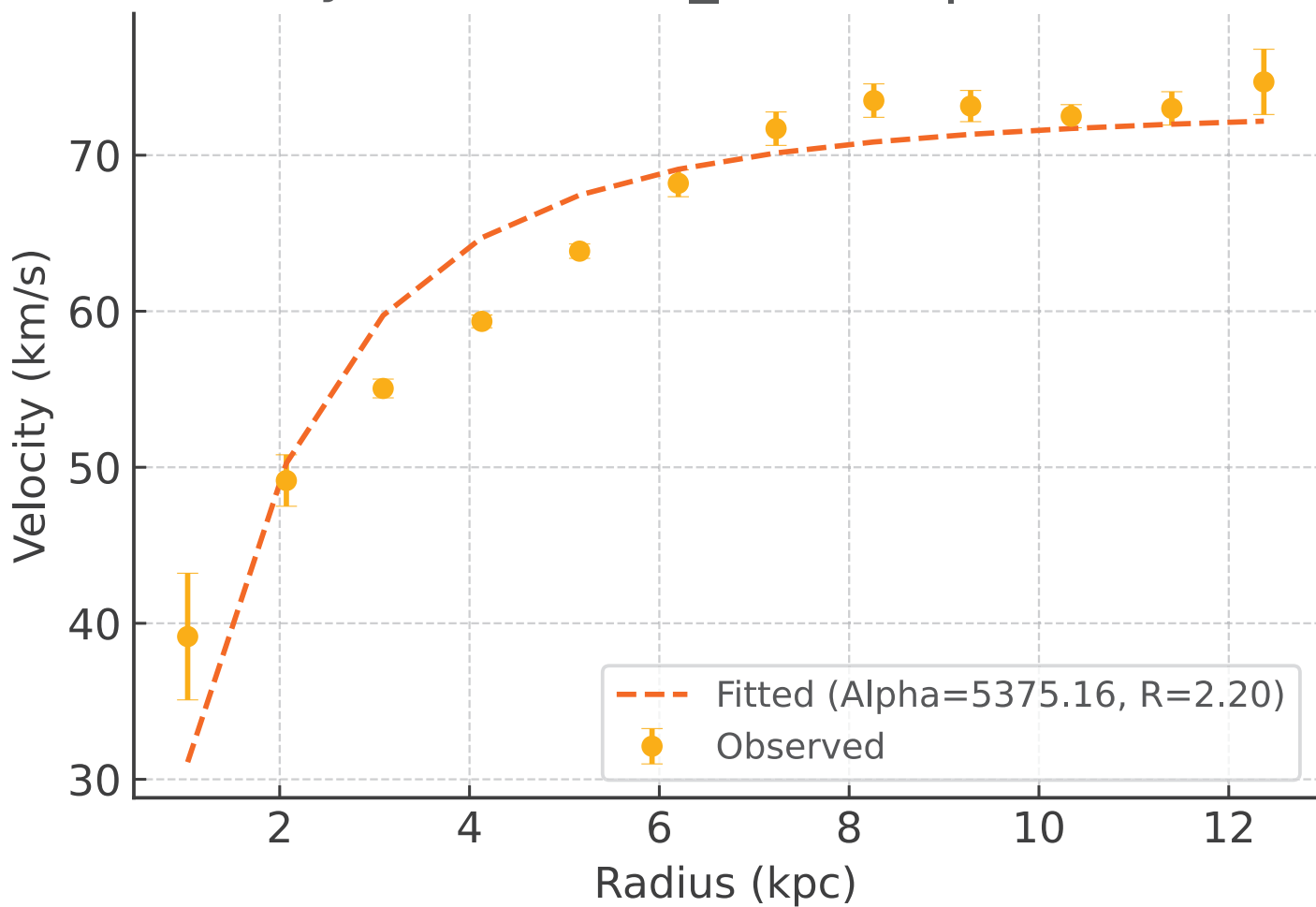
Galaxy: UGC05253\_rotmod |  $R^2 = -21.930$



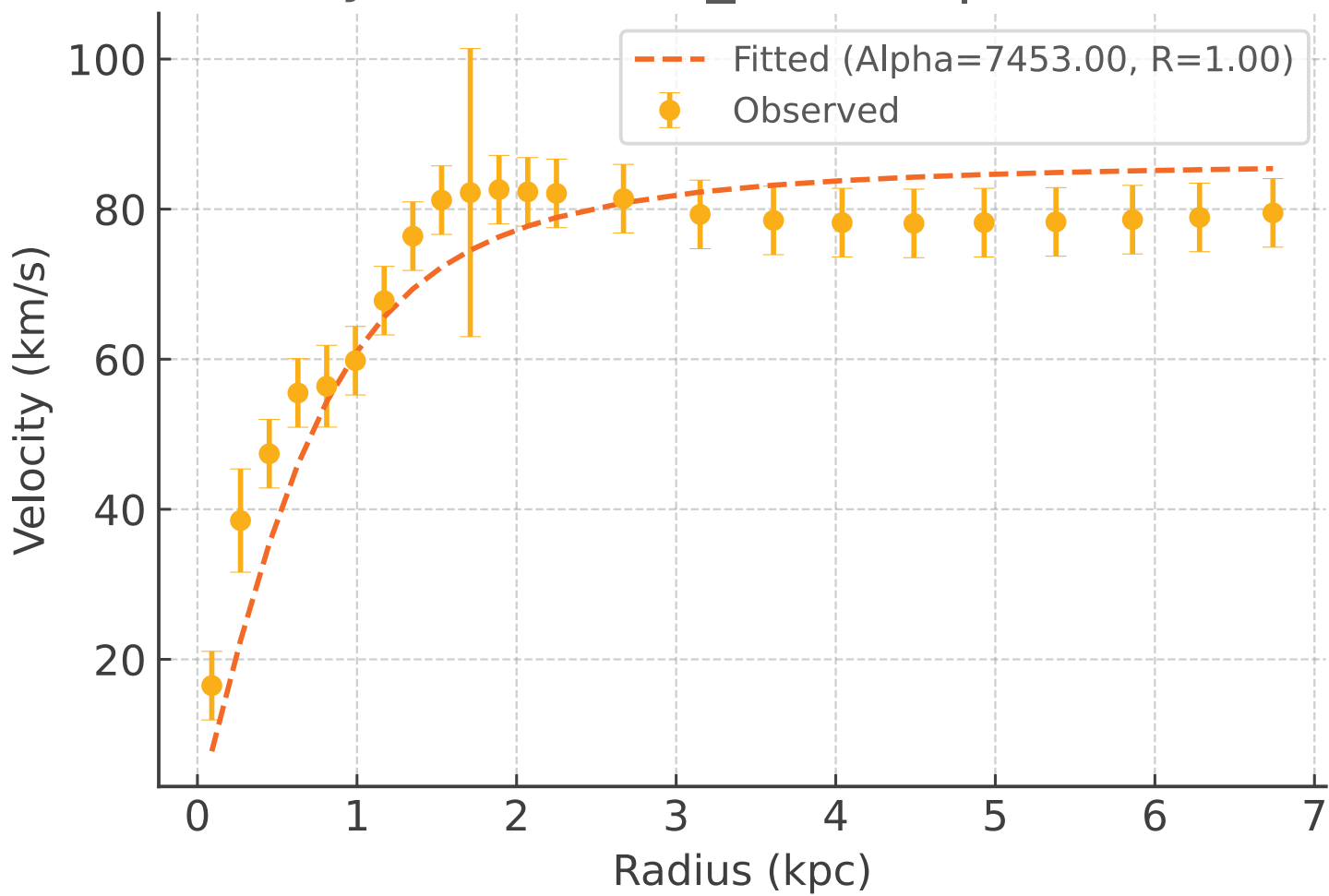
# Galaxy: UGC05414\_rotmod | $R^2 = 0.997$



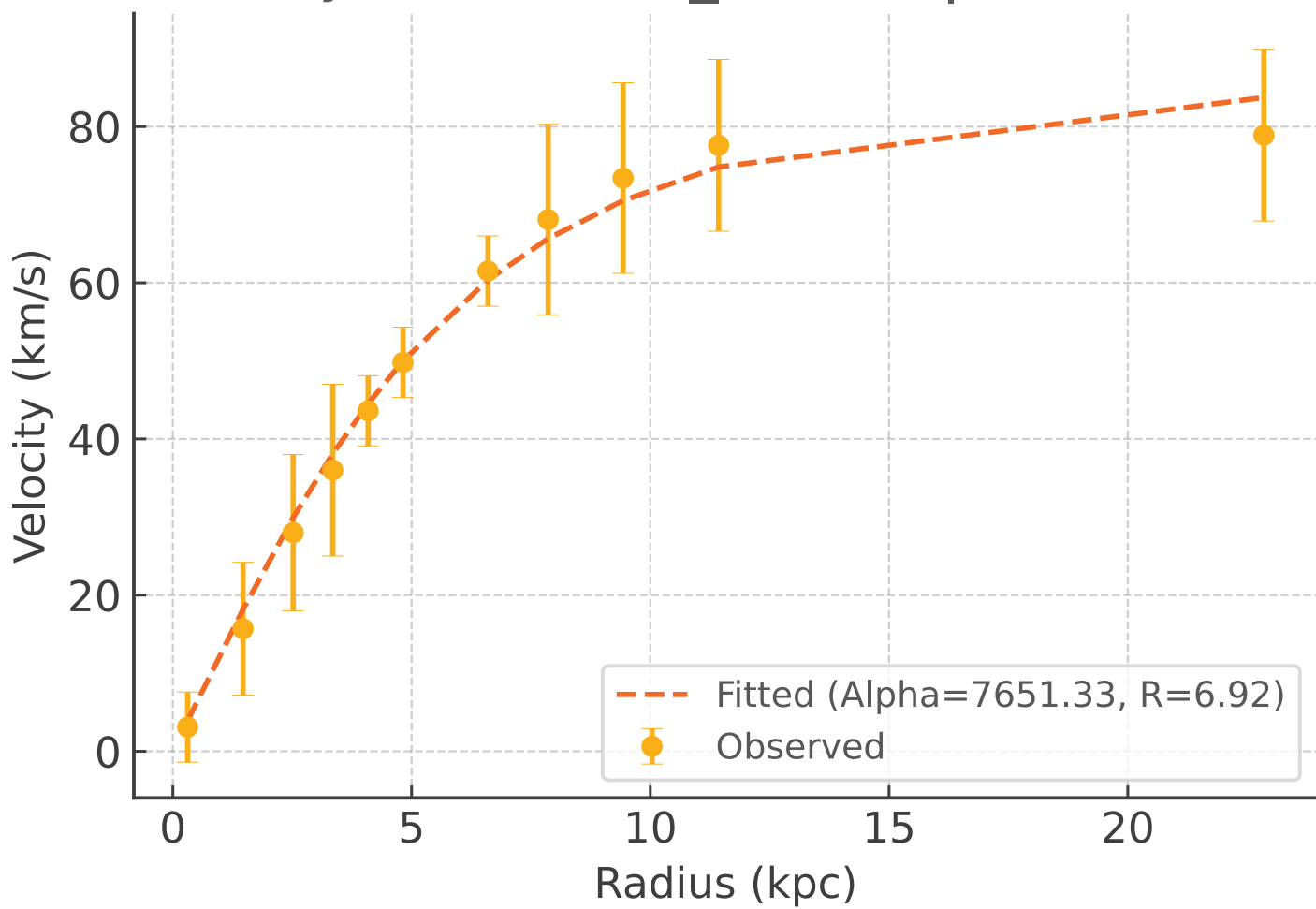
Galaxy: UGC05716\_rotmod |  $R^2 = 0.896$



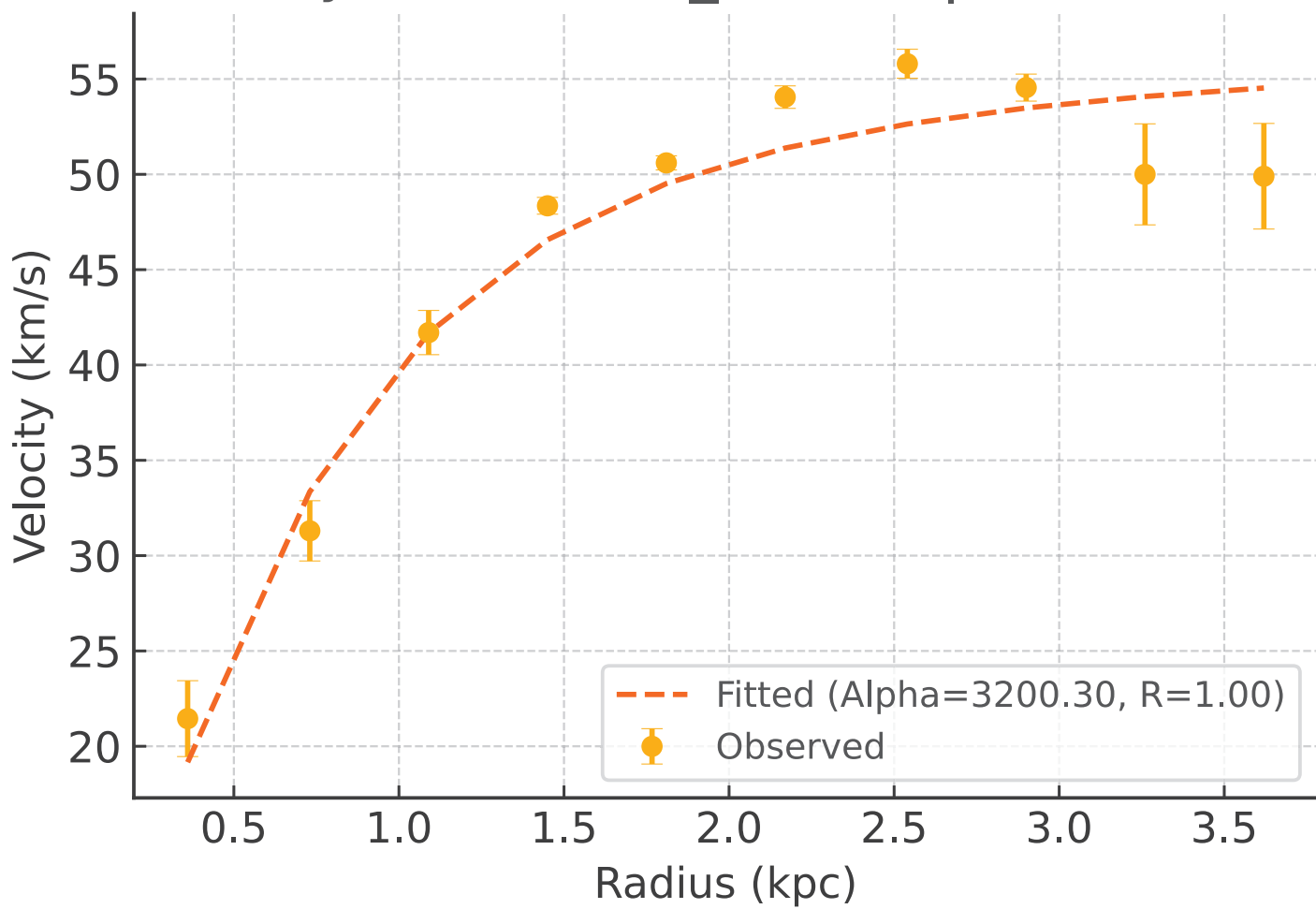
# Galaxy: UGC05721\_rotmod | $R^2 = 0.823$



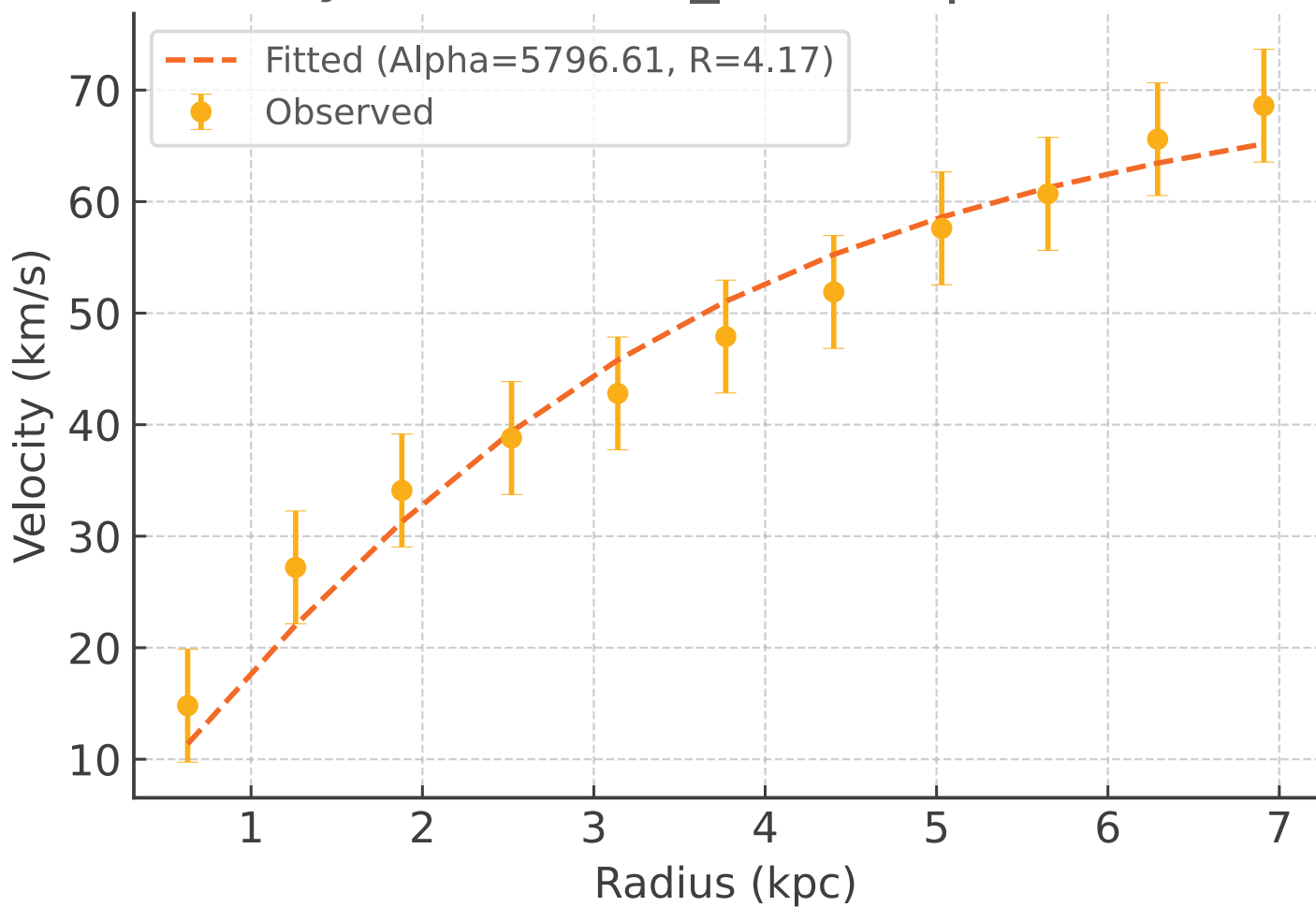
Galaxy: UGC05750\_rotmod |  $R^2 = 0.991$



Galaxy: UGC05764\_rotmod |  $R^2 = 0.938$

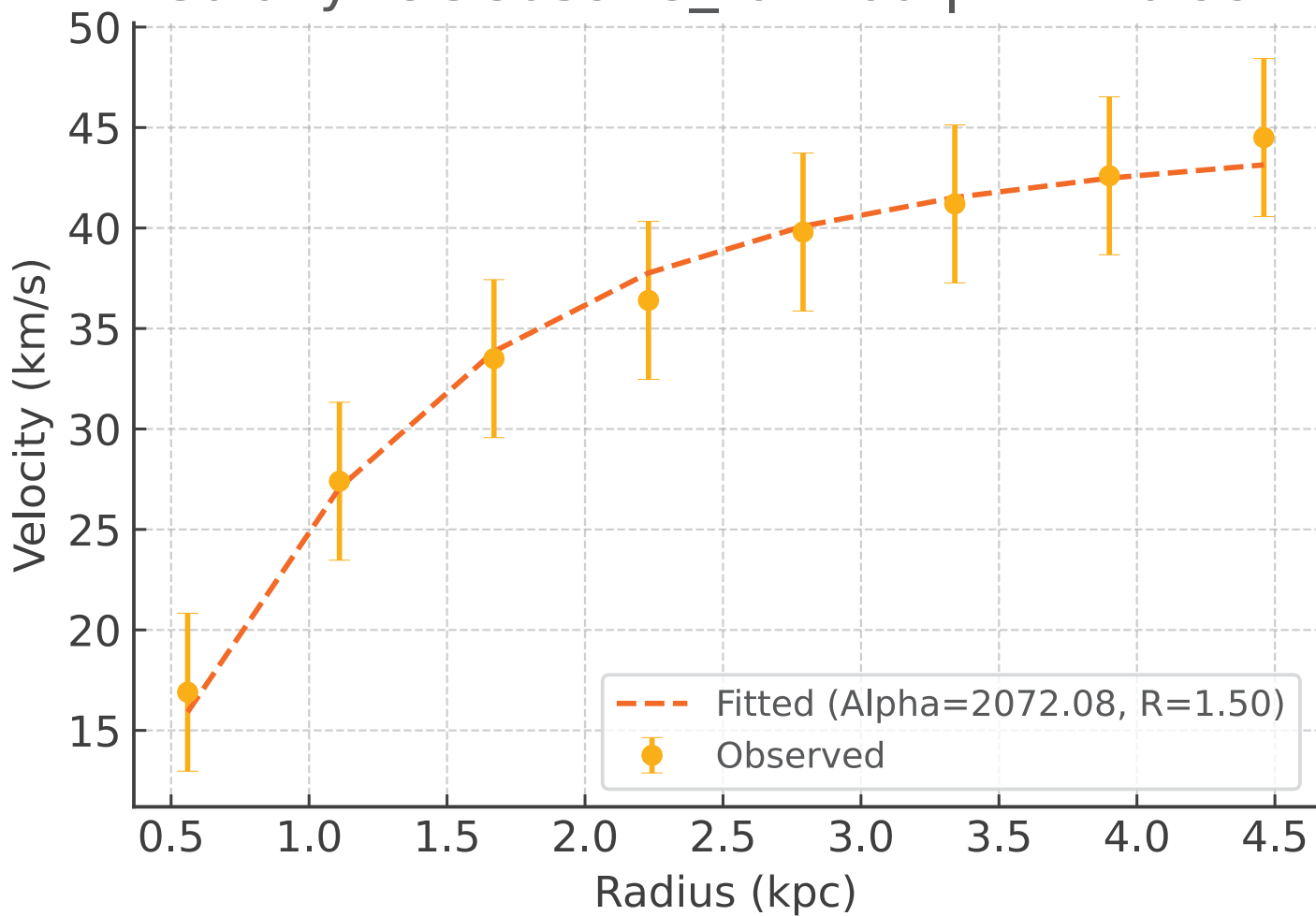


# Galaxy: UGC05829\_rotmod | $R^2 = 0.966$

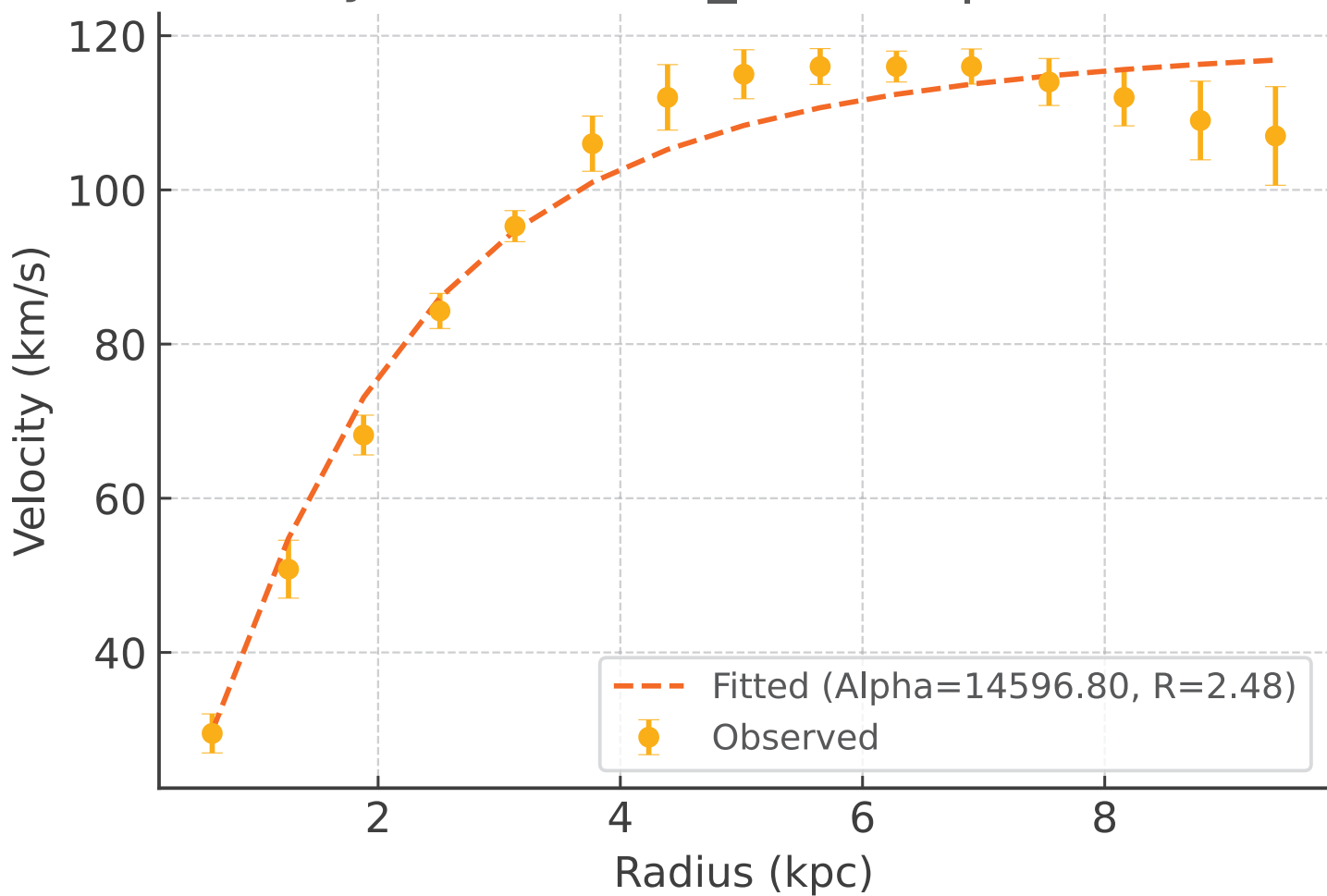




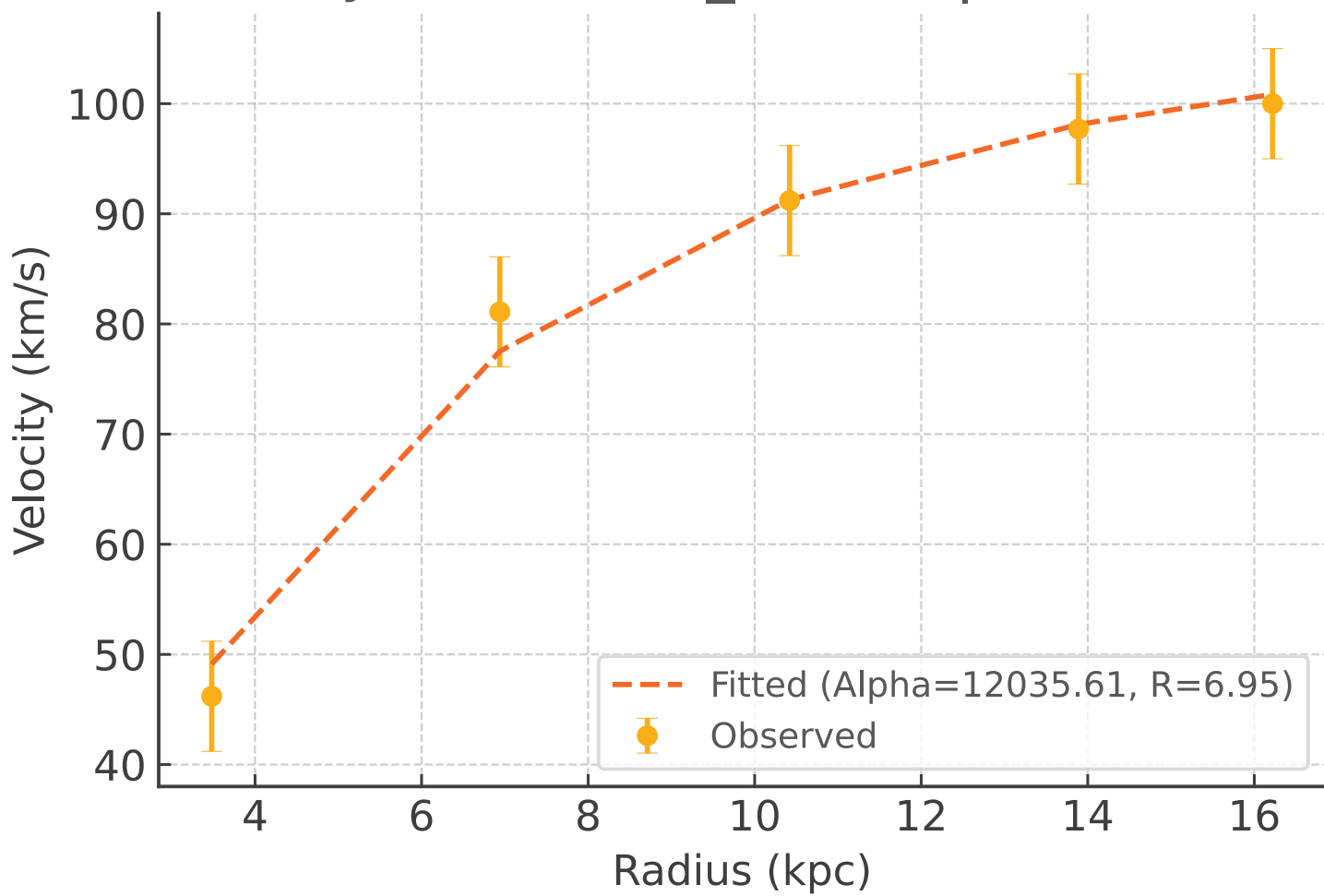
Galaxy: UGC05918\_rotmod |  $R^2 = 0.991$



