QUANTUM FUNDAMENTALS AND HEURISTICS

THE WORLD AROUND COMPLEX PROBABILITY AMPLITUDES, WAVE-ENSEMBLE PARITY, QUANTIZED FIELDS AND ALL THAT

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Reviewing quantum fundamentals and related subtleties, which typically are not discussed sufficiently – even if at all – in standard texts and discussing "paradoxes" arising as a result. The purpose is to expose quantum ideology in manifestly wholesome, yet possibly plain and simple way, without mysterious surrogates. Main topics include: general Quantum Mechanical framework as imaginary "diffusion"; superposition / interference of probability amplitudes, not particles; Statistical vs Copenhagen interpretations of Quantum Mechanics; quantum measurements as mapping from amplitudes to probabilities; wave-ensemble parity instead of wave-particle duality; quantum field theory as a manifestation of quantum vacuum and the ideas of Grand Quantum Canonical Ensemble; entanglement as a consequence of an integrity of the quantum system and "spooky actions" as a mystification of simple conditional probabilities.

Key words: classical mechanics, quantum mechanics, quantum field theory, wave function, principle of superposition, probability amplitude, conditional probability, wave-particle duality, wave-ensemble parity, field quanta

"I must take a stand with reference to the most successful physical theory of our period, viz. the statistical quantum theory"

A. Einstein

Introduction. **First 100 years with Quantum Mechanics.**

In 1923, about 100 years back, Louie de Broglie, reversing the trail, initially blazed by Planck and Einstein for massless photons, hypothesized a certain "wave-like" structure for massive microparticles. Shortly after, by efforts of the Gottingen group (Born, Heisenberg, Jordan) and Schrödinger, the Wave/ Quantum mechanics was created and began its triumphant journey in all aspects of our life.
Immediately thereafter the de Broglie conjecture, the most vexing and misleading paradigm of XX century – the so called wave-particle duality – was born and made its

way into innumerable publications and standard texts on Quantum Mechanics, and became a mantra and hallmark of the quantum theory. The obvious intuitive discomfort of the wave-particle duality arose from the apparent connotation between the distributed nature of a wave and spatially localized character of a particle / corpuscle. Not the least, it became a reason that Quantum Mechanics gained a reputation of the most counter-intuitive and confusing theory of our time. The most typical formulation of it – with slight variations here and there – declared that, depending on circumstances, microparticles exhibited either corpuscular or wave-like behavior, even though no one knew what a wave-like behavior of a one sinlge particle was supposed to mean. Later on, this formulation evolved to "microparticles taken individually are certainly not waves, but nether are they classical particles per se, with typical classical attributes such as trajectories and the like". As a matter of fact, this is way better and carries lots of truth, but still brings only a negative connotation and needs further clarification.

Another popular misconception in Quantum Mechanics arose from the proverbial wave function "collapse", blessed by the reputation of N. Bohr and his Copenhagen school. While never formally spelled out in specific terms, an impact of the so called "Copenhagen Interpretation" was so profound, that even in 90s of XXst century, reputable physical journals were publishing works of quite renown scientists on statistics of wave function "collapses", patterned, for instance, after Fokker-Planck diffusion and the like.

The gamut of quantum mysteries extends by the well alive and kicking "Schrödinger Cat" phantom: ironically, it was none other than the Quantum Mechanics founding father E. Schrödinger, who mentioned it in one of his groundbreaking works as quite a "strange" potentiality.

The latest addition to the glorious gallery of the alleged quantum vices was the so-called quantum non-locality, very much related to the currently emerging quantum computation technologies. Premised on the names of A. Einstein and his colleagues (typically billed as EPR paradox, Einstein *et al*, 1935) as well as on heroic efforts of J. Bell in a search for hidden parameters, this ill-famed fallacy continues appearing in all kind of the extant publications and stirring controversies, especially, in inexperienced minds.

Needless to say, albeit all of the above mysteries were adequately resolved away and long gone (in what follows they will be properly addressed, the brevity of paper permitting), the story of quantum paradoxes is extremely instructive in showing how persistent the stereotypes and prejudices can be, if not addressed timely and exhaustively.

In a fairness to current physics students, the bulk of the former quantum literature and its wisdom is so huge and formidable, there is no chance it can be followed and digested in a reasonable and timely manner by new entrants. To add an insult to injury, the overwhelming majority of standard quantum texts merely ignores the problem, and just copies and pastes long gone misconceptions. The harm inflicted by above paradoxes should not be taken lightly and /or underestimated. They confused and baffled generations of scholars (the author included!) in their student years, and had an disarming impact on a confident reasoning in Quantum Mechanics and related sciences.
It is then highly desirable to produce periodic reviews of existing and emerging "paradoxes" and promote transparent logical standards and sane judgments.
On a more general note, the quantum mechanics drastically differs, as it does, from the mechanics classical. As a result, there is a need for convenient hacks, short-cuts, and qualitative methods, complementing heavy technical tools and supporting efficient heuristics and reliable intuition.
Accordingly, the present notes aim to help bridging away that gap with a possibly compact and readable text.

The distribution of the material is as follows. Sec. 1 makes a cursory introduction to Quantum Mechanics emphasizing its stochastic nature caused by the quantum vacuum. Sec. 2 discusses the Principle of Superposition and its popular misconceptions. Sec. 3 describes general Quantum Measurement techniques in the context of the Principle of Superposition. Sec. 4 addresses paradoxes of wave function collapses in conjunction with the wave-particle duality conundrum. Sec.5 introduces ideas of Quantum Field Theory and, in particular, emergence of elementary particles as field quanta. Sec. 6 explains long distance quantum correlations in composite systems and demystifies away the so-called quantum non-locality and "spooky action at the distance". A tutorial Appendix to Sec. 5 supplies some additional didactical details regarding concepts of Quantum Field Theory and wave-ensemble parity vs wave-particle duality.

In what follows in the interest of compactness, a few common intuitive abbreviations are taken for most repetitive terms: **CM** – Classical Mechanics, **QM** – Quantum Mechanics, **QFT** – Quantum Field Theory, **CI** – Copenhagen Interpretation, **SI** – Statistical Interpretation, **WF –** wave function.

