MATHEMATICS OF SKATEBOARD QUARTER PIPE CONSTRUCTION

THOMAS GÜNTHER

MCG Bönen, Billy-Montigny-Platz 5, 59199 Bönen, Germany
E-mail adress: dr.thomas.guenther@gmail.com

Published in International Journal of Engineering and Applied Physics - Vol. 4, no. 3, pp. 1041–1054, 2024.

Available at: https://ijeap.org/ijeap/article/view/202

Abstract. The present article provides a detailed mathematical treatise on the geometry of skateboard transition ramps. These usually form a circular segment shape and are used in skateboarding for transitioning from a horizontal plane to another angle of incline. Transitions play a crucial role in the flow of skateparks and are required, for example, for quarter pipes, mini ramps, vert ramps, or jump ramps. Skateboarding has been an Olympic sport since 2023, yet many publicly funded skateparks still do not meet the demands of the sport. On one hand, there is often a lack of willingness from authorities to engage with the athletes beforehand; on the other hand, the people involved in the planning process often lack the necessary experience and mathematical expertise to perfectly fulfill the users' needs. Thoughtful planning is particularly crucial for ramps with curved surfaces to ensure the flow of the skatepark. In addition to the mathematical analysis, source codes for programs are provided to make the calculations as convenient as possible for all users.

1. Introduction

Already 20 years ago, it was noted in [7]: "Skateboarding is rapidly developing into the major informal sporting activity undertaken by young people [...]". As Borden writes in his book [10], "Skateboarders are an increasingly common feature of the urban environment - recent estimates total 40 million worldwide." Since "Skateboarders [...] make use of the urban fabric for their own activities", Woolley and Jones investigated in 2010 "Why they use particular locations within a city centre [...]", see [8]. In recent years, skateboarding has evolved into a mainstream sport. "In July 2021, before the Olympic games started in Tokyo, the President of World Skate Sabatino Aracu launched the Skateboarding Street and Park series to move onwards from the Olympic games into a new era of competitive skateboarding. [...] This signals that organized international skateboarding is developing into a more competitive phase.", cf. [9]. Due to this long-term development of skateboarding, there are now various studies that examine the cultural evolution of skateboarding, skateboarding in urban spaces or the integration of skateparks into urban spaces, [11, 14, 15, 16, 12]. Further there exists researche that "explore[s] how the sport has operationally evolved and how, as a major youth sport, Olympic inclusion has impacted on its organisational arrangements" [18]. And there are articles that address the pedagogical and social science aspects of skateboarding, see for example [19, 20].

But in scientific publications about skateboarding and skateparks, more detailed mathematical treatises on the geometry of skatepark elements, specifically the so-called skateboard ramps, are missing. As mentioned by Butz and Peters in 2018: "Skateboarding is not immediately associated with university research projects", cf. [13]. Even Whitleys "Public Skatepark Development Guide" [17] contains no mathematical treatises on ramp geometry. Indeed, there are many experience-based guides on the internet for constructing skateboard quarter pipes, but I have not found a comprehensive mathematical exposition on their geometry. For instance, the selection of the radius in the published guides is based on empirical values or left to visual estimation. Undoubtedly, the choice of a height and radius primarily depends on personal preferences. However, especially for non-professional ramp builders, it can be challenging to construct the perfect ramp for them. Visual estimation can be deceiving, and prefabricated dimensions may not necessarily align with individual preferences. The geometry of a quarter pipe significantly influences skating performance, particularly through a meaningful combination of height and radius. These two parameters determine the steepness angle at the highest point of the quarter pipe. To design a quarter pipe that aligns with personal preferences, it's advisable to be able to compare this steepness angle with other ramps. This article delves into the mathematical relationships of the critical parameters and also presents a method for determining them for comparison purposes with existing ramps.

Previous research in skatepark construction has primarily focused on pedagogical aspects related to student motivation, see for example [5], or has emphasized technical construction. In the realm of online resources, one can encounter works such as the bachelor thesis [2], which is concerned with the construction of a concrete skatepark. Although this work partly presents measurements of the examined concrete skatepark, it does provide a more profound mathematical examination of the geometry of the ramps. Same holds for [4], a master thesis in the field of landscape architecture. The latter work, which was also publised as a book, gives plenty information about skateparks and background information. Regarding the construction of transition radii, Poirer writes in [4] that "*Transitions are historically some of the most difficult features to design and build correctly for a number of reasons.*" Nevertheless, this work only deals with the topic on a very superficial level. While some recommendations or guidelines for radii are provided, a mathematical analysis is entirely missing. In my view, the most suitable information for constructing quarter pipes is provided by a YouTube video series, cf. [3]. However, this series also does not cover mathematical fundamentals but rather relies on experiential knowledge.

2. Research method

These investigations involve theoretical considerations conducted according to the usual scientific methods in the fields of mathematics and physics. The calculations and formulas are accompanied by algorithms written in the programming language Python and for the computer algebra system wxMaxima. These algorithms are accessible on GitHub, with corresponding links provided in the text. Error calculations follow the usual modeling based on Gauss's error propagation law.