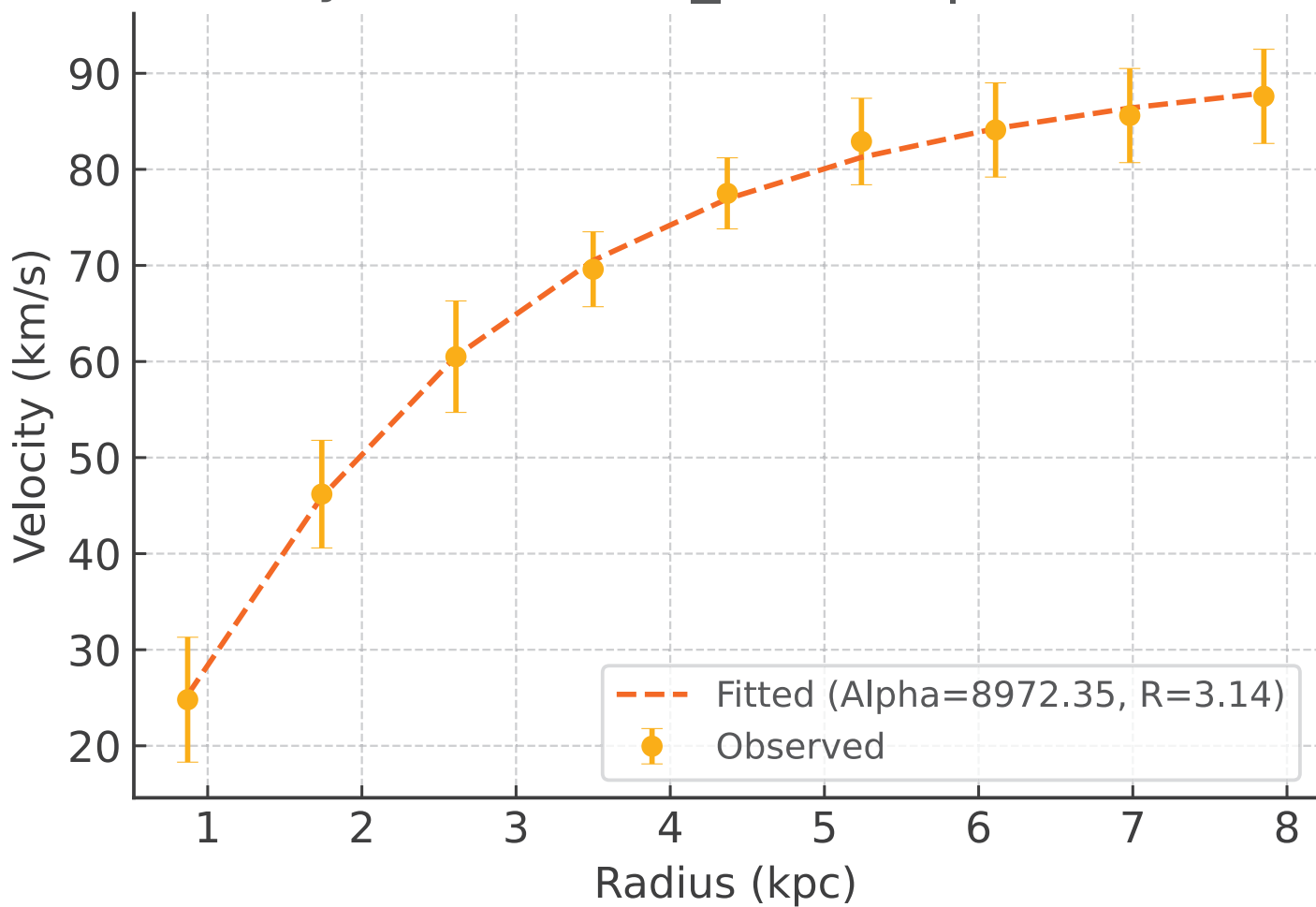
Galaxy: UGC05986\_rotmod |  $R^2 = 0.964$



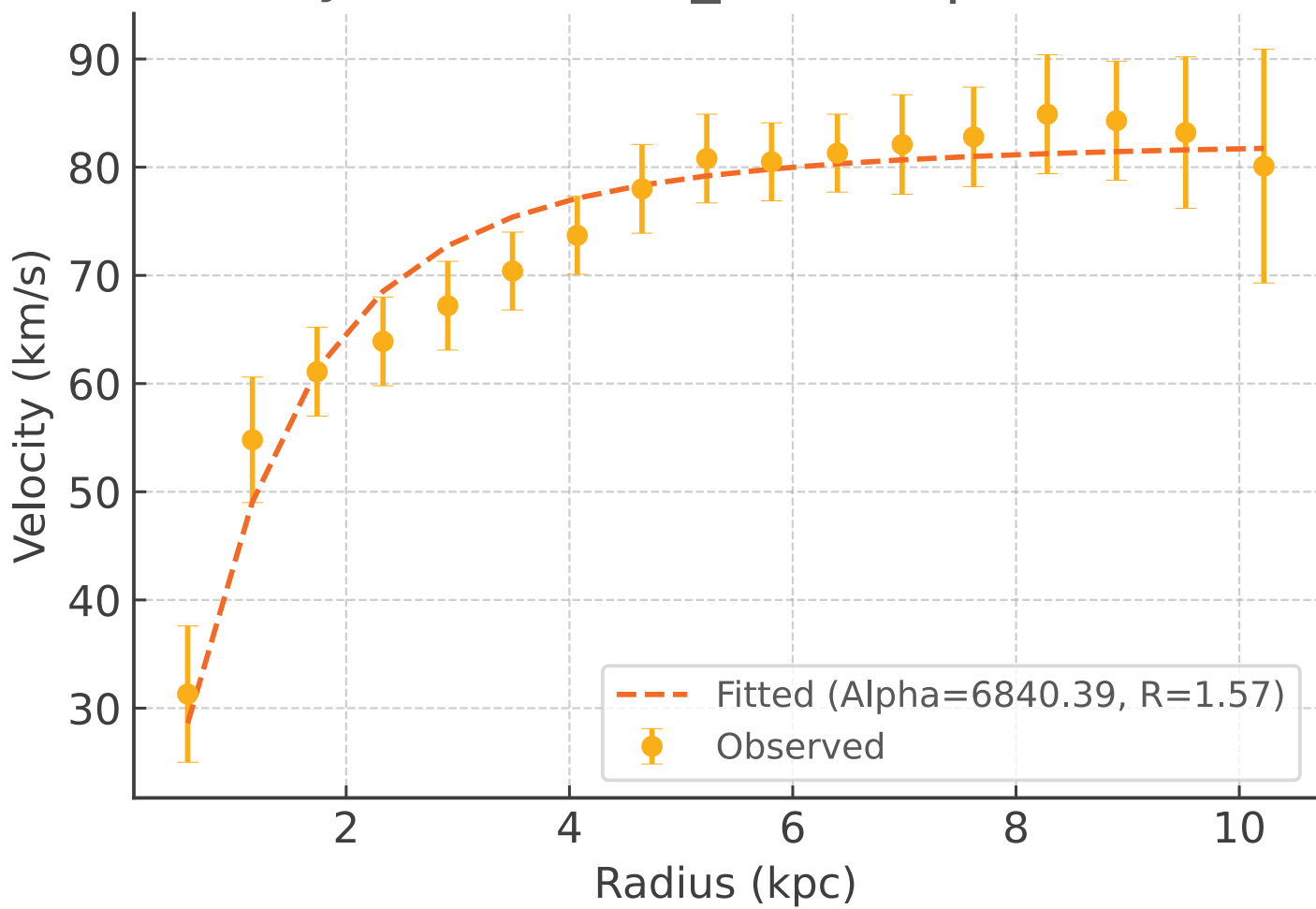
Galaxy: UGC05999\_rotmod |  $R^2 = 0.988$



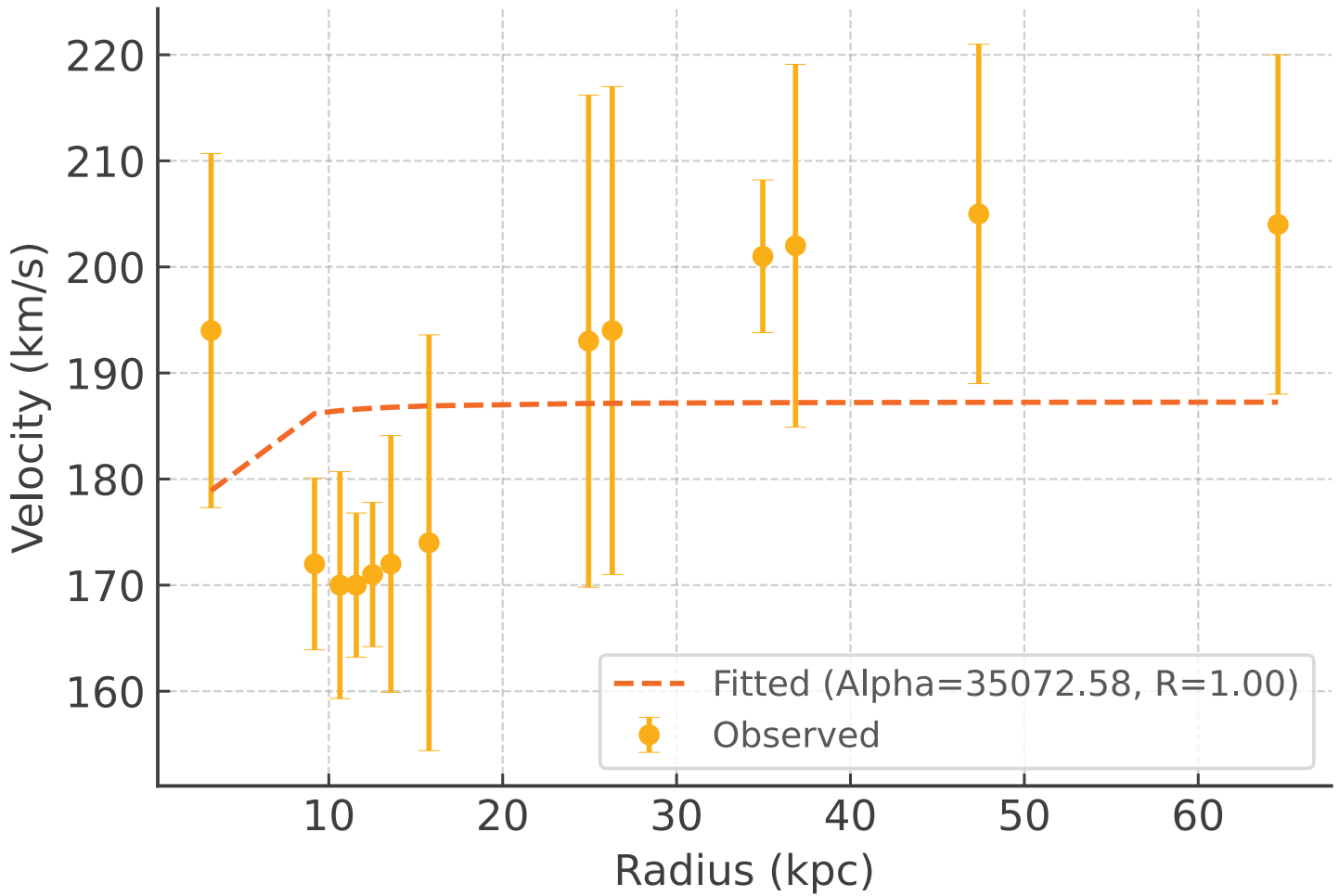
# Galaxy: UGC06399\_rotmod | $R^2 = 0.999$



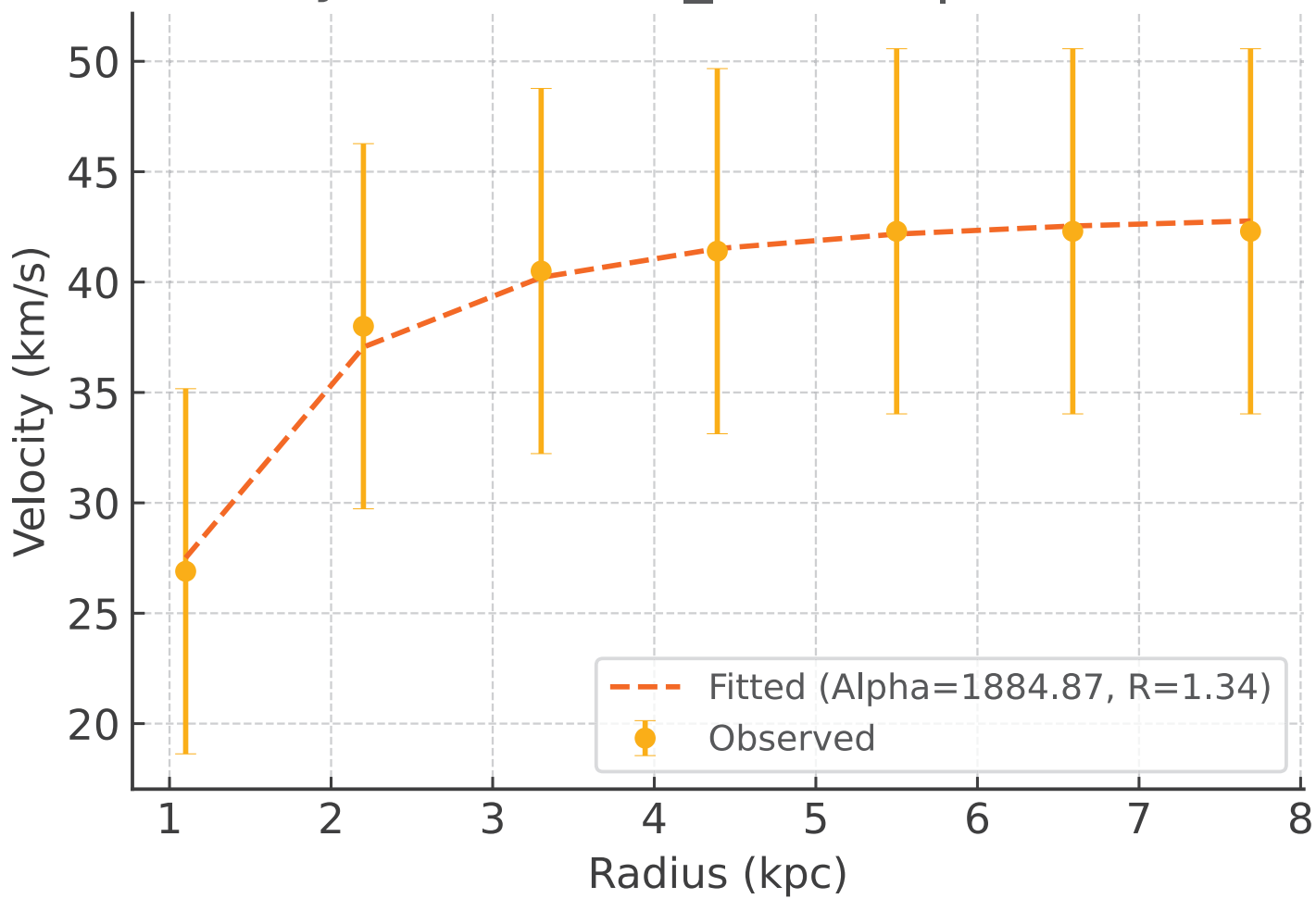
Galaxy: UGC06446\_rotmod |  $R^2 = 0.947$



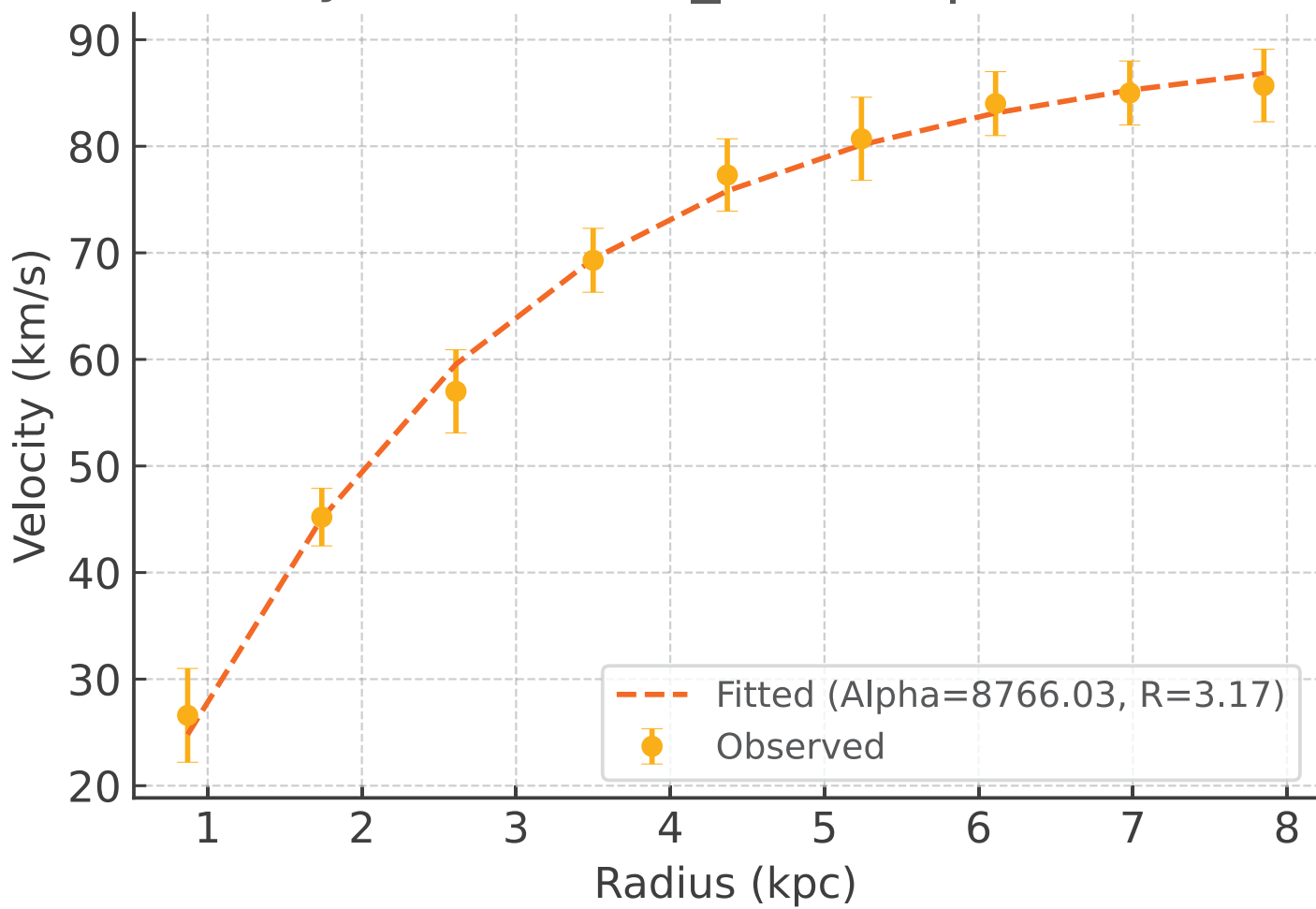
Galaxy: UGC06614\_rotmod |  $R^2 = -0.030$



Galaxy: UGC06628\_rotmod |  $R^2 = 0.991$

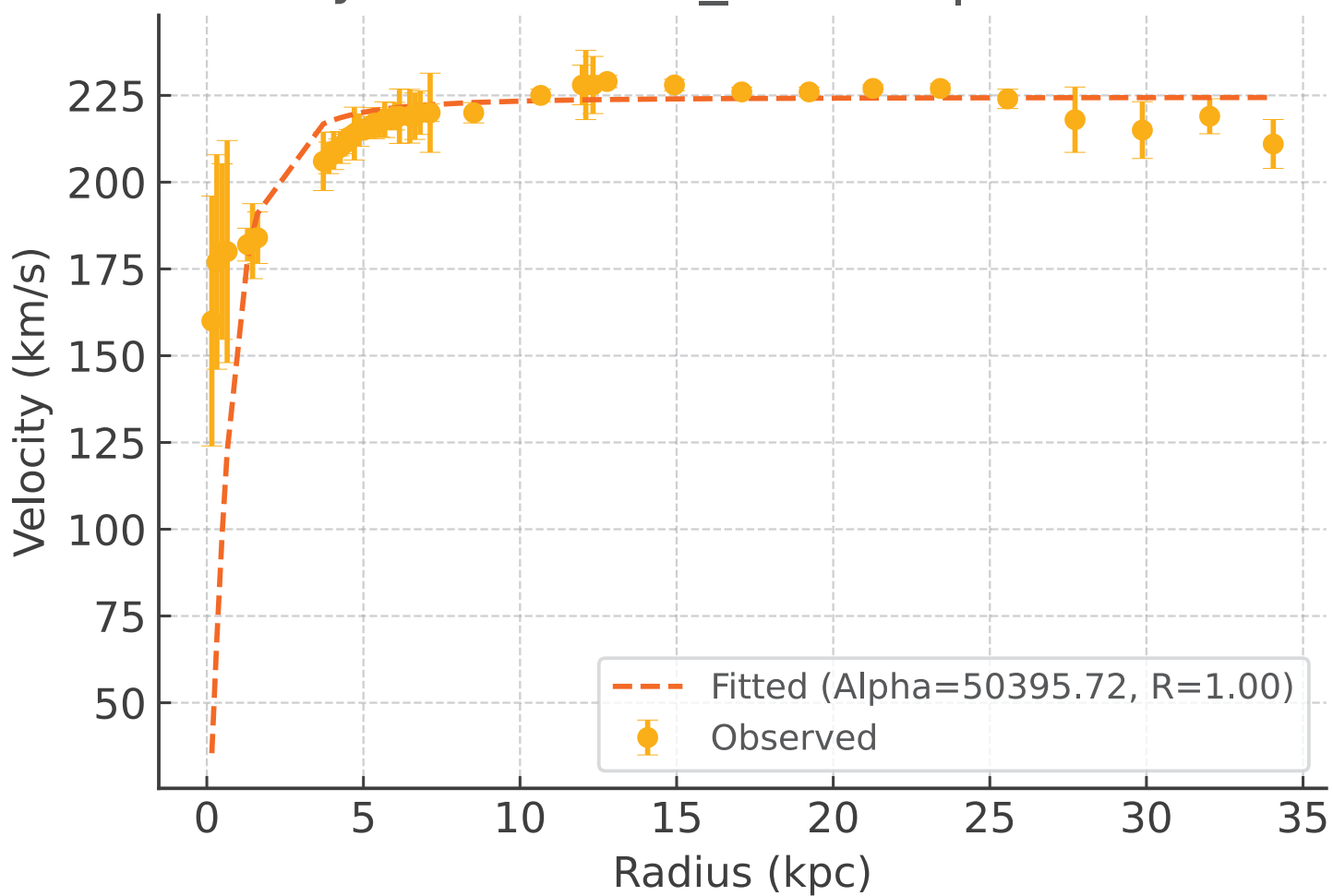


Galaxy: UGC06667\_rotmod |  $R^2 = 0.996$

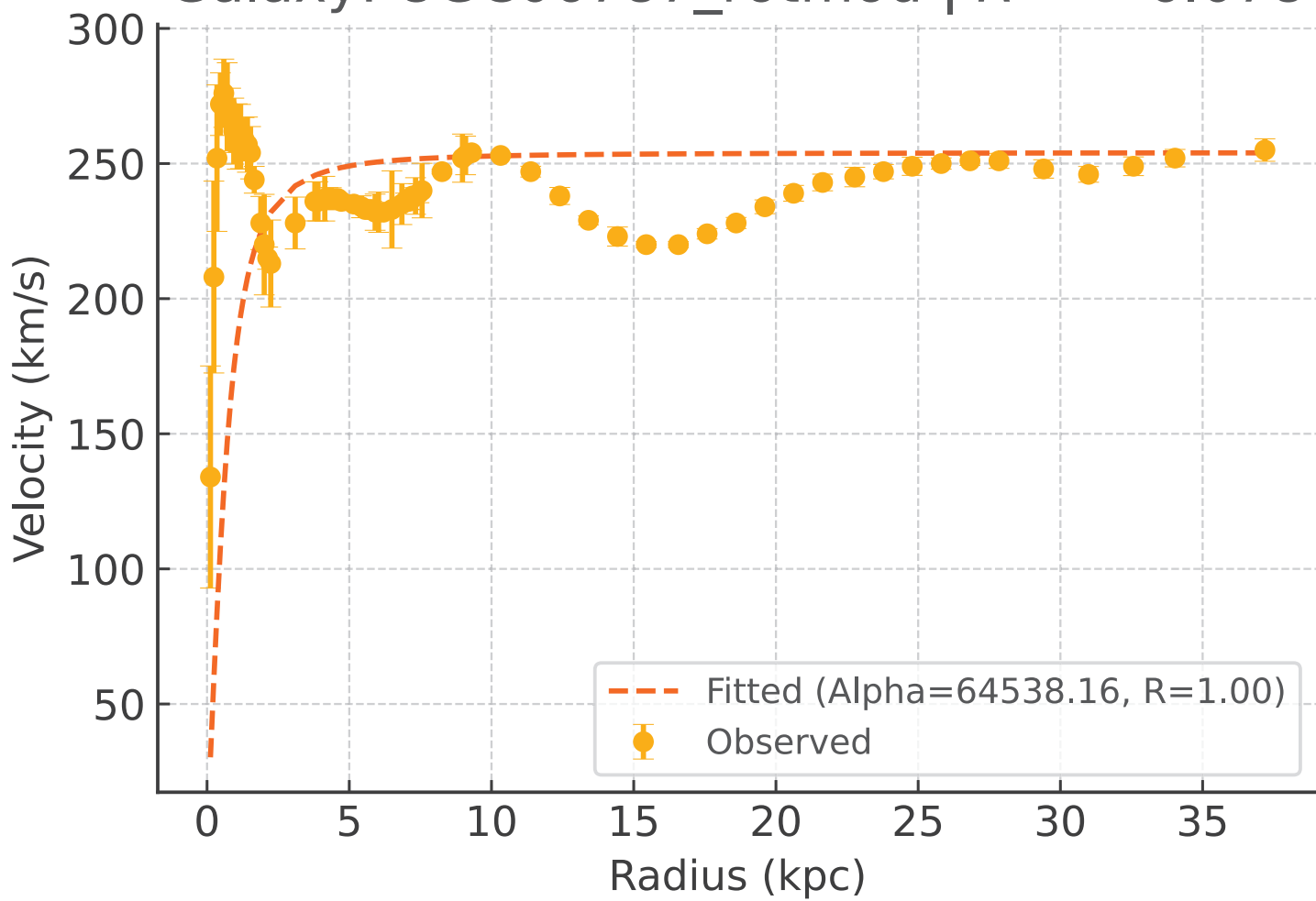




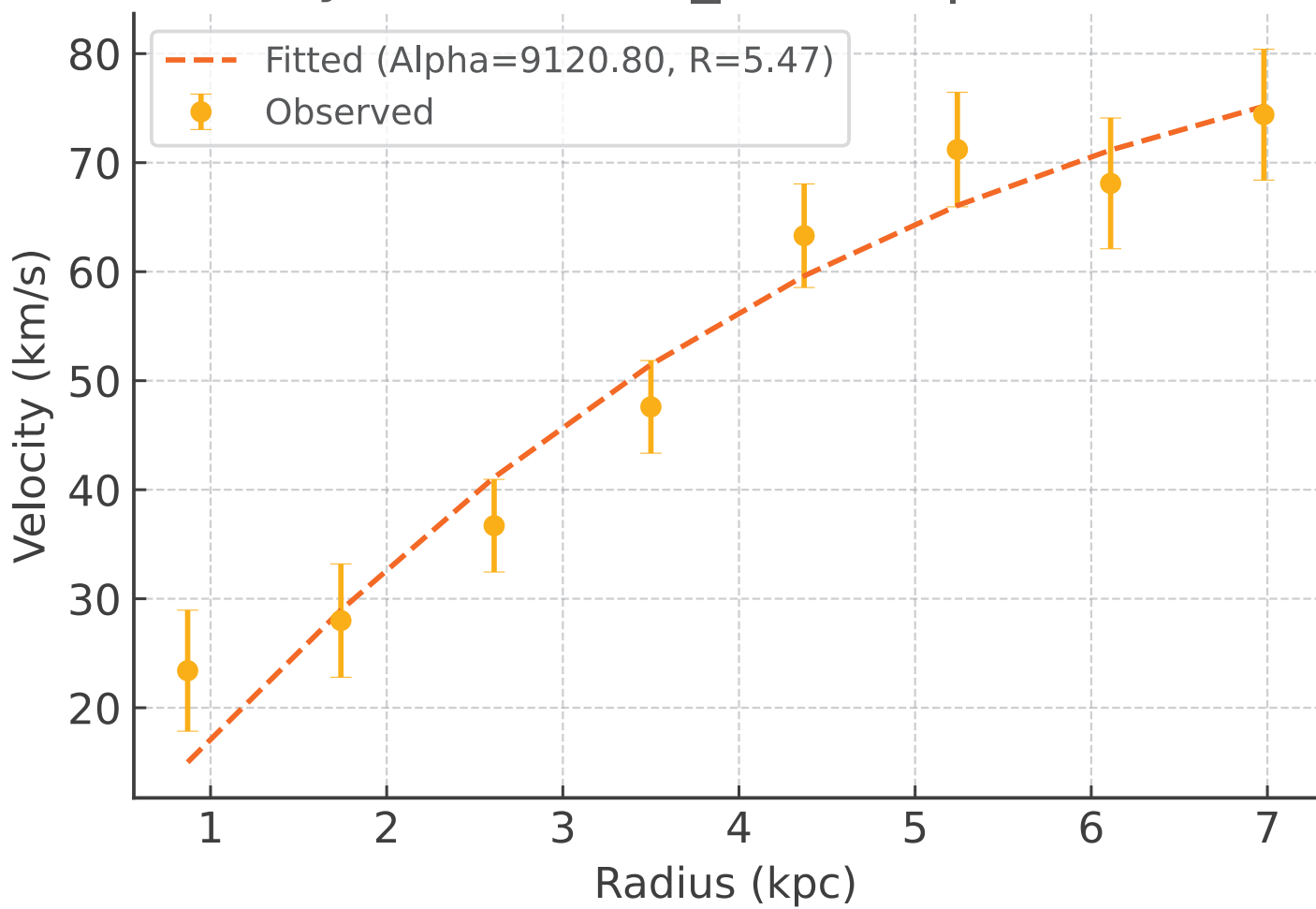
Galaxy: UGC06786\_rotmod |  $R^2 = -2.277$



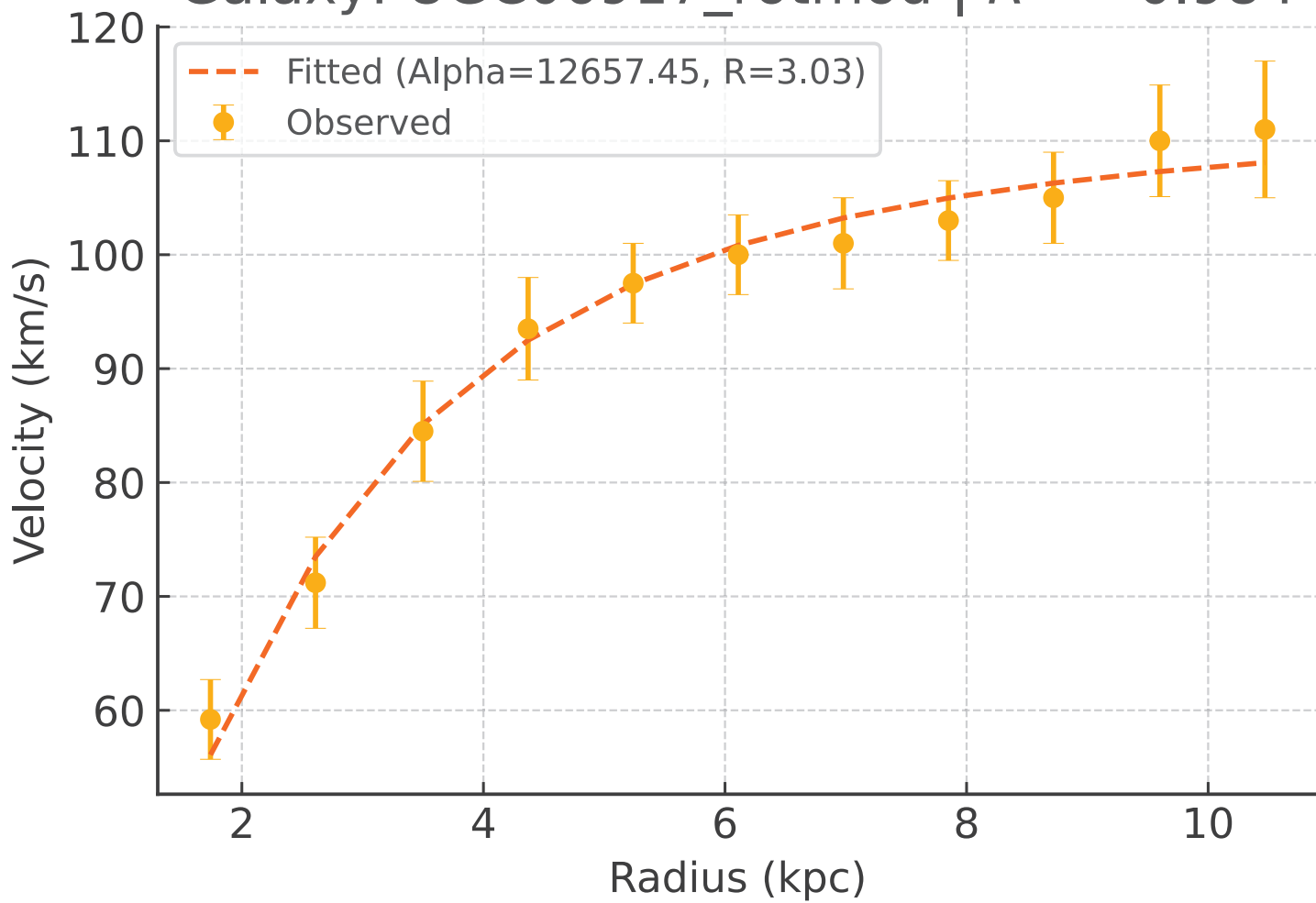
Galaxy: UGC06787\_rotmod |  $R^2 = -6.078$