References are arranged in an alphabetical order for an operational convenience.

1. The Statistical Framework of Quantum Mechanics.

"So if one asks what is the main feature of quantum mechanics, I feel inclined now to say that it is not noncommutative algebra. It is the existence of probability amplitudes which underly all atomic processes"

P. Dirac

It appears, the most logically and technically consistent introduction of Quantum Mechanics (QM) is simply an analytic extension of classical Wiener–Einstein diffusion to complex-valued Shrödinger – Born random motion via the transition from real to imaginary time t ->it and associated conversion of real valued classical probabilities to complex valued Probability Amplitudes. To that end, we make a cursory introduction to QM employing this approach, thereby emphasizing a statistical nature of QM. Specifically, starting with the replacement t -> it in the classical diffusion equation

$$C'_t(x,t) = D_{cl}C''_{xx}(x,t) \qquad (1.1)$$

where D_{cl} – classical diffusion coefficient, and setting the quantum diffusion coefficient $D = D_Q = \hbar/(2m)$, m – mass of a microparticle, we reproduce the Shrödinger equation for a free particle:

$$\Psi'_t(x,t) = i\, D_Q\Psi''_{xx}(x,t) \;\text{ or }\; i\hbar\, \Psi'_t(x,t) = -\,[\hbar^2/(2m)]\,\Psi''_{xx}(x,t) \qquad (1.2)$$

(Note in passing that as $\hbar$ turns very small, and m very large, quantum diffusion vanishes, as expected). Consistently with that, a classical diffusion probability (classical Green function –the solution to (1.1) for C(x.0) = δ(x))

$$G_{cl}(y, t; x ,0) = (4\pi Dt)^{1/2}\exp[-(y-x)^2/ (4Dt)] \qquad (1.3)$$

transforms to a quantum Probability Amplitude (quantum Green function – the solution to (1.2) for Ψ (x.0) = δ(x))

$$G_Q(y, t; x ,0) = (4\pi D_Q it)^{1/2}\exp[i(y-x)^2/ (4D_Q t)] \qquad (1.4)$$

In the same vein, classical composition law P(y, t' ← x, t) = ∫ P(y, t' ← z, T) dz P(z, T ← x, t) changes to composition of quantum Probability Amplitudes (Green functions G is a synonym for the Feynman amplitude K) G(y, t' x, t) = ∫ G(y, t' ← z, T) dz G(z, T ← x, t). Also, the replacement t -> it converts the Wiener real path integral measure into the complex-valued Feynman path integral measure (for more technicalities on this, please, see e.g. Roepstorf, 1994 and Nagasawa, 2000).

This short synopsis clearly hints at the stochastic nature of QM and indicates that a wave function (WF) plays a role of a Probability Amplitude distribution in an ensemble of quantum realizations, simply in a quantum ensembles (von Neumann, 1932; Blokhintsev, 1964 and 1968; Ballentine, 1970). More specifically, according to the Born stochastic interpretation of WFs, a randomness of a quantum motion (stochastic diffusion of a quantum particle) results from an impact of the quantum vacuum (as will be discussed in more detail in Sec.5 on QFT, the quantum vacuum – the lowest energy state of any system, i.e. without any particles / field quanta, yet having an infinite energy – underlies the state of any quantum system, and does have experimental manifestations!). In other words, heuristically we picture a quantum wandering as a diffusion under an impact of the vacuum field.

Now, according to the above, the de Broglie wave is nothing but the WF (Probability Amplitude) of a free particle with a linear momentum p in the q representation. It is identical to the Feynman amplitude (Feynman, Hibbs, 1965) $\exp[(i/\hbar)S(a,b)] \sim \exp(i x/ƛ)$, where $S(a,b)$ – is the classical action with $x = a - b$, and the reduced de Broglie wavelength $ƛ = \hbar/p$. In this context the de Broglie wavelength ƛ is the scale of oscillations of the Probability Amplitudes, i.e. the range, where wave aspects of quantum Probability Amplitudes are material. The well known illustrations include classic examples from wave optics: intensity oscillations near the edge of a semi-infinite screen, the diffraction on single and double slits, and so on (more on this - in the next Sec. 2).

We turn now to the Principle of Superposition in QM.

2. The Principle of Superposition.

The things that interfere in Quantum Mechanics are not particles. They are probability amplitudes for certain events"

R. Glauber

Of all linear equations of mathematical physics, obeying the Principle of Superposition, the Shrödinger equation was – and still is! – the most abundant source of innumerable and mysterious wonders, ensuing from erroneous interpretations. Suffices it to site a glaring Google response to the request "Quantum Superposition": "Superposition is the ability of a quantum system to be in a multiple states at the same time until its measured". Nothing can be farther from the truth than that! Here we discuss only one, the most typical formulation, that is: "in the state of the superposition a particle "simultaneously" resides in all

participating / superposing substates." (For other issues the reader is referred to classic works of D.I. Blokhintsev, 1964 and 1968).
On the contrary, and in the nut-shell: 1) it is the Probability Amplitudes of contributing states that interfere in the Principle of Superposition, not particles, and 2) this interference is by no means a signature of any kind of "simultaneity", but rather a consequence of the overlap of Probability Amplitudes.
First of all, we remind the reader of the epochal Tonomura 2-slit experiments for electron diffraction (Tonomura *et al*, 1989), which showed a pronounced interference pattern as a build-up result at the long temporal exposures, while for short exposures only random / irregular spots were observed.