3. Results and discussion

3.1. Transitions that taper down to zero degrees towards the ground.

The most important case describes transitions that exhibit an angle of zero degrees at the bottom to enable a continuously differentiable, thus seamless and kink-free transition to the flat ground. In order to mathematically describe the situation in this paper, a coordinate system is choosen as depicted in Figure 3.1. In this context, H represents the height, R the radius, and T the length or depth of the transition. As will be shown later, the angle ϕ, see Figure 3.1, also corresponds to the maximum slope angle at the highest point of the ramp to the horizontal. The quarter pipe tapering down to zero degrees is geometrically defined by the three parameters: radius R, height H, and depth T of the transition. Two of these parameters can be chosen within a reasonable range, while the third can be calculated accordingly. Excluding oververt ramps, it is obvious that $R \geq T \geq H$. In our choice of coordinate system, see Figure 3.1, the transition is part of the lower semicircle with radius R around the center $(0 \mid R)$. The lower semicircle is given by

$$y = -\sqrt{R^2 - x^2} + R \tag{3.1}$$

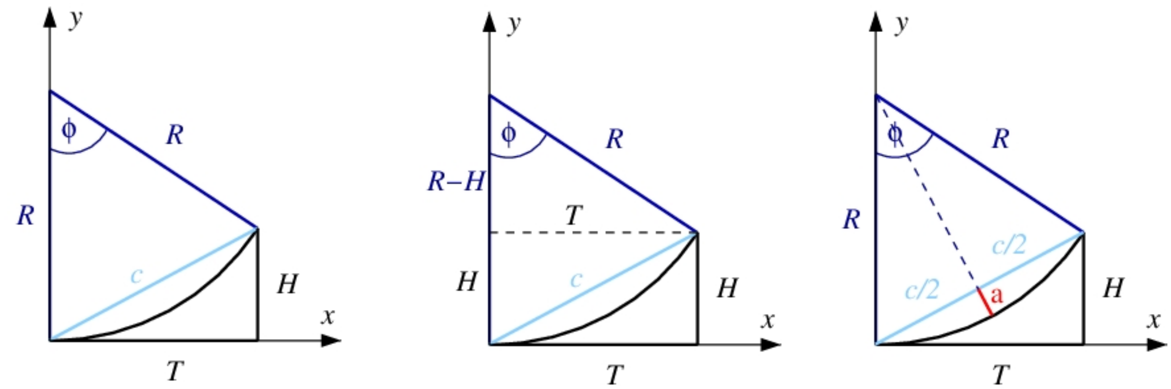

FIGURE 3.1. Transition that taper down to zero degrees. The three subfigures are referenced from left to right as a, b, and c.

Based on the assumption that the height and radius are predetermined, the depth of the transition tapering down to zero degrees can be derived from (3.1) as follows: From $H = -\sqrt{R^2 - T^2} + R$ one gets $(R - H)^2 = R^2 - T^2$ and thus

$$T = \sqrt{2RH - H^2}. \tag{3.2}$$

Certainly, this equation can also be solved for R, or as outlined above, for H. However, it seems rarely practical in practice to specify both the height and depth of the ramp and calculate the radius through $R = \frac{1}{2}\left(T^2 H^{-1} - H\right)$.

3.2. Slope angle and its maximum.

The incline angle at the highest point of the ramp decisively influences how the ramp behaves when skating on it. Therefore, it is advisable to calculate this angle during the planning of the quarter pipe and, as appropriate, compare it with the maximum incline angle of other ramps. This article also covers instructions on how to determine this angle from measurable quantities in existing ramps, see section 3.5.

Lemma1 demonstrates that the maximum incline angle ϕ_{max} coincides with the central angle ϕ depicted in Figure 3.1:

Lemma 1. *Maximum slope angle*
The maximum incline angle ϕ_{max} at the highest poit of the transition corresponds to the opening angle of the circular segment in Figure 3.1.

Proof. From equation (3.2) we recieve $\sqrt{R^2 - T^2} = \sqrt{R^2 - (2RH - H^2)} = R - H$. The maximum slope angle ϕ_{max} can be easily calculated through the derivative of the function (3.1), it is

$$\phi_{max} = \arctan\left(\frac{dy}{dx}\,|_{x=T}\right) = \arctan\left(\frac{T}{\sqrt{R^2 - T^2}}\right) = \arctan\left(\frac{T}{R - H}\right). \tag{3.3}$$

On the other hand, the trigonometric functions applied to the right-angled triangle in Figure 3.1b yield

$$\tan\phi = \frac{T}{R - H}. \tag{3.4}$$

Thus, it is demonstrated that $\phi = \phi_{max}$ holds true. □

We can combine equations (3.2) and (3.3) to determine the maximum slope angle ϕ_{max} from given radius and height:

$$\phi_{max} = \arctan\left(\frac{\sqrt{2RH - H^2}}{R - H}\right) \tag{3.5}$$

Another application of this formula is to calculate the slope angle at any given height. This is useful for constructing a smoothly differentiable transition to another straight ramp section, such as a so-called bank. This was used, for example, in the extension in Figure 3.7 on the right side. If h denotes the variable height, the function for the slope angle reads

$$\phi(h) = \arctan\left(\frac{\sqrt{2Rh - h^2}}{R - h}\right)$$

where R is the constant radius of the transition.

3.3. **Radius in dependence of given maximum slope angle and height.**
Sometimes it can be advantageous to specify the height and angle and calculate the remaining dependent parameters of a transition. Obviously, again from (3.2) and (3.3) or directly from (3.5) we get

$$\tan^2\phi = \frac{T^2}{(R-H)^2} = \frac{2RH - H^2}{(R-H)^2}$$

where $\phi = \phi_{max}$. From this one receives

$$0 = R^2 - 2RH + H^2 - \frac{2RH - H^2}{\tan^2\phi} = R^2 - 2RH\left(1 + \frac{1}{\tan^2\phi}\right) + H^2\left(1 + \frac{1}{\tan^2\phi}\right).$$