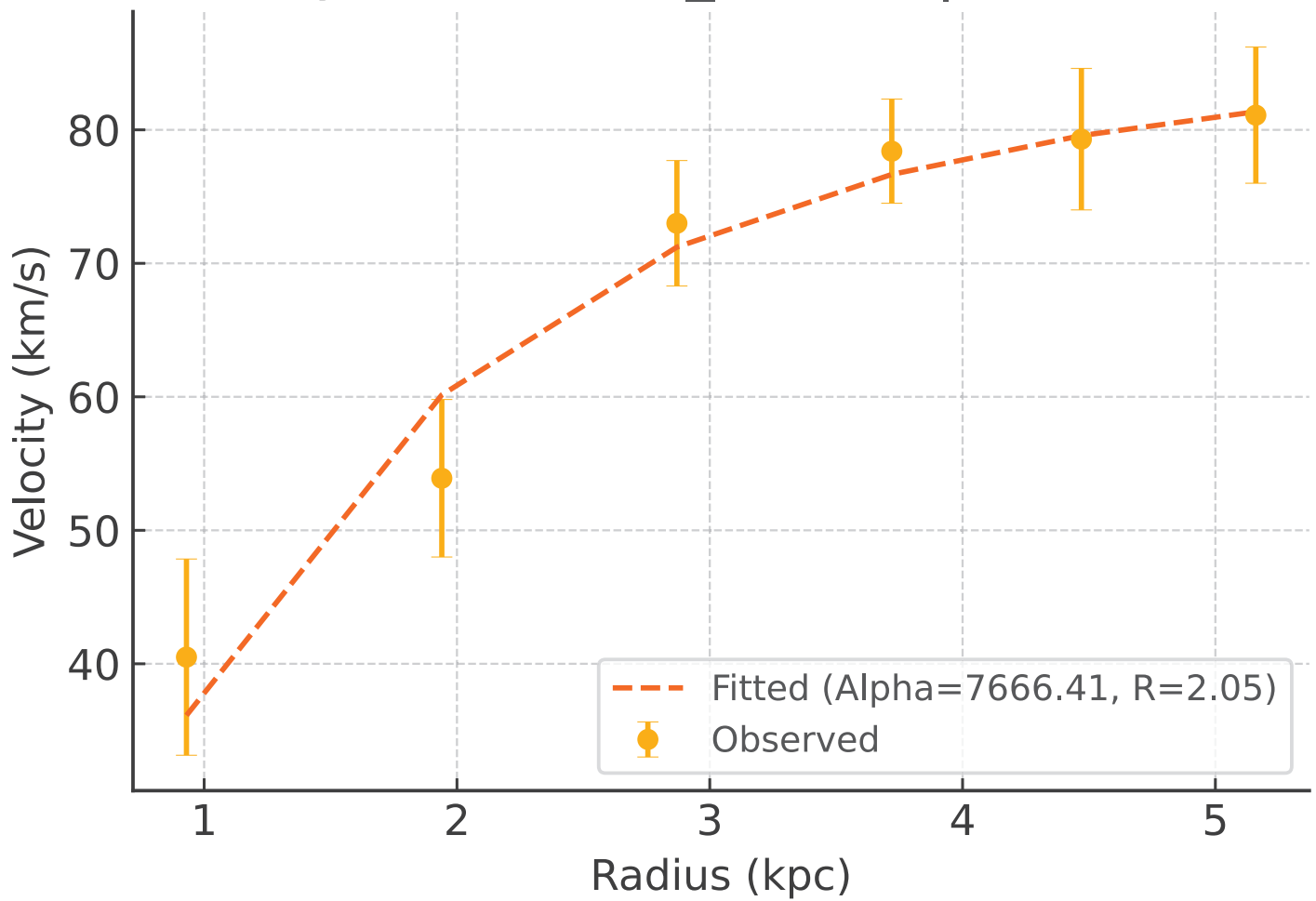
# Galaxy: UGC06818\_rotmod | $R^2 = 0.946$



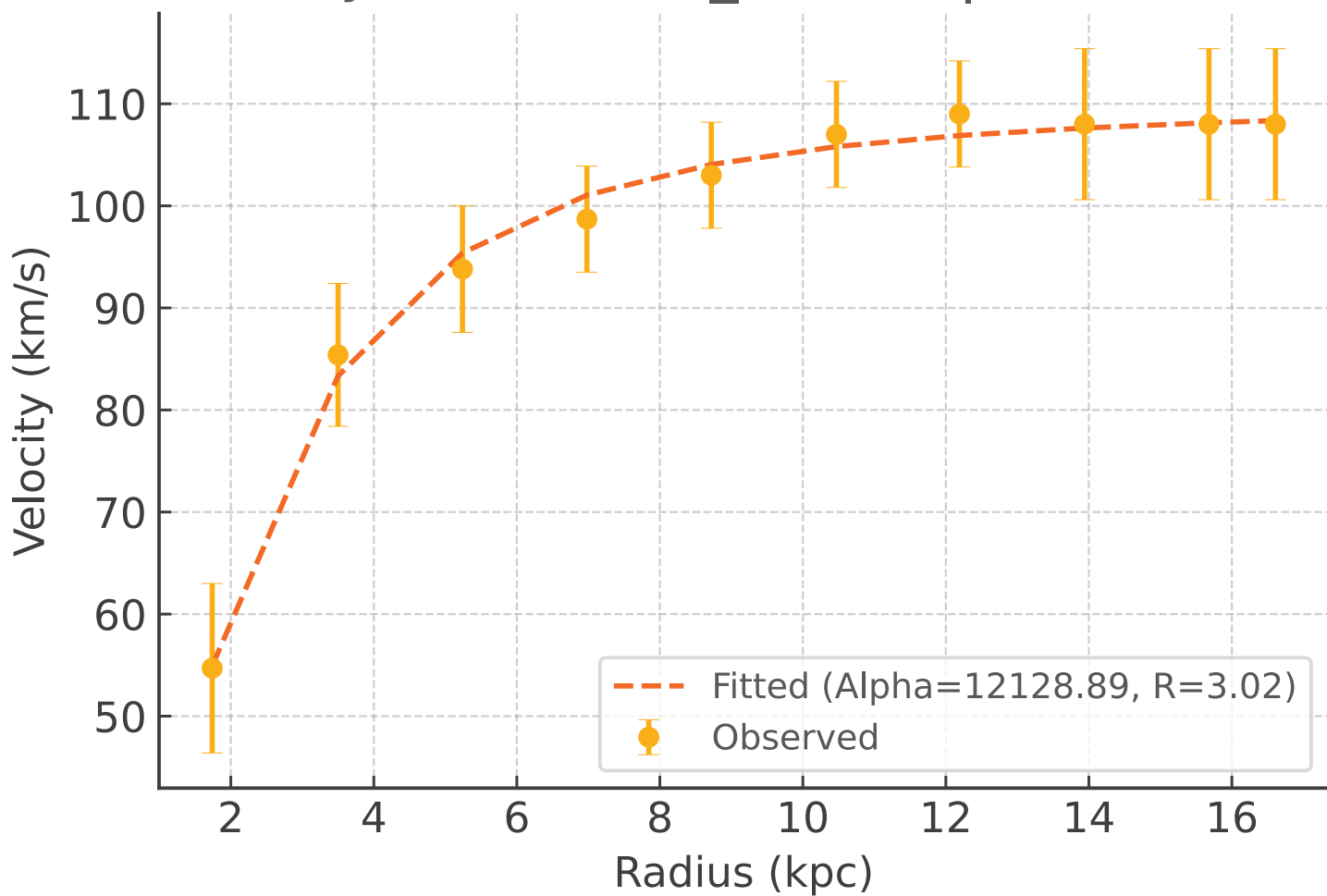
# Galaxy: UGC06917\_rotmod | $R^2 = 0.984$



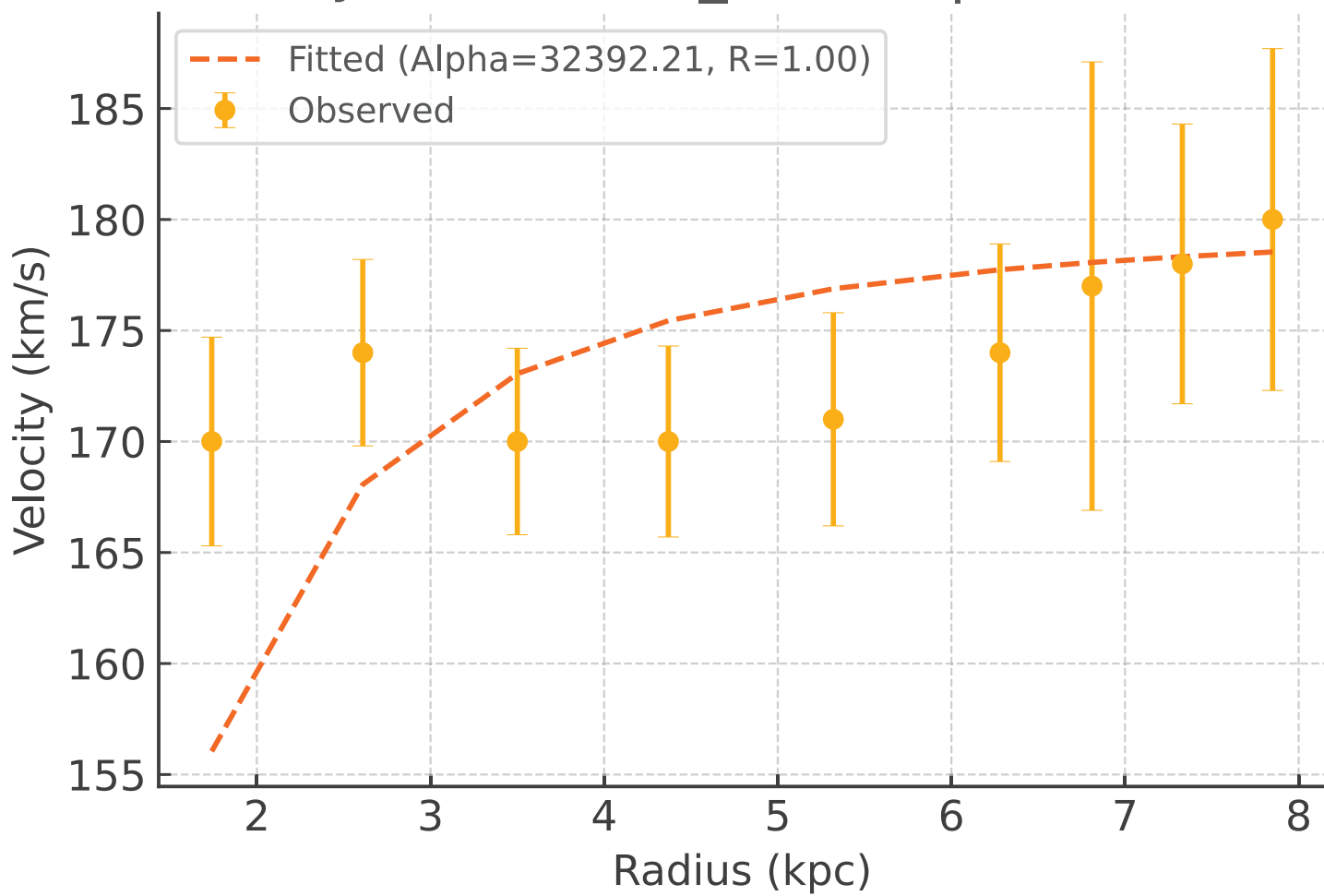
Galaxy: UGC06923\_rotmod |  $R^2 = 0.954$



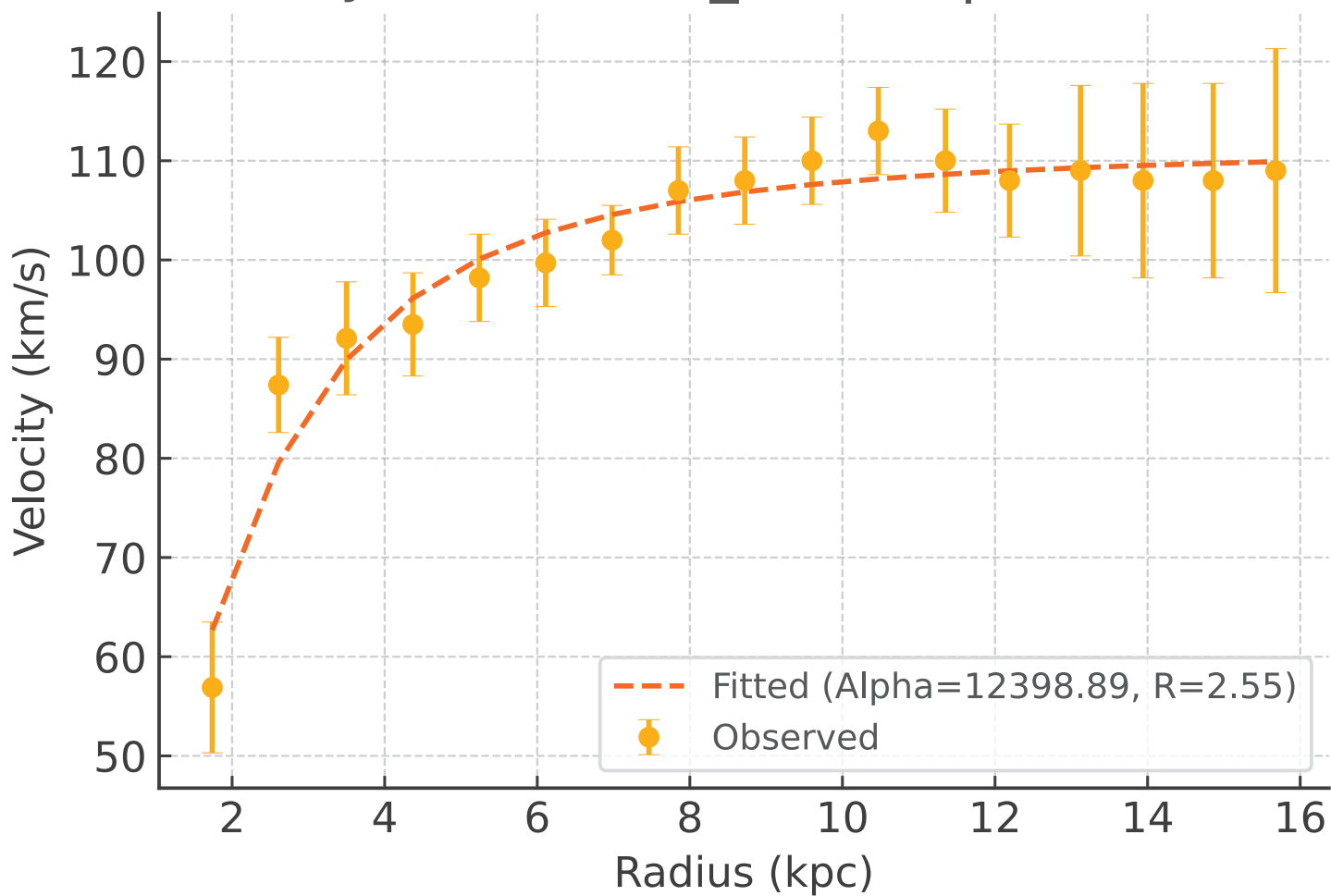
Galaxy: UGC06930\_rotmod |  $R^2 = 0.992$



# Galaxy: UGC06973\_rotmod | $R^2 = -1.730$

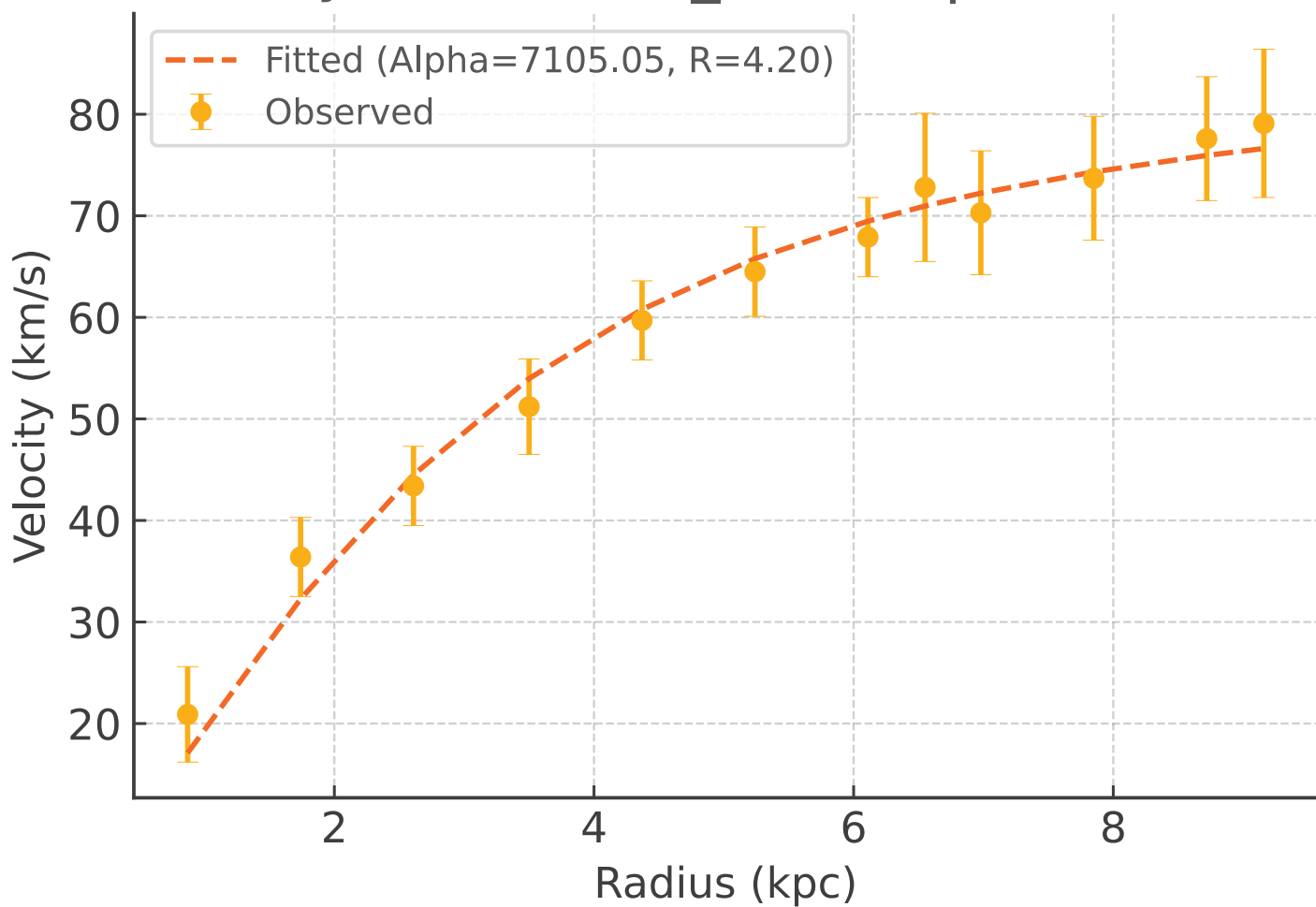


Galaxy: UGC06983\_rotmod |  $R^2 = 0.943$

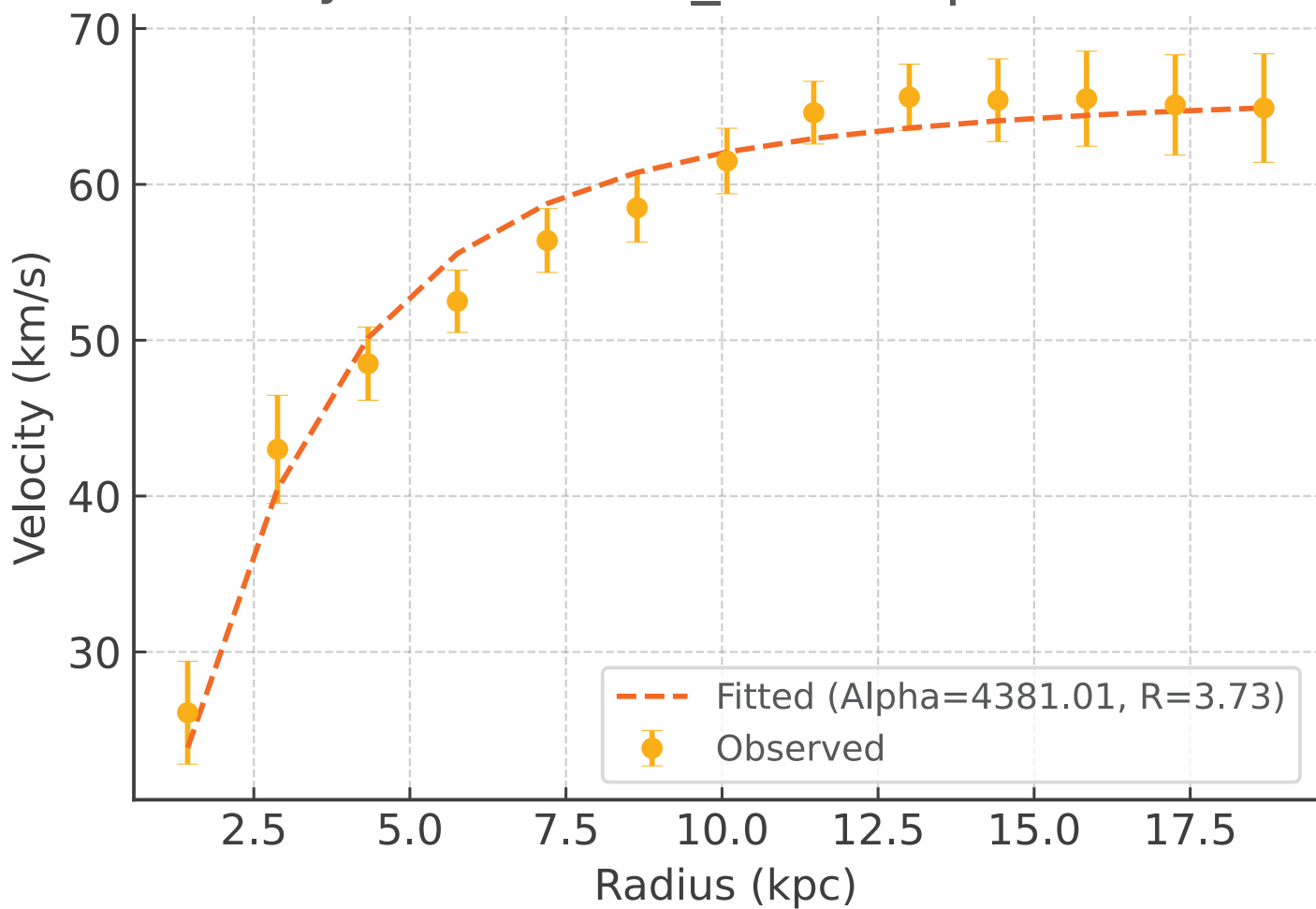




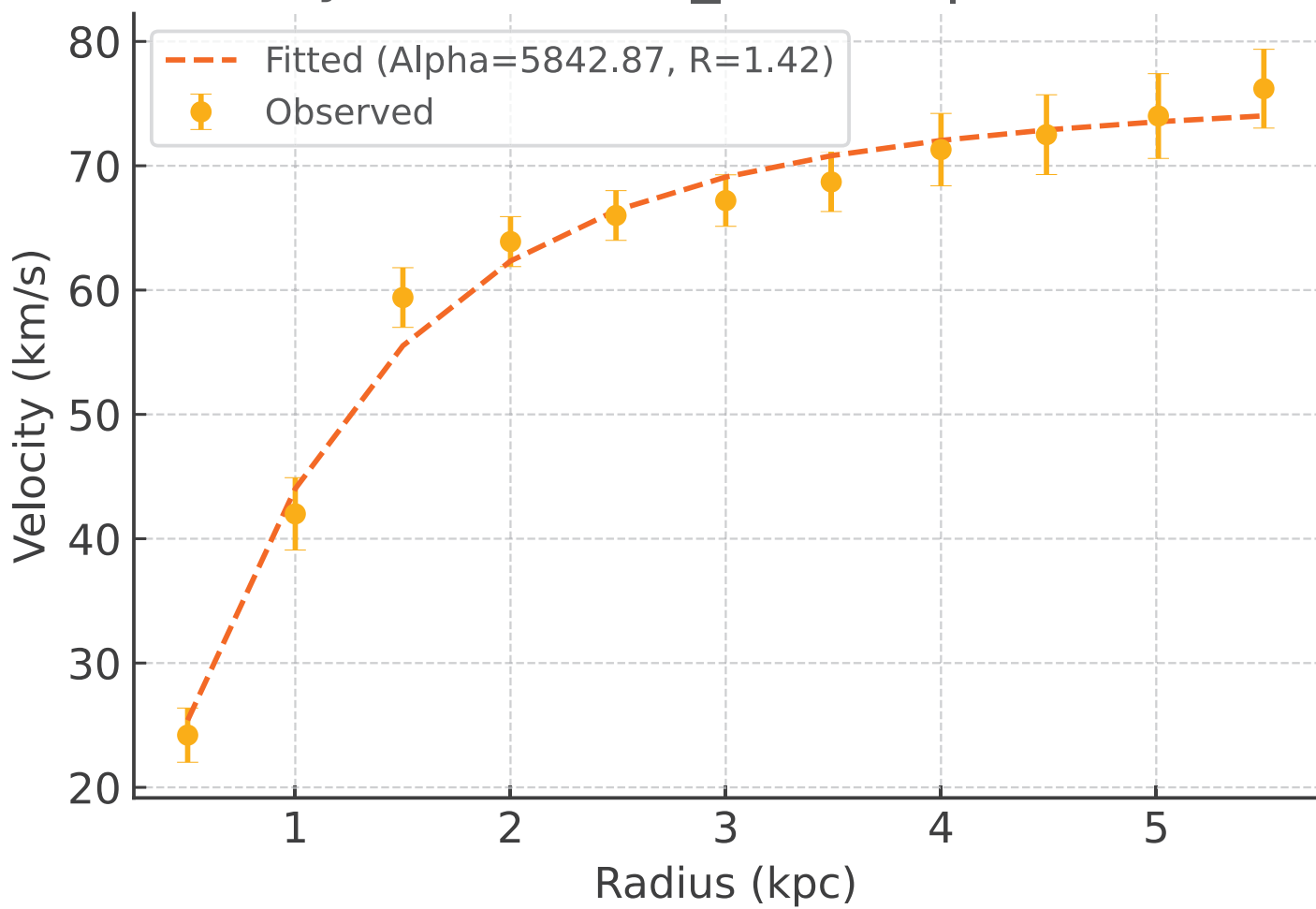
# Galaxy: UGC07089\_rotmod | $R^2 = 0.983$



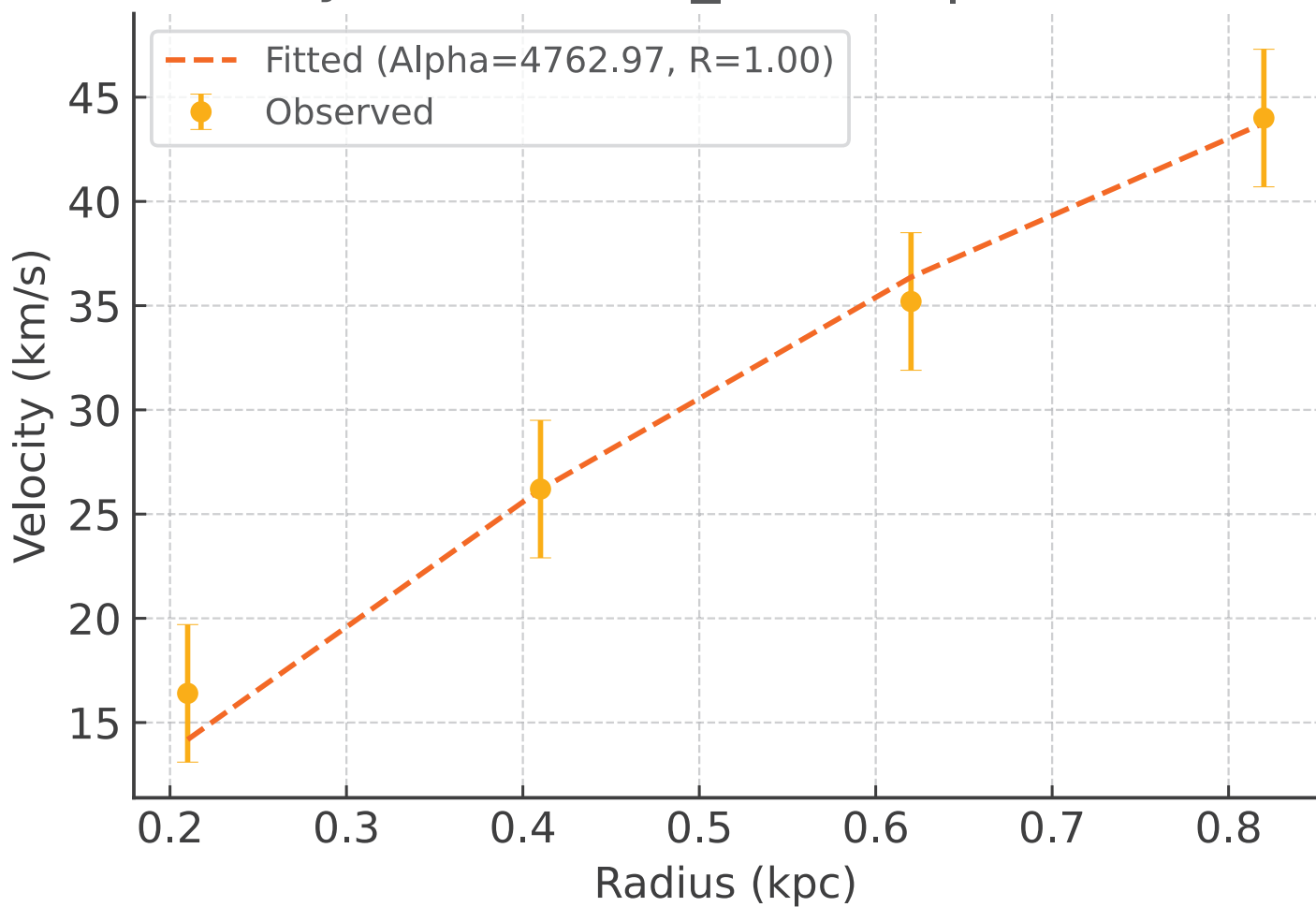
Galaxy: UGC07125\_rotmod |  $R^2 = 0.973$