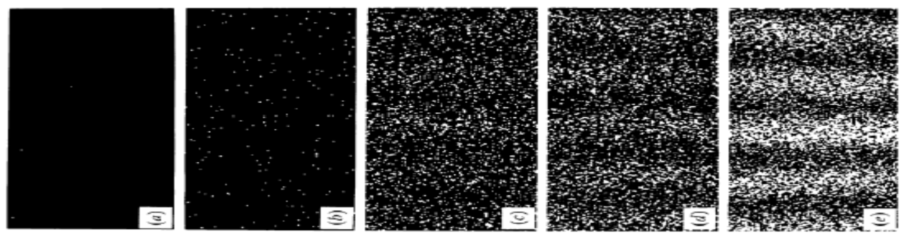
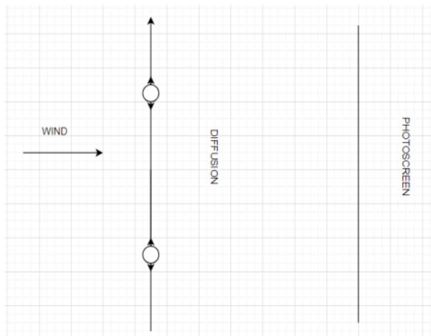


Fig. 2.1 The cumulative pattern generated by the "self-interference" of electrons sent one by one through a two-slit interferometer. Number of electrons: a) 10, b) 100, c) 3000, d) 20 000, e) 70 000 (from Tonomura *et al* (1989))

Inasmuch as the results similar to Tonomura et al. for electrons, were known also for photons since Taylor experiments in 1904 (Taylor, 1904), the build-up nature of diffraction patterns becomes evident for massive and massless microparticles alike, and lends a direct support to an interference of Probability Amplitudes, rather than interference of particles.
For an additional insight, consider now a classical analogue of the Tonomura set-up, namely: a vertical one-dimensional classical diffusion from two well separated sources of identical substances (depicted as small circles with short arrows up and down on Fig 2.2) with the drift (wind) pushing to the right in the horizontal direction (Fig. 2.2).



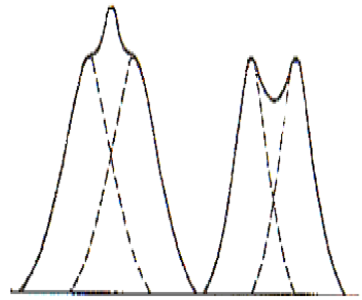


Fig. 2.2 2-slit experiment: classical analogue **Fig. 2.3 (**a, b**)** Density distribution at the screen
a) close sources, b) distant sources

At the beginning of the experiment (short exposures!) the spots on the screen will be totally random / chaotic, and not showing any systematic tendency – exactly as in the classic Tonomura quantum case. But as time progresses, the accumulation of spots will show more and more evidence of the smooth overlaid distributions (Fig. 2.3, a and b). The only difference from the original Tonomura set-up is the clear oscillatory interference pattern in the quantum case. We therefore come to a conclusion, that it is the short exposure / low-intensity limit that helps reveal the true source of interference: and that is the interwinding of random outcomes preventing from tracing back the origins of individual events. We emphasize that this effect exists in both classical and quantum cases alike, with the exception that quantum case is further embellished with intensity oscillations due to interference of complex-valued Probability Amplitudes.
To paraphrase: quantum superposition is simply an epitomized overlay of Probability Amplitudes, i.e. a trivial distribution overlap, plus the oscillations, ensuing from the complex nature of overlapping amplitudes.
This clearly demonstrates that intensity "waves' is nothing, but a build-up of elementary events into assemblages, distributed in a wave-like fashion. And there is no whatsoever interference of particles situating simultaneously around same spatial locations. This concludes the paradox resolution and staves off any mysteries arising from "simultaneity" in the quantum interference. It is exactly the reason why quantum measurements need the separation of "beams" as much as possible – compatible with an experimental equipment – to suppress the overlap of the superposition components (for more on this see the next section Sec. 3).

Now, the well known from classical optics diffraction patterns demonstrate what the superposition of quantum amplitudes produces in the limit of long exposure in terms of wave-like structures, to wit: the diffraction on and intensity oscillations by the edge of a semi-infinite opaque screen (Sömmerfeld, 1954), the diffraction on the single slit (any slit is a combination of two edges, and, depending on the distance between edges in comparison to the de Broglie $\bar{\lambda}$, the intensity pattern on the screen transforms from almost geometrical optics to a typical interferential wave distribution), the archetypal 2-slit diffraction, etc. The heuristic significance of these examples is in that they illustrate how the de Broglie scale $\bar{\lambda}$ interplays with other geometrical scales of experimental set-ups in classical optics (Figs. 2.4-2.6)

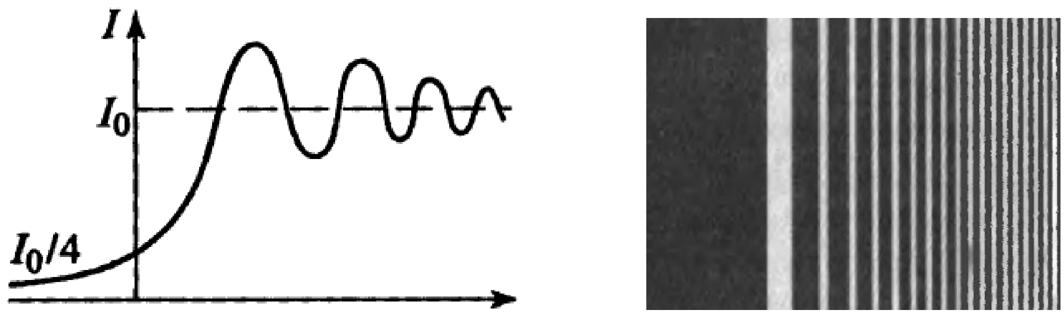


Fig.2.4 Diffraction on semi-plane (intensity distribution and photograph)

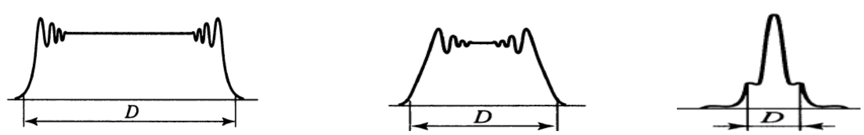


Fig.2.5 Intensity distribution for a diffraction on a slit, a) D >> $\bar{\lambda}$, b) D > $\bar{\lambda}$, c) D < $\bar{\lambda}$

In particular, on Fig. 2.4: for a semi-plane diffraction there is only one scale – the de Broglie wave-length $\bar{\lambda}$, affecting the profile. Fig. 2.5: two semi-planes at some distance D, forming thereby a slit of width D. The pattern depends on two scales, $\bar{\lambda}$ and D, a) D >> $\bar{\lambda}$ – geometrical optics, 2.5 b) D > $\bar{\lambda}$ – intermediate case, 2.5 c) D < $\bar{\lambda}$ – wave optics.