Since $1 + \tan^{-2}\phi = \sin^{-2}\phi$ the latter equation transformes into

$$R^2 - \frac{2H}{\sin^2\phi}R + \frac{H^2}{\sin^2\phi} = 0$$

Together with the identity $\sqrt{\sin^{-2}\phi - 1} = \cot\phi$ the solutions read:

$$R = \frac{H}{\sin^2\phi} \pm \sqrt{\frac{H^2}{\sin^4\phi} - \frac{H^2}{\sin^2\phi}} = \frac{H}{\sin^2\phi}\left(1 \pm \sqrt{\frac{1}{\sin^2\phi} - 1}\right) = \frac{1 \pm \cot\phi}{\sin^2\phi}H \tag{3.6}$$

Here, it remains to decide which sign models the real situation. Assuming a fixed height, a larger maximum incline angle must correspond to a smaller radius. Thus, we are seeking a monotonically decreasing function $R(\phi)$. Figure 3.2 shows the plot of both branches of the angle portion:

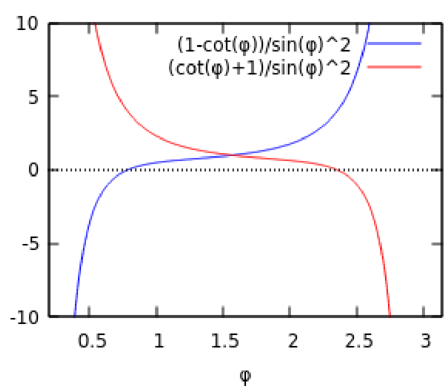

FIGURE 3.2. The function $f(\varphi) = (1 \pm \cot\varphi)\sin^{-2}\varphi$ represents the angle portion of (3.6).

Therefore, along with lemma 1, the radius as a function of height H and the maximum incline angle ϕ_{max} at the highest point of the quarter pipe can be calculated from

$$R(\phi, H) = \frac{1 + \cot \phi_{max}}{\sin^2 \phi_{max}} H. \tag{3.7}$$

3.4. Arc length of the transition.

When building a quarter pipe or mini ramp, it is helpful to be able to calculate the arc length of the transition, for example, to determine the amount of surfacing material needed in advance. The opening angle of the circular segment is given by (3.4), see lemma 1. Using equation (3.2), one receives

$$\phi = \arctan\left(\frac{T}{R-H}\right) = \arctan\left(\frac{\sqrt{2RH - H^2}}{R-H}\right). \tag{3.8}$$

Another way to calculate the opening angle arises from Figure 3.1b:

$$\phi = \arcsin\left(\frac{T}{R}\right) = \arcsin\left(\frac{\sqrt{2RH - H^2}}{R}\right) \tag{3.9}$$

The arc length L_{arc} of a circular segment with radius R and central angle ϕ can be calculated using polar coordinates $x = R\cos t$, $y = R\sin t$. Together with (3.9) one gets

$$L_{arc} = \int_0^{\phi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = R\phi = R \arcsin\left(\frac{\sqrt{2RH - H^2}}{R}\right) \tag{3.10}$$

see for example [1]. Analogously we receive $L_{arc} = R \arctan\left[\sqrt{2RH - H^2}\,(R-H)^{-1}\right]$ with (3.8). The angle ϕ is given in radians, so for the full angle of 2π, the circumference of the circle is $2\pi R$. This can be easily transformed by the connection $180\phi_{RAD} = \pi\phi_{DEG}$ of angle ϕ given in degree (ϕ_{DEG}), and angle ϕ given in radians (ϕ_{RAD}). With this, for example, the required surface area for a minramp[1] with height H and radius R, flat length L_f, and table length L_t can be calculated. Assuming the miniramp has width W, then the surface area is

$$S = W\left[L_f + 2L_t + 2L_{arc}\right] = W\left[L_f + 2L_t + 2R \arcsin\left(\frac{\sqrt{2RH - H^2}}{R}\right)\right]. \tag{3.11}$$

The above formula applies to ramps without verticals, but it can be easily adjusted by adding the term $2L_v$ inside the parentheses, where L_v represents the length of the vertical section.

Program codes implemented in wxmaxima [6] and Python, for the calculation of the depth or length, the curve length, and the maximum slope angle of a transition from given height and radius can be found here: https://github.com/tguent/math-quarter-pipe. Also, a Python code for determining the radius from a given height and maximum slope angle can be found here.

3.5. Determination of the transition radius through measurements on secants.

To achieve the desired geometry for a quarter pipe, it's useful to know the data from existing ramps to compare slope angles and radii. If blueprints are not available or do not exist, the radius must be determined through measurement. The simplest way to do this is by measuring the length of any secant and the distance from the secant's center to the quarter pipe. In Figure 3.1c, a specific secant is depicted. Note that any other secant within the transition can be chosen, although the length c should be chosen sufficiently large due to error propagation. From $R^2 = (c/2)^2 + (R-a)^2$ we get

$$R(a, c) = \frac{c^2}{8a} + \frac{a}{2}. \tag{3.12}$$

In practice, a plank can be placed inside the quarter pipe, from which one can determine the distance to the transition center.

[1] A typ of small halfpipe, mostly without vertical section.

3.6. Gaussian error propagation for the radius.

The measurements of the length of this plank as well as the distance from its center to the quarter pipe are subject to measurement errors. If we denote by Δa the measurement uncertainty of the length a, and by Δc the measurement uncertainty of the distance measurement for c, then the uncertainty ΔR for the value of the radius R can be estimated using the Gaussian error propagation law:

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial a}\Delta a\right)^2 + \left(\frac{\partial R}{\partial c}\Delta c\right)^2}$$

From (3.12) we get $\frac{\partial R}{\partial a} = \frac{1}{2} - \frac{c^2}{8a^2}$ and $\frac{\partial R}{\partial c} = \frac{c}{4a}$ it applies

$$\Delta R = \sqrt{\left(\frac{1}{2} - \frac{c^2}{8a^2}\right)^2 \Delta a^2 + \frac{c^2}{16a^2}\Delta c^2} \tag{3.13}$$

A Python / wxmaxima code for a program that determines the radius and its error is provided here: https://github.com/tguent/math-quarter-pipe.

