On *Limit* of Mathematical Analysis and Continuum Hypothesis

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Abstract

The limit of mathematical analysis is defined by $\varepsilon - \delta$. A concept of dynamic limit is proposed in the article, and the dynamic space of kernel numbers is established. This concept has been extended and studied in depth, yielding several results, including setting up shell-medium cluster, dynamic limit process and steps; kernel number clouds; introducing elfin number and elfin space which the elfin number is non-construct and extend of real number; discussions on the Continuum Hypothesis (CH) what is not contradiction with new dynamic space.

0. Introduction

First, let's state the definition of the classical limit with $\varepsilon - \delta$:

Definition 0.0 For real continuous functions $f(x)$, $\forall \varepsilon > 0, \exists \delta > 0$, if $|x - a| < \delta$, there must be

$$|f(x) - A| < \varepsilon$$

Then A is called the limit of the $f(x)$ at a, denoted by

$$\lim_{x \to a} f(x) = A$$

If $x - a > 0$ (< 0), then get the right (left) limit. When the left and right limits of a function at a point are equal, the function has a limit at that point.

This implies that ε, δ are real numbers to include later discussion.

1. Establishing Dynamic Limits and Dynamic Space

Here is a new Definition 0.0

Definition 1.1 For real continuous function, $f(x)$,

$$\left(\forall \varepsilon_q > 0 : \varepsilon_q \in \mathbb{Q}\right), \quad \left(\exists \varepsilon_r > 0 : \varepsilon_r \in \mathbb{R}\right), \ (\exists \delta > 0 : \delta \in \mathbb{R})$$

If $|x - a| < \delta$, must have

$$|f(x) - A| < \varepsilon_r < \varepsilon_q \tag{1.1}$$

Then A is called the limit of the $f(x)$ at a, denoted by

$$\lim_{x \to a} f(x) = A \tag{1.2}$$

If $x - a > 0$ (< 0), We get the right (left) limit. A function has a limit at a point when its left and right limits are equal.

If we regard the limit, $\lim\limits_{x \to a}$, as a dynamic process, then

Write ε_q down as $\varepsilon(q) \quad \left(\varepsilon(q) \in \mathbb{Q}\right)$;

Write ε_q down as $\varepsilon(r) \quad \left(\varepsilon(r) \in \mathbb{R}\right)$.

(Note: Here $\varepsilon(q)$, it is not the ε (x,y) equality symbol of first-order logic theory)

Definition 1.2 For real continuous functions, $f(x)$, according to the properties of rational numbers,

$$\left(\forall \varepsilon(q_i) > 0 : \varepsilon(q_i) \in \mathbb{Q}, i \in \mathbb{N}\right), \left(\exists \varepsilon(r_i) > 0 : \varepsilon(r_i) \in \mathbb{R}, i \in \mathbb{N}\right) (\exists \delta > 0 : \delta \in \mathbb{R})$$

There can be sorting:

$$\varepsilon(q_0) > \varepsilon(q_1) > \cdots \tag{1.3}$$
$$\varepsilon(r_0) > \varepsilon(r_1) > \cdots \tag{1.4}$$

and

$$\begin{aligned} &\varepsilon(q_0) > \varepsilon(r_0) > \varepsilon(q_1) \\ &\varepsilon(q_1) > \varepsilon(r_1) > \varepsilon(q_2) \\ &\cdots \end{aligned} \tag{1.5}$$

If $|x-a|<\delta$, there must be

$$|f(x)-A|<\varepsilon(r_i)<\varepsilon(q_i) \tag{1.6}$$

Then A is called the dynamic limit of the $f(x)$ at a, denoted by

$$f(x) \quad \lim_{x \to a} \quad A \tag{1.7}$$

Corollary 1.1 If $\left(\varepsilon(r_i) \in M_i : i \in \mathbb{N}\right)$ in Definition 1.2, and

$$M_i = \left\{\varepsilon(r) \,\middle|\, \varepsilon(r) \in \mathbb{R} : \varepsilon(q_{i+1}) < \varepsilon(r) < \varepsilon(q_i)\right\} \quad (i \in \mathbb{N})$$

Then Definition 1.2 also holds about M_i.

Proof: (omitted).

When formula (1.7) is called the dynamic limit, the corresponding formula (1.2) is called the static limit.

Since the sets, $\mathbb{Q}$ and $\mathbb{R}$ and M_i are infinite sets, the limit process in Definition 1.2 can be continued to form a dynamic limit. It will be used as a binary operation. The formula (1.7) is expressed as

$$f(x) \quad \lim \quad A \tag{1.7'}$$

Definition 1.3: In Definition 1.2, the A is called the kernel, denoted as A^k; $S=\left\{\varepsilon(q_i) : (i \in \mathbb{N}\right\}$ is the shell; $M_i (i \in \mathbb{N})$ in the Corollary 1.1 is the medium.

Definition 1.4: If real numbers and limits are regarded as a unit (Space), the real number (number point) becomes the kernel number of the shell-medium-kernel trinity. The limit, lim**,** is one-dimensional. It can be called a dynamic space of real kernel numbers. Assume that the shell-medium-kernel trinity is:

$$TNT = Trinity\left\{S, M, A^k\right\} \tag{1.8}$$

Definition 1.5: For any collection of mathematically studied objects, if the dynamic limit , lim, is used, then the kernel number of the shell-medium-kernel trinity can be defined. The limit, lim**,** is one-dimensional, It can be called a dynamic space of kernel numbers.

When dynamic limits, lim, move in one-dimensional, Then a cluster of shell-medium will form around the kernel. Like an egg with infinite shells.