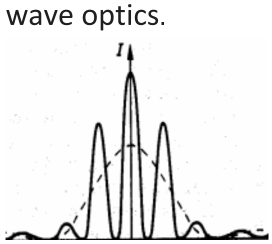


Fig. 2.6 2-slit diffraction (solid line, dashed line - one slit intensity), the case of D < B << $\bar{\lambda}$;

Fig.2.6. – two identical slits, and, therefore, the pattern depends now on 3 scales: $\bar{\lambda}$, D, and B – distance between slits. All of the above patterns arise from an

interference of the incident plane wave exp(ikx) with a wave, scattered off the obstacle. However – and this is of a key importance – for a sufficiently short exposures (how sufficiently – depends on the intensity!) all experiments would show only random spots, with practically NO traces of any regularity. Accordingly, the interference pictures on Figs. 2.4 – 2.6 become visible only after sufficiently long observations (long exposures, that is). This clearly indicates, that waves in QM (i.e. wave functions!) are but congregations of random elementary events, with distribution thereof modulated by wave profiles, and points in favor of the Statistical Interpretation (SI) of QM and WF as an Ensemble Measure, and not a measure related to individual particle (Blokhintsev, 1964 and 1968, Ballentine, 1970). Further, in the context of the Born postulate, Probability Amplitudes (WFs, that is) bridge, so to say, wave features of the ensemble with a behavior of individual particles, to wit: an amplitude is clearly a wave attribute, but applies to a probability, related to detecting an individual particle in an ensemble of similar particles.

A deeper technical grasp of the diffraction wave profiles can be obtained via a quasi-classical approximation, which aims to smooth the sharp transition from classical trajectories to quantal interference of Probability Amplitudes by treating the system as a combination of classical and quantum features. Specifically, the infinitesimally thin classical trajectories “swell” into Feynman “tubes” around them. And accordingly, the semiclassical wave function becomes $A(x)\exp[(i/\hbar)S_{cl}(x)]$, where $S_{cl}(x)$ is a classical action along the classical trajectory at the centre of the tube, and A(x) is a tube envelope (a transverse tube size is of the order of the de Broglie wavelength $\lambda\!\!\!^{-}$) to preserve a normalization of probability (Migdal, 1977). Thus, a tube becomes a microparticle channel, so that 1) the particle spatial spread appears as about $\lambda\!\!\!^{-}$, and, therefore, 2) quantum interference still appears as the interference of particles. Clearly, this is only a rather far-fetched make-shift, catering to reconcile the quantum interference with classical mechanics, and to make the feeling about quantum interference more palatable for the classical intuition. However, it is a surrogate of quite limited applicability, and, when used without a proper care, leads to well known non-sensical statements, such as “in a 2-slits diffraction single particle passes through both slits and interferes with itself” or “on a semi-transparent mirror particle splits into two pieces, which later on interfere in the interferometer legs” and so on. Especially pronounced these problems prove in interference of extremely faint beams. In this context, any phrases like “photons can / cannot interfere with each other or even with itself” or “one - or two - photon interference” is nothing but a bizarre rudiment of a classical thinking, a make-shift for dragging classical logic and symbols into quantum world, making no whatsoever sense for a consistent quantum framework. And even though it indeed can create - for a time being - some illusory comfort of understanding, it is helpful only so much and only up to the point. Relatedly, a well known Dirac dictum "photon can interfere only with itself and with nothing else" (Dirac, 1958) should be taken exclusively as that type of a pseudo-classical language and in this context only.

3. Basics of Quantum Measurements.

"By spatially separating beams with different momenta ***p****, the diffraction grating suppresses the interference between them…"*
D. Blokhintsev

The Principle of Superposition reduces a general quantum measurement to the measurements of superposing eigen states – for obvious reasons. In what follows, the short-cut "packet" stands for "a superposition of eigen states of a certain dynamic variable", e.g., linear or angular momentum packet, spin packet, etc. With that, the split of a packet into an only slightly spatially overlapping subcomponents / sub-beams, can be achieved either under static (direct) protocol, or under dynamic (indirect) protocol.

Direct (static) measurements. A set of external obstacles or fields split the incident packet into sub-beams of individual eigen states, e.g. the "sleeves" with a single linear momentum or spin, etc. This is what happens when the linear momentum packet hits the diffraction grating, or when the spin packet passes a magnetic field (Stern-Gerlach method), etc.
The conceptual scheme is given on Fig. 3. The related technicalities are found in the landmark paper (London and Bauer, 1930) and classic books (Blokhintsev, 1964 and 1968).

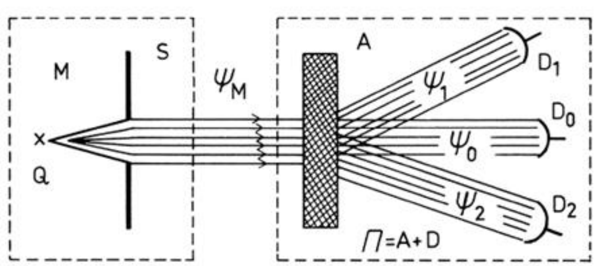


Fig. 3 Conceptul scheme of direct measurements

The initial packet prepared at S, proceeds to an analyzer (A), which splits the packet into sub-components, one of which, in turn, activates a particular detector (D), the “clicking” thereof signifies a particle “arrival”. In other words, an analyzer converts the packet WF into the sum of (almost) non-interfering partial WFs. This, obviously, is an asymptotic result in t -> ∞: the farther the detectors from the analyzer - the less is the overlap of the sub-beams, and the better is the measurement accuracy. That is, there always exists a residual overlap / interference, resulting in a so called false positives, which sets, thereby, a natural experimental limitations.