3.7. The order of magnitude of the measurement error in practice.

The radius of a quarter pipe is often in the range of 2 to 2.5 meters. For measurement purposes, a plank with a length around 1.5 meters should be used to account for measurement inaccuracies. Let's assume that the length of the secant, which is the length of the plank, can be determined with the same accuracy as the distance from the secant's center to the transition. In this case, we assume that the measurement uncertainty for length measurements is given by $\Delta l = \Delta a = \Delta c$. Equation (3.13) reduces to

$$\Delta R = \left[\left(\frac{1}{2} - \frac{c^2}{8a^2}\right)^2 + \frac{c^2}{16a^2}\right]^{\frac{1}{2}} \Delta l = \left[\frac{1}{4} - \frac{c^2}{8a^2} + \frac{c^4}{64a^4}\right]^{\frac{1}{2}} \Delta l$$

Let's say c is λ times longer than a, so $\lambda = c/a$, then this equation becomes

$$\Delta R = \left[\frac{1}{4} - \frac{\lambda^2}{8} + \left(\frac{\lambda^2}{8}\right)^2\right]^{\frac{1}{2}} \Delta l = \left[1 - \frac{8}{\lambda^2} + \frac{16}{\lambda^4}\right]^{\frac{1}{2}} \frac{\lambda^2}{8}\Delta l.$$

In practice, $c \gg a$, let's say c is for example about ten times as large as a. In zeroth-order approximation, the error for the radius can be estimated as follows:

$$\Delta R = \frac{\lambda^2}{8}\Delta l + \mathcal{O}\left(\lambda^{-2}\right) \approx \frac{\lambda^2}{8}\Delta l$$

For our example of $\lambda = 10$, this means that the uncertainty ΔR in determining the radius R is more than ten times (here $12,5$) as large as the measurement error Δl in length measurement. If you measure with an accuracy of one centimeter, the uncertainty in the radius is already over a decimeter. That illustrates the plank must have sufficient length, and the measurement must be carried out very accurately to obtain useful data.

3.8. Quarter pipe with incline at the bottom towards the horizontal.

Quarter pipes, especially those constructed as prefabricated elements or retrofitted, often feature a slight incline towards the horizontal at the bottom. This design choice is made to prevent the transition from reaching a zero-degree angle at the base, which would otherwise create a sharp edge where the surface meets the ground. By incorporating a gentle angle, such as that formed by a transition plate, with the horizontal, a smoother configuration is achieved when traversing the edge. Let us choose the coordinate system such that the origin marks the lowest part of the ramp in the cross-section of the quarter pipe. The highest point then lies at $(T \mid H)$, see figure 3.3.

The following theoretical derivation of the circular function for the transition radius can be directly applied to draw the transition on a rectangular plate with dimensions T and H. The vertices of the plate in the

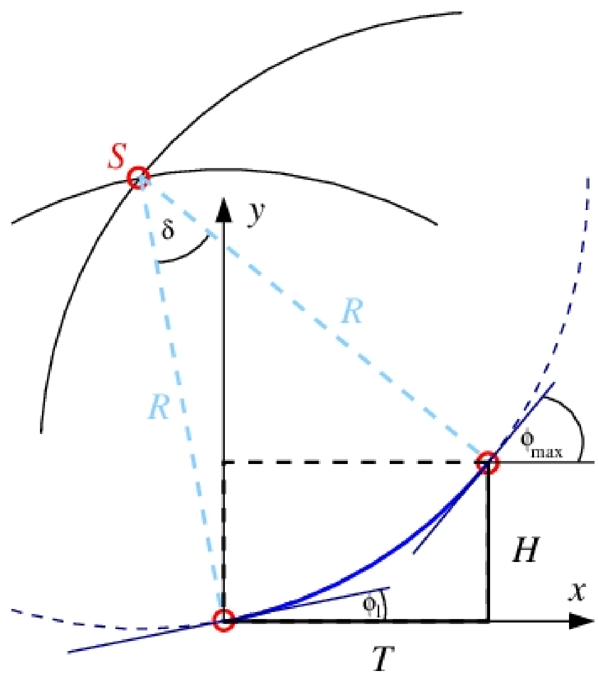

FIGURE 3.3. Quarter pipe with incline at the bottom towards the horizontal.

chosen coordinate system are located at $(0 \mid 0)$, $(T \mid 0)$, $(T \mid H)$ and $(0 \mid H)$. We draw a circle with radius R around the origin $(0 \mid 0)$ and another with the same radius around the center $(T \mid H)$. These two circles are described by the equations:

$$K_1 : \; x^2 + y^2 = R^2 \tag{3.14}$$

$$K_2 : \; (x - T)^2 + (y - H)^2 = R^2 \tag{3.15}$$

One of the intersection points of these circles forms the center of the transition circle. Setting 3.14 equal to 3.15 yields

$$x^2 + y^2 = x^2 - 2xT + T^2 + y^2 - 2yH + H^2$$

and therewith

$$y = -\frac{T}{H}x + \frac{T^2 + H^2}{2H} \tag{3.16}$$

Substituting this into equation 3.14, we obtain

$$\begin{aligned} R^2 = x^2 + \left(-\frac{T}{H}x + \frac{T^2 + H^2}{2H}\right)^2 &= \left(1 + \frac{T^2}{H^2}\right)x^2 - \frac{T^2 + H^2}{H^2}Tx + \frac{\left(T^2 + H^2\right)^2}{4H^2} \\ &= \frac{T^2 + H^2}{H^2}x^2 - \frac{T^2 + H^2}{H^2}Tx + \frac{\left(T^2 + H^2\right)^2}{4H^2}. \end{aligned}$$