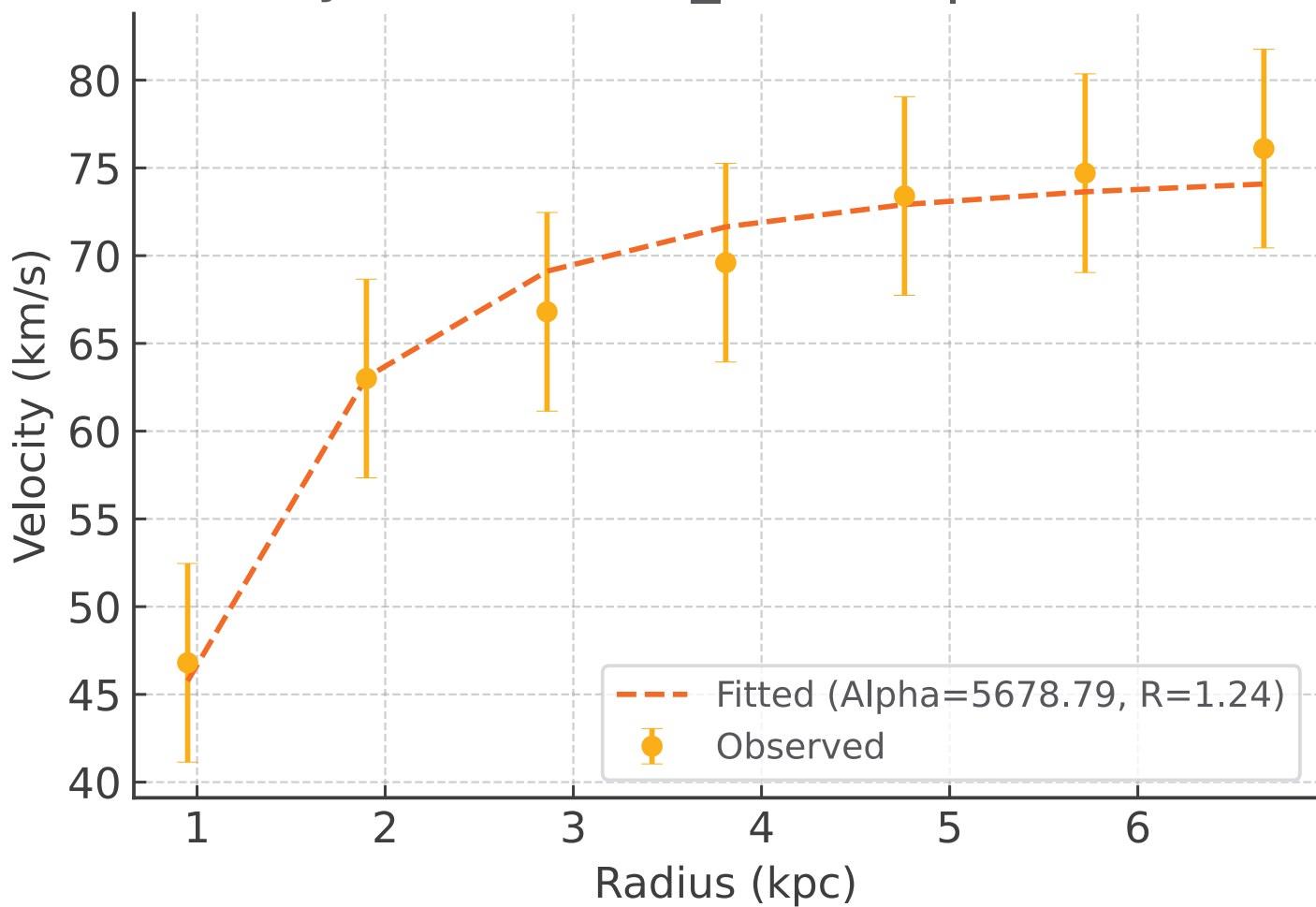
# Galaxy: UGC07151\_rotmod | $R^2 = 0.985$



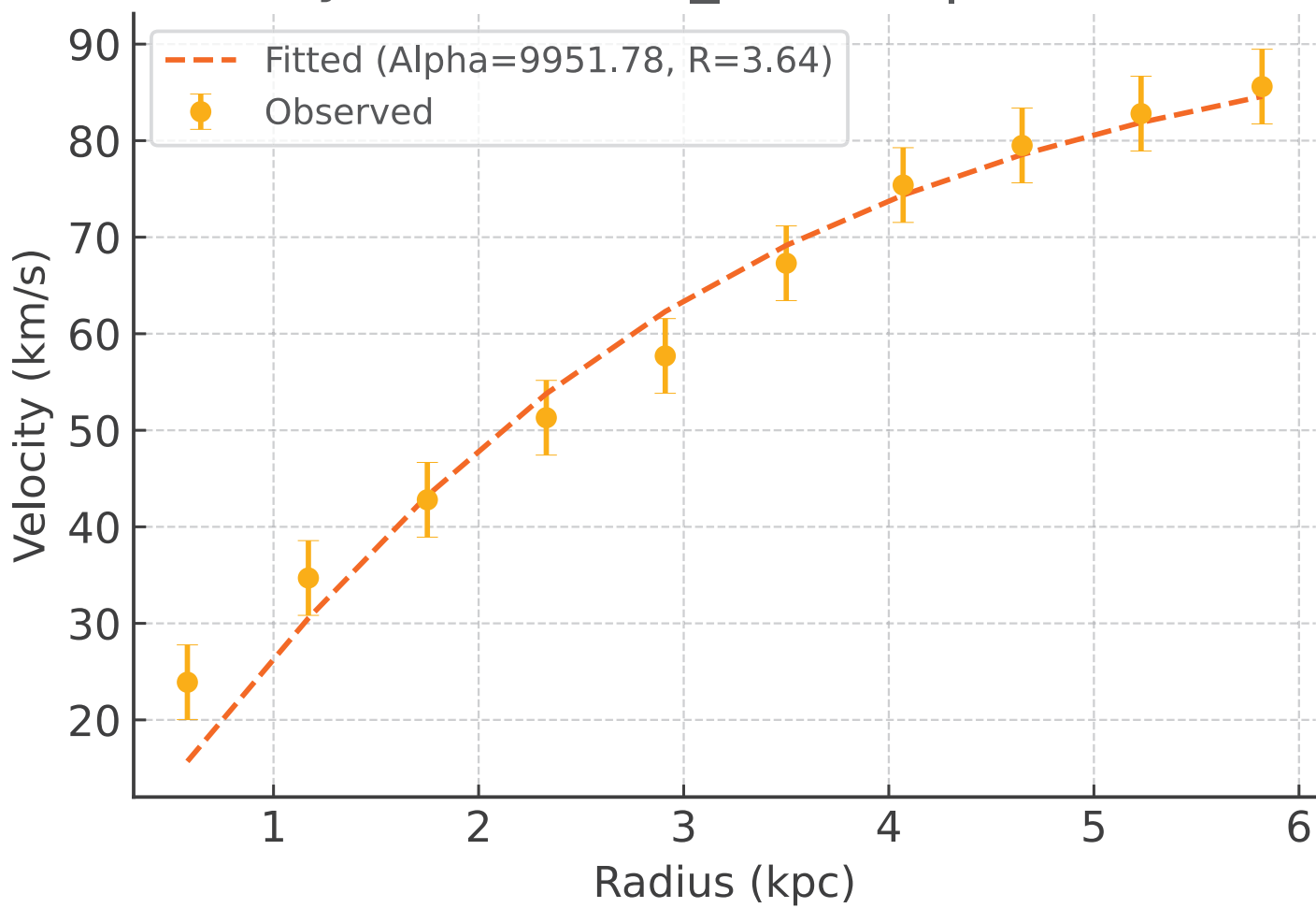
# Galaxy: UGC07232\_rotmod | $R^2 = 0.985$



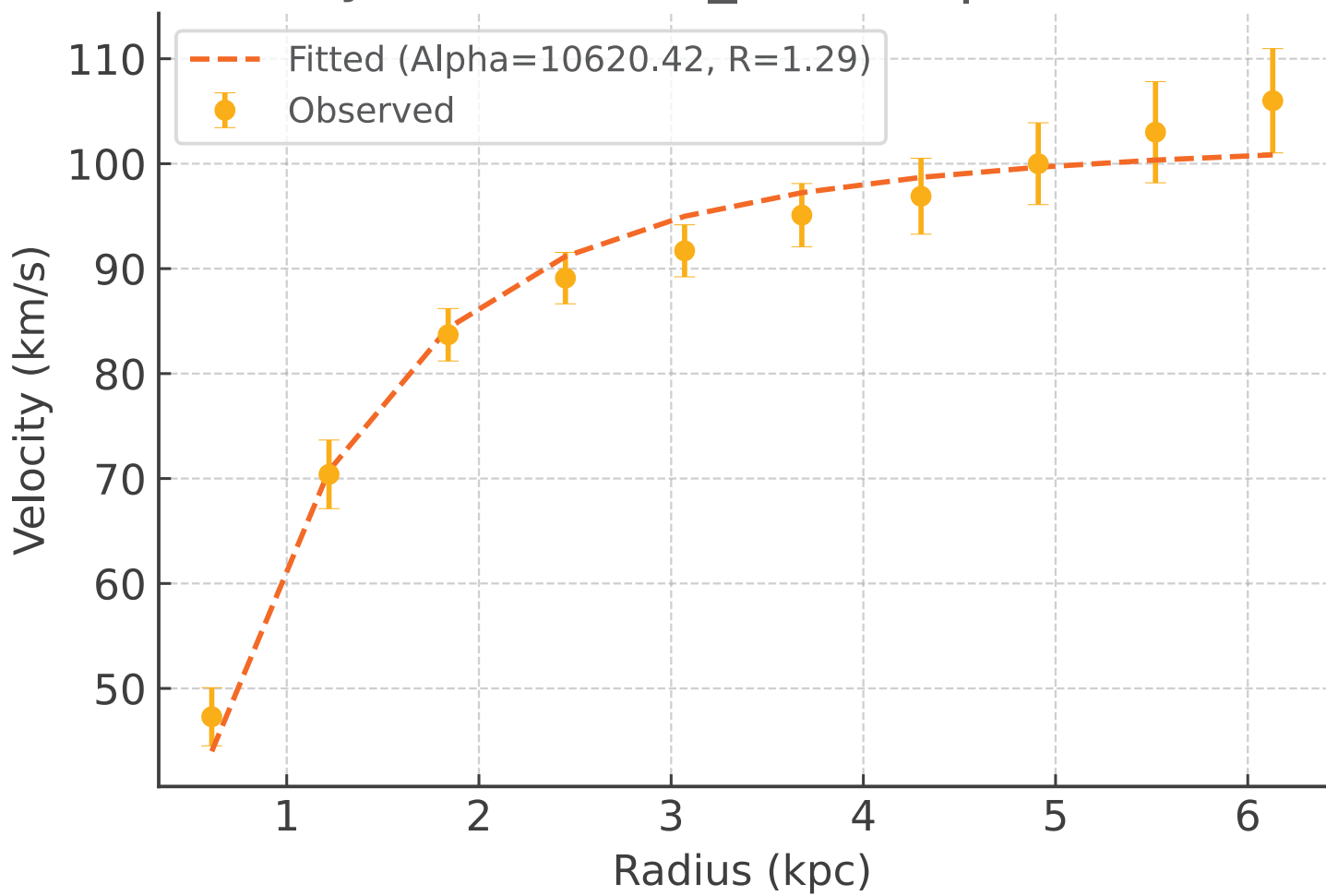
Galaxy: UGC07261\_rotmod |  $R^2 = 0.974$



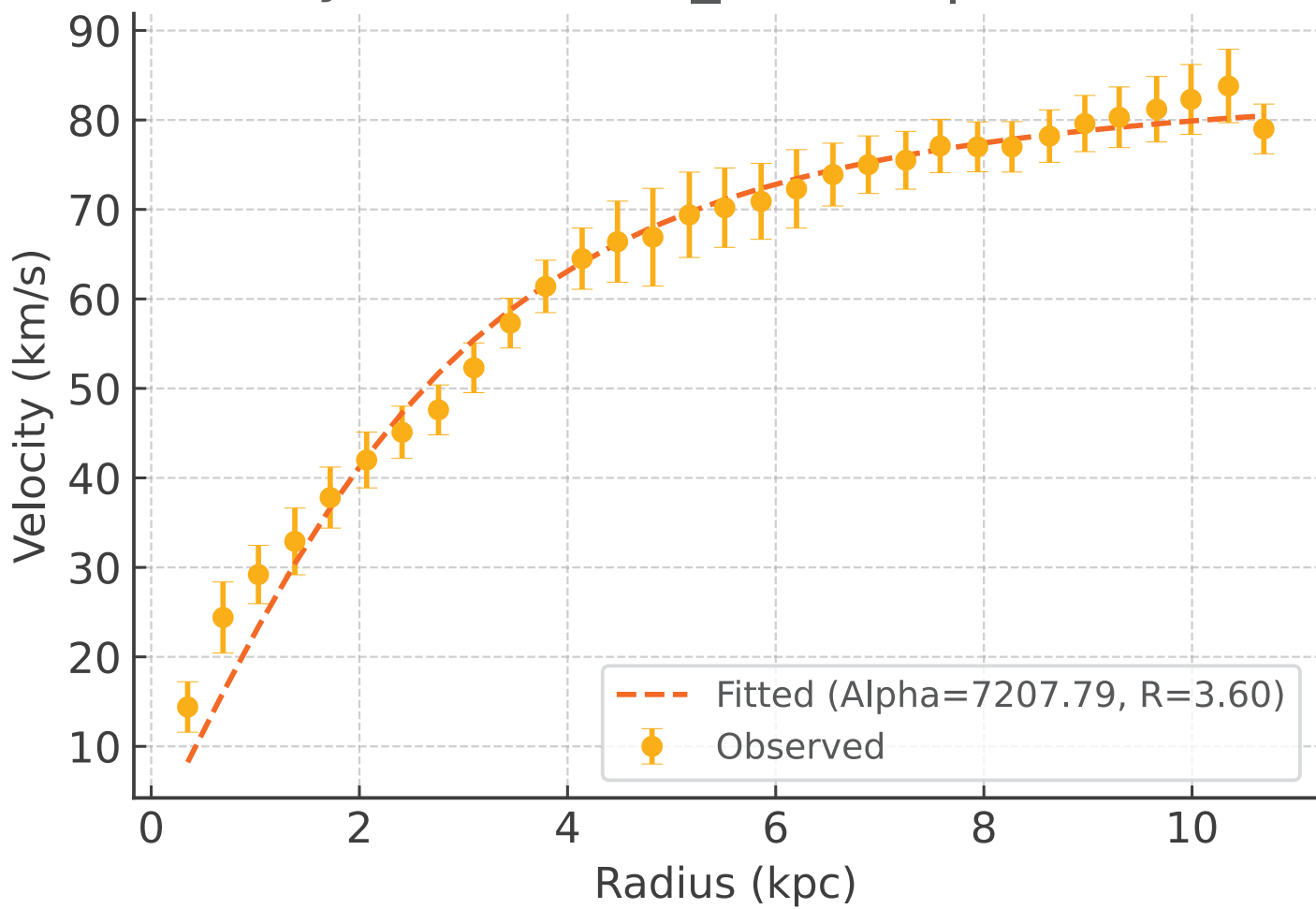
# Galaxy: UGC07323\_rotmod | $R^2 = 0.971$



# Galaxy: UGC07399\_rotmod | $R^2 = 0.976$

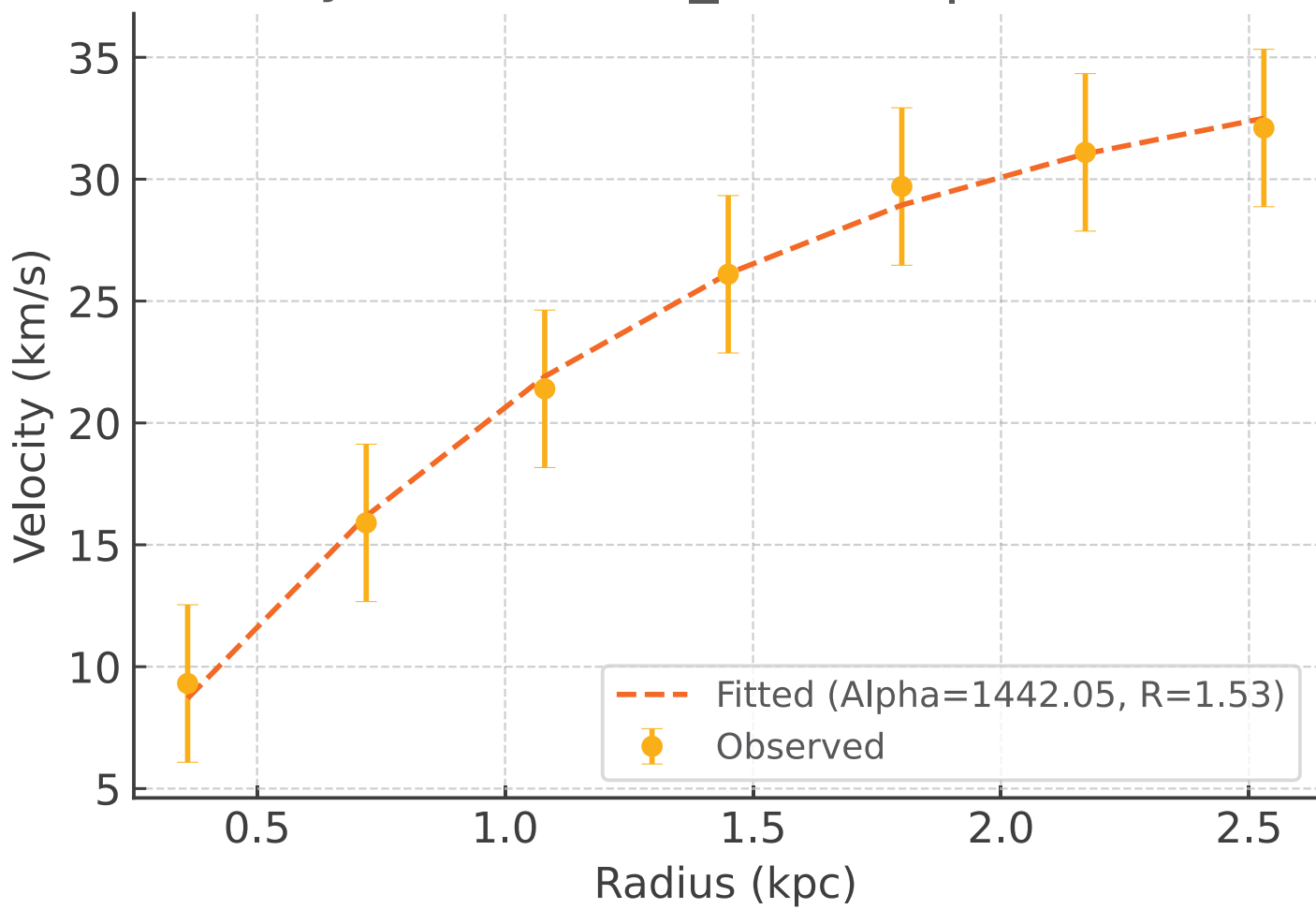


Galaxy: UGC07524\_rotmod |  $R^2 = 0.981$

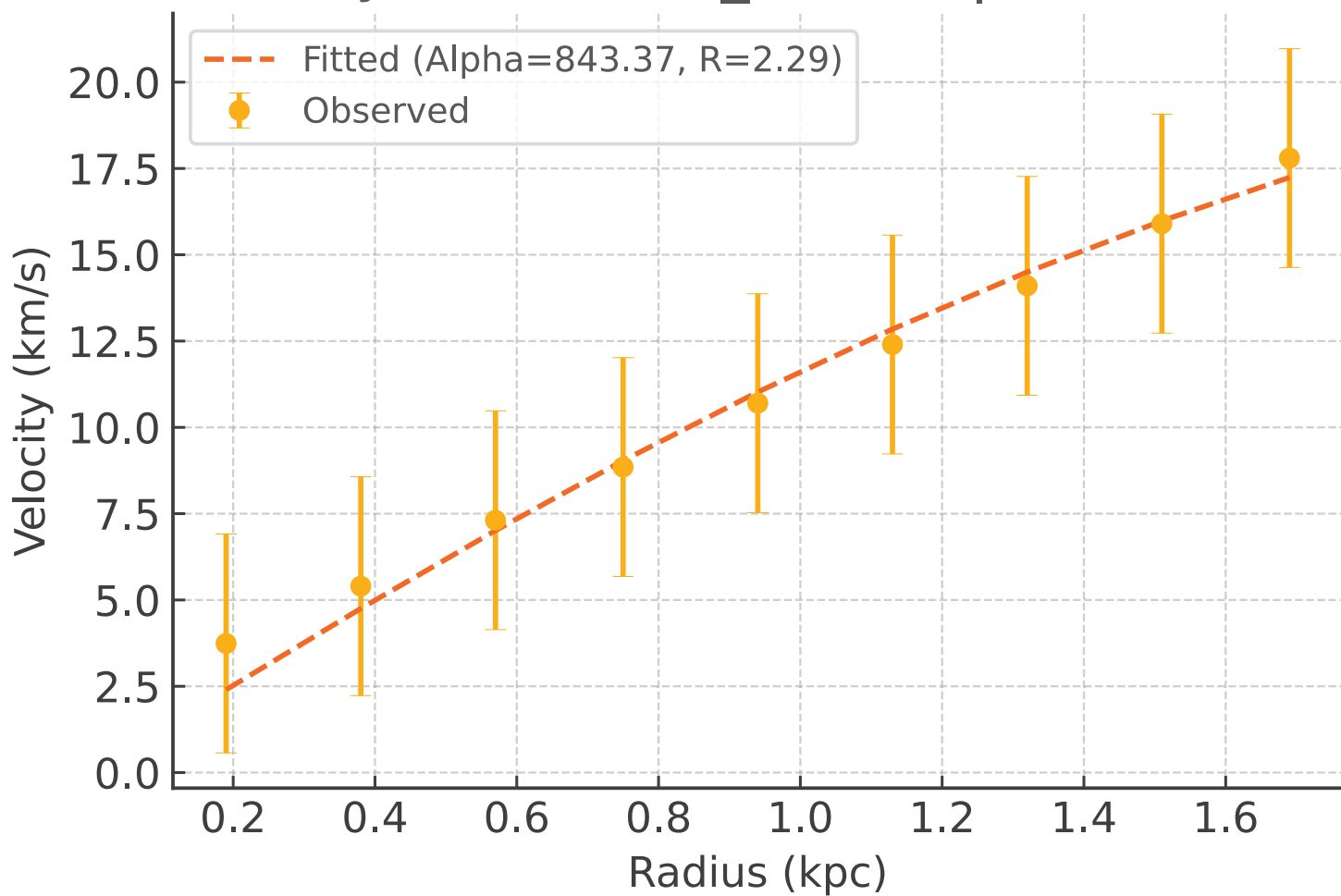




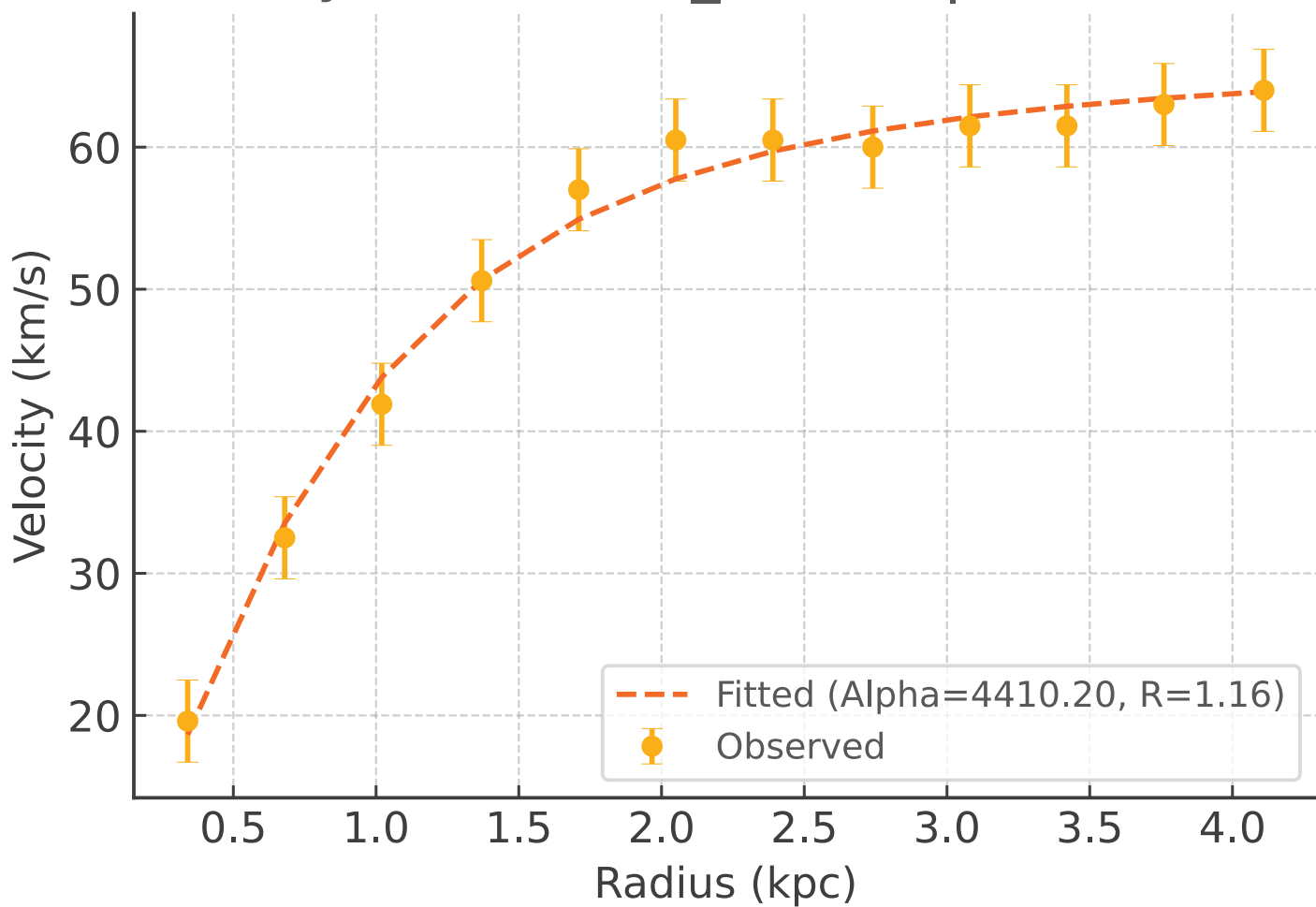
Galaxy: UGC07559\_rotmod |  $R^2 = 0.997$



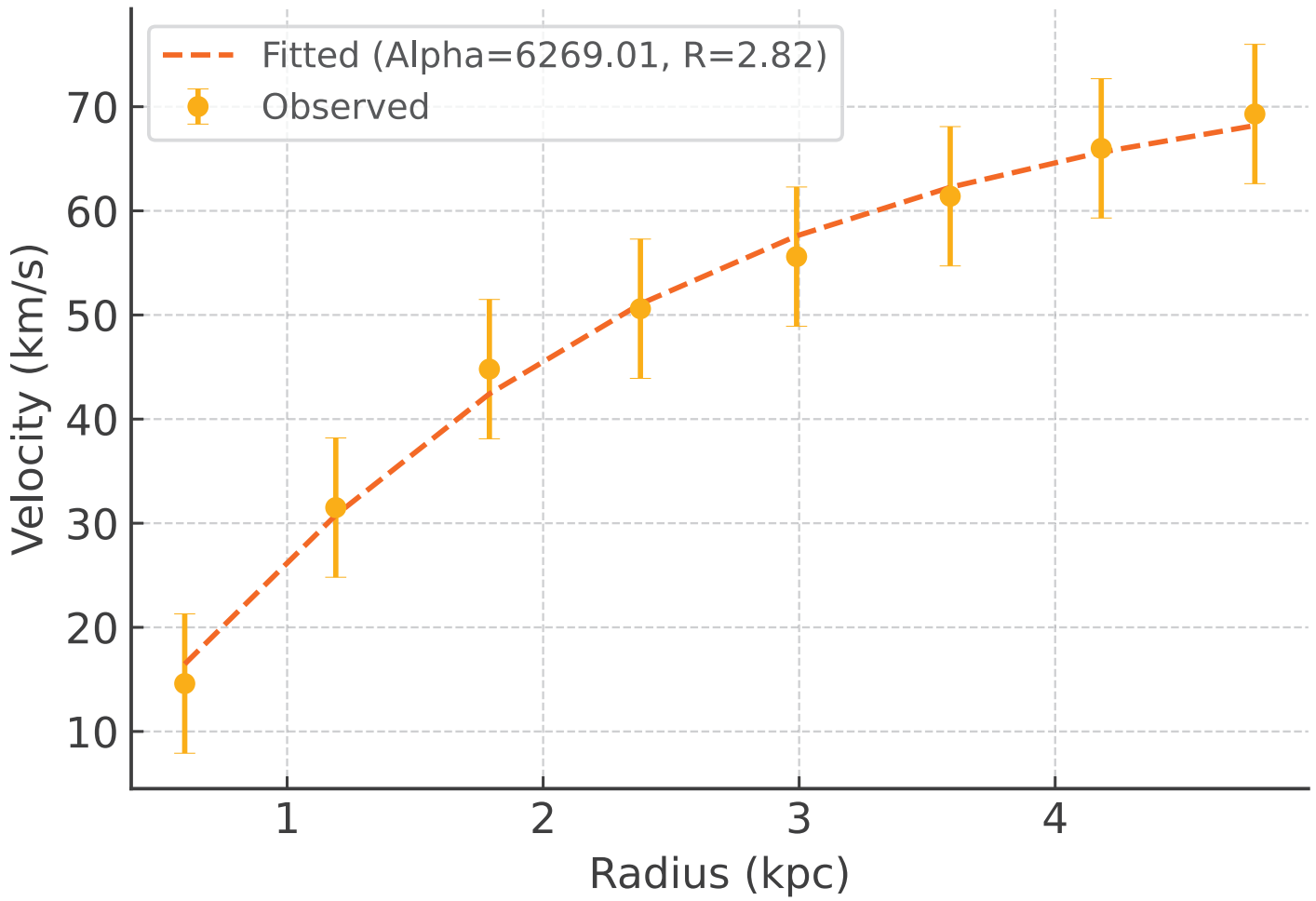
# Galaxy: UGC07577\_rotmod | $R^2 = 0.983$