***Indirect (dynamic) measurements*.** The same result can be achieved dynamically. Consider a one-dimensional linear momentum packet $\Psi(x) \sim \exp(ikx) + \exp(-ikx)$ (system I) (standing wave), which is set into an interaction with a classical object (system II, say, a classical heavy ball, which location is denoted by Q) initially at rest at Q = 0. Because **of the classicality of the scatterer**, the WF resulting from SE for the total system (I + II) $\Psi(x, Q, t)$ for t -> ∞ decays into a sum of two (almost) non-interfering summands: one is $\exp(ikx)\eta(Q)$, corresponding to a photon reflecting to the left off the ball, and the ball, accordingly, rolling to the right, and, conversely, $\exp(-ikx)\ \eta(-Q)$ - to a photon, reflecting to the right, and the ball, correspondingly, moving to the left (the η – function here is the smoothed version of the Heviside step function H(Q): 0, if Q<0, and 1, if Q >0).The interference term in the probability $|\Psi(x, Q, t)|^2$ is, then, effectively suppressed, as a product $\eta(Q)\eta(-Q)$ for t -> ∞ vanishes for all Q. In closing, we point out, that the direction of the classical object motion after the event of scattering serves as an indicator of the momentum of the microparticle immediately *prior* to the measurement.
The details of these calculations are found in (Blokhintsev, 1968 and Blokhintsev“Quantum Mechanics”, 5th ed, Sec. 25, 1976) and are extremely instructive to any beginners of Quantum Mechanics and well beyond.

4. **Wave manifestations in a microworld: wave function “collapse”, wave-particle duality and the like conundrums.**

*“The quantized system is at the same time a quantized vibrating continuum and quantized **collection** of particles”*
G. Mackey

The current section deals with paradigms of a WF “collapse”, and more generally, with wave-particle duality. Both paradoxes share the same root cause of a problem, namely, attributing the wave aspect not to the whole ensemble of particles, but to a one single particle (one ensemble member). In what follows, for brevity we will call this attribution a “primitive” one. Also, the current Sec. 4 and the following Sec. 5 show close overlaps and frequent reiterations: this is done deliberately and with didactic purposes.

We begin with a wave function "collapse" first.

Wave function "collapse".

According to the initial de Broglie conjecture, WFs were deemed as some "material" waves associated with real particles. (Later on, this idea had very much influenced the so called "Copenhagen Interpretation"). Then, when M. Born devised his Statistical Postulate, the WF became a strange hybrid of 1) a material wave, associated with one single particle, and 2) wave with probabilistic properties. The combination of 1) and 2) caused lots of troubles to the Copenhagen Interpretation (CI). Among other things, it led to a number of paradoxes, the most famous of which is the "collapse" of a wave function. More specifically, if a wave function is interpreted as a dynamic variable connected to an individual particle, then the "collapse" jump presents an obvious logical difficulty and a paradox to the CI. However, over the years, it became clear, that WF was not at all a material wave, but rather a wave of Probability, wave of a Probability Amplitudes, that is[1]. But the probabilistic nature assumes the pool of realizations, i.e. an ensemble, and this is the way a quantum ensemble emerges. (This is the essence of the "quantum ensemble" concept that D.I. Blokhintsev tirelessly argued for all along). In other words, the mysterious "collapse" becomes a trivial change of a probability from *a priori* to *a posteriori* values, (i.e. probability change from before to after measurements), which is a standard framework of a classical probability theory. That closes away the "collapse" paradox.
Needless to say, in an experimental context, the gradual build-up of the interference picture for low-intensity beams in conjunction with Born's statistical postulate lends a direct support to a view of the WF as a distribution function amplitude.

Wave-particle duality.

Now, more generally, the wave-particle duality is the natural extension of the WF "collapse" to a broader context.
As said just above, the key error of the initial naïve concept of wave-particle duality was a "primitive attribution". What's more, this error turned extremely contagious: the widely acclaimed Copenhagen (i.e. the Bohr school) Interpretation (CI) relates WF specifically to an individual particle. And because of the Bohr overwhelming –

[1] In that context, there is no wonder that R. Feynman made "probability amplitudes", complex-valued functions - with square modulus $|\Psi(x, t)|^2$ being a "normal" classical probability – the central element of his formulation of QM. Relatedly, and quite interestingly, in the Feynman parlance of complex probability amplitudes, the sum over all "trajectories" is nothing, but a Central Limit Theorem (CLT) for the sum of complex probability amplitudes.

and totally deserved! – reputation, the CI became (and still remains!) a major day in and day out source of problems and paradoxes in QM practices.

Initially, in the historical context, the wave-particle duality was stating that, depending on circumstances, microparticles exhibited either corpuscular or wave features / properties. Clearly, it is hard to fathom how a single spatially localized entity can possibly produce an interference picture in all configuration space. Then, because of these types of logical difficulties, and under the pressure of accumulating experimental evidence, the wave-particle duality formulation changed to: microparticles are neither corpuscles nor waves, but something else. This is somewhat better, but still vague and unsatisfactory. And only the SI brought the issue on the common sense footing: it is a "beam"/ensemble of particles that displays the wave behavior, and NOT an individual particle. And conversely, envisioning the discussion in the next Sec. 5 (from QM to QFT): a wave, treated quantum mechanically, can be categorized, as usual, via an excitation level and number of field quanta, populating that level and exhibiting particle properties. This is the essence of a modern understanding of the wave-ensemble parity, replacing the ill-famed wave-particle duality.

Bottomlining: wave patterns emerge only through particles, coalescing into assemblages, and, vice versa, the quantization of wave fields into its quanta produces particles. These two – seemingly independent and equally valid paradigms – become inherently related in the QFT, which capitalizes on this aspect, and gives to fields the status of the origin of all particles found the Universe.

The Historical Aside.

The saga of an erroneous perception of the WP duality is an instructive illustration against a thoughtless copying even from reputable authors and texts and misappreciating fundamentals in favor of formal technical manipulations. Specifically, while there were numerous indications to the wave nature of ensembles, and not single particles, these indications were largely missed.
To wit, as early as in 1935, Yu. Rumer in his "Introduction to Wave Mechanics" wrote:"…There exists no whatsoever analogy between the motion of a one single particle and wave. Meanwhile, being incautious, one speaks quite frequently about the wave nature of an electron, rather than of the wave nature of an electron beam".
On the same note, G. Mackey (cited at the epigraph to this section) in 1960 in his Harvard lectures on "The mathematical foundations of quantum mechanics" pointed

out that: ”The mathematical system comprises both the wave and particle aspects of electromagnetic radiation, and there is a paradox only when one tries to frame too naïve a physical picture”.
In the same vein, D. Blokhintsev underscored in his “Quantum Mechanics” in 1964: “Therefore, a particle state, characterized by a wave function, should be interpreted as an adherence of the particle to a certain quantum ensemble“.
The view of WF as a field of complex Probability Amplitudes is a good segue way transit point to QFT ideas.

