Improvement of the Harmonic Disturbance Rejection Performance in the Linear Active Disturbance Rejection Control System Using the m-*th* Order Extended State Observer with Frequency Estimation

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ABSTRACT

This paper proposes a method to improve the disturbance rejection performance and guarantee the high-precision tracking performance in the active disturbance rejection control(ADRC) system using the m-*th* order extended state observer(ESO) with frequency estimation. The ADRC system with the m-*th* order ESO becomes the system of the m-*th* order astatism, so the constant, ramp and parabolic disturbances are rejected perfectly, and the rejection performance on the harmonic disturbance is also improved according to the increase in the extended state order m. However, the rejection effect is not remarkably high on the harmonic disturbance because its components of derivatives increase according to the derivative order power of angular frequency, and the research to resolve such problems is not sufficient. The paper made a study of a method to raise the disturbance rejection performance and the stability of the ADRC system by applying the m-*th* order ESO with frequency estimation to the plant of canonical form which is represented in the integral chain structure, and analyzed and evaluated the disturbance rejection performance on the constant, ramp, parabolic and harmonic disturbances through the simulation. Results showed that the error could be reduced to 'zero' theoretically with respect to the aperiodic disturbances such as the constant, ramp and parabolic disturbances, and the rejection performance was also improved greatly on the harmonic disturbances. Using the disturbance rejection method by the m-*th* order ESO with frequency estimation, the rejection capability will be raised on any disturbance and the high-precision tracking performance can be improved.

Keywords: frequency estimation, disturbance rejection control, extended state observer, disturbance rejection performance

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1. Introduction

Nowadays it is of great importance to improve the disturbance rejection performance of control system to raise the tracking precision [1, 2]. 2-DOF control could meet the requirements of tracking precision enough by determining the parameters of controller appropriately, and the robust control and adaptive control are effective against the unstructured internal uncertainties with the bounded parameter variations [3-5]. Measurability and observability conditions of the reference signal or disturbance should be satisfied theoretically in the 2-DOF control. In the robust control the technological requirements should be considered accurately in setting the limitation of uncertainties or the required characteristics of robustness. In the adaptive control the real-time problem should be considered for the estimation of the internal uncertain parameters in case that the requirement on the control precision is high. The disturbance estimation method using the internal model principle and the observer was proposed in order to improve the disturbance rejection performance [6, 7]. In the internal model principle the disturbance could be rejected to some extent by determining the order of the integrator and parameters appropriately, however it is important to resolve the stability problem of control system. In case of using observer it should be considered to observe the disturbances and design the filter to remove the observation noise. Recently the active disturbance rejection control(ADRC) method has begun to interest the control system developers in rejecting the 'total' disturbance, and the method has the robustness against the parameter uncertainties, whole attenuation ability on the external and internal disturbances and the high adaptive ability on the linear and non-linear plants [8, 9]. A method representing the non-linear control plants in the canonical forms with the integral chain structure through some transformation was studied in [9, 10], and the stability of the active disturbance rejection control system was analyzed in the frequency-domain for the non-linear time-variant plant with uncertain dynamic characteristics. [11] analyzed the stability of the control system in the frequency-domain, and showed the active disturbance rejection control system has the high stability and robustness. The most references [12-15] studied the stability problem of the active disturbance rejection control system using the non-linear extended state observer based on the fal function or sat function and approved the convergence of the (n+1)*th* order active disturbance rejection control system for the plant of the unknown dynamic characteristics, and also analyzed the robustness and effects of the natural angular frequency of observer on the stability and the disturbance rejection when there exist the uncertain parameters in the active disturbance rejection control system. In [16, 17] the active disturbance rejection controller of the high precision servo system was designed to overcome the friction creeping phenomenon which is affecting the low speed performance, and the driving method of driving axis of MEMS gyroscope was proposed to resonate and regulate the output amplitude of axis by using the active disturbance rejection control method, and it was shown that the sinusoidal disturbance could be rejected and the ideal reference signal tracking could be implemented in the steady state by selecting the appropriate resonance frequency in [18]. In references [19-24] the control method was proposed based on the enhanced state observer to resolve the disturbance attenuation problems due to the mismatched uncertainties and the external disturbance in the systems without integral loop, and fractional-order active disturbance rejection control method was applied for the precise trajectory tracking and position decision control of the newly designed linear electric motor, as well the ADRC methods were proposed for the practical applications rather than the theoretical researches. [25] proposed a method to estimate optimally the parameters of the linear active disturbance rejection control system based on the available model information. With a very few model information, the active disturbance rejection control method could demonstrate the effective performance, but the performance could be raised more once the known model information is added. In the reference this method was described as 'generalized active disturbance rejection control'. However the reference did not propose the improving method of the active disturbance rejection control according to the detailed forms of disturbances. In [26] a method to estimate optimally the parameters of active disturbance rejection control system was proposed using the genetic algorithm. [27] suggested a parameter estimation method of the active disturbance rejection control applicable to the non-linear system where the sampling speed is not so fast. The reference suggested the parameter estimation method effective for the engineering practice based on one-chip processor, however the computing algorithm is complex, as well the method has certain limitations for the recent typical high performance processors. Regarding the disturbance error, the conventional active disturbance rejection control system could reduce the steady-state error to zero on the constant disturbance. In this case the differential term of the disturbance is assumed to be zero for the design of observer. Recently new methods are attracting attentions, where the order of the observer state variables is extended up to the higher order to raise the disturbance rejection performance [18]. That is, taking up to the (m-1)*th* order differential term of disturbance as the extended state value and including it in the design of observer, the steady-state error was reduced to zero with respect to the m-*th* order polynomial disturbance. In [28] the m-*th* order extended state observer was proposed to raise the efficiency of the extended state observer and analyzed the merits and demerits, and applied to stabilize the rubber deflection in the reinforcing stage of tyre. Also it researched to raise the disturbance rejection ability in case that the disturbance is periodic signal, especially the harmonic signal. The design method of the higher order extended state observer was proposed, but the performances of the higher order extended state observer and the higher order active disturbance rejection control system were not analyzed from the viewpoint of structure of control engineering. In [29] the resonant disturbance observer was proposed to remove the harmonic disturbance connected with the m harmonic waves, and applied it to the active filtering control of electric power. Here the parameter estimation was performed by combining the pole placement method with the optimal estimation method based on the Kalman-Bucy filtering. The feature of the ADRC is that it treats the external disturbances and the uncertainties of plant model totally, so that it depends on the model rarely and doesn't require the high gain, as well it estimates and compensates the real value of 'total disturbance' using the state observer [8, 9]. The structural fundamentals of the ADRC are the canonical plant of the cascade integrators and the state observer, and its core is the extended state observer [9]. However the (n+1)*th* order active disturbance rejection control system becomes the system of the 1st order astatism with respect to the disturbance while the (n+m)*th* order active disturbance rejection control system becomes the one of the m-*th* order astatism with respect to the disturbance.