Example 1.1: By the Definition 1.2, $A \in \mathbb{R}$. Let the kernel number be expressed in static space as,

$$A^k = A \tag{1.9}$$

Expressed in dynamic space as

$$A^k \supseteq A \tag{1.10}$$

Next, select the shell cluster, when $n \in \mathbb{N}, m \in \mathbb{N}$ there is

$$\frac{1}{n^m}, \frac{1}{(n+1)^m}, \frac{1}{(n+2)^m}, \cdots \tag{1.11}$$

Let

$$S_{n,m} = \left\{\frac{1}{n^m}, \frac{1}{(n+1)^m}, \frac{1}{(n+2)^m}, \cdots\right\} \tag{1.12}$$

For a shell (cluster), $S_{n,m}$, and can get medium(cluster),

$$M_{n,m} = \left\{\varepsilon(r) \,\middle|\, \varepsilon(r) \in \mathbb{R} : \frac{1}{(n+1)^m} < \varepsilon(r) < \frac{1}{n^m}\right\} \tag{1.13}$$

Get a shell-medium-kernel trinity,

$$TNT = \left\{Trinity\left\{S_{n,m}, M_{n,m}, A^k\right\}: i \in \mathbb{N}, \right\} \tag{1.14}$$

Corollary 1.2 The kernel of any dynamic space of kernel numbers can have multiple $\mathbb{TNT}$.

Definition 1.6: If the limit, lim, is a dynamic process, then the kernel number dynamic space can be written as

$$QUAT = Quaternary\left\{S, M, A^k, lim\right\} \tag{1.15}$$

If $S \subset \mathbb{R}, M \subset \mathbb{R}$ then a $QUAT$ is a real kernel number dynamic space.

Discussed below is the real kernel numbers dynamic space when $S \subset \mathbb{Q}, M \subset \mathbb{R}$,

$$QUAT^Q = Quaternary\left\{S, M, A^k, lim \,\middle|\, S \subset \mathbb{Q}, M \subset \mathbb{R}, A^k \in \mathbb{R}\right\} \tag{1.16}$$

For Example 1.1, its kernel number dynamic space is denoted by

$$QUAT^{Q}(n,m) = Quaternary\Big\{S_{n,m}, M_{n,m}, A(n,m)^k, lim \Big| \\ S_{n,m} \subset \mathbb{Q}, M_{n,m} \subset \mathbb{R}, A(n,m)^k \in \mathbb{R}\Big\} \tag{1.17}$$

To simplify the discussion, let $n = 0, \quad m = 0$, Then formula (1.12) becomes as follows

$$S_{0,0} = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots\} \tag{1.18}$$

From formula (1.17) we can get,

$$QUAT^{Q}(1,1) = Quaternary\Big\{S_{1,1}, M_{1,1}, A(1,1)^k, lim \Big| \\ S_{1,1} \subset \mathbb{Q}, M_{1,1} \subset \mathbb{R}, A(1,1)^k \in \mathbb{R}\Big\} \tag{1.17'}$$

According to set theory, there are infinite cardinal numbers of natural numbers $\aleph_0$; Cardinality of the set of uncountable ordinal numbers $\aleph_1$; $\cdots$. And

$$\aleph_0 < \aleph_1 < \aleph_2 < \cdots \tag{1.19}$$

Definition 1.7: For the real kernel numbers dynamic space, if the dynamic limit has $lim \in QUAT$,then its limit rank is.

The cardinality limit of the set of the natural numbers $lim(\aleph_0)$;
The cardinality limit of the set of countable ordinal numbers $lim(\aleph_0)$;
The cardinality limit of the set of uncountable ordinal numbers $lim(\aleph_1)$;
$\cdots$.

There can be a formula,

$$QUAT^{Q}(1,1) = Quaternary\Big\{S_{1,1}, M_{1,1}, A(1,1)^k, lim(\aleph_0) \Big| \\ S_{1,1} \subset \mathbb{Q}, M_{1,1} \subset \mathbb{R}, A(1,1)^k \in \mathbb{R}\Big\} \tag{1.17''}$$

Corollary 1.3: When solving for limit, the process of dynamic limit, $lim(\aleph_0)$, will continue to the process of dynamic limit, $lim(\aleph_1)$,; going on the dynamic limit process,

$lim(\aleph_1)$ to $lim(\aleph_{i+1}) \quad (i \geq 2, i \in \mathbb{N})$

Proof: Since solving for limit is an infinite process, we can get what the $lim(\aleph_0)$ dynamically connects to $lim(\aleph_1)$ by the characteristics of $\aleph_i (i \in \mathbb{N})$. It is going on what $lim(\aleph_1)$ dynamically connects to $lim(\aleph_2)$. $\cdots$

Theorem 1.1 The dynamic limit definition of the kernel numbers dynamic space ensures the limit definition of the static space.

Proof: It can be proved using the classical limit $\varepsilon - \delta$ method and Definition 1.1.

Corollary 1.4: In the dynamic limit $lim(\aleph_0)$, any shell has non-empty Medium.

Proof: $\forall n \in \mathbb{N}, \exists s = \frac{1}{n} > 0 : s \in S(n)_{1,1}$

For a shell $S(n)_{1,1}$, then a medium

$$M(n)_{1,1} = \left\{ \varepsilon(r) \,\middle|\, \varepsilon(r) \in \mathbb{R} : \frac{1}{(n+1)}, < \varepsilon(r) < \frac{1}{n} \right\}$$

Of course the $M(n)_{1,1}$ is not empty.

Theorem 1.2: $\forall M(n)_{1,1}$, kernel number $A(1,1)^k \subset M(n)_{1,1}$.

Proof: According to the definition of kernel $A(1,1)^k$, we can get it by $\varepsilon - \delta$ method.