5. From Quantum Mechanics (QM) to Quantum Field Theory (QFT).

“The assumption is made that for each type of elementary particle there exists an associated field of which the particles are the quanta”

E. Fermi

In this section we’ll address just a few, but fundamental concepts of QFT, usually not discussed in standard texts. For further technical details, please refer to extant literature.
As was mentioned in the previous section, the WF in ordinary QM can be interpreted as a probabilistic field of the Schrödinger Probability Amplitudes. However, this is still a classical field, i.e. it is a complex-valued function, existing at every point in configuration space, or linear momentum space, etc., depending on the representation. Advancing this field with relativity requirements, and then subjecting it to a standard second quantization procedure, paves the ideological way to a consistent QFT and, in particular, reproduces all technical results of QFT.

However, the full fledged QFT aims much higher: its ambition extends to trace down the origins of all elementary particles and to constructively explain away them via excitations / quanta of a primordial vacuum field.

The first version of a field quantization, known now as a canonical quantization, was formulated by Heizenberg and Pauli (Heizenberg, Pauli, 1929, see also Wentzel, 1949 and 2003) as early as only three years after the groundbreaking Schrödinger papers. Accordingly, an ordinary (coordinate representation) WF $\Psi(x)$ and its Fourier counterpart (momentum representation) $\Psi(p) = \boldsymbol{\Sigma}\Psi(x)e^{i(p/\hbar)x}$, become operators (operator-valued functions) acting on state vectors.

Reiterating once again: one should be mindful of the deep difference between classical and quantized fields. While the former are ordinary real-valued functions, solving classical equations of mathematical physics, the latter are operator-valued functions, with eigen states thereof containing particles.[1]

In a more layman parlance, QFT continues along the conceptual lines of ordinary QM: de Broglie waves and WFs do not signify any material waves, they stand for Probability Amplitude distributions, appearing as waves, and deliver the statistics in the assemblages of microparticles. Similarly, quantized fields are not fields per se: they are operators creating fields. More precisely, these operators act in the functional space of WFs and thereby create fields of Probability Amplitudes,(i.e., WFs) – exactly in the second quantization sense. Now, the key presumption of the QFT is that each type of particle originates from its own specific field, and as such, is the quantum of that field.

Accordingly, at the first instances of the Universe, the initial all-permeating primordial field decayed into a great variety of sub-fields, which, in turn, produced the whole gamut of known elementary particles. (Admittedly, as this is a quite an involved issue, still not fully understood, we will not delve into it any further.) The common production mechanism rests, essentially, on a picture of an elastic fabric, small vibrations of which around the equilibrium quantize into the eigen states of a global harmonic oscillator (with equidistant energy states $E = n\hbar\omega$, n being a level number, and $\hbar\omega$ – minimal energy quantum), or, equivalently, are associated with discrete wave modes (as, say, vibrating modes on the string).

The next key step is to view the n-th eigen mode as an ensemble of n elementary energy quanta / elementary excitations (similar to phonons for elastic medium, plasmons for plasma, magnons for spins ensembles, etc.). This is a point of a critical importance: it effectuates the phase transition from the underlying field to our world the way we know it. The technical foundation of this transition is formalized by the Heisenberg-Pauli correspondence: Ψ(x)-function -> **Ψ**(x)-operator (and equally so, Ψ(p)-function -> **Ψ**(p)-operator). Along the way, it replaces a global oscillator by an assemblage of local harmonic oscillators, all excited to the state n = 1, with energy $E_1 = \hbar\omega$. (This explains why a quantized field is often pictured by a field of harmonic oscillators, see the previous page footnote.)

As pointed out in Sec. 1, each excitation mode and ensuing quanta originate in the presence of the quantum vacuum, which, having an infinite supply of energy, emerge as a Grand thermostat (i.e. Grand Canonical Ensemble) as well as a source of

[1] For that reason quantized fields are oftentimes referred to as 'fields of quanta / particles" (Abrikosov *et al*, 1963) or, relatedly, "fields of harmonic oscillators, giving rise to elementary excitations / particles"(Polyakov, 1987).

randomness of microparticles. It is this quantum vacuum which coherently impacts the motion of all quanta to shape up their spatial Probability Amplitudes in a wave-like manner. In other words, while quanta DO NOT interact, the quantum vacuum serves as a common cause effectuating a wave-like quanta distribution. Relatedly, but in contrast: water molecules in water surface waves DO "communicate" via a surface tension, gluing them together and forcing to undulate. It is this tension that causes a wave propagation in a water.
Further, for quanta, originating from global vibrations with certain wave vector k and having the same wave vector, spatial locations can't be certain, but only random. This is consistent with the ordinary QM view at, say, a phonon with a certain wave vector k, (linear momentum $p = \hbar k$) as pertaining to an ensemble with the WF $\sim \exp(-ikx)$. Then the superposition of these plane wave amplitudes, say, $\boldsymbol{\Sigma}\ \Psi(k)\exp^{-ikx}$ decribes a more or less localized wave packet of phonons (equal $\delta(x)$ in the limit of all identical amplitudes $\Psi(k)$).