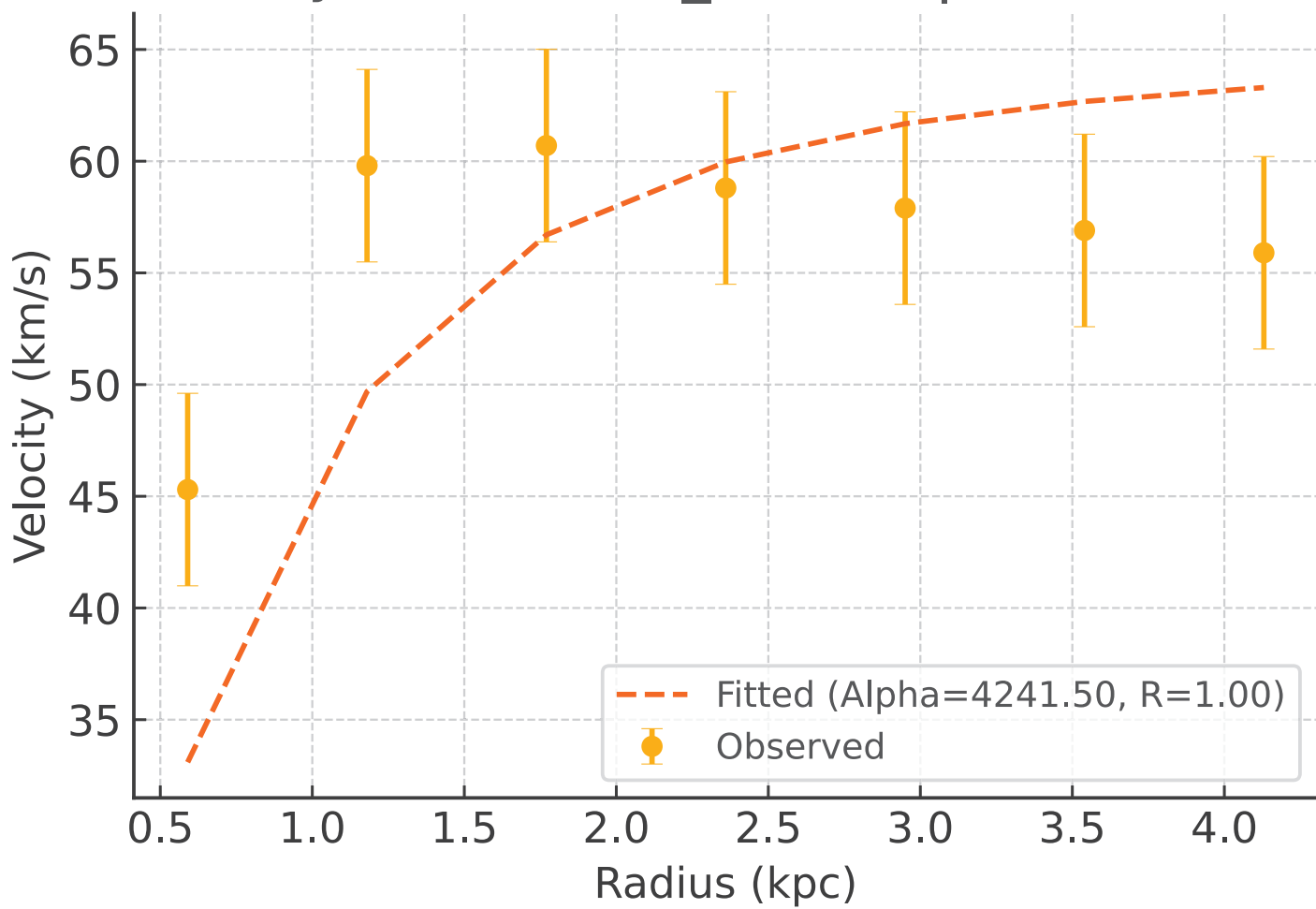
Galaxy: UGC07603\_rotmod |  $R^2 = 0.990$



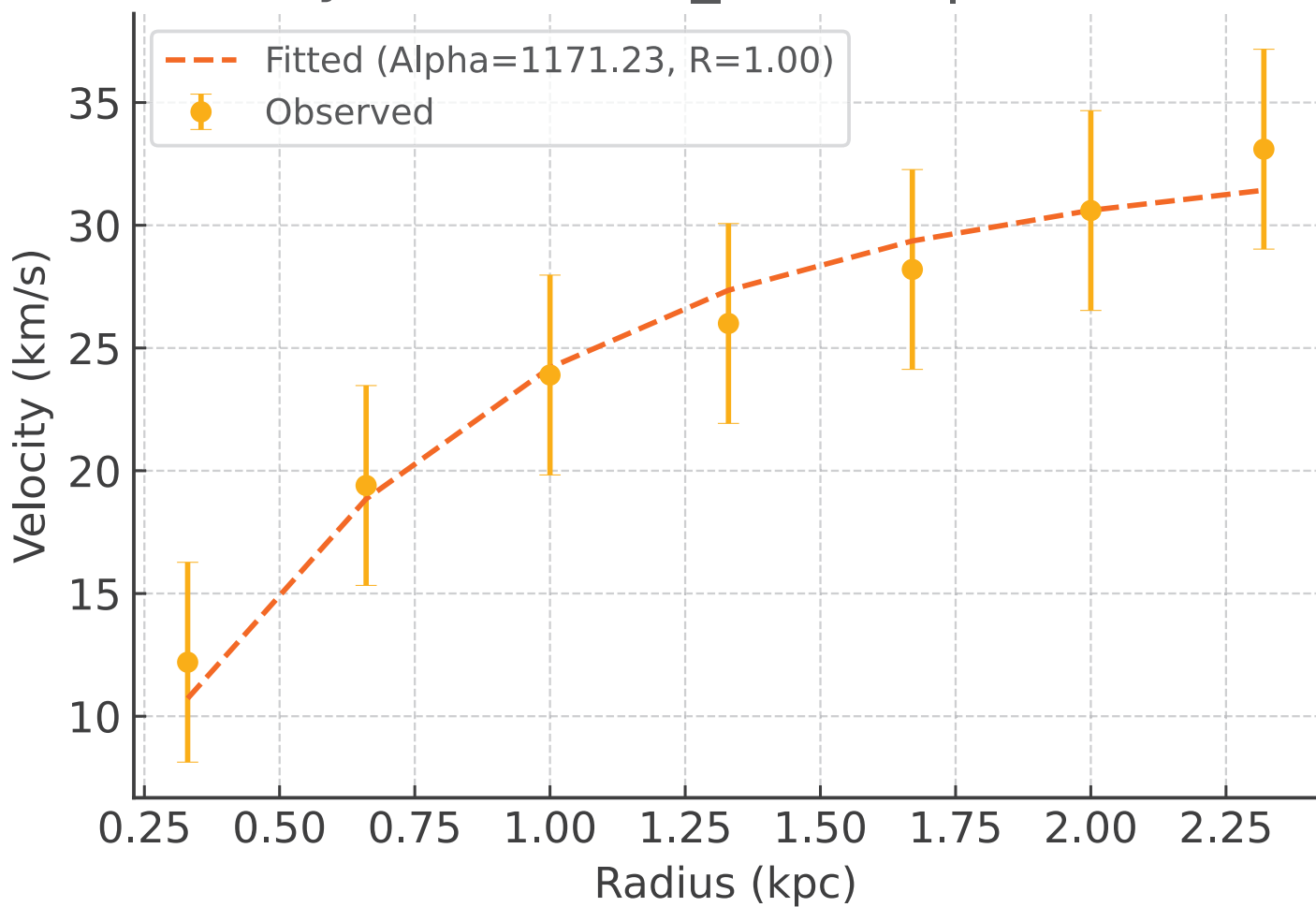
# Galaxy: UGC07608\_rotmod | $R^2 = 0.993$



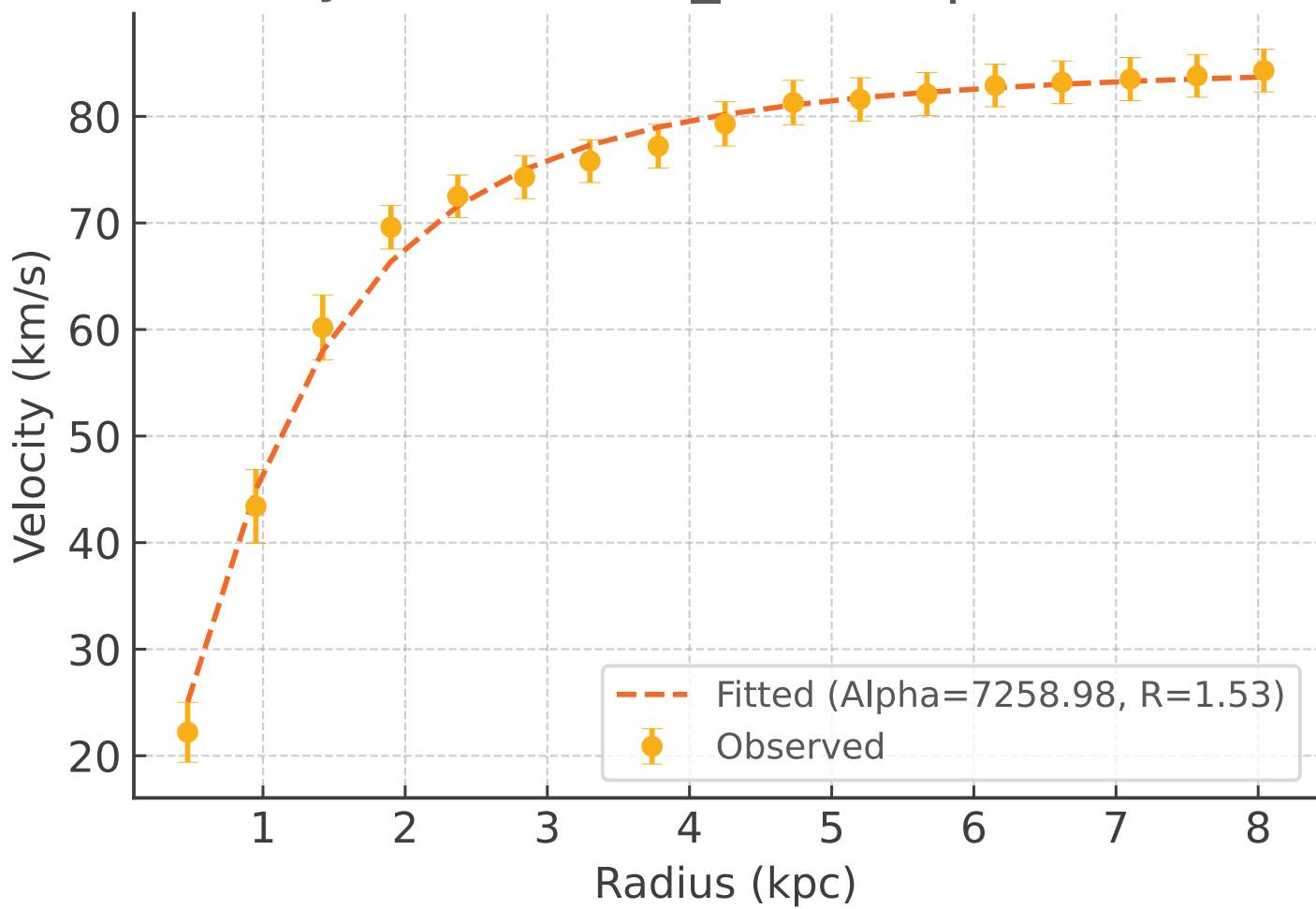
Galaxy: UGC07690\_rotmod |  $R^2 = -1.294$



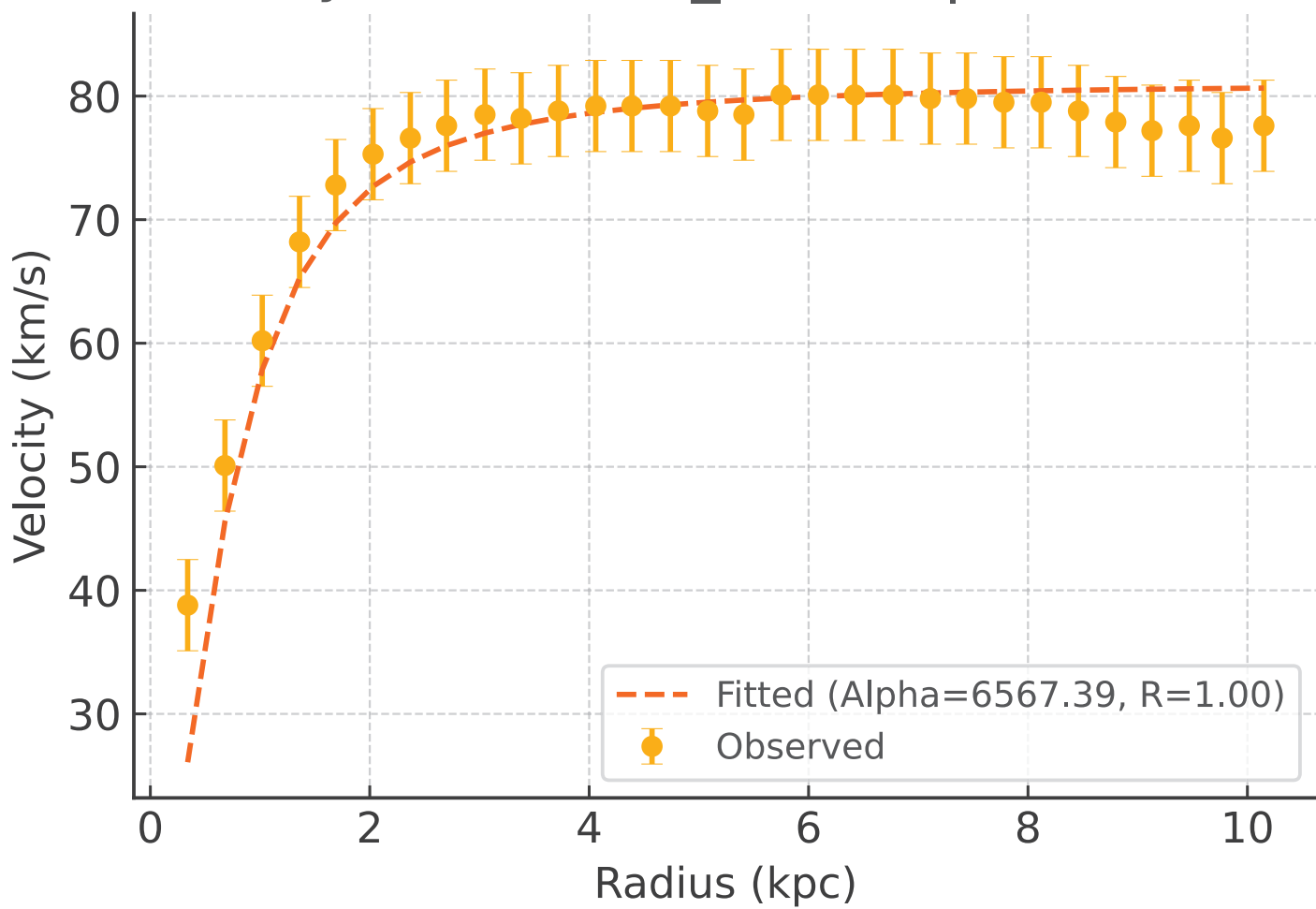
# Galaxy: UGC07866\_rotmod | $R^2 = 0.972$



Galaxy: UGC08286\_rotmod |  $R^2 = 0.992$

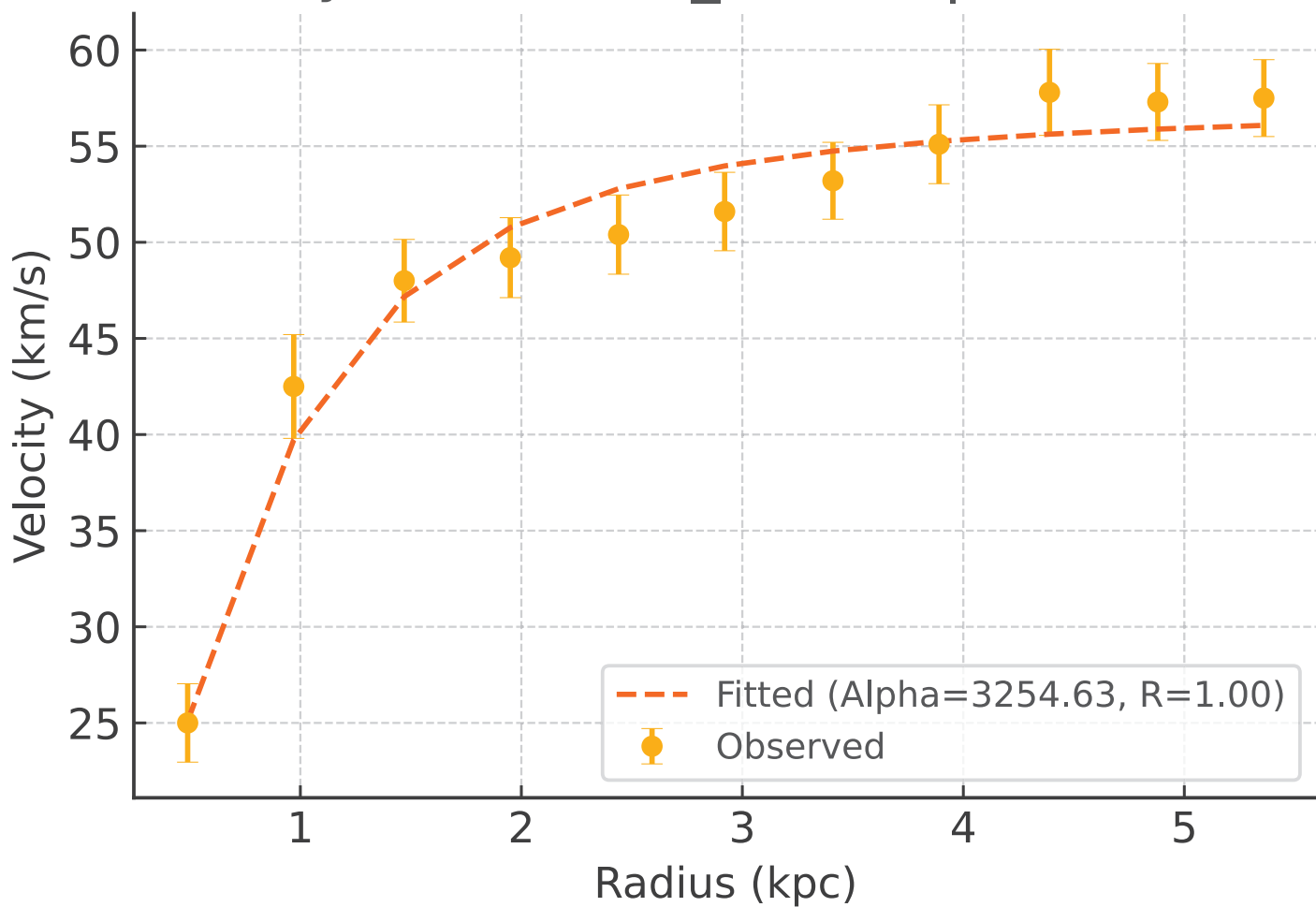


Galaxy: UGC08490\_rotmod |  $R^2 = 0.890$

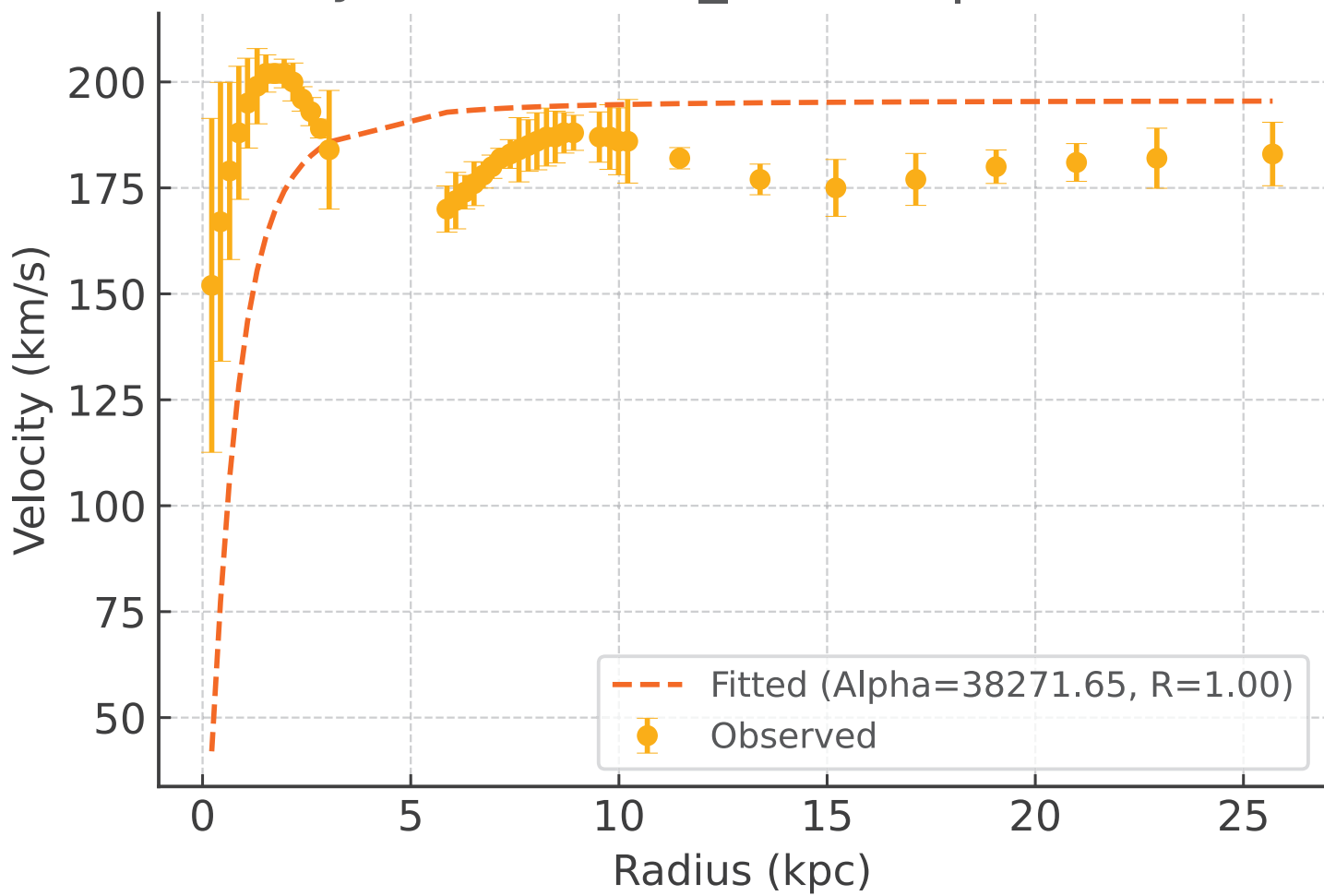




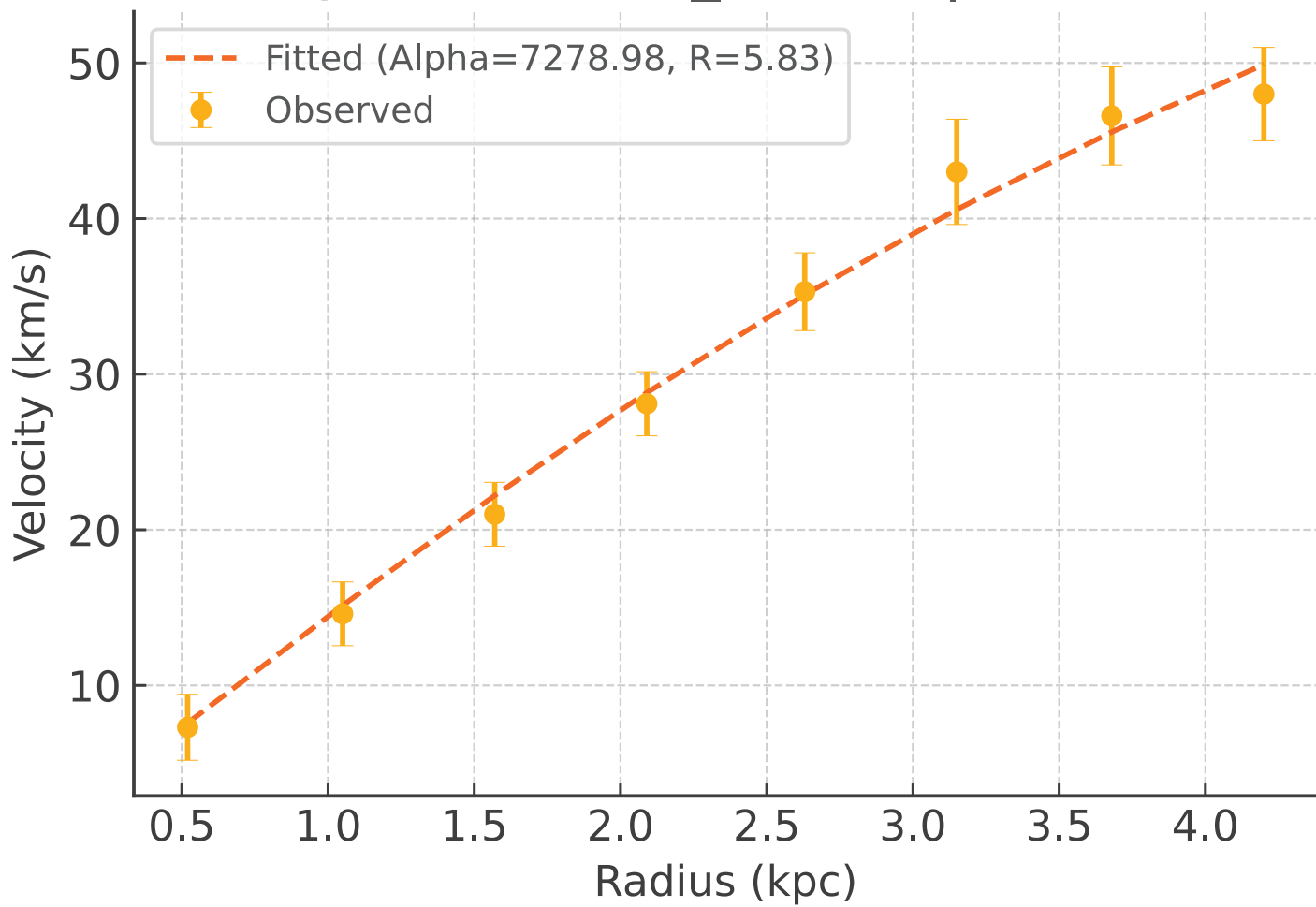
Galaxy: UGC08550\_rotmod |  $R^2 = 0.963$



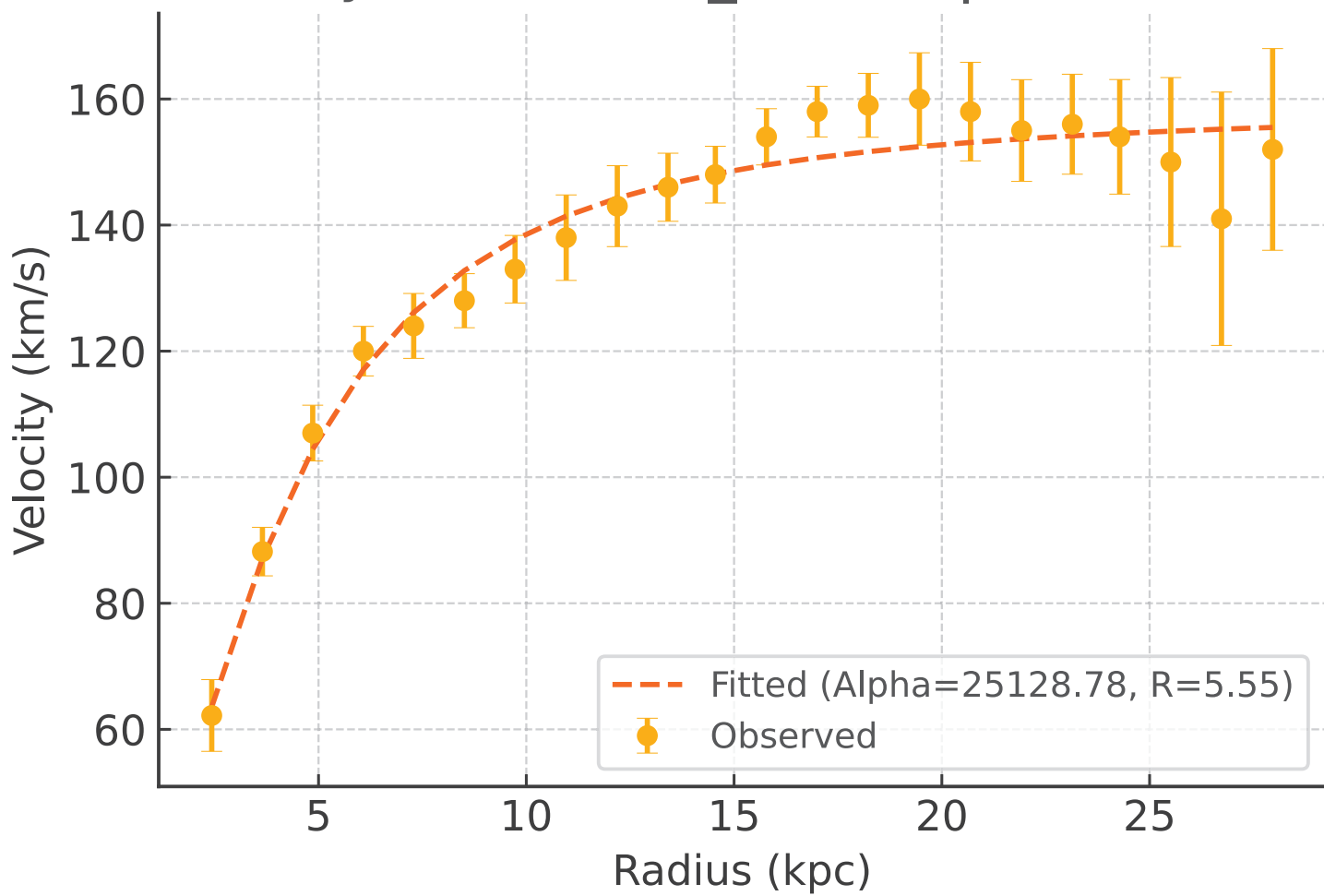
Galaxy: UGC08699\_rotmod |  $R^2 = -9.591$



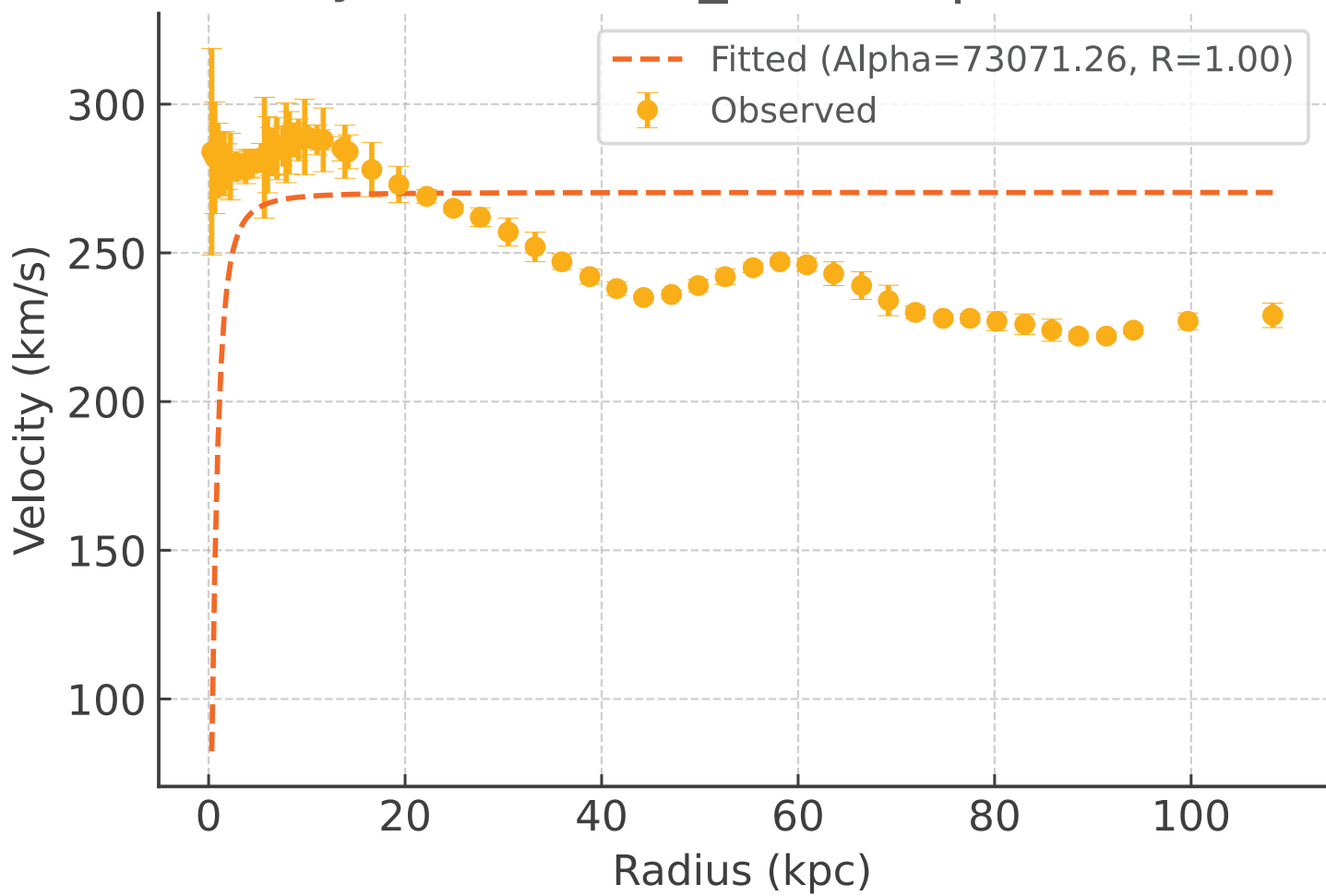
# Galaxy: UGC08837\_rotmod | $R^2 = 0.992$