We can get Corollary 1.5 from Theorem 1.2

Corollary 1.5: A cluster of shell-medium forms around the kernel, $A(1,1)^k$.

$$L(n)_{1,1} = \{(s, m) \,|\, s \in S(n)_{1,1}, m \in M(n)_{1,1}\}$$

Corollary 1.6: The real number $A(\in \mathbb{R})$ is also included in the medium $M(n)_{1,1}$ of the kernel number $A(1,1)^k$.

Proof: Using Definition 1.2 and method $\varepsilon - \delta$. (omitted)

This shows that the real number, $A(\in \mathbb{R})$, is contained by the kernel number, $A(1,1)^k$, and the medium, $M(n)_{1,1}$, in the dynamic limit $lim(\aleph_0)$. Now that

$$A \subset M(n)_{1,1} \quad A(1,1)^k \subset M(n)_{1,1}$$

Now we begin to discuss the limit, $lim(\aleph_0)$, based on cardinality of countable ordinal sets.

Corollary 1.7: In the dynamic limit $lim(\aleph_0)$, there is non-empty medium in any shell.

Proof: According to formula (1.16) and Definition 1.2, select the shell cluster.

$$\left(\forall \varepsilon(q_i) > 0 : \varepsilon(q_i) \in \mathbb{Q}, i \in \mathbb{N}\right), \quad \left(\exists \varepsilon(r_i) > 0 : \varepsilon(r_i) \in \mathbb{R}, i \in \mathbb{N}\right)$$

Let shell Cluster

$$S(q) = \left\{\varepsilon(q_i) \,\middle|\, \text{formula } (1.3),(1.4),(1.5) : q_i \in \mathbb{Q}, n \in \mathbb{N}\right\} \tag{1.20}$$

and medium cluster

$$M(r) = \left\{\varepsilon(r_i) \,\middle|\, \varepsilon(r_i) \in \mathbb{R} : \varepsilon(q_{i+1}) < \varepsilon(r) < \varepsilon(q_i), i \in \mathbb{N}\right\} \tag{1.21}$$

The proof is similar to that of Corollary 1.4.

From Corollary 1.7, we can get a kernel number dynamic space of,

$$\begin{aligned} &QUAT^Q = \\ &\qquad Quaternary\left\{S(q), M(r), A(q)^k, lim(\aleph_0) \,\middle|\, q \in \mathbb{Q}, r \in \mathbb{R}\right\} \end{aligned} \tag{1.22}$$

The same discussion can be applied to the limit, $lim(\aleph_1)$, of uncountable cardinality sets.

Example 1.2: Select the shell cluster in $\mathbb{R}$

According to the properties of real numbers:

$$\left(\forall \varepsilon(s_i) > 0 : \varepsilon(s_i) \in \mathbb{R}, i \in \mathbb{N}\right), \quad \left(\exists \varepsilon(r_i) > 0 : \varepsilon(r_i) \in \mathbb{R}, i \in \mathbb{N}\right)$$

Sorted:

$$\varepsilon(s_0) > \varepsilon(s_1) > \cdots \tag{1.23}$$

$$\begin{aligned} &\varepsilon(s_0) > \varepsilon(r_0) > \varepsilon(s_1) \\ &\varepsilon(s_1) > \varepsilon(r_1) > \varepsilon(s_2) \\ &\cdots \end{aligned} \tag{1.24}$$

Shell clusters are available,

$$S(r) = \left\{\varepsilon(s_i) \,\middle|\, s_i \in \mathbb{R}, n \in \mathbb{N}\right\} \tag{1.25}$$

Get medium clusters

$$M(r) = \left\{ \varepsilon(r_i) \,\middle|\, \varepsilon(r_i) \in \mathbb{R} : \varepsilon(s_{i+1}) < \varepsilon(r_i) < \varepsilon(s_i), i \in \mathbb{N} \right\} \tag{1.26}$$

And the kernel number dynamic space.

$$QUAT = Quaternary\left\{ S(r), M(r), A(r)^k, lim(\aleph_1) \right\} \,\middle|\, r \in \mathbb{R} \right\} \tag{1.27}$$

Definition 1.8: In the kernel number dynamic space $QUAT^Q(1,1)$, if obtains the limit value A of the real continuous function $f(x)$ according to the method $\varepsilon - \delta$ of the limit in the static space of Definition 1.0, then there is always,

$$A(1,1)^k \subset A, A \subset A(1,1)^k \text{ both } A(1,1)^k = A \tag{1.28-1}$$

That is, for the kernel dynamic space $QUAT^Q(1,1)$ only, the change after the dynamic limit $lim(\aleph_0)$ does not affect formula (1.28-1).

Corollary 1.8: In the dynamic limit, $lim(\aleph_0)$, of the kernel number dynamic space, $QUAT^Q$, there are

$$A(q)^k \subset A, A \subset A(q)^k \text{ as } A(q)^k = A \tag{1.28-2}$$

And in the dynamic limit, $lim(\aleph_1)$, of the kernel number dynamic space $QUAT$, there are also

$$A(r)^k \subset A, A \subset A(r)^k \text{ as } A(r)^k = A \tag{1.28-3}$$

Corollary 1.9: In the kernel number dynamic space, there are

$$A \subset A(r)^k \subset A(q)^k \subset A(1,1)^k \tag{1.28-4}$$

Corollary 1.10: In the kernel dynamic space, $QUAT^Q, QUAT$, for the kernel,

$$A^k(=A)$$

There are

$$\exists m \in M(r)$$

This m has the same $\varepsilon - \delta$ dynamic limit $lim(\aleph_0), lim(\aleph_1)$ function as A. And

$$m \neq A \tag{1.29}$$

Obviously $m(\in M)$ is dynamic. The medium of kernel number A^k look like cloud. Then the $A(\in \mathbb{R})$ of the static space corresponds to the medium M of the kernel number A^k in the dynamic space. It is called the number cloud.

Corollary 1.11: For the element of the medium $M(r)$ of the kernel number dynamic space, $QUAT^Q$, can be ordered or disordered.