Therefore, the (n+1)*th* order active disturbance rejection control system can reduce the disturbance error to 'zero' for the constant disturbance, but not for the other kinds of disturbance errors theoretically. The (n+m)*th* order active disturbance rejection control system can theoretically reduce the disturbance error to 'zero' for the constant, ramp and parabolic disturbances, but not for the harmonic disturbance theoretically. The most references, especially [9] introduced the evaluation on the observation error between the extended state value and extended state observation value, but mostly the error of system is not reduced even though the error between the extended state value and the extended state observation value is very small practically. The disturbance rejection performance of the (n+m)*th* order ADRC has been analyzed rarely on the harmonic disturbance as well. Thus this paper studies a method to improve the rejection performance on the harmonic disturbance for the active disturbance rejection control system using the m-*th* order extended state observer with the frequency estimation.

Firstly, it is shown that the canonical active disturbance rejection control system rejects the constant, ramp and parabolic disturbances perfectly according to the order m in case of using the m-*th* order extended state observer for the aperiodic disturbance.

Secondly, it is shown that the single-frequency harmonic disturbance will be rejected nearly perfectly in the active disturbance rejection control system using the extended state observer with frequency estimation.

The paper consists of the following topics.

Section 2 makes the model of the (n+m)*th* order active disturbance rejection control system and researches the disturbance rejection performance of the (n+m)*th* order active disturbance rejection control system with the frequency estimation. Section 3 investigates the stability of the (n+m)*th* order active disturbance rejection control system with frequency estimation, and Section 4 simulates and analyzes the results on the constant, ramp, parabolic and harmonic disturbances. Section 5 finds the corresponding conclusion.

It is verified that the disturbance rejection capability of system can be improved greatly in the control system using the active disturbance rejection control by the m-*th* order extended state observer with the frequency estimation.

2. Disturbance Rejection Performance of the (n+m)th Order Active Disturbance Rejection Control System with Frequency Estimation

To consider the problem simply let's describe the model of the control plant as

$$\left.\begin{array}{l} \dot{x}_i(t) = x_{i+1}(t), (i = \overline{1, n-1}) \\ \dot{x}_n(t) = F(x_1(t), x_2(t), \cdots x_n(t)) + bu(t) + d(t) \end{array}\right\} t \ge t_0 \qquad (1)$$
$$y(t) = x_1(t)$$
$$x_i(0) = 0$$

Where, $X(t)(X(t) \in R^n)$ is the state vector, $y(t)(y(t) \in R)$ is the observable output signal and $u(t)(u(t) \in R)$ is the control input.

Assumption 1. The known nominal value of b is b_0, and b is uncertain parameter satisfying $b > 0$ and $b \in [b_{\min}, b_{\max}]$.

Assumption 2. m is the extended order of system.

Assumption 3. Eq.(1) is controllable by the feedback control.

Assumption 4. External disturbance $d(t)$ is introduced from [18] and it can be considered to the extended order as follows.

$$\begin{aligned} &d(t) = d_1(t) + d_2(t) \\ &d_1 = \sum_{i=1}^{m} \alpha_{i-1} t^{i-1},\ |d_1(t)| \le \bar{d}_1, \\ &d_2 = A \sin \omega_d t,\ |d_2(t)| \le \bar{d}_2 \end{aligned} \tag{2}$$

Where $d_1(t)$ is the aperiodic disturbance, α_{i-1} is the maximum of derivative of $d_1(t)$, and the m-*th* order derivative of $d_1(t)$ is zero. $d_2(t)$ is the harmonic disturbance, and $A(|A| \le \bar{A})$ and $\omega_d(\omega_d \le \bar{\omega}_d)$ are amplitude and angular frequency of $d_2(t)$.