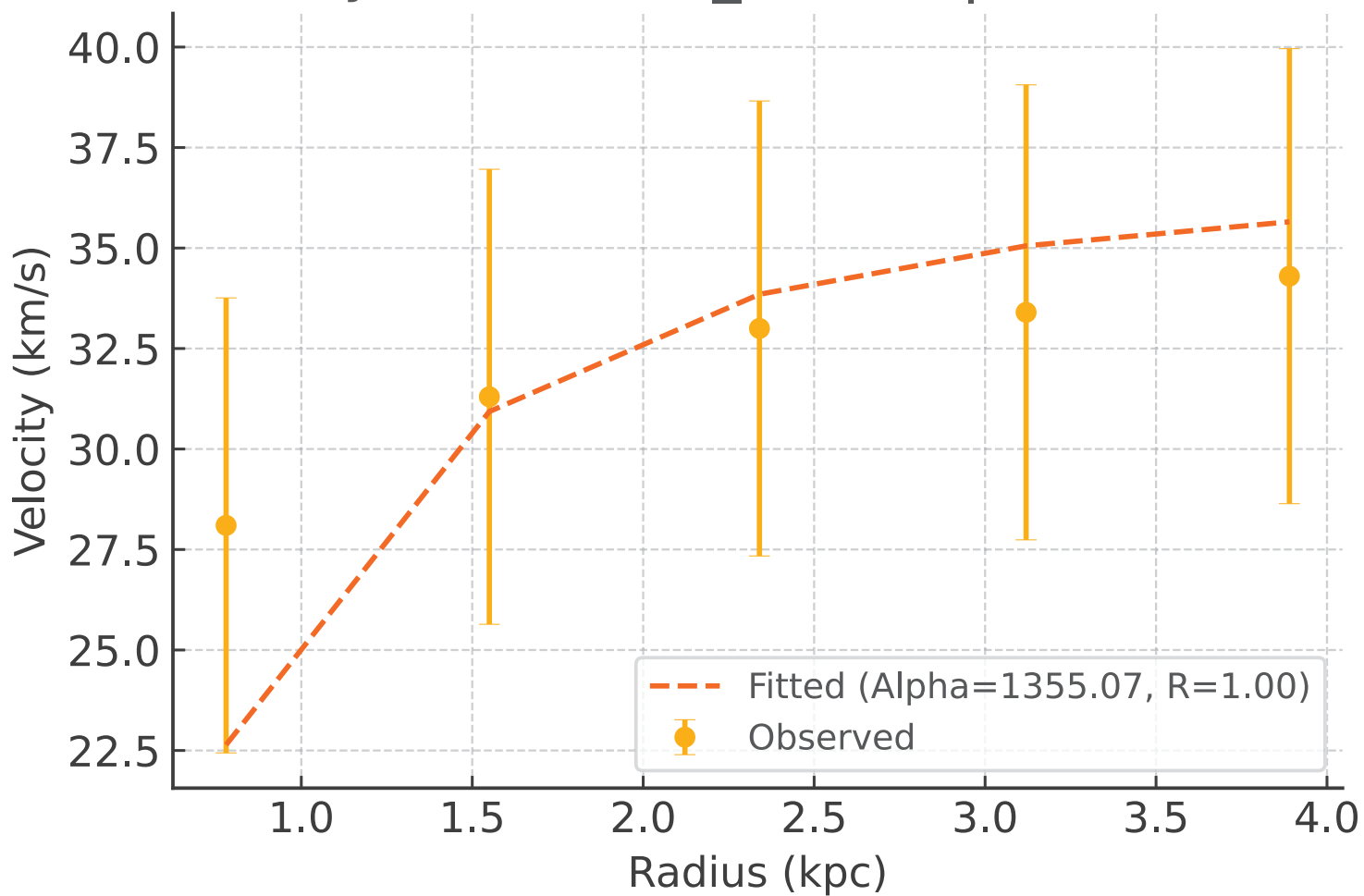
Galaxy: UGC09037\_rotmod |  $R^2 = 0.960$



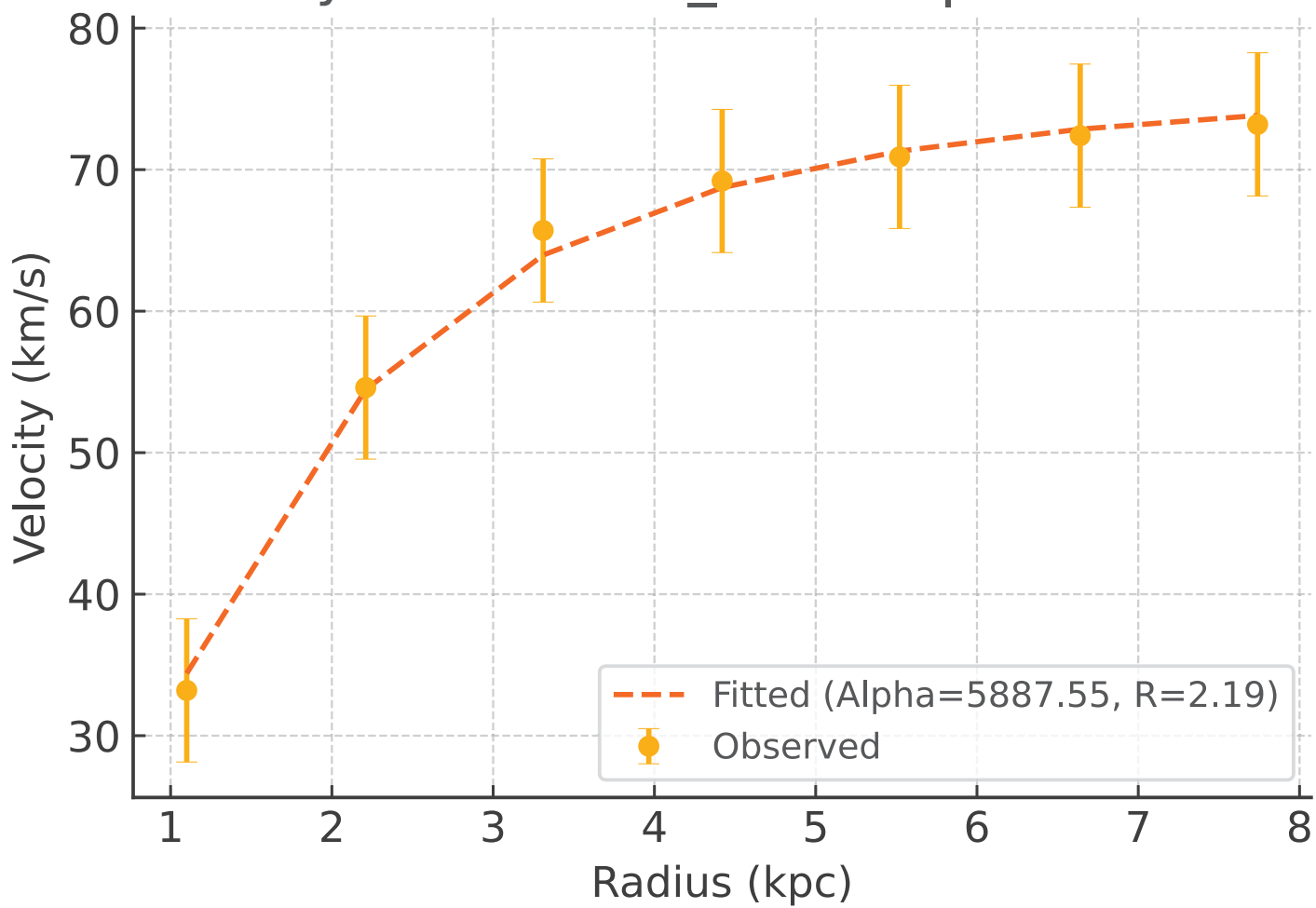
# Galaxy: UGC09133\_rotmod | $R^2 = -2.226$



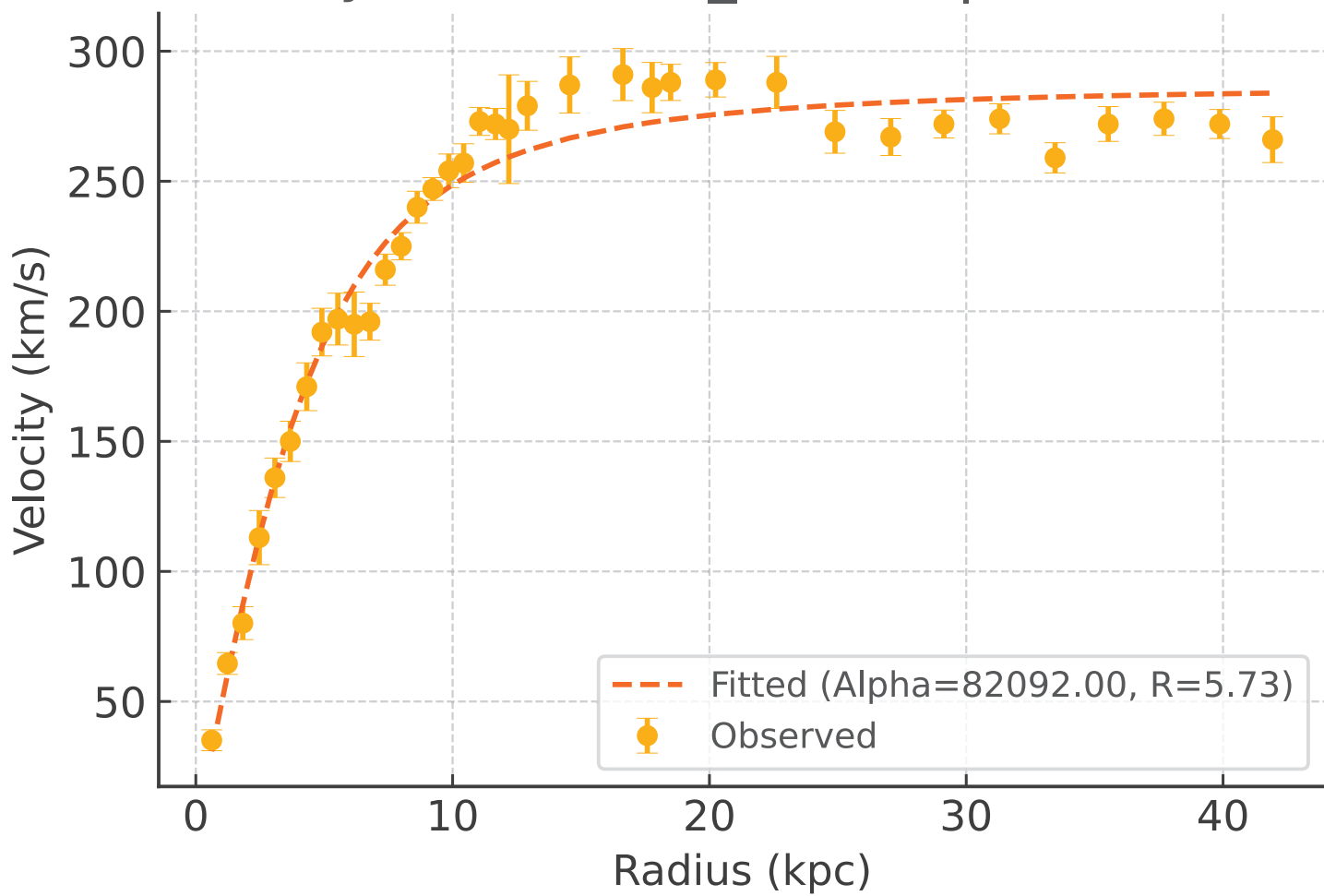
Galaxy: UGC09992\_rotmod |  $R^2 = -0.471$



Galaxy: UGC10310\_rotmod |  $R^2 = 0.996$

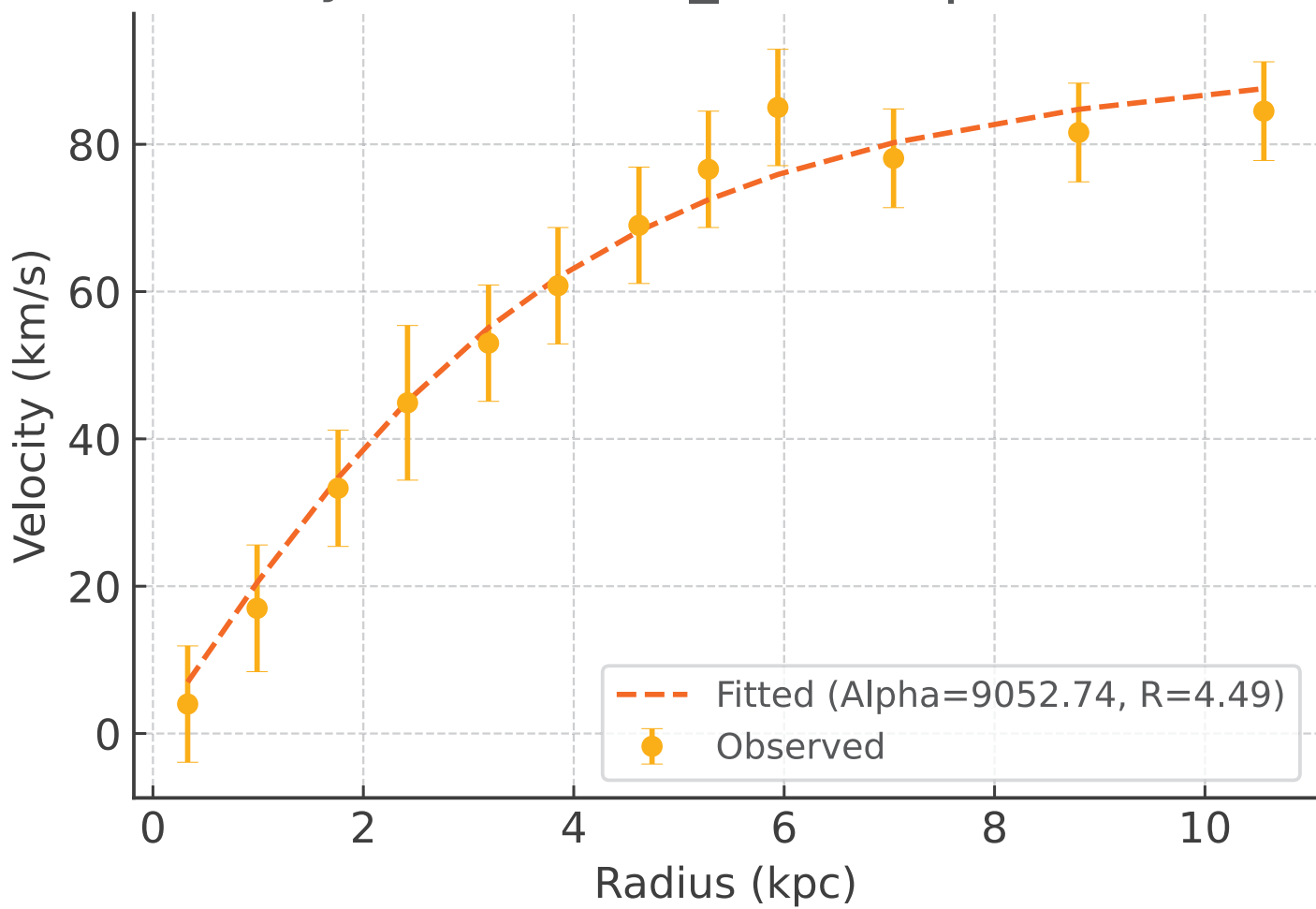


Galaxy: UGC11455\_rotmod |  $R^2 = 0.969$

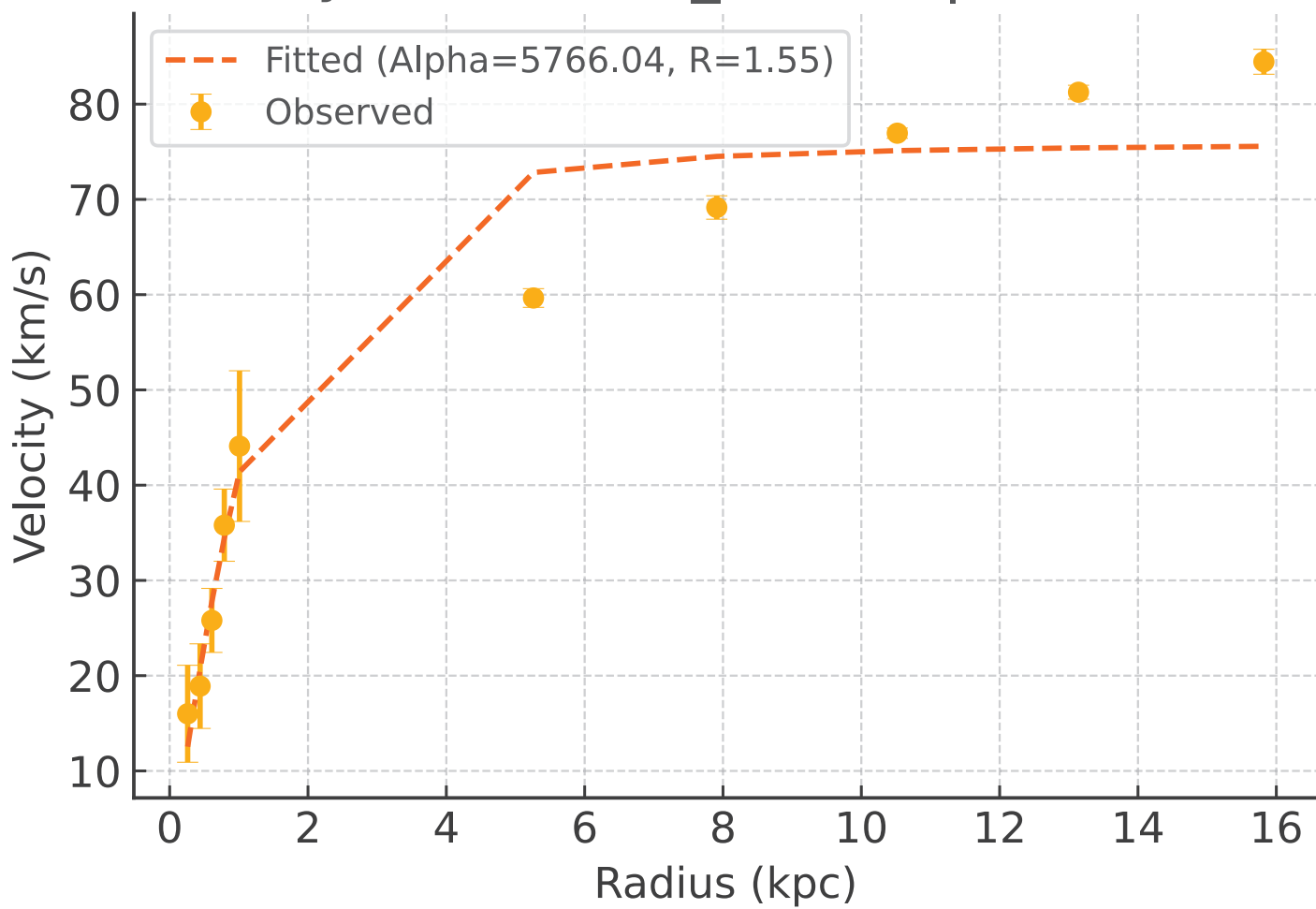




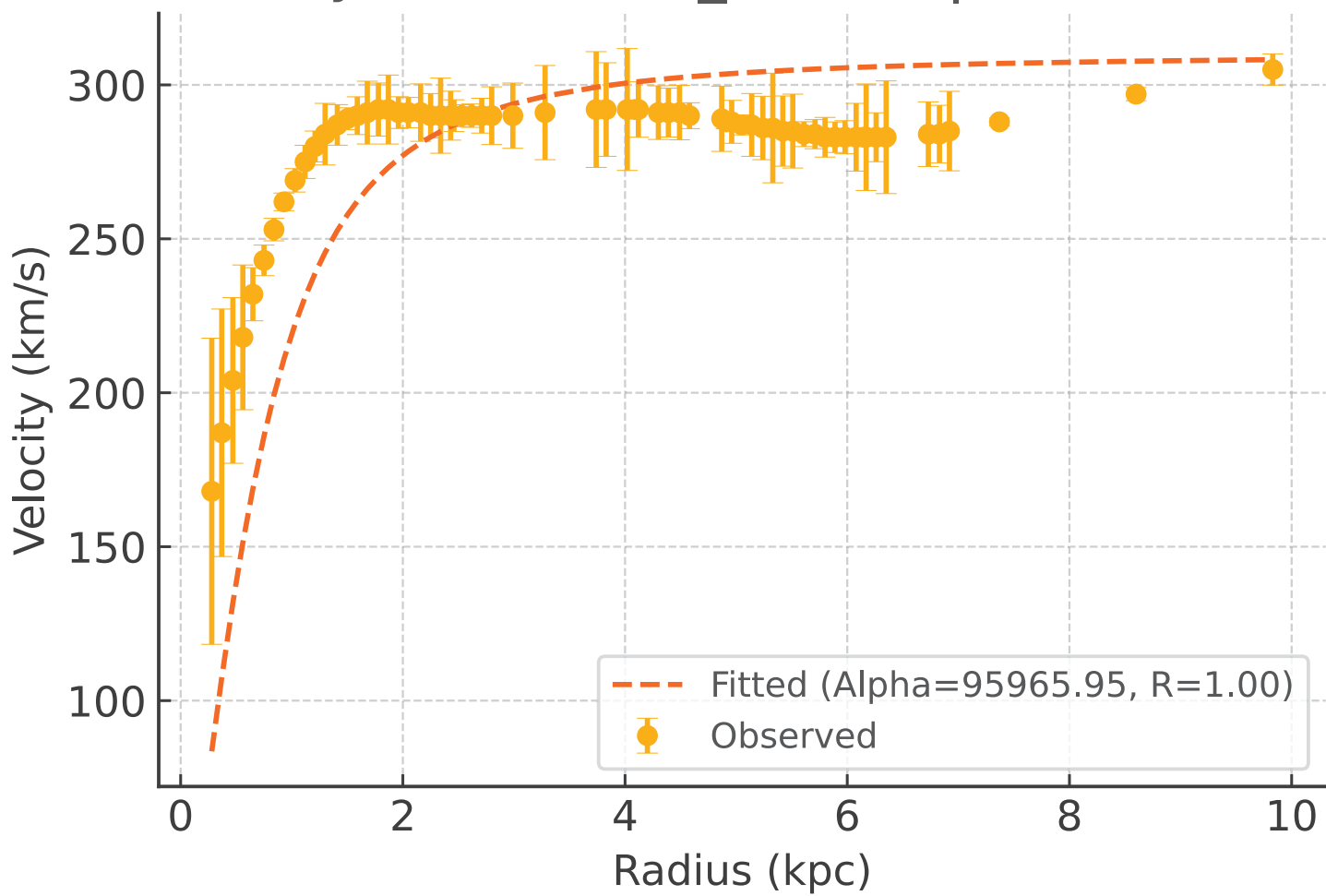
Galaxy: UGC11557\_rotmod |  $R^2 = 0.981$



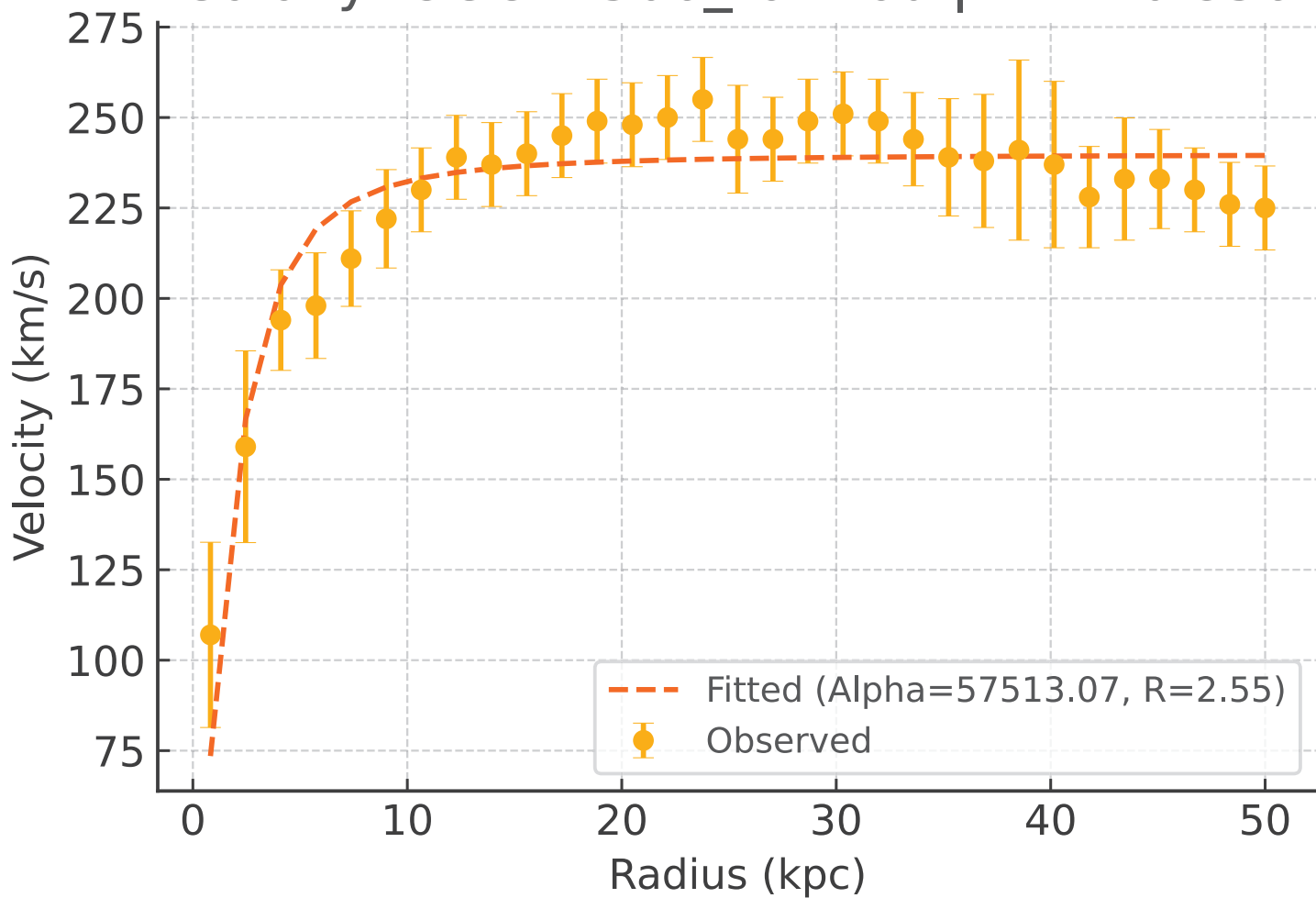
# Galaxy: UGC11820\_rotmod | $R^2 = 0.945$



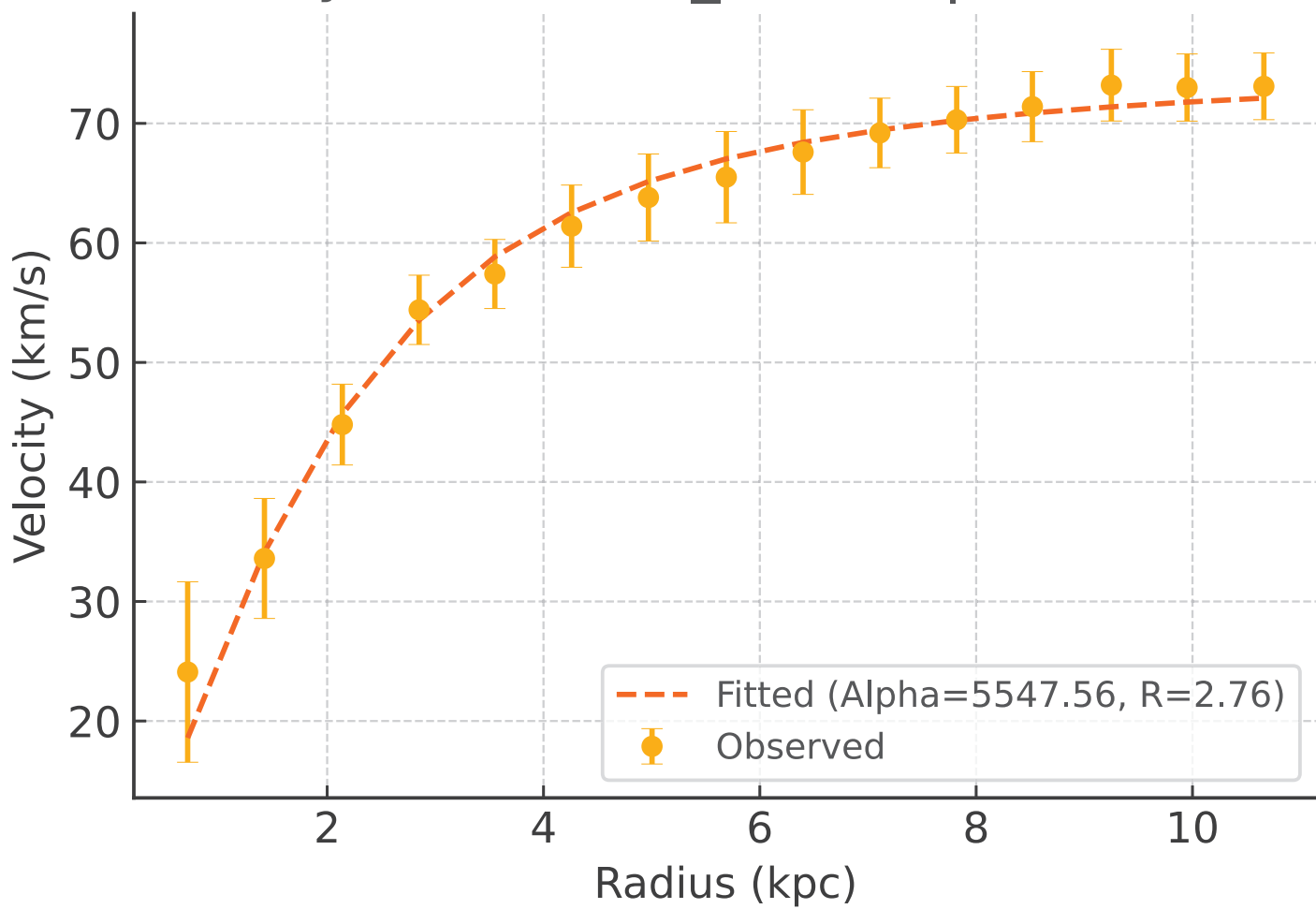
Galaxy: UGC11914\_rotmod |  $R^2 = -0.451$



