The Coupling Constant of Dirac Magnetic Monopoles is not 34.259 but 308.331

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Abstract

Textbooks tell us that the coupling constant of Dirac magnetic monopoles is 34.259. Here I show by using the quark hypothesis that at zero temperature the coupling constant is as high as 308.331. Moreover I show that it is a running coupling constant and is smaller than 0.5 at the Planck temperature.
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1 Introduction

In 1904 Thomson [1] has shown that the angular momentum generated by the Lorentz force between an electric charge and a magnetic charge is independent of their distance. As intrinsic spin and orbital angular momentum are quantized in units of (half-)integers times the reduced Planck constant $\hbar$, Dirac [2] concluded in 1931, that these conditions can be satisfied only if both electric charge and magnetic charge appear in discrete units only. Since then textbooks tell us that the coupling constant of Dirac magnetic monopoles is 34.259. Here I show that this conclusion is not correct. Electric charge appears not in multiples of the positron charge e, but in multiples of the quark charge $e/3$. At zero temperature the coupling constant of Dirac magnetic monopoles is 308.331. By using the weak energy condition I show that this coupling is a running coupling constant and becomes smaller than 0.5 at the Planck temperature.

2 Classical Dirac Magnetic Monopoles

I will use the natural units

$$\hbar = c = \varepsilon_0 = 1 \tag{1}$$

where $\hbar = h/2\pi$ denotes the reduced Planck constant, c the speed of light, and ε_0 the electric field constant.

The electric field strength generated by an electric charge Q resting in the center of the coordinate frame is

$$\mathbf{E} = \frac{Q}{4\pi r^2}\hat{\mathbf{r}} \tag{2}$$

where $\hat{\mathbf{r}} \equiv \mathbf{r}/r$ denotes the unit position vector, $\mathbf{r}$ the position vector, and $r \equiv \|\mathbf{r}\|$ its absolute value.

By analogy, the magnetic field strength generated by a magnetic charge q resting in the center of the coordinate frame is

$$\mathbf{B} = \frac{q}{4\pi r^2}\hat{\mathbf{r}} \tag{3}$$

In the classical (non-quantum mechanical) case the Lorentz force on a moving electric charge Q in the static magnetic field generated by a resting magnetic charge q is

$$m\ddot{\mathbf{r}} = Q\dot{\mathbf{r}} \times \mathbf{B} \tag{4}$$

Here $\dot{\mathbf{r}} \equiv \partial_t \mathbf{r}$ is the speed of the electric charge and $\ddot{\mathbf{r}} \equiv \partial_t \dot{\mathbf{r}} = \partial_t^2 \mathbf{r}$ is its acceleration. The rest mass of the electric charge is denoted by m. The rest mass of the magnetic charge is assumed to be much larger (infinity) than that of the electric charge, so that the magnetic charge can rest in the center of the coordinate frame.

By using

$$\begin{aligned} \partial_t \hat{\mathbf{r}} &= \partial_t \left(\mathbf{r}/\sqrt{\mathbf{r}\cdot\mathbf{r}}\right) \\ &= \frac{r\dot{\mathbf{r}} - \left(\frac{1}{2r}2\dot{\mathbf{r}}\cdot\mathbf{r}\right)\mathbf{r}}{r^2} \\ &= \frac{r^2\dot{\mathbf{r}} - (\dot{\mathbf{r}}\cdot\mathbf{r})\mathbf{r}}{r^3} \\ &= \frac{\mathbf{r}\times(\dot{\mathbf{r}}\times\mathbf{r})}{r^3} \end{aligned} \tag{5}$$

the orbital angular momentum of the electric charge generated by the Lorentz force

$$\mathbf{L} \equiv \mathbf{r} \times m\dot{\mathbf{r}} \tag{6}$$

gives

$$\begin{aligned} \partial_t\ \mathbf{L} &= \dot{\mathbf{L}} = \mathbf{r}\times m\ddot{\mathbf{r}} \\ &= Q\mathbf{r}\times\left(\dot{\mathbf{r}}\times\frac{q\mathbf{r}}{4\pi r^3}\right) \\ &= \frac{Qq}{4\pi}\partial_t\hat{\mathbf{r}} \end{aligned} \tag{7}$$

Subtraction gives

$$0 = \partial_t\left(\mathbf{L} - \frac{Qq}{4\pi}\hat{\mathbf{r}}\right) \tag{8}$$

Total angular momentum $\mathbf{J}$ is the sum of orbital angular momentum $\mathbf{L}$ and intrinsic spin $\mathbf{S}$,

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \tag{9}$$

Moreover it is conserved,

$$0 = \partial_t\mathbf{J} = \partial_t(\mathbf{L}+\mathbf{S}) \tag{10}$$

Comparison of equations (8) and (10) gives

$$\mathbf{S} = -\frac{Qq}{4\pi}\hat{\mathbf{r}} \tag{11}$$

Intrinsic spin is quantized in units of half-integers times the reduced Planck constant,

$$\|\mathbf{S}\| = \frac{n}{2} \tag{12}$$

where n denotes an arbitrary integer. Hence,

$$Qq = 2\pi n \tag{13}$$

This is the Dirac quantization condition. It requires that Q and q cannot have arbitrary values.