α_{i-1}, A and ω_d are uncertain quantities.

Assumption 5. In Eq.(1), considering the Assumption 1 and Assumption 2, lump disturbance

$$w(F, d, b, t) = F(x_1(t), x_2(t), \cdots x_n(t)) + (b(t) - b_0)u(t) + d(t) \tag{3}$$

reflecting the internal uncertainties and external disturbances satisfies Lipschitz condition.

Denoting the extended state variable $x_{n+1}(t)$ as

$$x_{n+1}(t) = w(F, b, d, t) = h(t) \tag{4}$$

then Eq.(1) can be written as follows.

$$\left.\begin{aligned} &\dot{x}_i(t) = x_{i+1}(t), (i = \overline{1, n-1}) \\ &\dot{x}_n(t) = w(F, b, d, t) + b_0 u(t) \end{aligned}\right\} t \ge t_0 \\ y(t) = x_1(t) \\ x_i(0) = 0 \tag{5}$$

Differentiating Eq.(4) m times and substituting it into Eq.(5), then it can be represented as the canonical m-*th* order extended state model.

$$\left.\begin{aligned} &\dot{x}_i(t) = x_{i+1}(t), (i = \overline{1, n-1}) \\ &\dot{x}_n(t) = x_{n+1}(t) + b_0 u \\ &\dot{x}_{n+j}(t) = x_{n+j+1}(t), (j = \overline{1, m-1}) \\ &\dot{x}_{n+m}(t) = h^{(m)}(t) \end{aligned}\right\} \\ y(t) = x_1(t) \tag{6}$$

where $h^{(m)}$ is the m-*th* derivative of $h(t)$.

2.1 Active Disturbance Rejection Method by the m-th Order Extended State Observer.

Let's consider the case of $d_2(t) = 0$, that is, $d(t) = d_1(t)$ in Eq.(2). Then Eq.(4) can be written as

$$x_{n+1}(t) = w(F, b, d_1, t) = h_{d1}(t) \tag{7}$$

From Eq.(7), Eq.(6) becomes as follows.

$$\left.\begin{array}{l} \dot{x}_i(t)=x_{i+1}(t),(i=\overline{1,n-1}) \\ \dot{x}_n(t)=x_{n+1}(t)+b_0u \\ \dot{x}_{n+j}(t)=x_{n+j+1}(t),(j=\overline{1,m-1}) \\ \dot{x}_{n+m}(t)=h_{d1}^{(m)}(t) \end{array}\right\} \quad (8)$$
$$y(t)=x_1(t)$$

Now the m-*th* order extended state observer with respect to the Eq.(8) can be written as follows.

$$\left.\begin{array}{l} \dot{\hat{x}}_i(t)=\hat{x}_{i+1}(t)+\beta_i(y(t)-\hat{y}(t)),(i=\overline{1,n-1}) \\ \dot{\hat{x}}_n(t)=\hat{x}_{n+1}(t)+b_0u(t)+\beta_n(y(t)-\hat{y}(t)) \\ \dot{\hat{x}}_{n+j}(t)=\hat{x}_{n+j+1}(t)+\beta_{n+j}(y(t)-\hat{y}(t)),(j=\overline{1,m-1}) \\ \dot{\hat{x}}_{n+m}(t)=\beta_{n+m}(y(t)-\hat{y}(t)) \end{array}\right\} t\ge t_0 \quad (9)$$
$$\hat{y}(t)=\hat{x}_1(t)$$

Where $\hat{x}_i(t)$ is the state vector of observer, $\hat{y}(t)$ is the output signal of observer, and β_i is the gain coefficient of observer whose characteristic polynomial $s^{n+m}+\sum_{i=1}^{n+m}\beta_i s^{n+m-i}$ satisfies the Hurwitz criterion.

If the disturbances on the control plant are possible to be observed from the extended state observer, then the control law will be denoted as follows.

$$u(t)=-\frac{\hat{x}_{n+1}(t)}{b_0}+u_0(t) \quad (10)$$

Where $\hat{x}_{n+1}(t)$ is the observation value of $x_{n+1}(t)$, and $u_0(t)$ is the control input when there’s no disturbance action.

In Eq.(10), let’s assume the control law u_0 is determined as follows.

$$u_0(t)=-\sum_{i=1}^{n}k_i x_i(t) \quad (11)$$

Where k_i is the state feedback coefficient.