Dividing by $\left(T^2 + H^2\right)H^{-2}$ leads to the follwing quadratic equation:

$$x^2 - Tx + \frac{T^2 + H^2}{4} - \frac{R^2H^2}{T^2 + H^2} = 0$$

Its solution is given by

$$x = \frac{T}{2} \pm \sqrt{\frac{T^2}{4} - \frac{T^2 + H^2}{4} + \frac{R^2H^2}{T^2 + H^2}} = \frac{T}{2} \pm \sqrt{\frac{R^2H^2}{T^2 + H^2} - \frac{H^2}{4}}.$$

As can be easily seen from Figure 3.3, we are interested in the negative branch of the solution. The x-coordinate of the center of the transition circle is therefore

$$x_s = \frac{T}{2} - H\sqrt{\frac{R^2}{T^2+H^2} - \frac{1}{4}}. \tag{3.17}$$

The y-coordinate is obtained by substituting (3.17) into equation (3.16):

$$y_s = -\frac{T}{H}x_s + \frac{T^2+H^2}{2H} = -\frac{T}{H}\left(\frac{T}{2} - H\sqrt{\frac{R^2}{T^2+H^2} - \frac{1}{4}}\right) + \frac{T^2+H^2}{2H} = \frac{H}{2} + T\sqrt{\frac{R^2}{T^2+H^2} - \frac{1}{4}}$$

In the chosen coordinate system, see Figure 3.3, the center of the transition circle is located at:

$$(x_s \mid y_s) = \left(\frac{T}{2} - H\sqrt{\frac{R^2}{T^2+H^2} - \frac{1}{4}} \;\middle|\; \frac{H}{2} + T\sqrt{\frac{R^2}{T^2+H^2} - \frac{1}{4}}\right) \tag{3.18}$$

From the equation of the circle

$$K: \, (x - x_s)^2 + (y - y_s)^2 = R^2, \tag{3.19}$$

we obtain the function $y = Q(x)$, which describes the transition circle

$$Q(x) = y_s - \sqrt{R^2 - (x - x_s)^2} \tag{3.20}$$

together with (3.18). It is evident again from Figure 3.3 that the negative branch of the solutions must be taken, as indicated by the blue circular segment line describing the transition circle.

3.9. **The upper and lower angles of the quarter pipe.**

From (3.20), we obtain the slope function

$$\frac{dQ}{dx} = \frac{x - x_s}{\sqrt{R^2 - (x - x_s)^2}}$$

and therewith the angle

$$\phi(x) = \arctan\left|\frac{x - x_s}{\sqrt{R^2 - (x - x_s)^2}}\right|.$$

The lower angle ϕ_l and the maximum slope angle ϕ_{max} of the quarter pipe are given by:

$$\phi_l = \phi(0) = \arctan\left|\frac{-x_s}{\sqrt{R^2 - x_s^2}}\right|, \quad \phi_{max} = \phi(T) = \arctan\left|\frac{T - x_s}{\sqrt{R^2 - (T - x_s)^2}}\right| \tag{3.21}$$

3.10. **Determination of the radius.**

When constructing a quarter pipe that does not taper to zero degrees at the bottom, it is helpful to calculate the radius based on the given angles at the top and bottom, as well as the height. To do this, a nonlinear system of equations can be set up and solved numerically using the equations (3.21) and the following geometric considerations.

From Figure 3.4 we infer that $\alpha = 180° - (\phi_{max} - \phi_l)$. Consider the quadrilateral with interior angles δ, α, and two $90°$ angles. Since the sum of the angles must be 360 degrees, it follows that $\alpha + \delta = 180°$. From this we receive

$$\delta = 180° - \alpha = \phi_{max} - \phi_l \tag{3.22}$$

The opening angle delta of the circular segment can be determined using the trigonometric functions. But in this case, R and T are not orthogonal to each other. Thus we can not use (3.8) or (3.9). Consider Figure

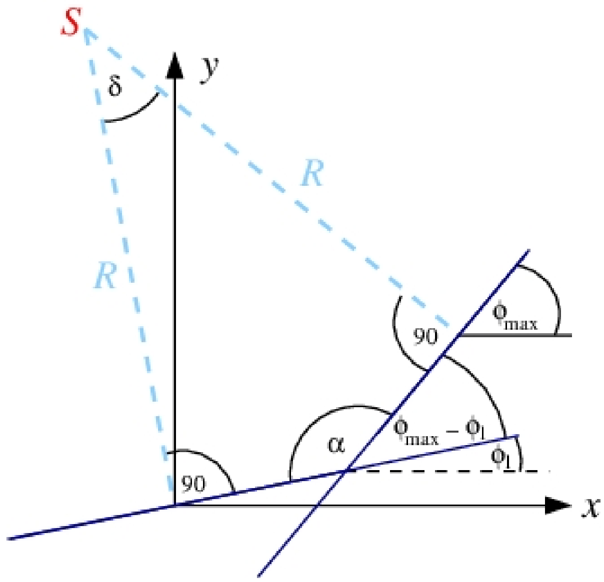

FIGURE 3.4. Quarter pipe with incline at the bottom towards the horizontal.