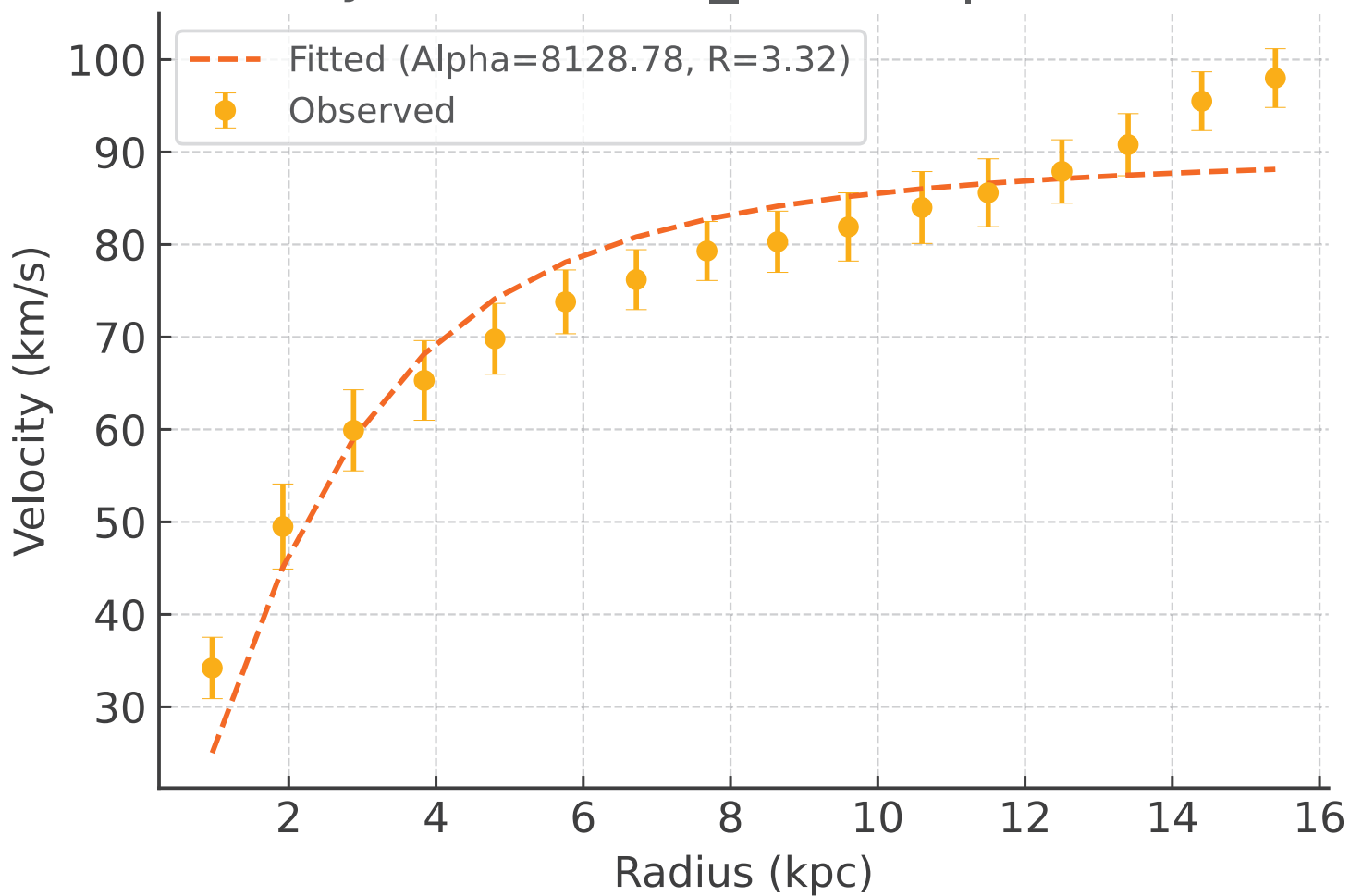
Galaxy: UGC12506\_rotmod |  $R^2 = 0.856$



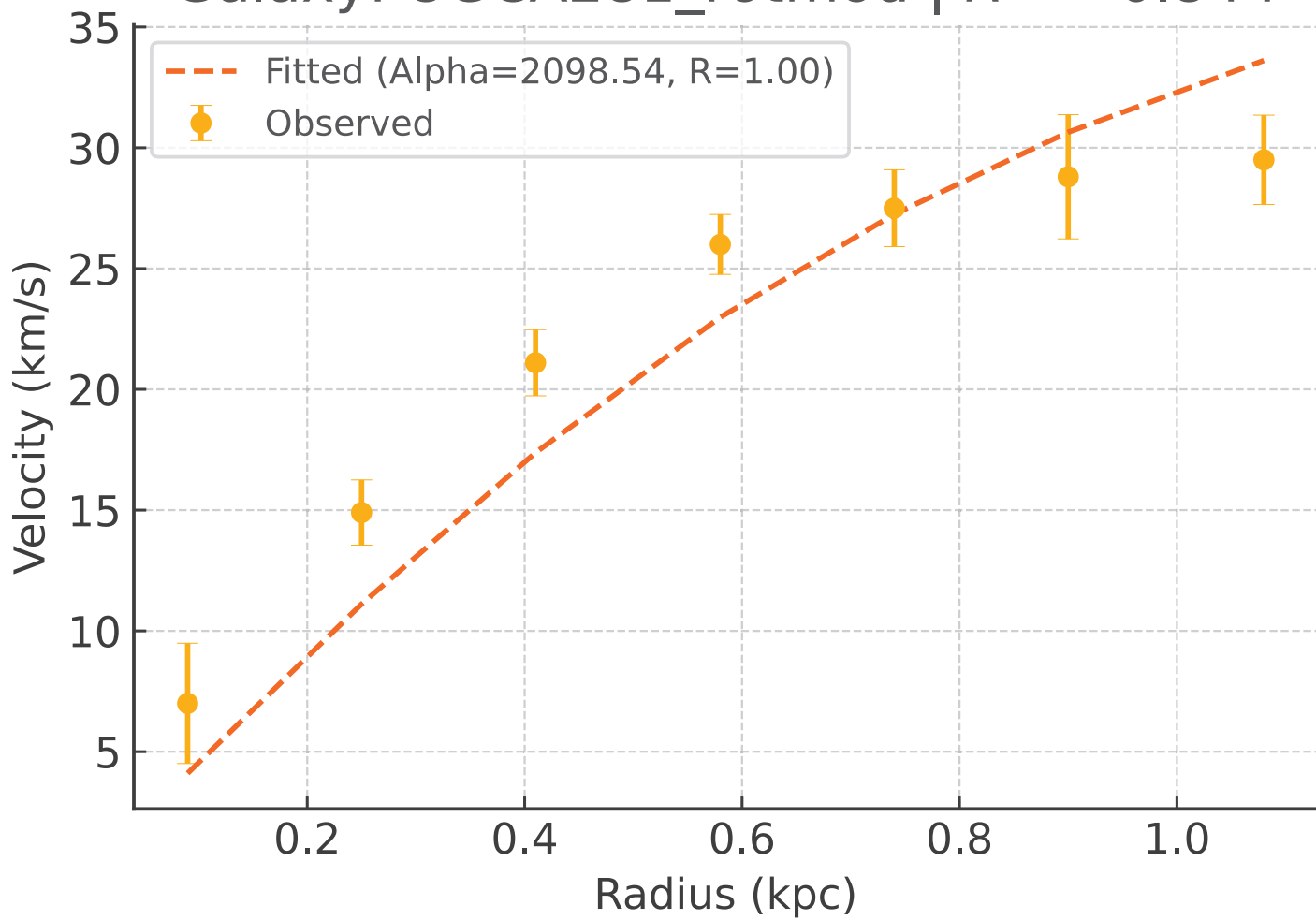
Galaxy: UGC12632\_rotmod |  $R^2 = 0.985$



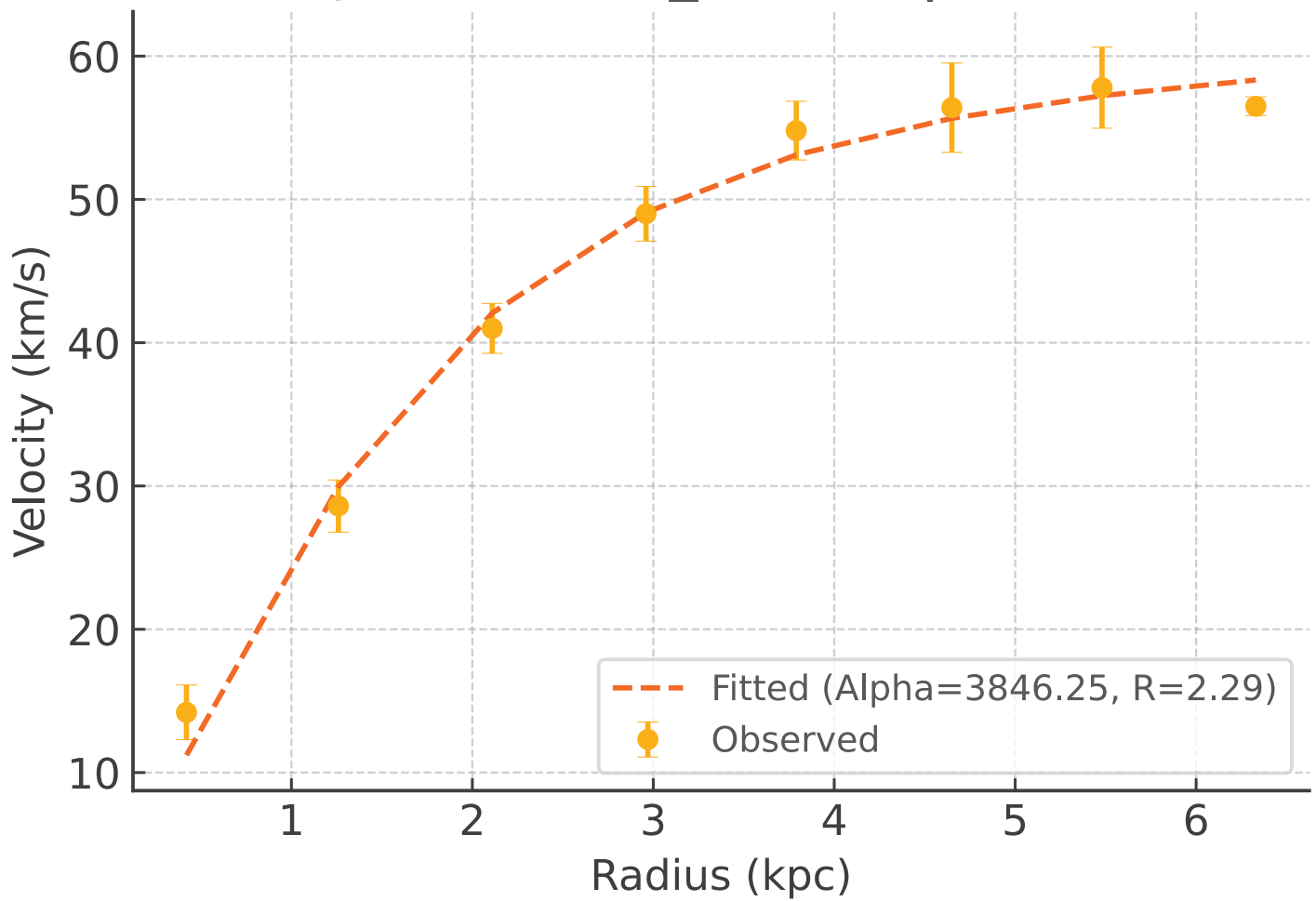
# Galaxy: UGC12732\_rotmod | $R^2 = 0.912$