Thus the control law of the active disturbance rejection control system is found as follows.

$$u(t)=-\frac{\hat{x}_{n+1}(t)}{b_0}-\sum_{i=1}^{n}k_i x_i(t) \quad (12)$$

Substituting Eq.(12) into Eq.(5), then

$$\left.\begin{array}{l} \dot{x}_i(t)=x_{i+1}(t),\ (i=\overline{1,n-1}) \\ \dot{x}_n(t)=w(F,b,d_1)-b_0\sum_{i=1}^{n}k_i x_i(t) \end{array}\right\} t\ge t_0 \quad (13)$$
$$y(t)=x_1(t),\quad x_i(0)=0$$

Also, substituting Eq.(12) into Eq.(9), then

$$\left.\begin{aligned}&\dot{\hat{x}}_i(t)=\hat{x}_{i+1}(t)+\beta_i(y(t)-\hat{y}(t)),(i=\overline{1,n-1})\\&\dot{\hat{x}}_n(t)=\beta_n(y(t)-\hat{y}(t))-b_0\sum_{i=1}^{n}k_i x_i(t)\\&\dot{\hat{x}}_{n+j}(t)=\hat{x}_{n+j+1}(t)+\beta_{n+j}(y(t)-\hat{y}(t)),(j=\overline{1,m-1})\\&\dot{\hat{x}}_{n+m}(t)=\beta_{n+m}(y(t)-\hat{y}(t))\\&\hat{y}(t)=\hat{x}_1(t)\end{aligned}\right\}\ t\geq t_0 \tag{14}$$

Using the Laplace transformation of Eq.(13), $E(s)$ can be written as follows.

$$E(s)=\frac{\hat{X}_{n+1}(s)-W(s)}{\left(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1}\right)} \tag{15}$$

Where $E(s)$ is the Laplace transformation of $e(t)=r(t)-y(t)$, and $r(t)(r(t)=0)$ is the reference signal, and $\hat{X}_{n+1}(s)$ and $W(s)$ are the Laplace transformations of $X_{n+1}(t)$ and $W(F,b,d_1,t)$.

Let's denote the observer error as follows.

$$\hat{e}(t)=y(t)-\hat{y}(t)=r(t)-e(t)-\hat{y}(t) \tag{16}$$

Using the Laplace transformations of Eq.(14) and Eq.(16), the observer error can be written as follows.

$$\hat{E}(s)=\frac{-(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1})E(s)}{s^n+\sum_{i=1}^{n}\beta_i s^{n-i}} \tag{17}$$

Also, from Eq.(14) the extended state $\hat{X}_{n+1}(s)$ can be found as follows.

$$\hat{X}_{n+1}(s)=\frac{\left(\sum_{j=1}^{m}\beta_{n+j}s^{m-j}\right)}{s^m}\hat{E}(s) \tag{18}$$

Substituting Eq.(17) into Eq.(18), then

$$\hat{X}_{n+1}(s)=\frac{\left(\sum_{j=1}^{m}\beta_{n+j}s^{m-j}\right)}{s^m}\cdot\frac{-(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1})E(s)}{s^n+\sum_{i=1}^{n}\beta_i s^{n-i}} \tag{19}$$

Now substituting Eq.(19) into Eq.(15) and arranging it, then the error model of the linear active disturbance rejection control system using the m-*th* order extended state observer is found as follows.

$$E(s)=\frac{-s^m\left(s^n+\sum_{i=1}^{n}\beta_i s^{n-i}\right)W(s)}{s^m\left(s^n+\sum_{i=1}^{n}\beta_i s^{n-i}\right)\left(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1}\right)+\left(\sum_{j=1}^{m}\beta_{n+j}s^{m-j}\right)\left(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1}\right)} \tag{20}$$

Where $E(s)$ is the error on the disturbance $W(s)$.

From Eq.(20) the error transfer function from the lump disturbance to error is as follows.

$$\Phi(s)=\frac{s^m\cdot\left(s^n+\sum_{i=1}^{n}\beta_i s^{n-i}\right)}{s^m\left(s^n+\sum_{i=1}^{n}\beta_i s^{n-i}\right)\left(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1}\right)+\left(\sum_{j=1}^{m}\beta_{n+j}s^{m-j}\right)\left(s^n+b_0\sum_{i=1}^{n}k_i s^{i-1}\right)} \tag{21}$$

Where $\Phi(s)$ is the transfer function of the disturbance error.

In Eq.(21), the error coefficient is written as follows.

$$C_i=\frac{\partial}{\partial s}\Phi(s)\bigg|_{S=0},\ i=\overline{0,m} \tag{22}$$

Where C_i is the error coefficient and $\Phi(s)$ is the transfer function from disturbance to error.

The following results can be found from Eq.(21).

The canonical (n+m)*th* order linear active disturbance rejection control system becomes the system of the m-*th* order astatism with respect to the disturbance according to the extended state order m, thus the system can reduce the error to 'zero' theoretically on the (m-1)*th* order disturbance.

2.2 Harmonic Disturbance Rejection Method by the 2nd Order Extended State Observer with Frequency Estimation.

Let's consider the case of $d_1(t)=0$, that is, $d(t)=d_2(t)$ in Eq.(2).