3.3 and draw a connecting line between the points $(0 \mid 0)$ and $(T \mid H)$. Let c be the length of this line, it holds $c = \sqrt{H^2 + T^2}$. Analogous to Figure 3.1c, the angle δ can now be determined by

$$\sin\left(\frac{\delta}{2}\right) = \frac{c}{2R} = \frac{\sqrt{H^2+T^2}}{2R}. \tag{3.23}$$

With (3.22) and (3.21) we receive

$$2R\sin\left(\frac{\phi_{max}-\phi_l}{2}\right) = \sqrt{H^2+T^2}\,,\ \tan(\phi_l) = \frac{-x_s}{\sqrt{R^2-x_s^2}}\,,\ \tan(\phi_{max}) = \frac{T-x_s}{\sqrt{R^2-(T-x_s)^2}}$$

where x_s is given by (3.17). These equations form the following system of nonlinear equations:

$$2R\sin\left(\frac{\phi_{max}-\phi_l}{2}\right) - \sqrt{H^2+T^2} = 0 \tag{3.24}$$

$$\sqrt{R^2-x_s^2}\cdot\tan(\phi_l) + x_s = 0 \tag{3.25}$$

$$\sqrt{R^2-(T-x_s)^2}\cdot\tan(\phi_{max}) + x_s - T = 0 \tag{3.26}$$

The set of (3.24) and (3.25) was used in a wxmaxima [6] code, see https://github.com/tguent/math-quarter-pipe, for a program that calulates the Radius R and the depth T. The input parameters are both angles ϕ_l and ϕ_{max} as well as the height of the transition.

3.11. **The arc length.**

For example, for precise surface calculations, it is necessary to know the curve length of the quarter pipe. We have already performed such calculations in section 3.4. However, the formula used for the curve length there cannot be applied here. In this case, the arc length δR, cf. (3.10), can be calculated using equation (3.23):

$$L_{arc} = (\phi_{max}-\phi_l)\,R = \delta R = 2R\arcsin\left(\frac{\sqrt{H^2+T^2}}{2R}\right)$$

A quarter pipe with an angle to the horizontal should only be used when a transition to zero degrees at the bottom is not possible, for example, when a ramp plate has to be used for retrofitted quarter pipes.

This geometry is not used for pipes. The surface area of a single quarter pipe, in comparison to (3.11), here consists only of

$$S = W\left[L_t + L_{arc}\right] = W\left[L_t + 2R\arcsin\left(\frac{\sqrt{H^2+T^2}}{2R}\right)\right].$$

Again, L_t represents the table length and W the width of the quarter pipe.

3.12. Applications of the theory.

At the end of this article, a brief insight will be provided on how the preceding considerations can be concretely applied in ramp construction.

Visualizing quarter pipe geometry with LibreCAD.

In order to have a model for visualization prior to construction, let's briefly discuss how to create the side view of a quarter pipe using LibreCAD[2]. Here, the modeling in LibreCAD is done according to the choice of the coordinate system in Figure 3.2. At first choose and calculate the parameters radius R, height H, depth T and the maximum slope angle ϕ_{max} of the transition (cf. https://github.com/tguent/math-quarter-pipe).

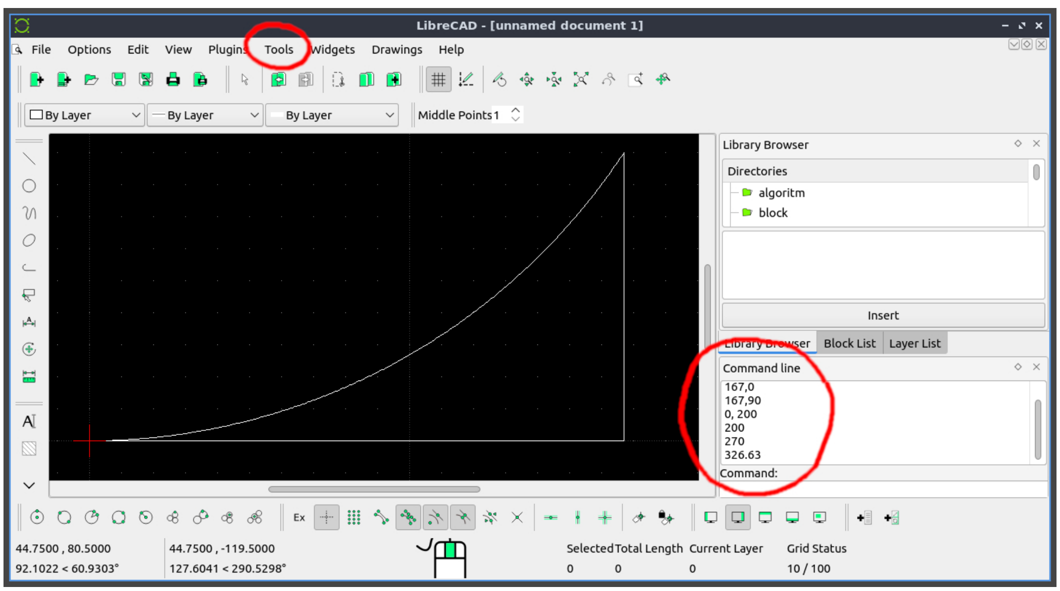

FIGURE 3.5. Visualizing quarter pipe geometry with LibreCAD

In LibreCAD choose 'Line' and '2 points' from the 'Tools' menu. Use the command line for the input $(0,0)$ and $(T,0)$. Repeat the procedure for a line with the points $(T,0)$ and (T,H). Alternatively, the three points $(0,0)$, $(T,0)$ and (T,H) can also be entered successively. Now we have to add a suitable circle segment. For this purpose, select 'Curve' with 'Center, Point, Angles' from the 'Tools' menu. Use again the command line for the input of the data: Center $(0,R)$, radius R, start angle 270, end angle $270+\phi$. Figure 3.5 shows the result for a transition with height 90 and radius 200.