# Galaxy: UGCA281\_rotmod | $R^2 = 0.844$

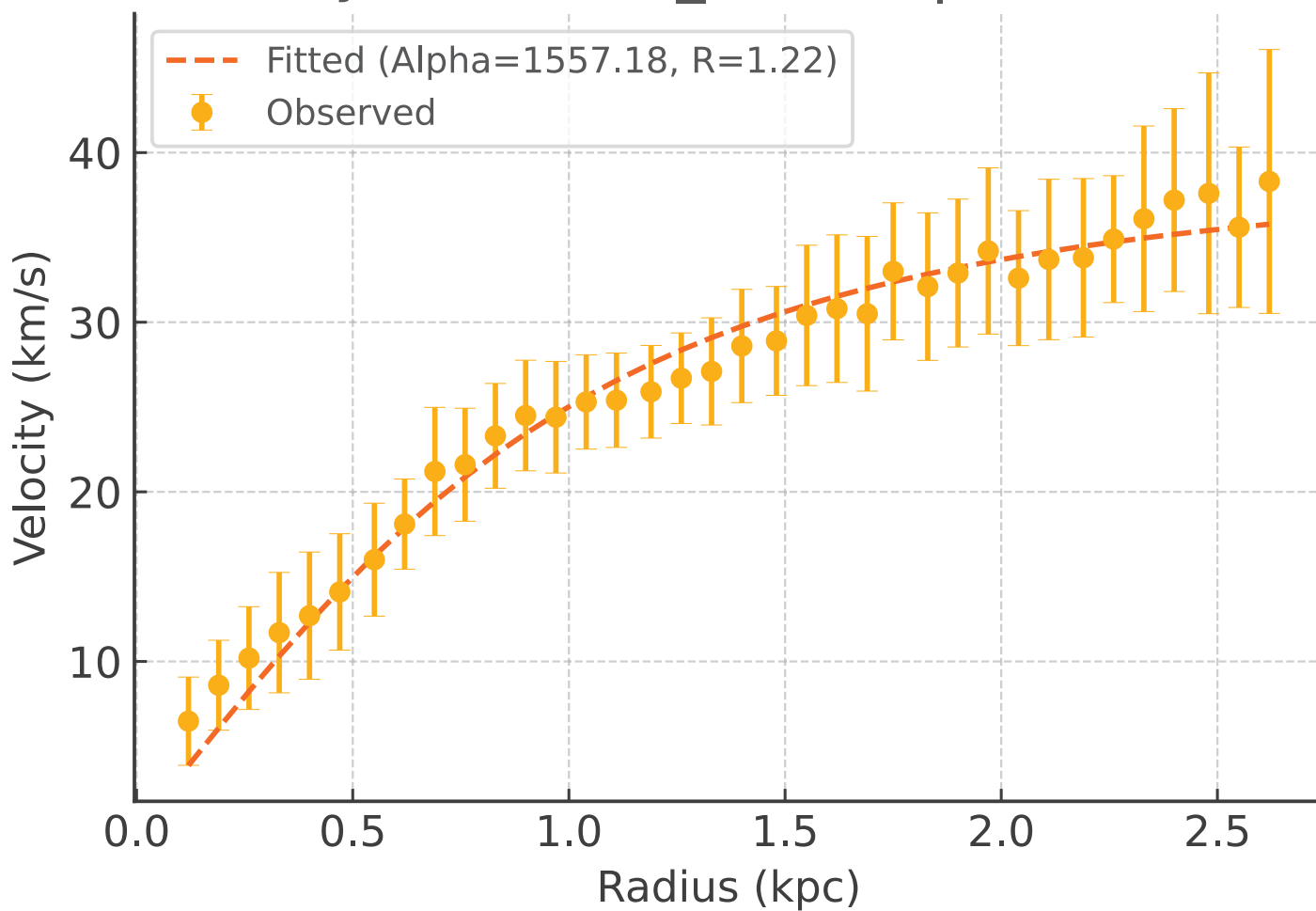


Galaxy: UGCA442\_rotmod |  $R^2 = 0.989$

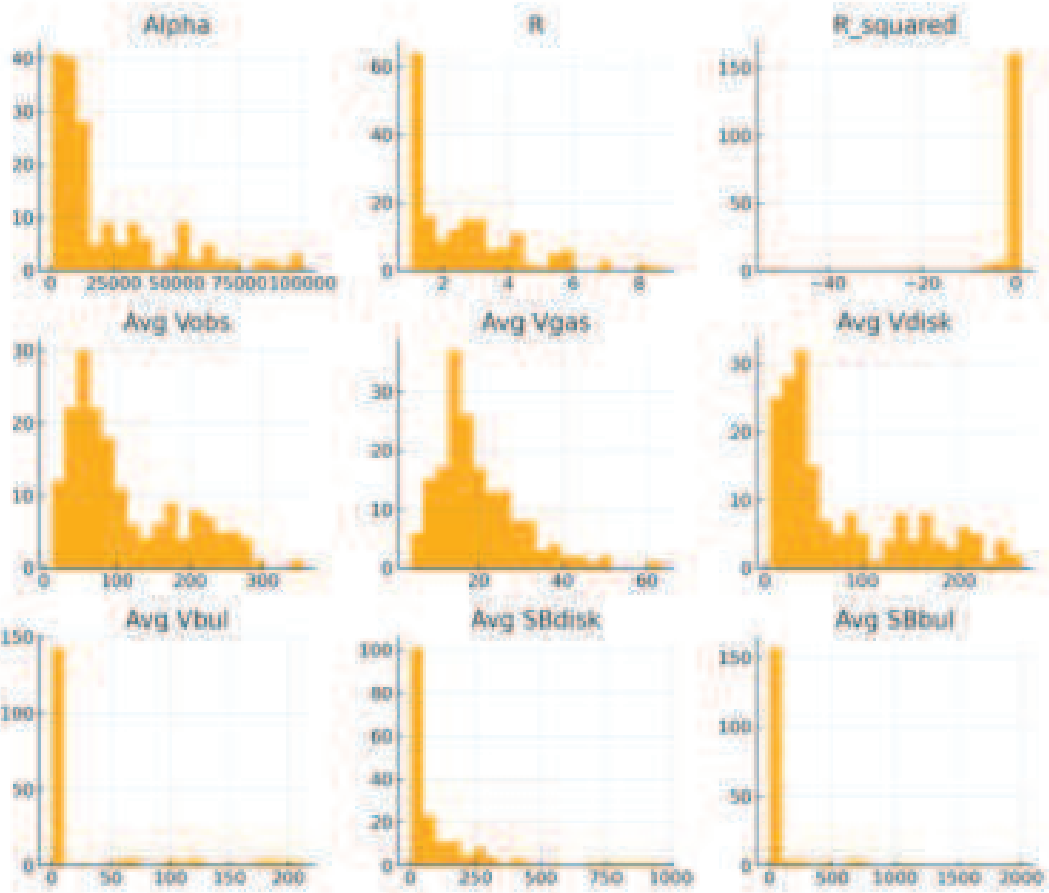




# Galaxy: UGCA444\_rotmod | $R^2 = 0.977$

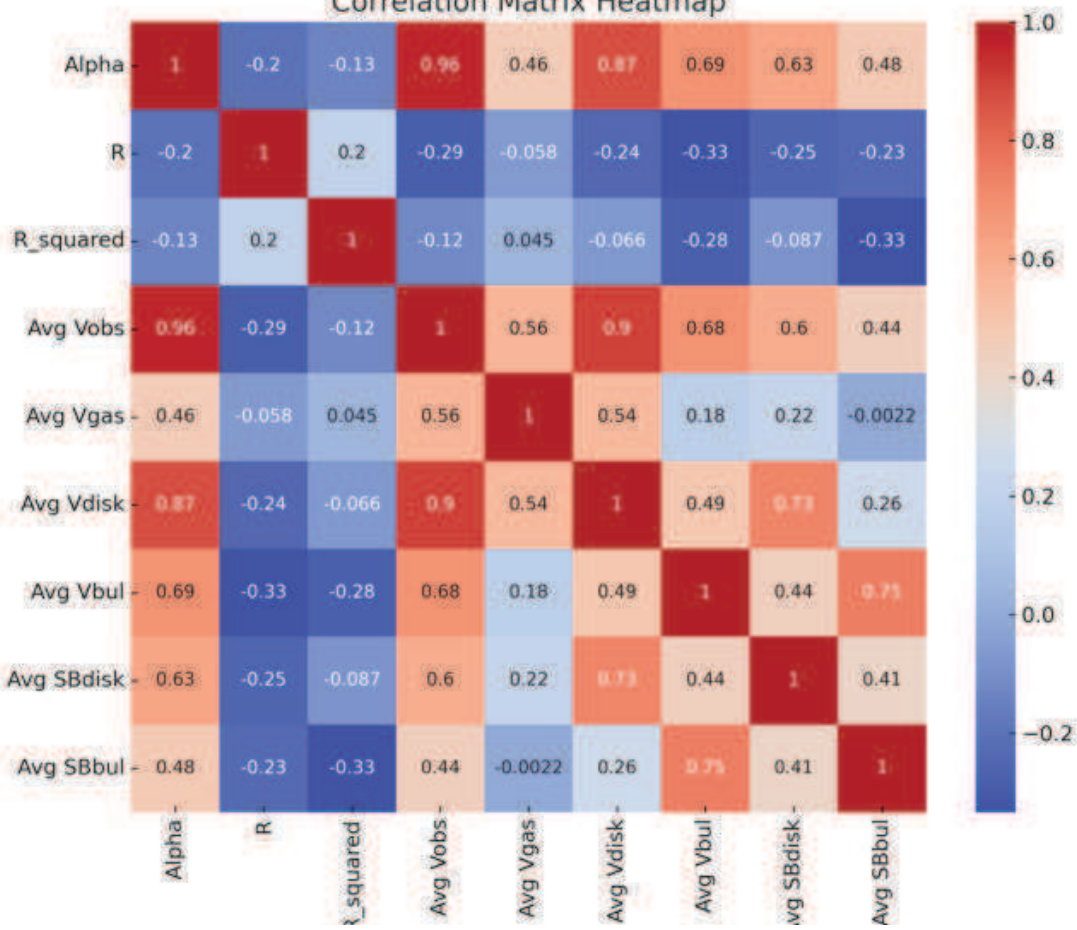


### Histogram of Variables

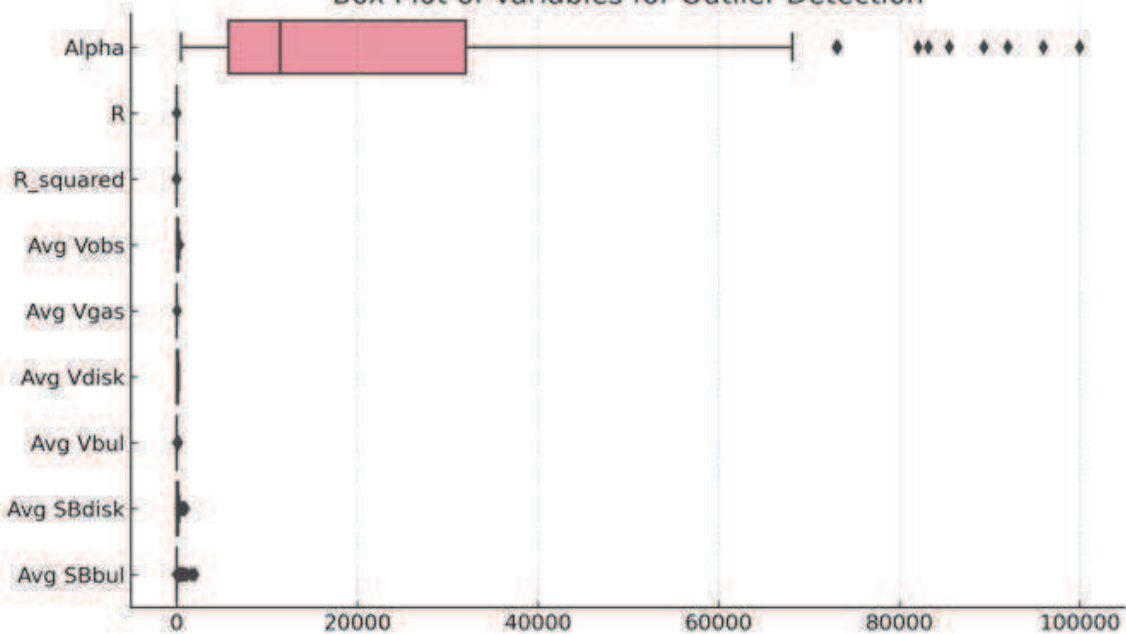




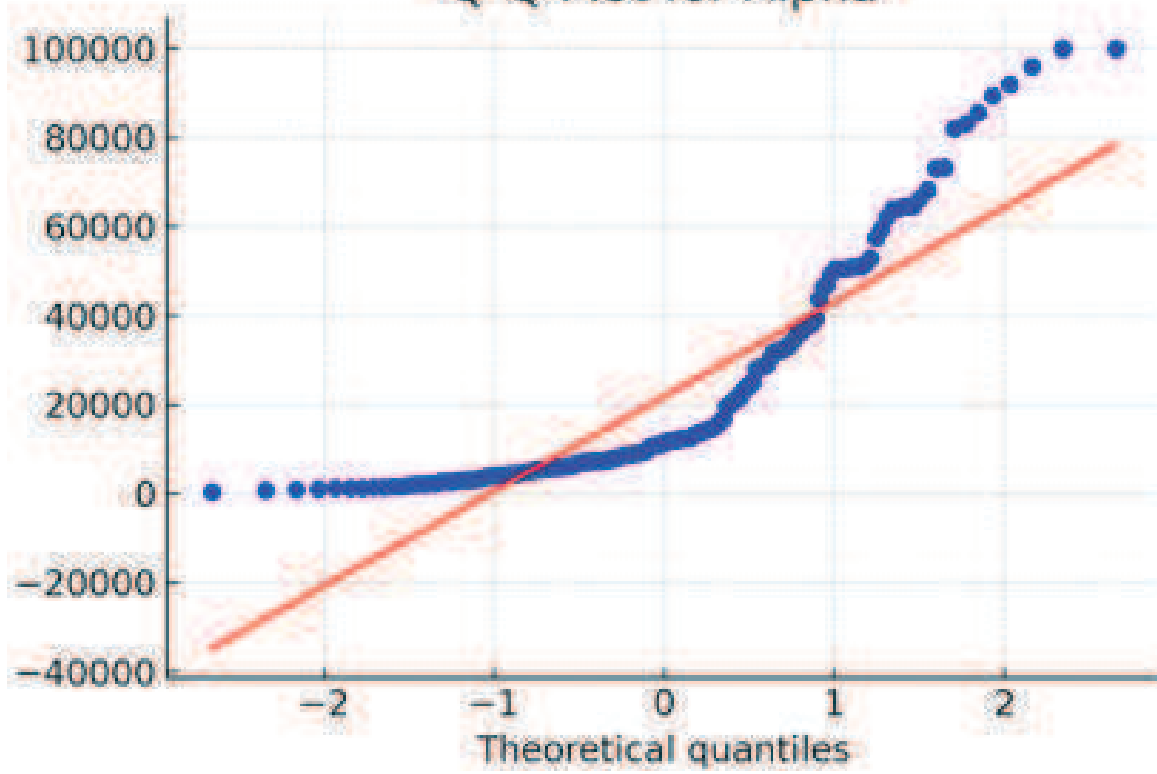
Correlation Matrix Heatmap

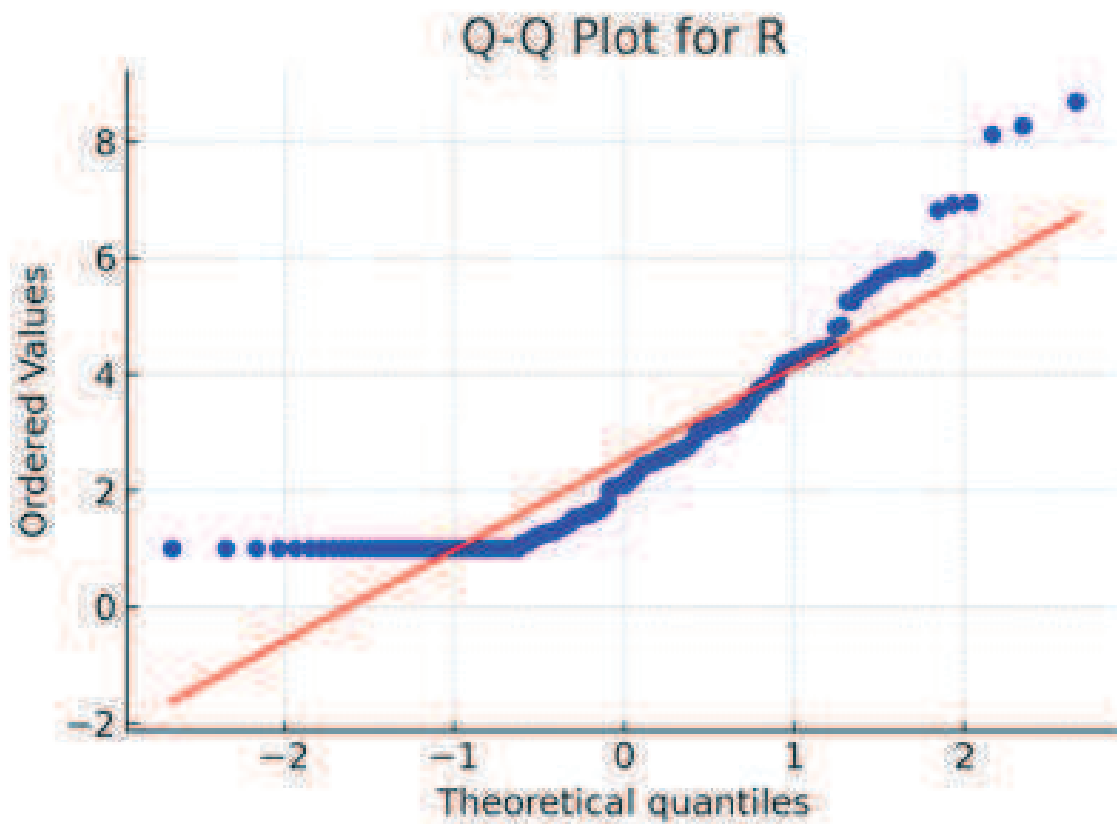


Box Plot of Variables for Outlier Detection

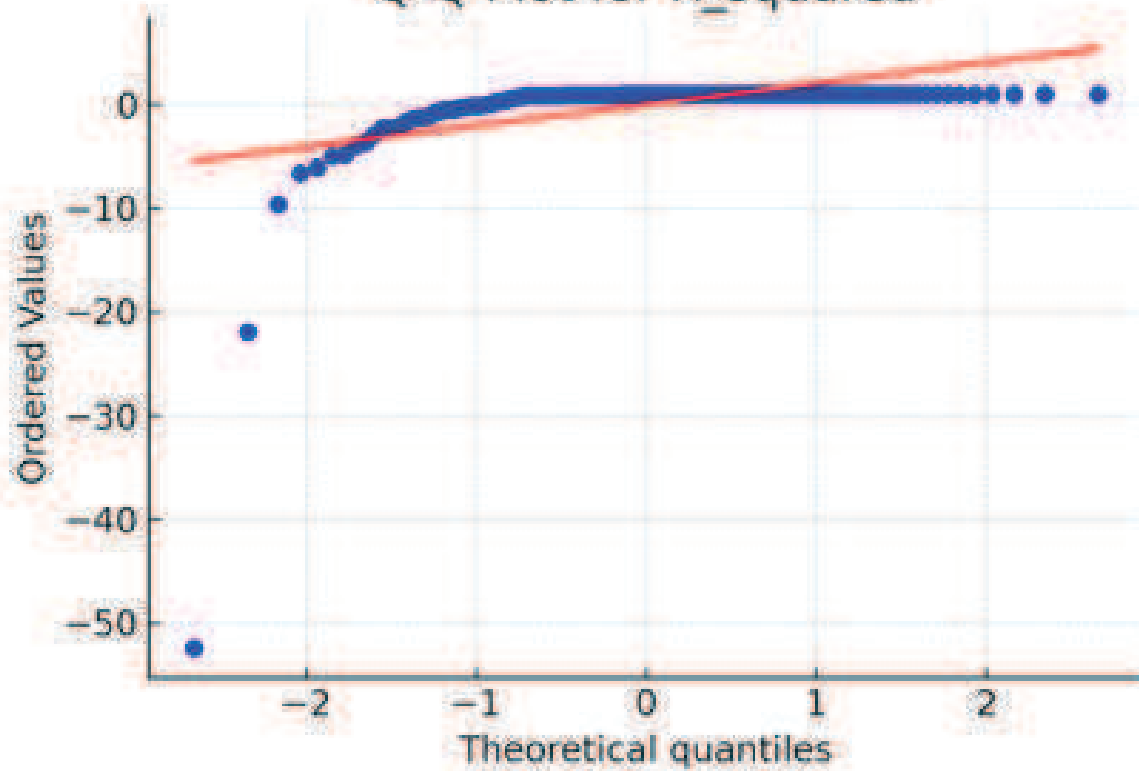


Q-Q Plot for Alpha



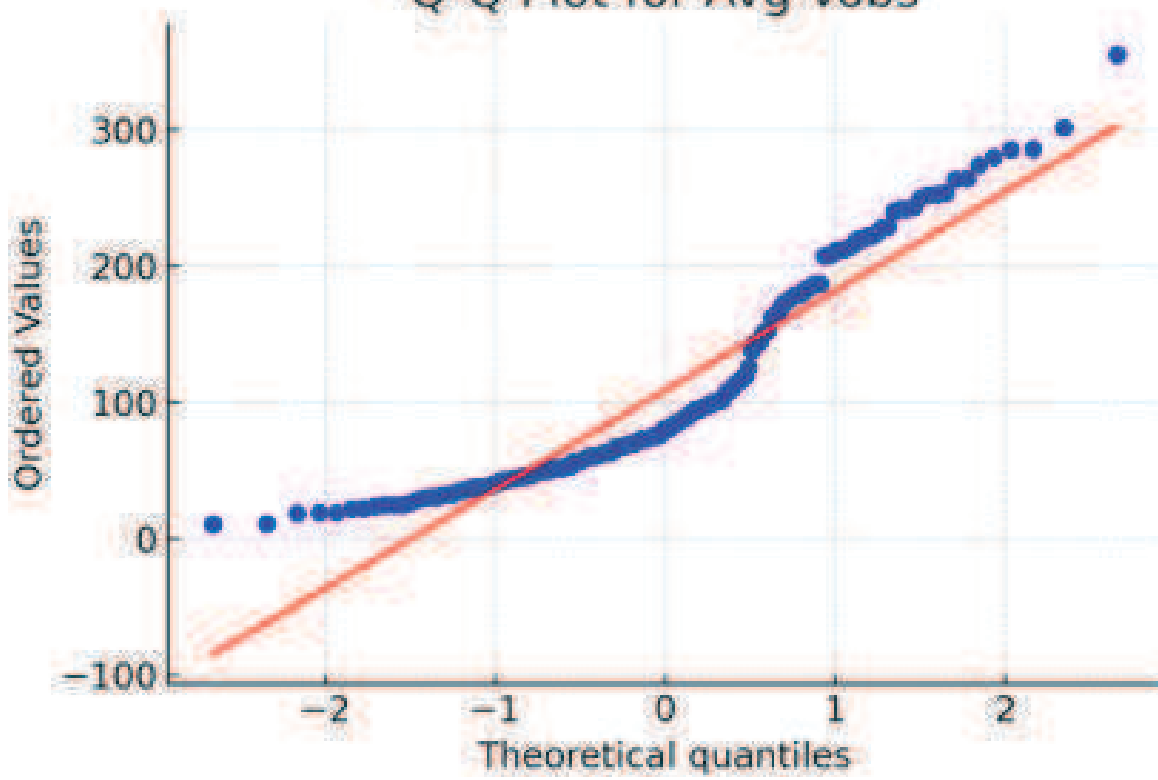


Q-Q Plot for R\_squared

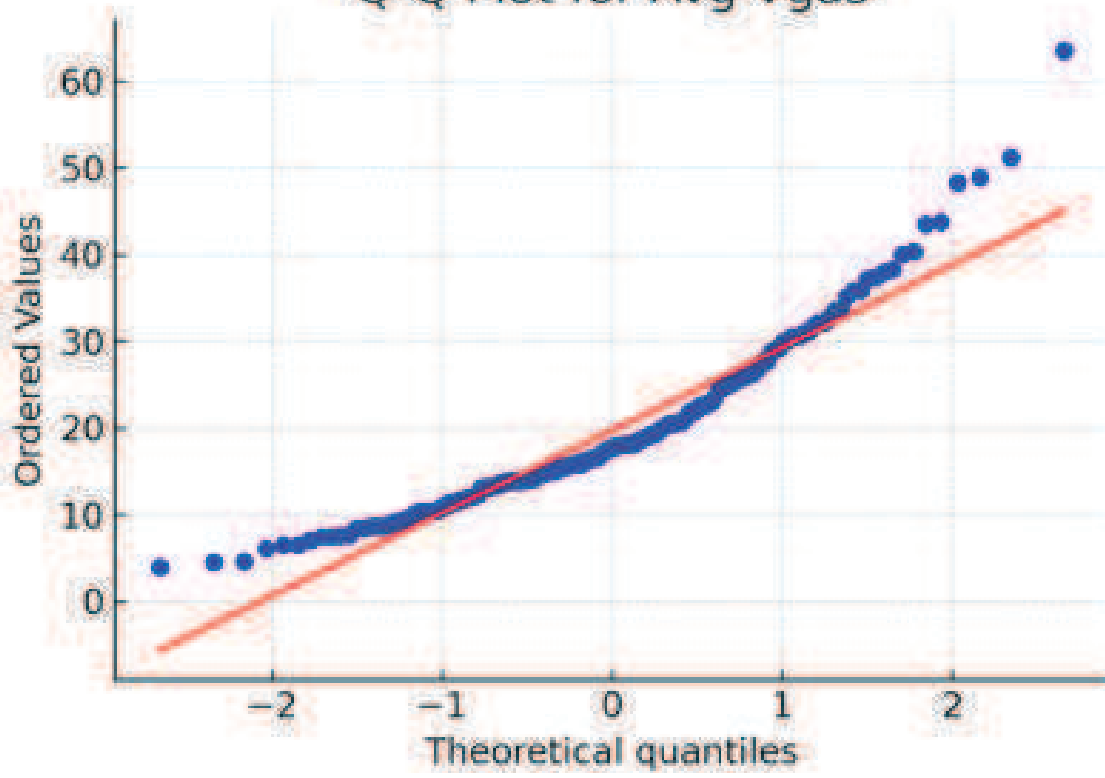




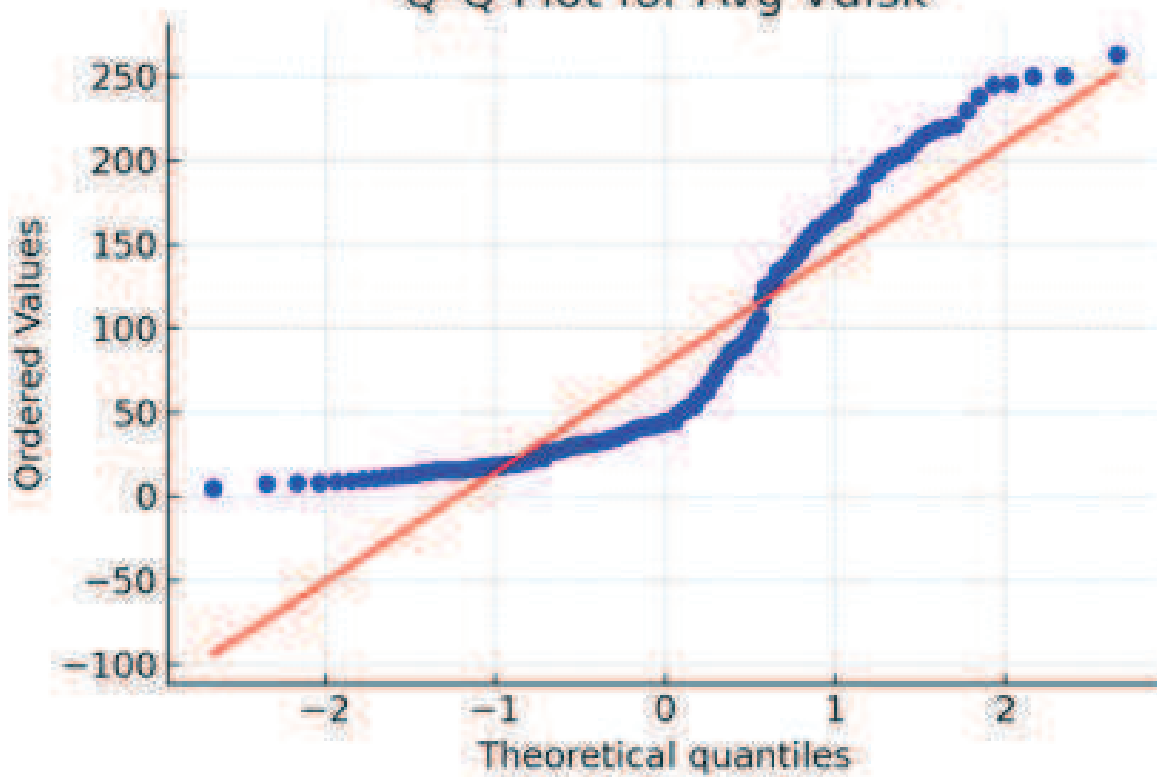
Q-Q Plot for Avg Vobs



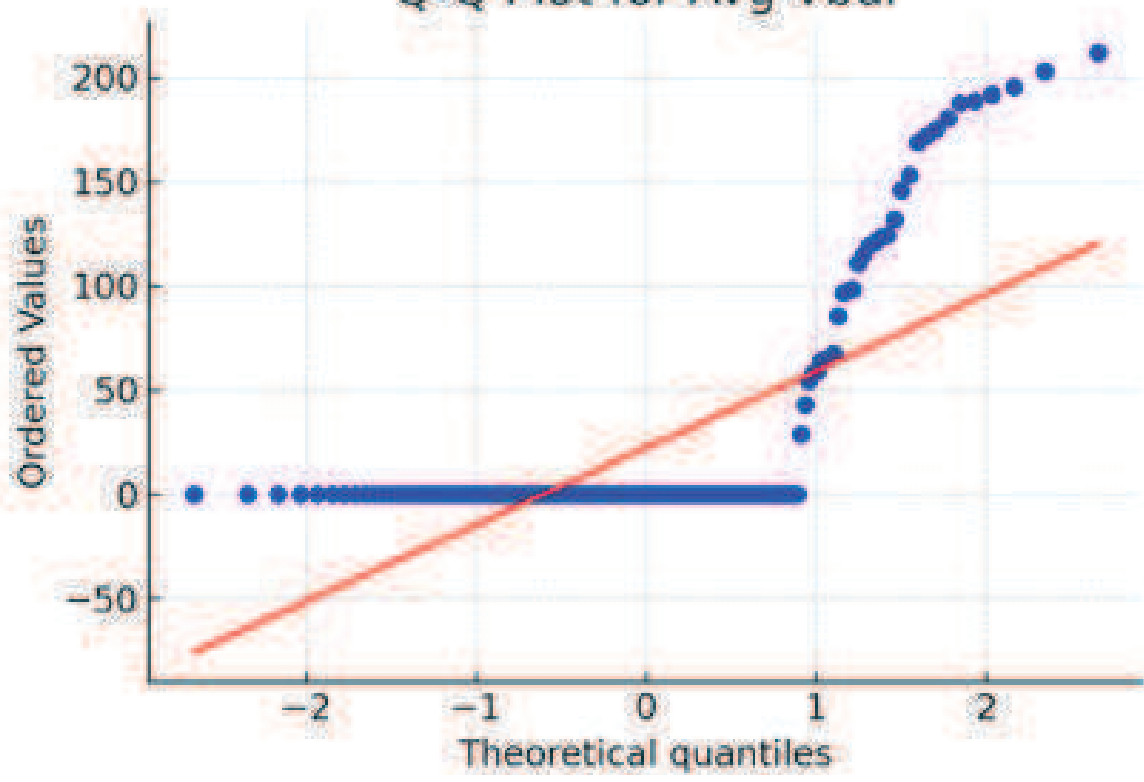
Q-Q Plot for Avg Vgas



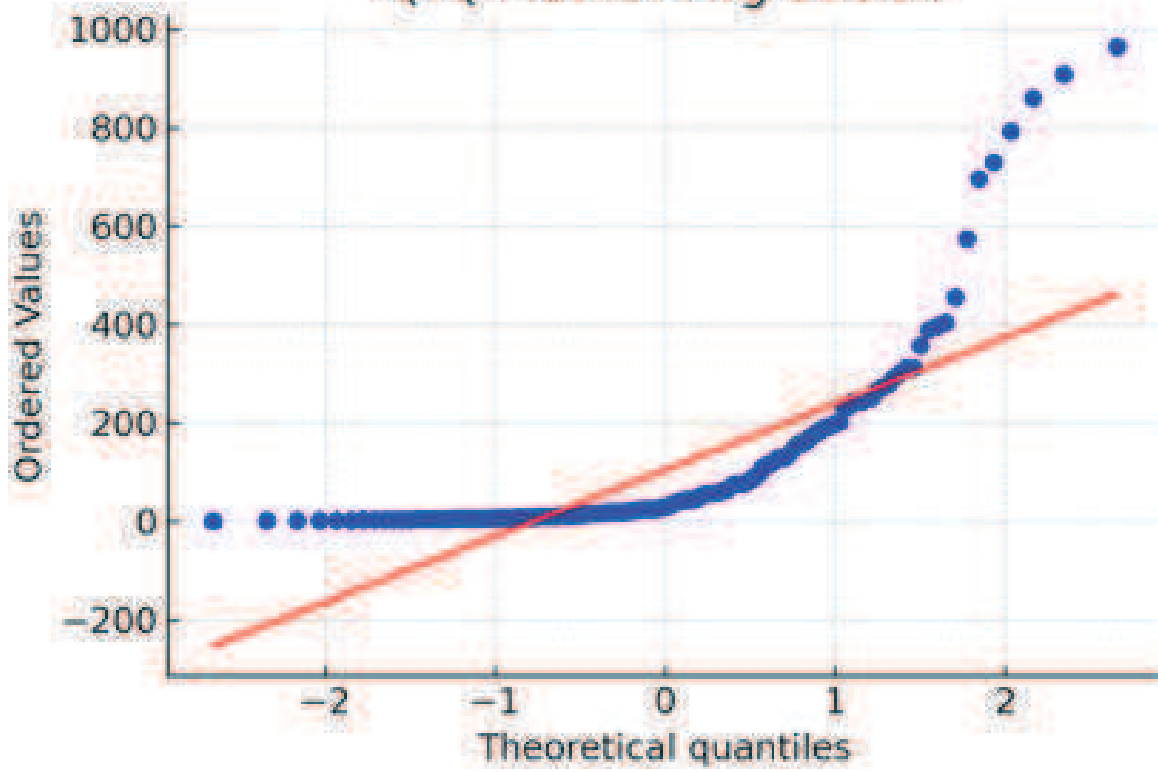
Q-Q Plot for Avg Vdisk



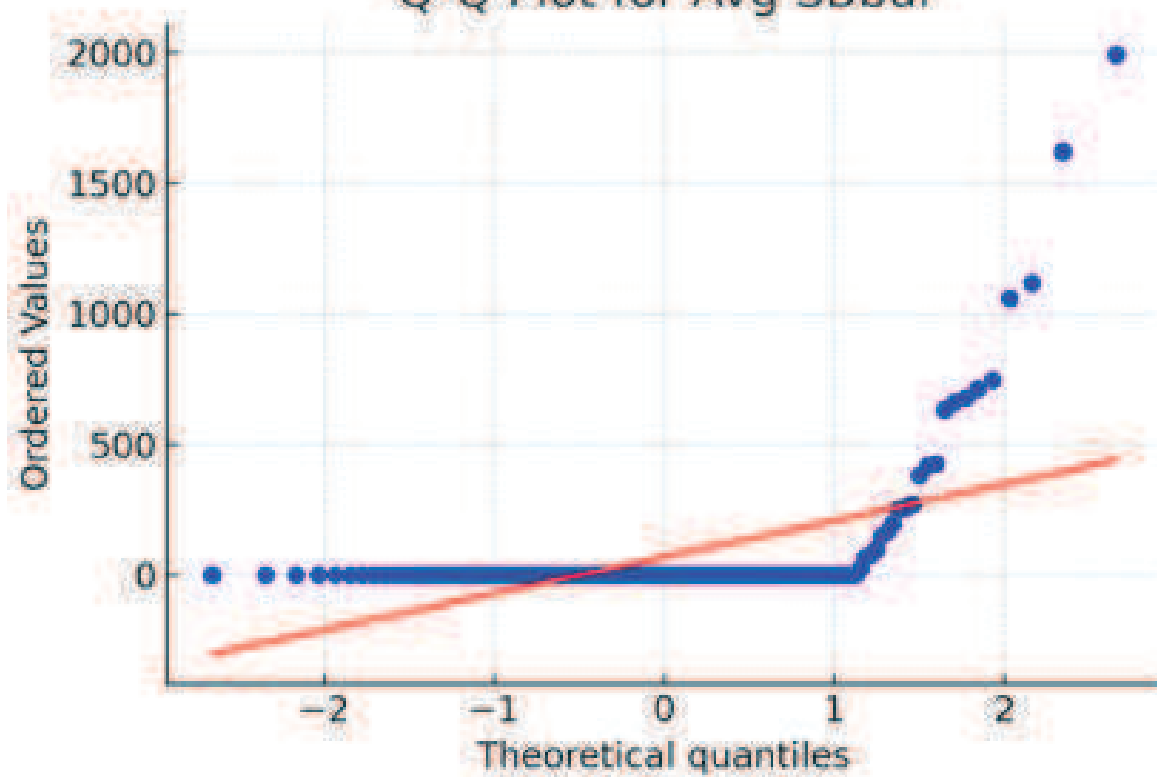
Q-Q Plot for Avg Vbul



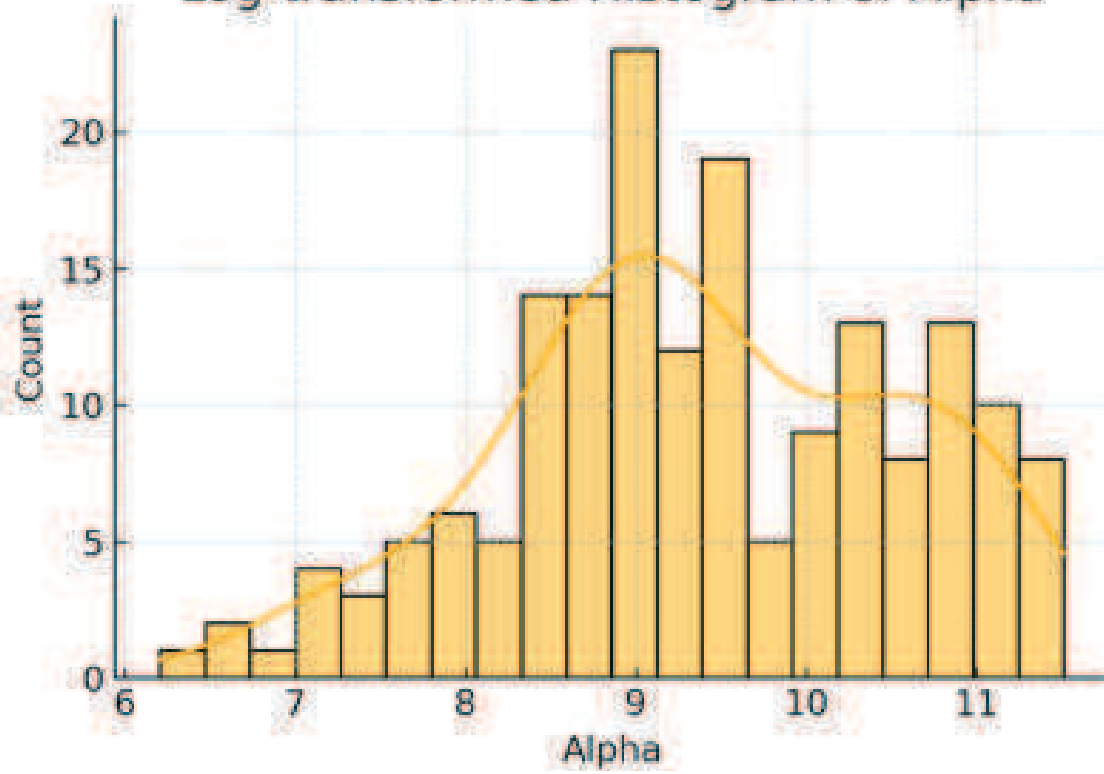
Q-Q Plot for Avg SBdisk



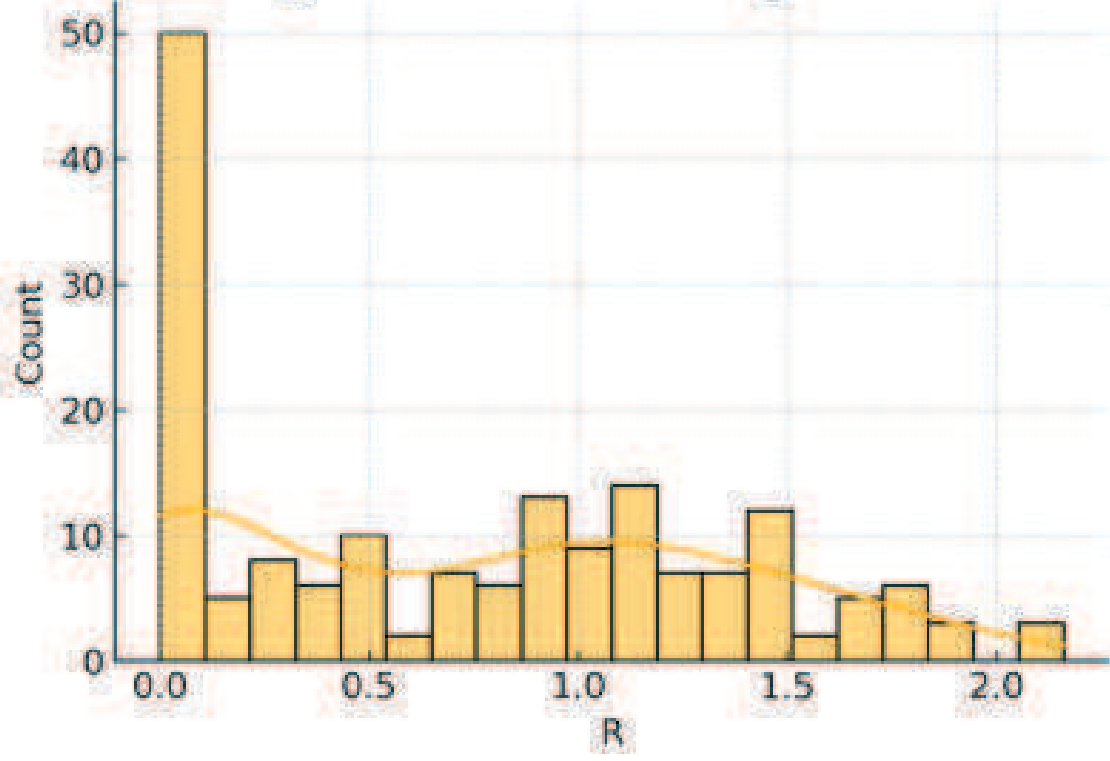
Q-Q Plot for Avg SBbul



Log-transformed Histogram of Alpha

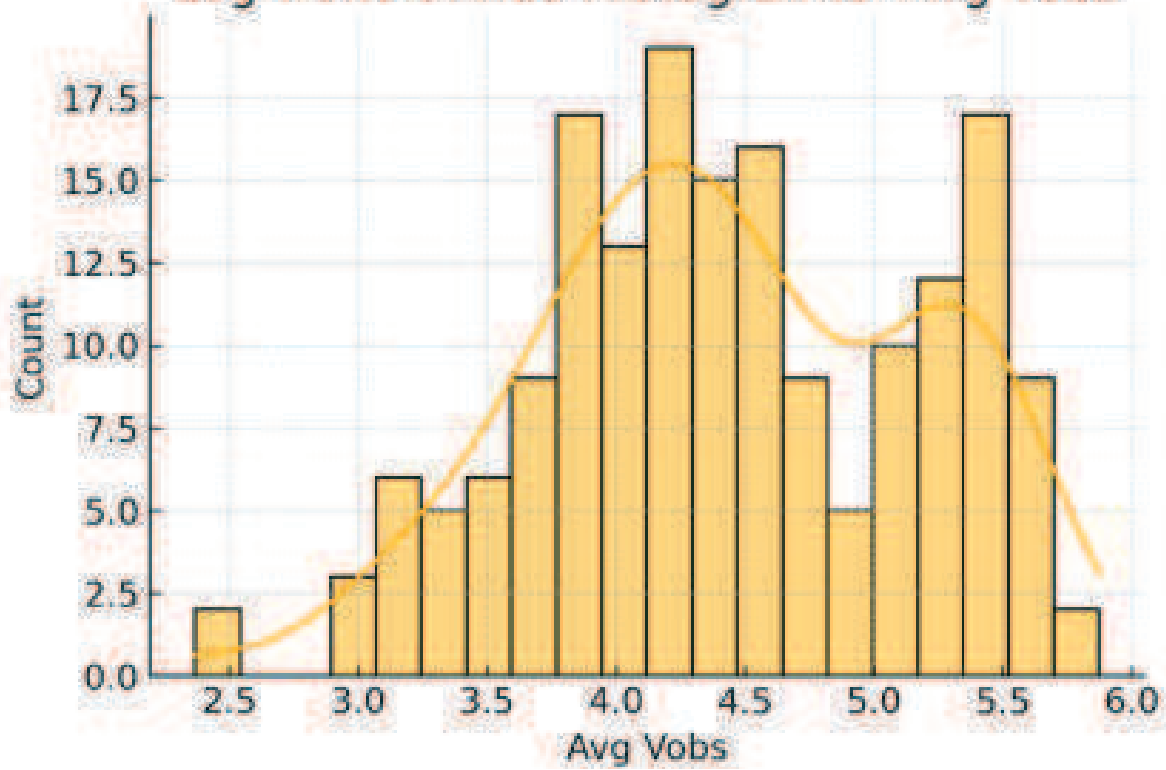


Log-transformed Histogram of R

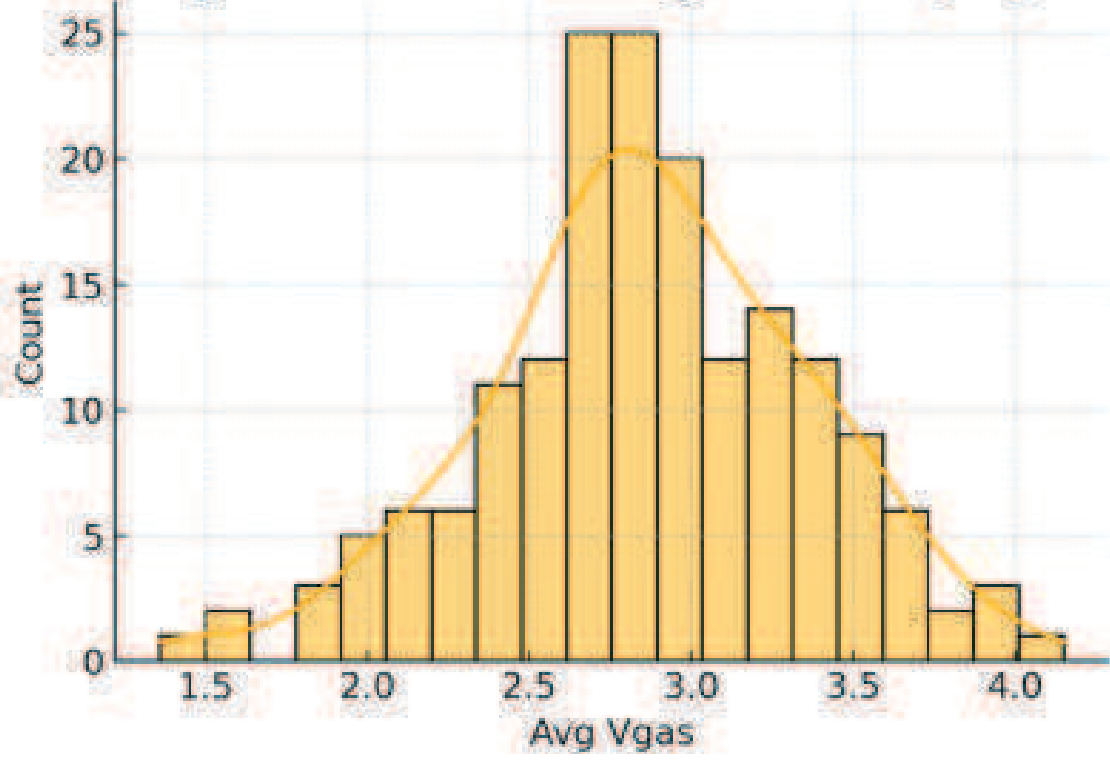




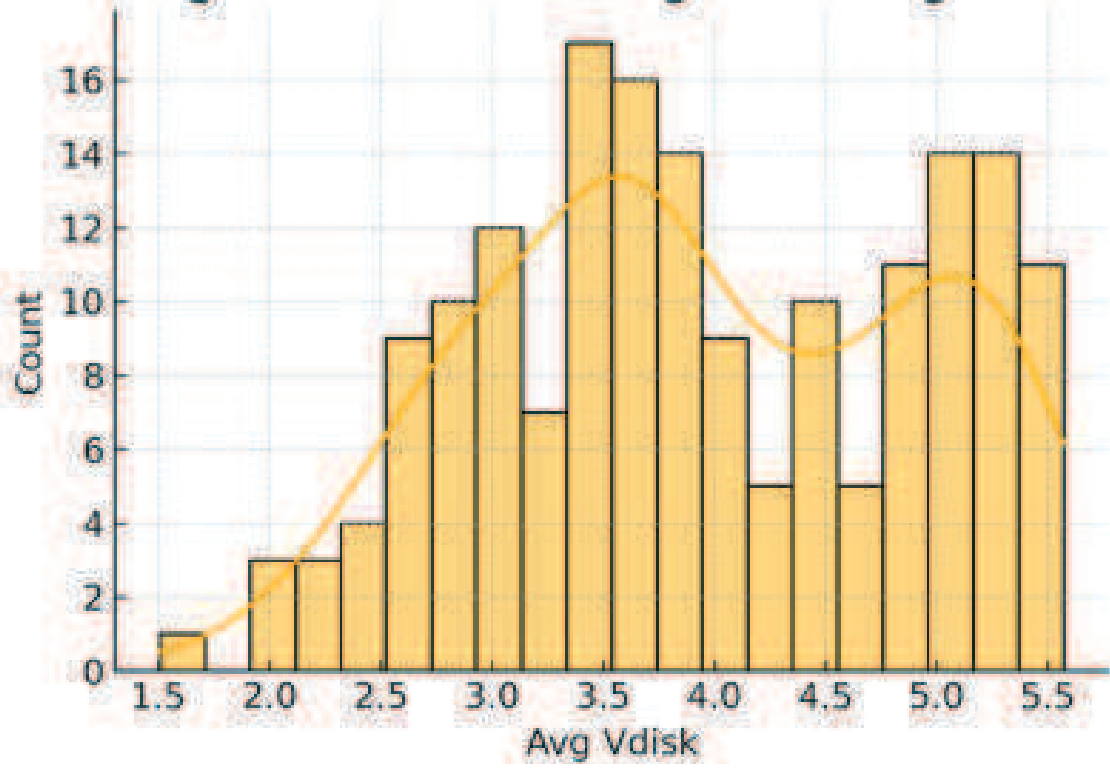
Log-transformed Histogram of Avg Vobs



Log-transformed Histogram of Avg Vgas



Log-transformed Histogram of Avg Vdisk



Log-transformed Histogram of Avg SBdisk