Then Eq.(7) is written as follows.

$$x_{n+1}(t)=w(d_2,t)=A\sin\omega_d t=h_{d2}(t) \tag{23}$$

Let's differentiate the Eq.(23) twice continuously, then

$$\left.\begin{aligned}&h_{d2}(t)=A\sin\omega_d t=x_{n+1}\\&\dot{h}_{d2}(t)=A\omega_d\cos\omega_d t=x_{n+2}\\&\ddot{h}_{d2}(t)=-A\omega_d^2\sin\omega_d t=-\omega_d^2 h_{d2}(t)=x_{n+3}\\&h_{d2}^{(i)}(t)=-\omega_d^2 h_{d2}^{(i-2)},i=\overline{3,m}\end{aligned}\right\} \tag{24}$$

Substituting Eq.(24) into Eq.(6), then

$$\left.\begin{aligned}&\dot{x}_i(t)=x_{i+1}(t),(i=\overline{1,n-1})\\&\dot{x}_n(t)=x_{n+1}(t)+b_0u\\&\dot{x}_{n+i}(t)=x_{n+i+1},(j=\overline{1,m-1})\\&\dot{x}_{n+m}(t)=-\omega_d^2 x_{n+m-2}(t)\end{aligned}\right\} \tag{25}$$

$$y(t)=x_1(t)$$

Now let's denote the disturbance frequency to be estimated as $\theta=-\omega_d{}^2$, then Eq.(25) can be written as follows.

$$\left.\begin{aligned}&\dot{x}_i(t)=x_{i+1}(t),(i=\overline{1,n-1})\\&\dot{x}_n(t)=x_{n+1}(t)+b_0u\\&\dot{x}_{n+i}(t)=x_{n+i+1}(t),(j=\overline{1,m\text{-}1})\\&\dot{x}_{n+m}(t)=\theta\cdot x_{n+m-2}(t)\end{aligned}\right\}\quad y(t)=x_1(t) \tag{26}$$

Then the extended state observer for Eq.(26) is as follows.

$$\left.\begin{aligned}&\dot{\hat{x}}_i(t)=\hat{x}_{i+1}(t)+\beta_i(y(t)-\hat{y}(t)),(i=\overline{1,n-1})\\&\dot{\hat{x}}_n(t)=\hat{x}_{n+1}(t)+\beta_n(y(t)-\hat{y}(t))+b_0u(t)\\&\dot{\hat{x}}_{n+i}(t)=\hat{x}_{n+i+1}(t)+\beta_{n+i}(y(t)-\hat{y}(t)),(j=\overline{1,m-1})\\&\dot{\hat{x}}_{n+m}(t)=\hat{\theta}\cdot\hat{x}_{n+m\text{-}2}(t)+\beta_{n+m}(y(t)-\hat{y}(t))\end{aligned}\right\}\quad \hat{y}(t)=\hat{x}_1(t) \tag{27}$$

From Eq.(27) and Eq.(26) the error dynamics of the system can be found as follows.

$$\left.\begin{aligned}&\dot{x}_{ei}=x_{e(i+1)}-\beta_i x_{e1},\quad (i=\overline{1,n+m-1})\\&\dot{x}_{e(n+m)}=\theta\,x_{n+m-2}-\hat{\theta}\,\hat{x}_{n+m\text{-}2}-\beta_{n+m}x_{e1}\end{aligned}\right\} \tag{28}$$

Where $x_{ei}=\hat{x}_i-x_i$, and $\tilde{\theta}$ is the parameter estimation error, $(\tilde{\theta}=\hat{\theta}-\theta)$.

Adding and subtracting $\hat{\theta}\,x_{n+i-2}$ from the second term of Eq.(28), and considering $\tilde{\theta}=\hat{\theta}-\theta$, then the following can be found.

$$\left.\begin{aligned}&\dot{x}_{ei}=x_{e(i+1)}-\beta_i x_{e1},\qquad (i=\overline{1,n+m-1})\\&\dot{x}_{e(n+m)}=\hat{\theta}\,x_{e(n+m)}+\tilde{\theta}(\,\hat{x}_{n+m-2}-x_{e(n+m-2)}\,)-\beta_{n+m}x_{e1}\end{aligned}\right\} \tag{29}$$

Then Eq.(29) can be written as the following state model.

$$\dot{\tilde{X}}=(A_1+\hat{\theta}A_2-LC)\tilde{X}+\tilde{\theta}A_2(\hat{X}-\tilde{X}) \tag{30}$$

Where

$$\tilde{X}=(x_{e1}\ x_{e2}\ \cdots\ x_{e(n+m)})^T,$$

$$L=(\beta_1\ \beta_2\ \cdots\ \beta_{n+m})^T,$$

$$A_1=\begin{pmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\cdots&0&1\\0&0&\cdots&0&0\end{pmatrix}_{(n+m)\times(n+m)},$$

$$A_2=\begin{pmatrix}0&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&1&0\end{pmatrix}_{(n+m)\times(n+m)},$$

$$C=(1\ 0\ 0\cdots0)_{1\times(n+m)}$$

To determine the estimation law, let’s introduce the Lyapunov function as follows.

$$V = \frac{1}{2}\tilde{X}^T P\tilde{X} + \frac{1}{2}\tilde{\theta}^2 \quad (31)$$

Where P is the positive definite matrix, and λ is the weight coefficient.