In other words, field modes thus act – in the Heisenberg-Pauli sense – as generators of field quanta ensembles. Then again, it is only natural for WF to serve as a distribution in that ensemble (Blokhintsev, 1968). All that convincingly indicates that a sufficiently populated ensemble of quanta, arising from a certain wave field mode, is an alternative, but equivalent representation of this mode. Paraphrasing this in a short-cut way: there is a parity between a field wave and ensuing assemblage of particles. Then, there is no wonder that the Probability Amplitude distribution in an ensemble – i.e. the WF of an ensemble – exhibits clear wave features. What's more, the well-known interference patterns found in intensive light and/ or electron beams, scattering off standard obstacles (wedges, single and double slits, etc., Sec. 2), – i.e. in the classical limit – show a striking, yet natural resemblance of elastic waves scattering off those objects. This is a wave / ensemble side of the story.
On the other hand, one should be cognizant of wave concept limitations regarding individual quantum / particle aspects. Indeed, strictly speaking, the quantized wave / field, which is a source of "particles", is not an ordinary function, ranging in the whole space. Rather, as it was indicated above, it is a field of harmonic oscillators, with a quantization thereof giving rise to particles. And only when particles accumulate, different modes and their WFs become apparent.
Recappping: an identification of elementary particles with field quanta affords the applicability of elementary QM to the latter, and view them as entities in transit from instances of fields to coalescences, underlying classical objects of the macro-world.

6. **Preservation of conservation induced correlations in extended systems and "spooky action at the distance".**

"There is no physical connection between
two subsystems at large distances.
Yet, ***the conditional*** *probability,*
does depend on which state of
the either subsystem we select"
A. Migdal

Measurements in extended quantum systems (such as, e.g., coherent pairs of photons) in conjunction with conservation laws naturally lead to the concept of entanglement. Based on standard conditional probabilities, this concept, in and of itself, does not imply any "spooky" effects. And yet, it is the underestimation of these conditional probabilities, that leads to a mysterious quantum "non-locality" and "spooky action at the distance", that continues to overwhelm an extant literature.

In a bit more detail, consider in this context photon pairs coherently produced in a radiative double-atomic decay or PRDC (parametric down-conversion) process. Such conditions are found in well known experiments by Freedman and Clauder (Freedman and Clauder, 1972), Aspect et al (Aspect, Dalibard, Roger, 1982), etc. Similar conditions are realized in thought experiments with spins, analyzed by D. Bohm (Bohm, 1951), and the like. The correlation of data, obtained at both ends of a pair, unequivocally points to conservation of related variables, and yet, this became a source of innumerable and wordly discussions about an alleged non-locality of QM, routing back to the well-known EPR paper (Einstein *et al*, 1934) and the Einstein "spooky action at the distance". In the mean time, the "paradox" arises trivially from the confusion in elementary probability theory and resolves equally so.

We begin with measuring scalar variables. Consider in this regard a generic set-up for detecting, say, charges within an electron-positron pair e-p (emerging in the process, reversed to a two-photon annihilation of an electron-positron pair). Inasmuch as the pair emerges (t = 0) via the local (point-wise) interaction, the latter trivially enforces a correlation between the pair components. **However, we access (measure!) the related information only later on, when the components moved away from each other, and that is what creates an impression of "non-locality".** More specifically, prior to the measurement, component charges (or other dynamic variables) cannot be ascertained: because of the interaction at the pair creation ($t = 0$), products of the component individual eigen functions are not eigen for the whole e-p pair (the latter is a superposition: an interaction at $t = 0$ mixes $\psi^{e}_{detector1}(-)\psi^{p}_{detector2}(+)$ with $\psi^{p}_{detector1}(+)\psi^{e}_{detector2}(-)$ at the proportion 50/50). Therefore, there are equal chances for positive or negative charges to be detected at either end. But once the result at any end is obtained, the outcome at the opposite end is ascertained instantaneously (conservation induced correlation!), and regardless of the spatial separation between components. (At large distances, the electron-positron Coulomb intraction is negligible, and the correlation arises only because of the conservation of charge). It is this situation that creates an illusion of "non-locality" and oftentimes is interpreted as the "spooky action at the distance". However, what's missed in this consideration, is that the result is premised on the **CONDITIONAL**, not absolute probability: that is, once the outcome at one end is, say, – e (+ e), then the **CONDITIONAL** probability of getting + e (– e) at the other end, is immediately 100%. In other words, for spatially extended systems the outcome of the second measurement is always CONDITIONAL of the first measurement, regardless of the distance. And obviously, it is this CONDITIONAL probability that is always non-local in spatially extended systems, as it deals with spatially separated entities.

We can as well reverse this set-up to consider correlations between two photons (ensuing from the annihilation of an e-p pair), say, in terms of momenta of emerging photons: quite analogously, these correlations will again obey CONDITIONAL probabilities. The same logic readily applies to measuring other vector dynamic variables (e.g. spin, etc.) by adding a spatial orientation to arguments of conditional probabilities.
In other words, it is our framework that is non-local, and not QM. That closes the "paradox" and the whole issue of "spooky actions".

Tutorial Appendix to Sec. 5

This Appendix gives an extra exposure to a few matters, essentially simple, yet confusing novices and seasoned professionals alike. It is, strictly speaking, not necessary, but comes quite desirable, given a long history of related misconceptions.

***About "field quanta"*.** The term "field quantum", strictly speaking, is a misnomer, and might be confusing if applied literally and without caution. In particular, the field quantum is certainly NOT a “thin” slice (extending to all space) of any classical field, say, Schrödinger Probability Amplitude fields, i.e. Ψ(x) or Ψ(p) fields, etc. Rather, as indicated above, it “visualizes” some vibrating substance and elementary excitations, emerging from these vibrations. The field quantum is, in fact, a “field **energy** quantum”, i.e. a minimal excitation energy for a harmonic oscillator, embodying this vibration, and it is elementary particles that are these field quanta in disguise.