Proof: According to formulas (1.24) and (1.25), any s_i, s_{i+1} in

$$s_i \in S(r), s_{i+1} \in S(r), i \in (N)$$

There are m_1, m_2 in

$$m_1 \neq m_2, m_1 \in M(r), m_2 \in M(r)$$

So that

$$s_{i+1} < m_1 < s_i, \quad s_{i+1} < m_2 < s_i$$

Obviously, whether $m_1 < m_2$ or $m_1 > m_2$, the result of dynamic limit, $lim(\aleph_1)$, using $\varepsilon - \delta$ is the same.

It can be disordered.

In addition, if there exists $r_m \in \mathbb{R}$ such that

$$m_1 \in M(r) \cap \{m_r : s_{i+1} < m_r < r_m\}, m_2 \in M(r) \cap \{m_r : r_m < m_r < s_i\}$$

Then denoted by

$$m_1 < m_2.$$

The result of the dynamic limit $lim(\aleph_1)$ of the $\varepsilon - \delta$ method is also unchanged.

It has to be orderly.

If we assume that the dynamic limit, lim, moves towards infinity ∞ at a constant speed.

Definition 1.9: For any infinite cardinality $\aleph_i (i \in \mathbb{N})$, set limit step

$$\xi_i = \xi(\aleph_i) \quad (i = 0,1,2\cdots) \tag{1.30}$$

And

$$\Xi = \{\xi_i \,|\, \xi_i = \xi(\aleph_i) : i = 0,1,2\cdots\} \tag{1.31}$$

According to the definition of kernel number dynamic space, let the limit step be from a shell of a certain layer through the corresponding medium to the shell of the lower layer, as,

$$\xi_i(s_j \to m_j \to s_{j+1}) \quad (i = 0,1,2\cdots, j \in \mathbb{N}) \tag{1.32}$$

Or

$$\xi_i(sms_j) \quad (i = 0,1,2\cdots, j \in \mathbb{N}) \tag{1.33}$$

Corollary 1.12: ξ_0, ξ_1 exist.

Proof: According to the definitions of shell and medium in $QUAT^Q$ and $QUAT$, the corresponding limit step can be constructed.

Corollary 1.13: The limiting step ξ_0, is countable; the limiting step ξ_1 is uncountable.

$$\xi_0 < \xi_1 \tag{1.34}$$

Definition 1.10: The dynamic limit of the kernel dynamic space, $QUAT^Q$, is denoted by,

$$lim(\xi_0) \tag{1.35}$$

The limiting process is denoted as,

$$lim(\xi_0)_{s_j \to m_j \to s_{j+1}} \quad (j \in \mathbb{N}) \tag{1.36}$$

In short,

$$lim(\xi_0)(sms_j) \quad (j \in \mathbb{N}) \tag{1.37}$$

or

$$lim(\xi_0)_{\to} \qquad (1.37)'$$

Similarly, the dynamic limit of the kernel dynamic space, $QUAT$, is denoted as

$$lim(\xi_1) \tag{1.38}$$

The limiting process is denoted as,

$$lim(\xi_1)_{s_j \to m_j \to s_{j+1}} \quad (j \in \mathbb{N}) \tag{1.39}$$

In short,,

$$lim(\xi_1)(sms_j) \quad (j \in \mathbb{N}) \tag{1.40}$$

or

$$lim(\xi_1)_{\to} \tag{1.40'}$$

Corollary 1.14: In the dynamic limit, $lim(\xi_0)$, of the kernel dynamic space, $QUAT^Q$. The limit of a real function, $f(x)$, is the kernel, $A(q)^k(=A)$, and its medium, $M(r)$, or cloud in Definition 1.1.

Proof: It can be proved by method, $\varepsilon - \delta$.

Corollary 1.15: Any $m_q \in M(q) \quad (M(q) \in QUAT^Q)$, irrational numbers exist

$m_r \in M(r) \quad (M(r) \in QUAT)$**,**

there is a

$m_r \notin M(q)$.

So that the kernel dynamic space, $QUAT$, has an uncountable subset of the medium,

$$M(r)_{\mathbb{R}-\mathbb{Q}} = \{M(r) - M(q) \,|\, M(r) \in QUAT, M(q) \in QUAT^Q\} \tag{1.41}$$

And the limiting process, $lim(\xi_1)(sms_j) \quad (j \in \mathbb{N})$, runs on $M(r)_{\mathbb{R}-\mathbb{Q}}$.

Proof: According to the definition of $\aleph_0, \aleph_1$ and shell, if m_r does not exist, then the limit process can only have countable limit steps, $lim(\xi_0)_{\to}$, and cannot have uncountable limit steps, $lim(\xi_1)_{\to}$, which is a contradiction. So the medium element exists.

Obviously $M(r)_{\mathbb{R}-\mathbb{Q}}$ is existence and not countable.

Definition 1.10: If we consider a real two-dimensional space

$$(x, y) = (x, f(x))(x \in \mathbb{R}) \tag{1.42}$$

There is a real dynamic space,

$$(x, y, l) = (x, f(x), lim(\xi)) \quad (x \in \mathbb{R}, lim(\xi) \in QUAT) \tag{1.43}$$

and dynamic limit

$$f(x) \quad \lim_{x \to a}(\xi) \quad A^k(= A) \tag{1.44}$$

or

$$f(x) \quad lim(\xi_1) \quad A^k(= A) \qquad (1.44)'$$

Definition 1.11: Let the dynamic space represented by formula (1.43) be $3DL$ (Dimension-Limit). And during the dynamic limit process, the plane (x, y) slides along the dynamic limit axis according to the limit step.