[2]The software is freely available at https://librecad.org/.

Example.
An existing mini ramp is to be extended by adding a slightly steeper quarter pipe with a smaller radius. Opposite the quarter pipe, a bank is to be constructed. The builder of the mini ramp copied the radius from the quarter pipe of a public skatepark, but the exact radius is unknown to the builder. First, we determined the radius of the already existing ramp. If a square metal tube with a lenght of $c = 150\,cm$ is placed in the transition of the mini ramp, the distance from its center to the transition is $a = 11.7\,cm$. We assume that the length of the edge pipe is known with an accuracy of up to $\Delta c = 1\,mm$, and the distance measurement was performed with an accuracy of $\Delta a = 2\,mm$, see section 3.5. Now the program provided on https://github.com/tguent/math-quarter-pipe can be used for radius calculation. The caculation results in $R = (246 \pm 4)\ cm$, cf. Figure 3.6. During the construction of the pipe, cross slats with a thickness of 3 cm were laid on the transition side panels, with 9 mm thick surface boards placed on top. Consequently, the radius is reduced by approximately 4 cm compared to the template from which the transition was traced. Within the measurement accuracy, it can be assumed that the template has a radius of 2.5 meters. Since the template is a prefabricated quarter pipe, this seems to be a quite realistic value.

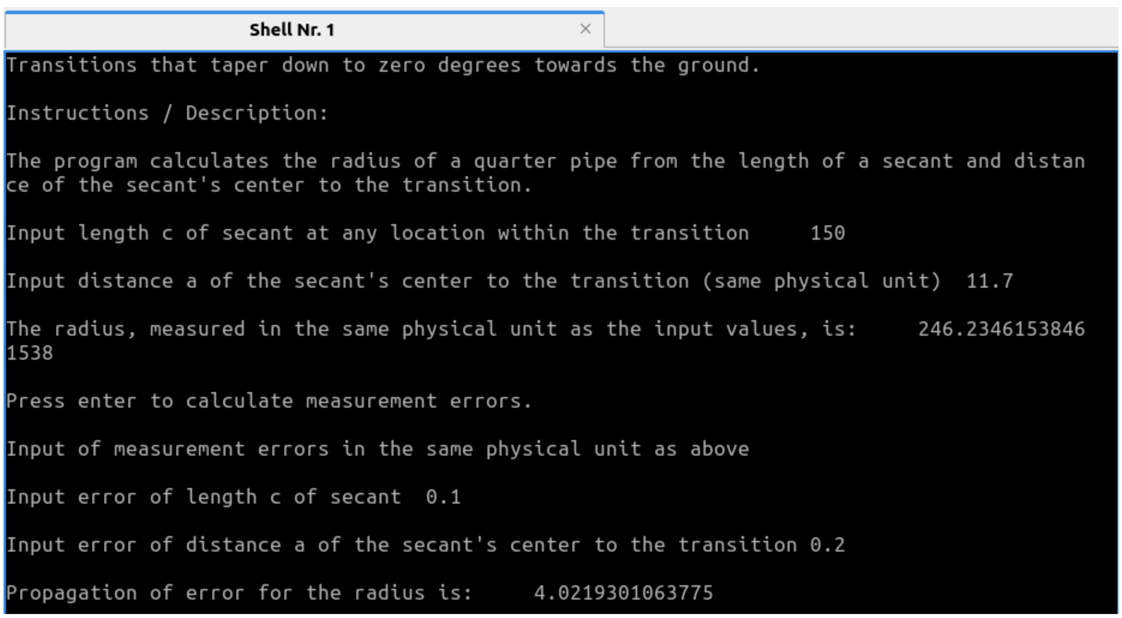

FIGURE 3.6. Measurement of the radius.

Next, along with its height, we can determine the maximum slope angle of the existing mini ramp from (3.5). The latter formular is also implemented in a Python as well as a wxMaxima code provided. Due to space constraints, further screenshots of the program are omitted here. For the maximum gradient angle, with a height of 97 cm and a radius of 246 cm, the value is approximately 53 degrees. The quarter of the side extension was choosen to have a height of 90 cm and a radius of 200 cm. Since the quarter pipe for the side extension is to be built in the same construction style as the mini ramp, an additional 4 cm must be accounted for cross beams and surface decking. Therefore, the cut-out side panel must have a height of 86 cm and a radius of 204 cm to achieve the desired dimensions in the end. The final geometry with 90 cm height and a radius of 200 cm results in a maximum gradient angle of about 57°. Figure 3.7 shows the ramp under construction.

FIGURE 3.7. Pipe and side extension in the company's hall of Stamm & Ast

4. CONCLUSION

The present article provides a mathematical description of the fundamentals of geometry for transitions in skateboard ramp construction. It bridges the gap with the many online construction guides for ramps, which typically adhere to predefined measurements. The described methods were tested a few times in practice, with the result that the pre-calculated ramps exactly met expectations. By providing the corresponding software codes, the theoretical models developed here can be used by any ramp builder who is interested. Through precise calculations in the planning of ramps, the needs of athletes can be addressed more accurately.

Acknowledgements
The author would like to thank Stefan Beer from the Company STAMM & AST for the opportunity to implement the theoretical foundations developed here into practice. Further thanks go to Tim Schulze, a professional skateboard ramp builder, with whom I have had many interesting conversations about quarter pipes. Last but not least, I thank all the people with whom I have worked on our own little private ramp projects over the nearly 40 years of skateboarding.