Now let's derive the frequency estimation formula from the convergence condition of Eq.(31).

$$\begin{aligned} \dot{V} &= \tilde{X}^T P\dot{\tilde{X}} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= \tilde{X}^T(\overline{A}^T P + P\overline{A})\tilde{X} + \tilde{\theta}(X^T A_2^T P\tilde{X} + \tilde{X}^T PA_2 X + \frac{1}{\lambda}\dot{\tilde{\theta}}) \\ &= -\tilde{X}^T Q\tilde{X} + \tilde{\theta}(X^T A_2^T P\tilde{X} + \tilde{X}^T PA_2 X + \frac{1}{\lambda}\dot{\hat{\theta}}) \end{aligned} \quad (32)$$

Where $\overline{A} = A_1 + \hat{\theta}A_2 - LC$ and $X = \hat{X} - \tilde{X}$.

In Eq.(32) let's denote as

$$\overline{A}^T P + P\overline{A} = -Q \quad (33)$$

Now let's consider that there exists the positive definite matrix Q, and the positive definite matrix P is determined.

If we formulate as

$$X^T A_2^T P\tilde{X} + \tilde{X}^T PA_2 X + \frac{1}{\lambda}\dot{\tilde{\theta}} = 0 \quad (34)$$

then $\dot{V} \le 0$, that is, the extended state observer will be stable. Thus, from Eq.(34) the frequency estimation law can be written as follows.

$$\dot{\tilde{\theta}} = -\lambda(X^T A_2^T P\tilde{X} + \tilde{X}^T PA_2 X) \quad (35)$$

Denoting $\dot{\tilde{\theta}} = \dot{\hat{\theta}} - \dot{\theta}$ and assuming $\dot{\theta} \approx 0$ with respect to the change of time in Eq.(35), the following can be written.

$$\dot{\hat{\theta}} = -\lambda(X^T A_2^T P\tilde{X} + \tilde{X}^T PA_2 X) \quad (36)$$

Thus the following results is found from Eq.(36).

If $\dot{\hat{\theta}}$ is determined, the frequency can be estimated, therefore, the harmonic disturbance can be rejected effectively.

2.3 Active Disturbance Rejection Method by the m-th Order Extended State Observer with Frequency Estimation.

Substituting the frequency estimation terms of Eqs.(35) and (36) into Eqs.(26) and (27), the extended state model of the control plant and extended state observer model is written as follows.

$$\left.\begin{aligned} &\dot{x}_i(t) = x_{i+1}(t), (i = \overline{1, n-1}) \\ &\dot{x}_n(t) = x_{n+1}(t) + b_0 u \\ &\dot{x}_{n+j}(t) = \dot{x}_{n+j+1}(t) + \theta \cdot x_{n+i-2}(t), (j = \overline{1, m-1}) \\ &\dot{x}_{n+m}(t) = \theta \cdot x_{n+m-2}(t) + h_{d1}^{(m)}(t) \end{aligned}\right\} \quad (37)$$
$$y(t) = x_1(t)$$

$$\left.\begin{aligned}&\dot{\hat{x}}_i(t)=\hat{x}_{i+1}(t)+\beta_i(y(t)-\hat{y}(t)),(i=\overline{1,n-1})\\&\dot{\hat{x}}_n(t)=\hat{x}_{n+1}(t)+b_0u(t)+\beta_n(y(t)-\hat{y}(t))\\&\dot{\hat{x}}_{n+j}(t)=\hat{x}_{n+j+1}(t)+\beta_{n+j}(y(t)-\hat{y}(t)),(j=\overline{1,m-1})\\&\dot{\hat{x}}_{n+m}(t)=\hat{\theta}\cdot\hat{x}_{n+m\text{-}2}(t)+\beta_{n+m}(y(t)-\hat{y}(t))\end{aligned}\right\}\qquad(38)$$
$$\hat{y}(t)=\hat{x}_1(t)$$

Now if it is possible to estimate the frequency from Eq.(36) and the disturbance on control plant can be observed from the extended state observer, then Eq.(10) can be used for the law of active disturbance rejection control.

Finally the active disturbance rejection control system using the m-*th* order extended state observer can reject the aperiodic disturbance and the harmonic disturbance effectively by means of Eqs.(36), (37) and (38).

3. Stability of the (n+m)th Order Active Disturbance Rejection Control System with Frequency Estimation

The stability of the entire system can be investigated as follows.

Firstly, the control plant is linear, and the extended state observer is linear on the aperiodic disturbance, thus the stability of the entire system is dependent on the stability of the extended state observer.

From Eq.(8) and Eq.(9), the dynamic characteristic of the extended state observer is as follows.

$$\left.\begin{aligned}&\dot{x}_{e\,i}(t)=x_{e_i(i+1)}(t)-\beta_i x_{e1}(t),\ (i=\overline{1,n-1})\\&\dot{x}_{e\,n}(t)=\hat{x}_{e(n+1)}(t)-\beta_n x_{e1}(t)\\&\dot{x}_{e(n+j)}(t)=x_{e(n+j+1)}(t)-\beta_{n+j}x_{e1}(t),(j=\overline{1,m-1})\\&\dot{x}_{e(n+m)}(t)=h^{(m)}(t)\end{aligned}\right\}\qquad(39)$$
$$\hat{e}(t)=x_{e1}(t)$$

Where $x_{e\,i}(t)(x_{e\,i}(t)=x_i(t)+\hat{x}_i(t))$ is the state variable of the observer error.