Corollary 1.16: The kernel number, $A(1,1)^k$, contains the shell-medium subset, $S(q)_{Q-N}, M(r)_{Q-N}$, of the kernel number,$A(q)^k$, and the dynamic limit, $\lim(\xi_0)$. As same before, The kernel number, $A(q)^k$, contains the shell-medium subset, $S(r)_{R-Q}, M(r)_{R-Q}$, of the kernel number,$A(r)^k$, and the dynamic limit, $\lim(\xi_1)$. So the kernel number and its medium form a number cloud, that is, in the dynamic limit,$\lim(\xi)$, the limit is a number cloud.

Proof: (Omitted).

Definition 1.12: The kernel numbers have

$$A(1,1)^k = A(\xi_0)_N^k; A(q)^k = A(\xi_0)_Q^k; A(r)^k = A(\xi_1)^k \tag{1.45}$$

and

$$A(\xi_i)^k \quad (i = 2,3,\cdots) \tag{1.46}$$

Corollary 1.17: Any kernel number in the dynamic limit process contains a higher-order kernel number

$$A(\xi_0)_N^k \supset A(\xi_0)_Q^k \supset A(\xi_i)^k \cdots \quad (i \in N) \tag{1.47}$$

Proof:

From Corollary 1.16, Corollary 1.17 and Definition 1.12, we have formula (1.47).

2. High Cardinality Dynamic Limit and the Corresponding Kernel Dynamic Space

In Section 1, the basic set discussed is the real numbers, $\mathbb{R}$. The three cardinal dynamic limits are constructed on which the kernel number dynamic spaces. Then, the sequence and measurability of the real number set, $\mathbb{R}$, (there is a distance between two numbers) ensures that the $\varepsilon - \delta$ method can be used. The existence of the $\aleph_0, \aleph_1$ cardinality ensures that the dynamic limit process $lim(\xi_0)_{\rightarrow}, lim(\xi_1)_{\rightarrow}$ can run. The dynamic limit process ensures the existence of the shell and medium that "encapsulate" the kernel.

Definition 2.1: Let the cardinality dynamic limit

$$\{lim(\xi_i) \,|\, i \geq 2, i \in \mathbb{N})\} \tag{2.1}$$

Limit process

$$\{lim(\xi_i)_{\rightarrow} \,|\, i \geq 2, i \in \mathbb{N})\} \tag{2.2}$$

Let

$$\alpha_0 = \mathbb{Q}, \alpha_1 = \mathbb{R}, \alpha_2, \alpha_3, \cdots \tag{2.3}$$

And the corresponding kernel number dynamic space

$$QUAT(\aleph_i) = Quaternary\left\{S(\alpha_i), M(\alpha_i), A(\alpha_i)^k, lim(\xi_i)\right\} \Big| i \geq 2, i \in \mathbb{N}\Big\} \tag{2.4}$$

Corollary 2.1: In high-cardinality kernel dynamic space

$$QUAT(\aleph_i)(i \geq 2 : i \in \mathbb{N})),$$

To be able to execute its dynamic limit process

$$\{lim(\xi_i)_{\rightarrow} \,|\, i \geq 2 : i \in \mathbb{N})\} \tag{2.5}$$

and the elements of the set are sequence and measurable.

Proof: When $i = 2$

By Corollary 1.15 the kernel number, $A(\xi_1)^k$, has a shell-medium subset, $S(r)_{R-Q}, M(r)_{R-Q}$. They form a cloud of numbers in the dynamic limit, $lim(\xi_1)$. If the cardinality, $\aleph_2$, exists, then the dynamic limit process, $lim(\xi_2)_{\rightarrow}$, exists. By formula (2.2) and the $\varepsilon - \delta$ method, if any $s \in S(\alpha_2)$ then there exists $m \in M(\alpha_2)$ such that $s \neq m$. And the shell-medium subset, $S(\alpha_2), M(\alpha_2)$, is not empty, and the elements in them are sequence and measurable.

When $i > 2$, the same logic can be proved.

Corollary 2.2: There exists some element

$$e_s \in S(\alpha_2), e_m \in M(\alpha_2)$$

and

$$e_s \notin \mathbb{R}, e_m \notin \mathbb{R}.$$

Proof: Because the cardinality of $S(r)_{R-Q}, M(r)_{R-Q}$ is $\aleph_1$, and the cardinality of $S(\alpha_2), M(\alpha_2)$ is $\aleph_2$, and

$$\aleph_1 < \aleph_2$$

If any $e_s, e_m (e_s \in S(\alpha_2), e_m \in M(\alpha_2))$, there are $e_s \in \mathbb{R}, e_m \in \mathbb{R}$ then there is

$$\aleph_1 = \aleph_2$$

Contradiction.

Definition 2.2: Assume

$$E(-R)_2 = \{e \,|\, e \in S(\aleph_2) \text{ or } e \in M(\aleph_2), e \notin \mathbb{R}\} \qquad (2.6)$$

$E(-R)_2$ exists in the shell-medium of the kernel number dynamic space, $QUAT(\aleph_2)$. Currently it is not constructible like real numbers, $\mathbb{R}$. But it can be the limit, such as A of static space. $\mathbb{Q}, \mathbb{R}, E(-R)_2$ are all mixed in the number cloud of the kernel number, $A(\alpha_2)^c$.

Definition 2.3: The cardinality of a set is,

$$|E| \qquad (2.7)$$

Corollary 2.3: In the kernel dynamic space, $QUAT(\aleph_2)$, CH (Continuum hypothesis) is irrelevant.