Further, one should be mindful that manifestations of quantum fields are quite different from classical, i.e. quantum fields never exhibit themselves as one wholesome pattern, extending over all space as a piece (even a pale one!) of the total diffraction pattern. Quite to the contrary, the cohesive diffraction / interference pattern is assembled, as a mosaic, via spatially localized foot-prints of “quanta / particles”, distributed according to ordinary QM WFs. That is, reiterating what has been already said, the quantum field surfaces up only via elementary excitations in disguise, i.e. in the form of minimal energy portions (“quanta”) which are also localized in space a la classical corpuscles. (Again, it is these quanta / corpuscles whose spatial Probability Amplitudes distribution are traditional QM WFs, coinciding with classical field amplitudes at high quanta densities). And once again, in the parlance of elementary excitations, a quantum field – which formally is an operator-valued function – can be loosely patterned after connected harmonic oscillators, the latter filling up all configuration space, as it was pictured in earlier models of canonical field quantization. That hints that in the microworld, we do not see omnipresent sine or cosine profiles, characteristic of classical waves, as, say, wave on the water surface. The wave intensity, or $|\Psi(x)|^2$, spoken in QM, is a foot-print of a high-density quanta flow in the classical limit, i.e. the accumulation of particle foot-prints reproduces the classical Huygens – Fresnel interference patterns. On a broader note, our cognition and intuition are shaped up by and rest on classical macroscopic world. That is why patterns of, say, water surface waves, waves on strings, etc. dominate our heuristics of discrete entities / particles in the Universe. In reality though, we should think of classical fields as converted (secondly quantized) into quanta producing operator-valued functions – generators of particles.

About non-zero rest mass**.** An important caveat regarding the mass generation in the described scenario is that almost all elementary particles (except photons) have a non-zero rest mass, while collective excitation of the elastic fields (similar to collective modes in many-body systems: phonons, plasmons, magnons, etc.) all have zero rest mass and “gapless” linear dispersion spectrum $\omega = kv$. The traditional way of rectifying this defect in the early mechanistic interpretations of QFT was to complement by “hand” the usual elastic harmonic modes with additional inertial vibration terms – incorporating the rest mass m into thus nonlinear dispersion spectrum $\hbar\omega = ((\hbar k)^2 + (mc^2)^2)^{1/2}$ – which appears then somewhat far-fetched and rather catering to a desired result. The alternative mechanism – and more convincing! - came through the gauge Yang-Mills fields (the so-called Higgs mechanism): the spontaneous symmetry breaking converts merely a parameter in the “Mexican hat” field potential into the mass of generated quanta, see, for instance (Cheng, Li, 1984). Also, mindful should one be that in the relativistic QFT there is no impenetrable “Great Chinese wall” separating the rest mass energy from kinetic energy and vice versa: these two are both natural components of a field energy quantum, complementing each other in various situations (e.g. electron-positron pair annihilation into photon pairs, etc.).

About “wave-ensemble parity” vs “wave-particle duality”. The discussions in Secs. 3-5 illustrate an ensemble meaning of the WF and its interpretation as, in essence, wave-ensemble parity (or parity between field waves and ensembles of field quanta), whichever caters better to our classical minds. And does it explain the key root cause of all wave-particle duality problems: it is an attempt to combine two fundamentally different essentialities – individual and collective, quantum and classical – under one single umbrella. (At some initial point, in 1927, there was even a special name, coined by Sir A. Eddington, for that umbrella – wavicle – which survived even until nowerdays; R. Feynman mentioned it in his lectures “The Character of the Physicsl Law”, [Feynman, 1965]). Indeed, a particle aspect of the wave-particle duality is about an individual entity, while another – wave aspect, is, essentially, collective. They do not overlap, and that is why the whole concept is obsolete and clearly is a misnomer: there is no such thing as a duality per se. That resolves this old “paradox”. Also, as a by-product, we understand now, why and how particles in QM conform to the wave attribute – the amplitude $\Psi(x)$ – via the genial Born postulate.
Only can we admire the Mother Nature ingenuity in building such an elegant construction: emergence of excitation quanta (particles) as a transitory substance *en route* from a hidden vacuum field phase to a phase manifestly classical.

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professional mastery of quantum theory and avoid vexing and impeding “paradoxes” in the future.

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References

Abrikosov, A.A., Gor’kov, L.P., Dzyaloshinskii, I.E., Methods of Quantum Field Theory in Statistical Physics, Prentice Hall, 1963

Aspect, A., Dalibard, J., Roger, J., Phys. Rev. Lett., **49**, 1804 (1982)

Ballentine, L., Rev. Mod. Phys., **42**, 358 (1970)

Blokhintsev, D.I., Quantum Mechanics, Reidel, 1964; The Philosophy of Quantum Mechanics, Reidel, 1968, reprint - 2010; Statistical ensembles in quantum mechanics. In: Quantum Mechanics, determinism, causality, and particles, Reidel, 1976; Quantum Mechanics, 5th ed, Sec. 25, 1976. https://vixra.org/pdf/2207.0018v1.pdf

Bohm, D., Quantum Mechanics, Prentice Hall, 1951 and Dover, 1979

Cheng, Ta-Pei, Li, Ling-Fong, Gauge Theory of elementary Particle Physics, Clarendon, 1984

Dirac, P.A.M. The Principles of Quantum Mechanics, Clarendon, 1958

Einstein, A., **Podolsky, B., Rosen, N**. Phys. Rev., **47**, 777 (1935)

Feynman, R.P, The Character of Physical Law, Modern Library, 1965

Feynman, R.P., Hibbs, A.R., Quantum Mechanics and Path Integrals, McGraw Hill, 1965

Freedman, S, and Clauder, J, Phys.Rev. Lett., **28**, 938, (1972)

Heisenberg, W., **Pauli, W**., Z.Physik, **56**, 1 (1929)

London, F., **Bauer, E.,** The theory of observation in Quantum Mechanics,1939 / In: J.Wheeler and W. Zurek, Quantum Theory and Measurement, Princeton University Press, 1982

Migdal, A.B., Qualitative Methods in Quantum Theory, CRC Press, 1977

Nagasawa, M., Stochastic processes in quantum physics, Springer, 2000

Von Neumann, J., Mathematische Grundlagen der Quantenmechanik, Springer, 1932; English edition: Mathematical Foundations of Quantum Mechanics, Princeton University Press, 1955

Polyakov, A.M., Gauge Fields and Strings, Routledge, 1987

Roepstorf, G., Path Integral approach to quantum physics, Springer, 1994

Sömmerfeld, A., Lectures on Theoretical Physics: Optics, v. IV, Academic Press, 1954

Taylor, G., Proc. Camb. Phil. Soc., **15**, 114 (1909)

Tonomura, A., **Endo, J., Matsuda,T., Kawasaki,T**., Am. J. Phys., **57**, 117 (1989)

Wentzel, G., Quantum theory of fields, Intersciense Publishers, 1949 and Dover, 2003