The characteristic equation of Eq.(38) is as follows.

$$D_C(s)=s^{n+m}+\sum_{i=1}^{n+m}\beta_i s^{n+m-i}\qquad(40)$$

Where $D_C(s)$ is the characteristic equation of observer.

From the characteristic polynomial

$$D_{0C}(s)=(s+\omega_C)^{n+m}\qquad(41)$$

by which Eq.(40) satisfies the Hurwitz criterion, the gain coefficients of observer can be represented as follows.

$$\beta_i=\alpha_i\omega_C^i\qquad(42)$$

Where α_i is the binomial coefficient and ω_C is the natural angular frequency of observer.

Using Eq.(12) and Assumption 3, with respect to the aperiodic disturbance the stability analysis of the active disturbance rejection control system combined with the m-*th* order extended state observer can be found from [18].

Secondly, with respect to the periodic disturbance the stability of the active disturbance rejection control system using the m-*th* order extended state observer results in the convergence of the frequency estimation.

The convergence of the frequency estimator is related to the evaluation of Hurwitz criterion of Eq.(30).

In Eq.(30), when $\tilde{\theta} = 0$, let's denote the characteristic equation as follows.

$$D(s) = \left| sI - (A_1 + \hat{\theta}A_2 - LC) \right| = s^{n+m} + \beta_1 s^{n+m-1} + (\beta_2 - 1)s^{n+m-2} + \\ + (\beta_3 - \beta_1)s^{n+m-2} + \cdots + (\beta_{n+m-1} - \beta_{n+m-3})s + \beta_{n+m} \qquad (43)$$

The gain coefficients of observer determined by Eq.(42) satisfy the following condition.

$$\beta_1 < \beta_2 < \cdots \beta_i \cdots < \beta_{n+m} \qquad (44)$$

Thus Eq.(30) satisfies the Hurwitz criterion from Eq.(44).

Finally the (n+m)*th* active disturbance rejection control system with frequency estimation is stable.

4. Simulation Results

4.1 Feature Simulation of the Disturbance Error according to the Extended Order

To simulate and analyze let's make the model of control plant as follows.

$$\left.\begin{aligned} \dot{x}_1 &= u + d \\ y &= x_1 \end{aligned}\right\} \qquad (45)$$

From Eq.(2) the disturbance d includes the constant, ramp, parabolic and harmonic disturbances.

$$\begin{aligned} d &= d_1 + d_2 \\ d_1 &= f_0 + f_1 t + f_2 t^2 \\ d_2 &= A\sin(\omega_d t) \end{aligned} \qquad (46)$$

Taking $x_2 = d$ and $m = 3$, the m-*th* order extended state model is as follows.

$$\left.\begin{aligned} \dot{x}_1 &= u + x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= d^{(3)} \\ y &= x_1 \end{aligned}\right\} \qquad (47)$$

Then the extended state observer is as follows.

$$\left.\begin{aligned}\dot{\hat{x}}_1 &= u + \hat{x}_2 + \beta_1(y - \hat{y}) \\ \dot{\hat{x}}_2 &= \hat{x}_3 - \beta_2(y - \hat{y}) \\ \dot{\hat{x}}_3 &= \hat{x}_4 - \beta_3(y - \hat{y}) \\ \dot{\hat{x}}_4 &= \beta_4(y - \hat{y}) \\ \hat{y} &= \hat{x}_1\end{aligned}\right\} \tag{48}$$

Using the proposed design method the active disturbance rejection control law and the regulator's control law are designed as follows.

$$\left.\begin{aligned}u &= -x_2 + u_0 \\ u_0 &= k_1(r - y) + k_2\dot{r}\end{aligned}\right\} \tag{49}$$

Where r is the reference signal, and k_1 and k_2 are the transfer coefficients of the regulator, that is, $k_1 = 100$, $k_2 = 1$.

According to the extended state order the values of β_i are as follows when $\omega_C = 50\ 1/s$.

$$\beta_1 = 4\times 50, \beta_2 = 6\times 50^2, \beta_3 = 4\times 50^3, \beta_4 = 50^4 \tag{50}$$

Fig 1 shows the simulation block diagram of the system.

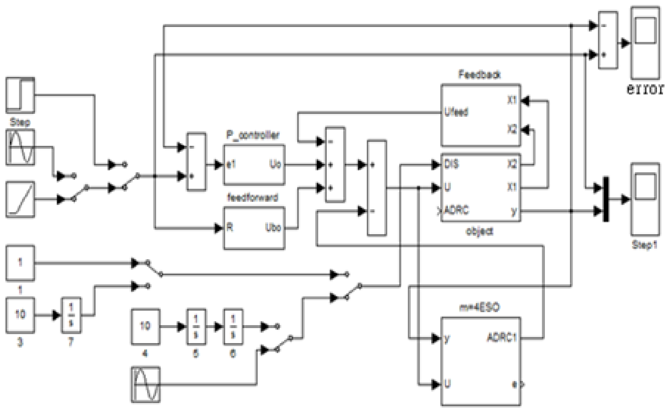

Figure 1. Block diagram of the system designed by the proposed method

From Eq.(46) the simulation results with respect to the aperiodic disturbance $f_0 = 1$, $f_1 = 10\ 1/s$ and $f_2 = 10\ 1/s^2$ are as follows.