Proof: From Corollary 2.2 it can be concluded that

$$E(-R)_2 \neq \emptyset$$

Then, the number cloud of the static limit, A , has elements that

$$e \in E(-R)_2$$

make

$$e \notin \mathbb{R} \wedge e \notin \mathbb{Q}$$

Since in the cloud of the kernel, $A(\alpha_2)^k$, there are elements other than $\mathbb{Q}$ with the cardinality of $\aleph_0$ and $\mathbb{R}$ with the cardinality of $\aleph_1$。 That is to say, according to the definition of shell and medium, $S(\alpha_2), M(\alpha_2)$, such that

$$S(\alpha_2) \subset (\mathbb{Q} \cup \mathbb{R} \cup E(-R)_2)$$
$$M(\alpha_2) \subset (\mathbb{Q} \cup \mathbb{R} \cup E(-R)_2)$$

Because $e \notin \mathbb{R} \wedge e \notin \mathbb{Q}$, therefore the cardinality of $E(-R)_2$

$$\aleph_0 = |\mathbb{Q}| < |\mathbb{Q} \cup E(-R)_2| \leq |\mathbb{R} \cup E(-R)_2|$$

Moreover, under the dynamic limit process, $lim(\xi_2)_{\rightarrow}$,

$$|\mathbb{Q} \cup \mathbb{R} \cup E(-R)_2| = |\mathbb{R} \cup E(-R)_2| = \aleph_2$$

Let the set of all proper subsets H_{2S} of $E(-R)_2$,

$$H_{2S} = \{\eta_{2s} | \eta_{2s} \subset E(-R)_2, |\eta_{2s}| \leq \aleph_1\} - \{E(-R)_2\}.$$

Suppose

$$H_{2S} \uparrow = \{\eta_{2s} | \eta_{2s} \subset H_{2S}, |\mathbb{Q} \cup \eta_{2s}| < |\mathbb{R}| = \aleph_1, |\eta_{2s}| < \aleph_1\},$$

$$H_{2S} \downarrow = \{\eta_{2s} | \eta_{2s} \subset H_{2S}, |\mathbb{Q} \cup \eta_{2s}| > |\mathbb{Q}| = \aleph_0, |\eta_{2s}| < \aleph_1\}.$$

Let

$$H_{2S}(max) = \cup_{\eta_{2s} \subset H_{2S}\uparrow} \eta_{2s},$$

and

$$H_{2S}(min) = \cap_{\eta_{2s} \subset H_{2S}\downarrow} \eta_{2s}$$

To be able to obtain

$$H_{2S}(max) \subset H_{2S}, \quad |\mathbb{Q} \cup H_{2S}(max)| < \aleph_1$$

and

$$H_{2S}(min) \subset H_{2S}, \quad |\mathbb{Q} \cup H_{2S}(min)| > \aleph_0$$

And any $e \in E(-R)_2, \quad |\{e\} \cup \mathbb{Q}| < \aleph_1$ both

$$H_{2S}(max) \neq \emptyset$$

1. If $H_{2S}(min) \neq \emptyset$

Prove it on a case-by-case basis,

1.1

If $H_{2S}(min) = H_{2S}(max)$ has

$$\aleph_0 < |\mathbb{Q} \cup H_{2S}(min)| = |\mathbb{Q} \cup H_{2S}(max)| < \aleph_1$$

1.2

If $H_{2S}(min) \cap H_{2S}(max) \neq \emptyset$ and $|H_{2S}(min) \cap H_{2S}(max)| < \aleph_1$ then

$$H_{2S}(min) \cap H_{2S}(max) \in H_{2S}\uparrow$$

or

$$H_{2S}(min) \cap H_{2S}(max) \in H_{2S}\downarrow ,$$

Both

if $H_{2S}(min) \cap H_{2S}(max) \subset H_{2S}(min)$,
then $H_{2S}(min) \cap H_{2S}(max) = H_{2S}(min)$

or

if $H_{2S}(min) \cap H_{2S}(max) \subset H_{2S}(max)$,
then $H_{2S}(min) \cap H_{2S}(max) = H_{2S}(max)$

Must have

$$H_{2S}(min) = H_{2S}(min) \cap H_{2S}(max) = H_{2S}(max)$$

both

$$\aleph_0 < |\mathbb{Q} \cup H_{2S}(min)| = |\mathbb{Q} \cup H_{2S}(max)| < \aleph_1$$

1.3

If

$$H_{2S}(min) \cup H_{2S}(max) \in H_{2S} \downarrow$$

then must have

$$\aleph_0 < |\mathbb{Q} \cup H_{2S}(min) \cup H_{2S}(max)| < \aleph_1$$

1.3.1

If

$$H_{2S}(min) \cup H_{2S}(max) \in H_{2S} \uparrow$$

then must have

$$H_{2S}(min) \cup H_{2S}(max) \subset H_{2S}(max)$$

Have to

$$H_{2S}(min) \cup H_{2S}(max) = H_{2S}(max)$$

in that case

$$H_{2S}(min) = H_{2S}(max)$$

contradictions

the reason why $H_{2S}(min) \cap H_{2S}(max) \neq \emptyset$, Same as 1.2

1.3.2

If

$$H_{2S}(min) \cup H_{2S}(max) \in H_{2S} \downarrow$$

then must have

$$\aleph_0 < |\mathbb{Q} \cup H_{2S}(min) \cup H_{2S}(max)| < \aleph_1$$

CH is not true.

Note: $|E(-R)_2|$ is an unreachable cardinality.

2. $H_{2S}(min) = \emptyset$

Then

$$\aleph_0 = |\mathbb{Q} \cup H_{2S}(min) \cup H_{2S}(max)| < \aleph_1$$

CH is true.

Definition 2.4: Assume

$$\mathbb{R}_1 = P(\mathbb{R}) = P(\mathbb{R}_0)$$
$$\mathbb{R}_i = P(\mathbb{R}_{i-1}) \quad (i > 2, i \in \mathbb{N}) \tag{2.8}$$

$$E_i = \{e \,|\, e \in S(\aleph_i) \text{ or } e \in M(\aleph_i), e \notin \mathbb{R}_{i-1}, i \in \mathbb{N}\} \tag{2.9}$$

Corollary 2.4: In $QUAT(\aleph_i)(i \geq 2, i \in \mathbb{N})$, GCH (General Continuum hypothesis) is irrelevant.