List of abbreviations

(1) *Skateboard quarter pipe:* A skateboard quarter pipe is a type of ramp used in skateboarding that features a curved surface transitioning from a horizontal plane to a vertical plane, or the transition continues at another angle of incline. Typically, this is followed by a horizontal section at a higher elevation. At the top edge of the ramp, there is often a metal pipe, with a diameter typically in the range of around 6 cm, called coping. This allows skateboarders to perform grinds and stalls. Many quarter pipes are made from wood, which provides a smooth and consistent surface. These can be found in both indoor and outdoor skate parks. For more permanent installations, concrete is used. Concrete quarter pipes are durable and provide a different feel compared to wood, often preferred for large public skate parks.

(2) *Transition:* The primary feature of a quarter pipe is its curved transition, which usually forms a circular segment shape. Transitions also form the crucial element in jumpramps and skateboard pipes like mini ramps or vert ramps.

(3) *skateboard pipes, mini ramps or vert ramps:* When two quarter pipes are placed facing each other, typically separated by some distance, referred to as flat.

Author's contributions
The entire study is the author's contribution.

Availability of data and materials
Data used in this research work are referenced within the article.

Funding
This research did not receive any external funding.

Declarations Competing interests
The author declares no competing interests

References

[1] I. N. Bronstein, "Taschenbuch der Mathematik", B.G.Teubner Stuttgart-Leibzig, 1996.

[2] T. Daskalov, "Concrete skateparks - Design and construction of a skateboarding recreational facility", Bachelor's thesis, Häme University of Allpied Sciences, 2015.

[3] B. Degros, "Building a Mini-Ramp (Part 1) Choosing the Height and Radius", https://youtu.be/nfRWPDdMCg4?feature=shared, 2023.

[4] D. Poirier: Skate Parks: A guide for landscape architects and planners, Master thesis, https://books.google.de/books?id=2hAGkAEACAAJ, Kansas State University, 2008.

[5] W. H. Robertson, R. D. Meyer, T. L. Wilkerson: The Mathematics of Skateboarding: A Relevant Application of the 5Es of Constructivism, Journal of Education and Learning, Vol. 1, No. 2, 2012, ISSN 1927-5250 E-ISSN 1927-5269, Journal of Education and Learning, Vol. 1, No. 2, 2012.

[6] A. Vodopivec: wxMaxima 18.02.0. http://andrejv.github.io/wxmaxima/.

[7] C . Freeman, T. Riordan, "Locating Skateparks: The Planner's Dilemma", Planning, Practice & Research, Vol. 17, No. 3, https://doi.org/10.1080/026974502200005682, 2002.

[8] H. Woolley, R. Johns, "Skateboarding: The City as a Playground.", Journal of Urban Design, 6(2), 211–230, https://doi.org/10.1080/13574800120057845, 2001.

[9] Å. Bäckström, S. Blackman, "Skateboarding: From Urban Spaces to Subcultural Olympians." YOUNG, 30(2), 121-131, https://doi.org/10.1177/11033088221081944, 2022.

[10] I. Borden, "Skateboarding, Space and the City: Architecture and the Body", Bloomsbury Publishing, 2001.

[11] I. Borden, "The spaces of skateboarding", Architects' Journal, https://www.architectsjournal.co.uk/practice/culture/the-spaces-of-skateboarding, 2014.

[12] I. Borden, I. "Southbank skateboarding, London and urban culture: The undercroft, Hungerford Bridge and House of Vans." In K. J. Lombard (Ed.), Skateboarding: Subcultures, sites and shifts (pp. 91-107). London, England: Routledge, 2016

[13] K. Butz, C. Peters, "Skateboard Studies", Koenig Books, ISBN 9783960983415, https://books.google.de/books?id=QzD4twEACAAJ, 2018.

[14] S. Holsgens, "Skill Acquisition and Korean Landscape Architecture: An Ethnographic Account of Skateboarding in Seoul, South Korea", Journal of Sport and Social Issues, DOI: 10.1177/0193723519832460, 2019.

[15] B. Glenney, S. Mull, "Skateboarding and the Ecology of Urban Space.", Journal of Sport and Social Issues, 42(6), 437-453. https://doi.org/10.1177/0193723518800525, 2018.

[16] C. Chiu, "Contestation and Conformity: Street and Park Skateboarding in New York City Public Space.", Space and Culture, 12(1), 25-42. https://doi.org/10.1177/1206331208325598, 2009.

[17] P. Whitley, "Public Skatepark Development Guide: Handbook for Skatepark Advancement", Skaters for Public Skateparks, ISBN 9780692000809, https://books.google.de/books?id=ppBttwAACAAJ, 2009.

[18] M. Batuev, L. Robinson, "How skateboarding made it to the Olympics: an institutional perspective.", International Journal of Sport Management and Marketing, 17 (4-6), pp. 381-402. https://doi.org/10.1504/IJSMM.2017.087446, 2017

[19] D. Buckingham, "Skate Perception: Self-Representation, Identity and Visual Style in a Youth Subculture." In: Buckingham, D., Willett, R. (eds) Video Cultures. Palgrave Macmillan, London. https://doi.org/10.1057/9780230244696_7, 2009.

[20] K. Sorsdahl, et al. "Experiences and Perceived Benefits of a Youth Skateboarding Program in South Africa: From the Physical to Emotional and Beyond.", Journal of Adolescent Research, 39(3), 770-795. https://doi.org/10.1177/07435584211052983, 2024.