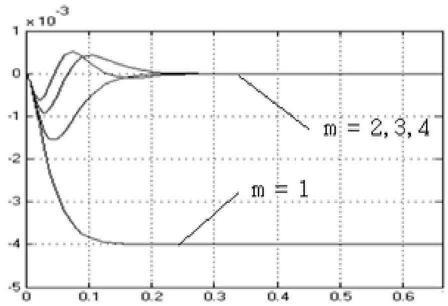

Figure 2. Rejection features on the constant ramp disturbance according to m

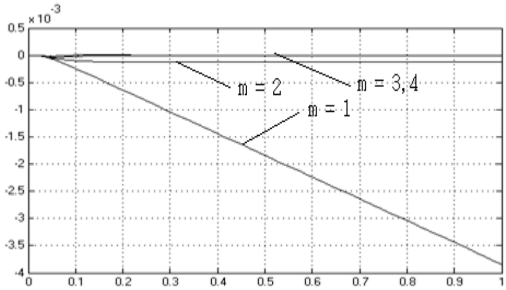

Figure 3. Rejection features on the constant parabolic disturbance according to m

Fig 2 shows that the 1st order extended state observer becomes the system of the 1st order astatism with respect to the disturbance while Fig 3 shows that the 2nd order extended state observer becomes the one of the 2nd order astatism with respect to the disturbance.

4.2 Feature Simulation of the Harmonic Disturbance Error according to the Extended Order.

From Eq.(36) the frequency estimation law is as follows when $n=1$, $m=3$ and $\boldsymbol{P}=\boldsymbol{I}$.

$$\begin{aligned}\dot{\hat{\theta}} &= -\lambda(X^T A_2^T P\tilde{X} + \tilde{X}^T P A_2 X) \\ &= -\lambda x_3 \tilde{x}_4 = -\lambda(\hat{x}_3 - \tilde{x}_3)\tilde{x}_4\end{aligned} \tag{51}$$

1) For the single-frequency harmonic disturbance

In the simulation the disturbance is assumed to be $d_2 = 5\sin(30t)$.

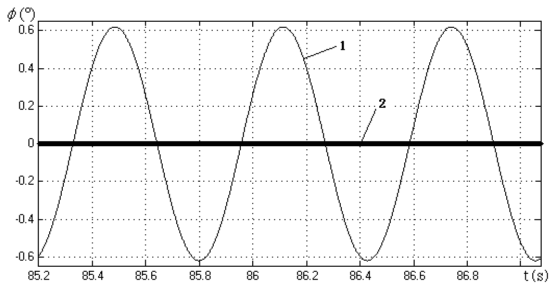

Figure 4. Error with respect to the single-frequency harmonic disturbance (1-error when the frequency is not estimated, 2-error when the frequency is estimated)

Fig 4 shows that the disturbance rejection performance is very high when the single-frequency harmonic disturbance is acting, that is, the error is 0.64°($11.717 mrad$) when the frequency is not estimated, but the error is 0.0001°($1.745\times10^{-3} mrad$) when the frequency is estimated.

2) For the multifrequency harmonic disturbance

In the simulation the disturbance is assumed to be $d_2 = 1\sin(10t) + 2\sin(20t)$.

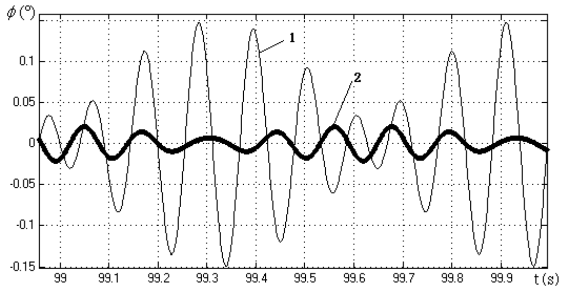

Figure 5. Error with respect to the multifrequency harmonic disturbance (1-error when the frequency is not estimated, 2-error when the frequency is estimated)

Fig 5 shows that the disturbance rejection performance is very high when the multifrequency harmonic disturbance is acting, that is, the error is 0.14°($2.4mrad$) when the frequency is not estimated, but the error is 0.02°($0.34mrad$) when the frequency is estimated, therefore the performance is high up to about 7 times.

5. Conclusion

This paper proposes a method to improve the disturbance rejection performance of the active disturbance rejection control system using the m-*th* order extended state observer with frequency estimation in order to guarantee the high-precision tracking performance.

The following conclusion has been reached from the proposed method.

Firstly, it was shown that the canonical active disturbance rejection control system could reject the constant, ramp and parabolic disturbances perfectly according to the order m in case of using the m-*th* order extended state observer for the aperiodic disturbance.

Secondly, it was shown that the single-frequency harmonic disturbance could be rejected nearly perfectly and the disturbance rejection performance is considerably improved on the multifrequency harmonic disturbance in the active disturbance rejection control system using the extended state observer with frequency estimation.

Thus it was verified that the disturbance rejection capability of the active disturbance rejection control system could be improved greatly by using the m-*th* order extended state observer with frequency estimation.

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