Proof: Same as the proof of Corollary 2.3.

and there exists a nonempty subset $E_{iS}(i > 2, i \in \mathbb{N}) \subset E(-E_{i-1})$ with

$$\aleph_{i-1} < |\mathbb{R}_{i-1} \cup H_{iS}(min) \cup H_{iS}(max)| < \aleph_i \quad (i > 2, i \in \mathbb{N}),$$

So GCH is not true.

or

$$\aleph_{i-1} = |\mathbb{R}_{i-1} \cup H_{iS}(min) \cup H_{iS}(max)| < \aleph_i \quad (i > 2, i \in \mathbb{N}),$$

So GCH is true.

Finish proof.

Corollary 2.5: When $E_i \neq \emptyset$, the static space limit, A, (see Definition 1.0) exists for the kernel dynamic space.

Proof : Can be defined using the $\varepsilon - \delta$ method and the shell-medium of the kernel dynamical space. (omitted).

Corollary 2.6: When $E_i \neq \emptyset$ and GCH is true, the static space limit, A, (see Definition 1.0) exists for the kernel dynamic space.

Proof : Can be defined using the $\varepsilon - \delta$ method and the shell-medium of the kernel dynamical space. (omitted).

By Corollary 2.6.

Definition 2.5: There is a kind of dark element in the cloud of the $A^k = A \quad (A \in \mathbb{R})$. It is called as "elfin" element.

$$e \in E = E_0 \tag{2.10}$$

Corollary 2.7: When $(i \in \mathbb{N}, i > 3)$, The presence of some element

$$e_s \in S(\alpha_i), e_m \in M(\alpha_i)$$

makes

$$e_s \notin E_{i-1}, e_m \notin E_{i-1}$$

Proof: similar to the proof of Corollary 2.2, (omitted).

As in paragraph 1, Definition 1.8.

Axiom: If the kernel dynamic space $QUAT(\aleph_i)(i \in \mathbb{N})$ yields the limit value,A of a real continuous function $f(x)$ for any limit $\varepsilon - \delta$ method in the static space according to Definition 1.0, then there is always.

$$A(\alpha_i)^k \subset A, A \subset A(\alpha_i)^k \text{ both } A(\alpha_i)^k = A \tag{2.11}$$

That is, for the kernel dynamic space $QUAT(\aleph_i)(i \in \mathbb{N})$ only, the change after the dynamic limit $lim(\aleph_0)$ does not affect formula (2.11). And the kernel number limit is unique. Call the dynamics limit,$lim(\xi_i)$,of the kernel number dynamics space Terminating.

Now the kernel number dynamical spaces exist simultaneously as rational numbers , irrational numbers,$\mathbb{R} - \mathbb{Q}$, elfin numbers , and dynamical limits,$lim(\xi_i)$.

Similar to Definition 1.10 and Definition 1.11

Definition 2.6: If we consider real $n(n = 3,4,\cdots)$ dimensional spaces and the formula (2.1)

$$(x_1, x_2, \cdots) \quad (x_i \in \mathbb{R} : i = 1,2,3,\cdots) \tag{2.12}$$

The shells and medium in the dynamical space, $QUAT$, of the real kernel are based on formula (2.12). Assume,

$$QUAT_i \quad (i = 1,2,3,\cdots) \tag{2.13}$$

with real dimensional dynamic space,

$$\begin{aligned}&(l_j, x_1, x_2, \cdots) = (lim(\xi_j), x_1, x_2, \cdots)\\ &\qquad (x_i \in \mathbb{R}, lim(\xi_j) \in QUAT_i : i = 1,2,3,\cdots, j = 1,2,3,\cdots)\end{aligned} \tag{2.14}$$

and dynamic limits

$$f(x_1, x_2, \cdots) \quad \lim_{x\to\infty} (\xi_j) \quad \infty^c(=\infty) \quad (j = 1,2,3,\cdots) \tag{2.15}$$

Definition 2.7: Denote the dynamic space represented by formula (2.14) as nDL(Dimension-Limit). And in the dynamic limit process, the space $(x_1, x_2, \cdots)$ is sliding along the dynamic limit axis, $l_j(j = 1,2,3,\cdots)$, in a limit step, $lim(\xi_j)(j = 1,2,3,\cdots)$.

Definition 2.8: The set of all elfin of the kernel dynamical space is $\mathbb{E}$, and the set of all elfin of the real n dimensional dynamical space are

$$\mathbb{E}_n \quad (n = 1,2,\cdots) \tag{2.16}$$

To summarize, in a static space based on the real numbers, there are functions, $f(x)(x \in \mathbb{R})$ that are continuous, i.e., there are no gaps between the real numbers, or simply put, the real axis is continuous

And based on the formula (1.43) in Definition 1.10, if there is a function, $f(x)(x \in \mathbb{R})$ that iterates over all the

$$y \quad (y \in \mathbb{R} \text{ or } y = A(\alpha_i)^k),$$

In the kernel dynamical space of formula (2.4), there are an infinite number of elfin, $e \in \mathbb{E}_1$ around $A(\alpha_i)^k$. Then the function

$$f(x)(x \in \mathbb{Q} \cup \mathbb{R}) \in \mathbb{Q} \cup \mathbb{R}$$

is equivalent to

$$f(x)(x \in \mathbb{R}) \in \mathbb{R}$$

There is a gap in the kernel dynamic space. When added with a elfin number, then the function

$$f(x)(x \in \mathbb{Q} \cup \mathbb{R} \cup \mathbb{E}_1) \in \mathbb{Q} \cup \mathbb{R} \cup \mathbb{E}_1$$

is equivalent to

$$f(x)(x \in \mathbb{R} \cup \mathbb{E}_1) \in \mathbb{R} \cup \mathbb{E}_1$$

and has "continuity" in the kernel dynamic space.