In the days of Dirac [2], that is is 1931, quarks were not known. So he assumed that electric charge is quantized in units of the positron charge e. Therefore he assumed the unit magnetic charge g' so as to satisfy

$$eg' = 2\pi \tag{14}$$

By using the Sommerfeld fine-structure constant

$$\alpha_E \equiv \frac{e^2}{4\pi} \simeq \frac{1}{137.036} \tag{15}$$

this gives the coupling constant

$$\frac{g'^2}{4\pi} = \frac{1}{4\pi}\left(\frac{2\pi}{e}\right)^2 = \frac{1}{4\alpha_E} \simeq 34.259 \tag{16}$$

However, since the prediction [3] and observation [4, 5] of quarks we know that electric charge is quantized in units of $e/3$. Hence, the unit magnetic charge g is given by

$$\frac{e}{3}g = 2\pi \tag{17}$$

This gives the magnetic coupling constant

$$\alpha_M \equiv \frac{g^2}{4\pi} = \frac{1}{4\pi}\left(\frac{6\pi}{e}\right)^2 = \frac{9}{4\alpha_E} \simeq 308.331 \tag{18}$$

3 Non-Classical Dirac Magnetic Monopoles

Because of the introduction of the Lorentz force, the calculation above was made for classical (non-quantum mechanical) objects with electric and magnetic charge. However, since 1997 I am argueing [6, 7] that the quantum field theoretical interaction between electric and magnetic charges requires the introduction of a velocity operator which allows the definition of a Lorentz force. So the calculation above is valid also for the quantum physical case.

More precisely, the introduction of the velocity operator requires the existence of absolute speed which is defined by the finite light cone generated by the Hubble effect. The velocity operator is therefore an effect not only of quantum field theory, but also of gravitation theory. The velocity operator is therefore an effect of quantum gravity.

Moreover the calculation above gives the Dirac quantization condition only if equation (3) is used. It requires that both the Einstein electric photon [8] and the Salam magnetic photon [9] have zero rest mass. A massive magnetic photon would require a Yukawa potential whose exponential term would destroy the Dirac quantization condition.

4 Running Coupling Constant

In 2024 I predicted [10] the existence of elementary fermions with magnetic charge $q = g$ (called hanselons and gretelons) and also with $q = 2g$ (gretelons). The binding energy of two gretelons with opposite magnetic charge $\pm 2g$ which are separated by the distance r would become quite large,

$$E_b = -\frac{q^2}{4\pi r} = -\frac{(2g)^2}{4\pi r} \tag{19}$$

The total energy of two bound gretelons would be

$$E \simeq M_1 + M_2 + \frac{1}{r} - \frac{g^2}{\pi r} - \frac{GM_1M_2}{r} \tag{20}$$

where M_1 and M_2 are the rest masses of the gretelons, $1/r$ is their zero point energy given by the uncertainty principle, $-g^2/\pi r$ is their magnetic binding energy, and $-GM_1M_2/r$ is their gravitational binding energy. G denotes the Newtonian gravitational constant.

The weak energy condition states that there cannot exist any negative energy densities. Hence, $E \geq 0$. Moreover the rest masses of elementary particles cannot be larger than the Planck mass $M_P = G^{-1/2}$, because otherwise their Schwarzschild radius would become larger than the Planck length $l_P = 1/M_P = G^{1/2}$. Hence,

$$0 \leq E \leq 2M_P + \frac{1}{r} - \frac{g^2}{\pi r} - \frac{GM_PM_P}{r} \tag{21}$$

In principle, the mutual distance of the two (elementary) gretelons can become as small as the Planck length,

$$r \geq l_P = 1/M_P \tag{22}$$

Hence, in the case $r = 1/M_P$ it is

$$0 \leq E \leq 2M_P - \frac{g^2}{\pi}M_P = M_P\left(2 - \frac{g^2}{\pi}\right) \tag{23}$$

This can be satisfied only if

$$\alpha_M(r = l_P) = \frac{g^2(r)}{4\pi} \leq \frac{1}{2} \tag{24}$$

Hence, $\alpha_M(r)$ must be a running coupling constant. It must decrease with distance and increase with energy. – That $\alpha_E(r)$ is a running coupling constant is known since the work of Gell-Mann and Low in 1954 [11].

5 Summary

By using the quark hypothesis I have shown that at zero temperature the magnetic coupling constant is as high as 308.331. By using the weak energy condition and applying it on a system of two bound magnetic charges, I have shown that the magnetic coupling is a running coupling constant and becomes smaller than 0.5 at the Planck scale.

References

[1] J. J. Thomson, *Elements of the Mathematical Theory of Electricity and Magnetism* (Cambridge University Press, 1904).

[2] P. A. M. Dirac, Quantised Singularities in the Electromagnetic Field, *Proceedings of the Royal Society of London A* **133**, 60–72 (1931).

[3] M. Gell-Mann, A Schematic Model of Baryons and Mesons, *Physics Letters* **8**, 214–215 (1964).

[4] J. J. Aubert et al., Experimental Observation of a Heavy Particle J, *Physical Review Letters* **33**, 1404–1406 (1974).

[5] J. E. Augustin et al., Discovery of a Narrow Resonance in e^+e^- Annihilation, *Physical Review Letters* **33**, 1406–1408 (1974).

[6] R. W. Kühne, A Model of Magnetic Monopoles, *Modern Physics Letters A* **12**, 3153–3159 (1997).

[7] R. W. Kühne, Quantum Field Theory with Electric-Magnetic Duality and Spin-Mass Duality but Without Grand Unification and Supersymmetry, *African Review of Physics* **6**, 165–179 (2011).

[8] A. Einstein, Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt, *Annalen der Physik* **17**, 132–148 (1905).

[9] A. Salam, Magnetic Monopole and Two Photon Theories of C-Violation, *Physics Letters* **22**, 683–684 (1966).

[10] R. W. Kühne, Conservation of Baryon and Lepton Number is an Effect of Electric and Magnetic Charges, *Qeios* (2024), doi:10.32388/DGIRV1 .

[11] M. Gell-Mann and F. E. Low, Quantum Electrodynamics at Small Distances, *Physical Review* **95**, 1300–1312 (1954).