A similar discussion can be made for the real n dimensional dynamic space of Definition 2.6.

Thus, in the kernel dynamical space $\mathbb{R} \cup \mathbb{E}_1$, there are positions (e.g., sizes, distances, etc.) about their elements. And there are no gaps in the dynamical space, there is both "continuity". The cardinality of the set of numbers is then a "quantity". This quantity can be accumulated into "potential", and potential is used to distinguish the level of the quantity.

So the CH expression is: There is no set whose cardinality (potential) is absolutely greater than the set of countable (quantity) and absolutely smaller than the set of real numbers (quantity).

3. Static Space limit ∞

Let's analyze the static space limit value, $\infty(=A)$. To be precise, the ∞ cannot be a value, but a process.

Similar to Definition 1.1 of section 1.

Definition 3.1 For real continuous function, $f(x)$,

$$\left(\forall \varepsilon_q > 0 : \varepsilon_q \in \mathbb{Q}\right), \quad \left(\exists \varepsilon_r > 0 : \varepsilon_r \in \mathbb{R}\right), \ (\exists \delta > 0 : \delta \in \mathbb{R})$$

If $|x^{-1}| < \delta$, must have

$$|f(x) - A| < \varepsilon_r < \varepsilon_q \tag{3.1}$$

Then A is called the limit of the $f(x)$ at ∞, denoted by

$$\lim_{x \to \infty} f(x) = A \tag{3.2}$$

If $x > 0$ (< 0), We get the right (left) limit. A function has a limit at a point when its left and right limits are equal.

Definition 3.2 For real function, $f(x) = \begin{cases} f(x) & f(x) \neq 0 \\ f(x)^{-1} = 0 & f(x) = 0 \end{cases}$,

$\left(\forall \varepsilon_q > 0 : \varepsilon_q \in \mathbb{Q}\right)$, $\left(\exists \varepsilon_r > 0 : \varepsilon_r \in \mathbb{R}\right)$, $(\exists \delta > 0 : \delta \in \mathbb{R})$

If $|x - a| < \delta$, must have

$$|f(x)^{-1}| < \varepsilon_r < \varepsilon_q \tag{3.3}$$

Then ∞ is called the limit of the $f(x)$ at a, denoted by

$$\lim_{x \to a} f(x) = \infty \tag{3.4}$$

If $x - a > 0$ (< 0), We get the right (left) limit. A function has a limit, ∞, at a point,a, when its left and right limits are equal.

Definition 3.3 For real function, $f(x) = \begin{cases} f(x) & f(x) \neq 0 \\ f(x)^{-1} = 0 & f(x) = 0 \end{cases}$,

$\left(\forall \varepsilon_q > 0 : \varepsilon_q \in \mathbb{Q}\right)$, $\left(\exists \varepsilon_r > 0 : \varepsilon_r \in \mathbb{R}\right)$, $(\exists \delta > 0 : \delta \in \mathbb{R})$

If $|x^{-1}| < \delta$, must have

$$|f(x)^{-1}| < \varepsilon_r < \varepsilon_q \tag{3.5}$$

Then ∞ is called the limit of the $f(x)$ at ∞, denoted by

$$\lim_{x \to \infty} f(x) = \infty \tag{3.6}$$

If $x > 0$ (< 0), We get the right (left) limit. A function has a limit, ∞, at a point, ∞, when its left and right limits are equal.

Only Definition 3.3 will be discussed below. Refer to Definition 1.6.

Definition 3.4: If the limit, lim, is a dynamic process, then the kernel number dynamic space can be written as

$$QUAT^{\infty} = Quaternary\{S, M, \infty^k, lim\} \tag{3.7}$$

If $S \subset \mathbb{R}, M \subset \mathbb{R}$ then a $QUAT$ is real kernel number dynamic space.

Then the shell-medium and number clouds discussed in section 1 and section 2 also apply to the kernel,∞, dynamic space, $QUAT^{\infty}$. Under the action of the dynamic limit, $lim(\xi_i)$, and the limit process, $lim(\xi_i)_{\rightarrow}$, in Definition 2.1, the shell-medium and cloud of the kernel ∞ are more intuitive. The continuation of the limit process, $lim(\xi_i)_{\rightarrow}$, or the continuation of the process of the limit step on the limit axis, allows the limit ∞ to be open to infinity, or it can be a closed loop. The following Definition 3.5 is a mirror image of Definition 3.3

Definition 3.5 For real continuous function, $f(x)$,

$$\left(\forall \varepsilon_q > 0 : \varepsilon_q \in \mathbb{Q}\right), \quad \left(\exists \varepsilon_r > 0 : \varepsilon_r \in \mathbb{R}\right), \ (\exists \delta > 0 : \delta \in \mathbb{R})$$

If $|x| < \delta$, must have

$$|f(x)| < \varepsilon_r < \varepsilon_q \tag{3.7}$$

Then 0 is called the limit of the $f(x)$ at 0, denoted by

$$\lim_{x \to 0} f(x) = 0 \tag{3.8}$$

If $x > 0$ (< 0), We get the right (left) limit. A function has a limit, 0, at a point, 0, when its left and right limits are equal.

Therefore the limit, 0, can be used as the closed-loop connection point of the limit, ∞. On the contrary, the limit, ∞, can also be used as the closed-loop connection point of the limit, 0. To form closed loops in two directions. On the limit axis, the limit process, $lim(\xi_i)_{\rightarrow}$, also begins. Choosing either an open or closed loop configuration does not make the $\varepsilon - \delta$ method contradictory.

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