Toward a Human-Centric Metaverse: Novel Causal Decision Models for Supply Chain Risk Management

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Abstract

This study addresses the complexities of selecting the optimal virtual reality (VR) platform for risk management in Supply Chain Management (SCM), emphasizing the significance of human-centric attributes in this decision-making process. As SCM encompasses the strategic coordination of suppliers, manufacturers, and distributors, the integration of advanced technologies, including VR, becomes essential for enhancing operational efficiency and resilience in today's dynamic market environments. This paper proposes a novel MADM model that incorporates the R.Graph method to account for the interactions between criteria. We developed two distinct algorithms: the first directly calculates ranks based on attribute interactions, while the second modifies weights to reflect these interactions. By focusing on user experience, accessibility, collaboration features, and other relevant attributes, the model aims to facilitate a comprehensive evaluation of VR platforms. The application of qualitative input data allows for a more nuanced analysis, particularly in scenarios where quantitative data is limited. This research contributes to the understanding of how VR technologies can be leveraged to enhance risk management within supply chains, ultimately fostering greater resilience and adaptability. The findings underscore the importance of aligning technology with organizational objectives and user needs, paving the way for innovation and improved performance in the metaverse. The selected platforms for this study are Bentley Synchro XR and Augmentir**,** which emerged as the best VR technologies based on our evaluation.

Keywords: Multiple Criteria Analysis, Supply Chain Management, Virtual Reality, R.Graph

1. Introduction

Supply Chain Management (SCM) is a critical and multifaceted field encompassing the strategic coordination and oversight of networks involving suppliers, manufacturers, and distributors. This management approach is essential for ensuring the effective delivery of products (Christopher, 2022). SCM integrates processes such as procurement, production, logistics, and distribution to guarantee that products reach end-users in optimal condition. The holistic nature of SCM is vital for maintaining the integrity and performance of the supply chain, especially in technologically intensive industries. Modern SCM practices increasingly leverage data analytics, artificial intelligence, and machine learning to optimize operations, enhance forecasting accuracy, and improve decision-making processes. Additionally, the integration of advanced technologies like the metaverse enhances transparency and traceability within supply chains, creating resilient and responsive systems capable of adapting to dynamic market conditions. In this context, Virtual Reality (VR) plays a pivotal role in enhancing operational efficiency within the metaverse, a digital space that represents the convergence of enhanced physical realities and persistently existing virtual environments (Fig. 1). The metaverse, made possible through VR, augmented reality (AR), and internet technologies, allows users to engage in immersive virtual experiences, fundamentally changing business interactions with consumers and operational management. Major investments from leading companies, such as Meta, into metaverse platforms underscore the transformative potential of VR, expected to revolutionize digital interactions and commerce, leading to the emergence of novel virtual economies, reshaping social interactions, and creating innovative avenues for businesses to connect with customers (Kye et al., 2021). As the metaverse expands, it significantly broadens the scope and scale of digital ecosystems, promising a more interconnected and immersive digital realm where virtual assets and digital identities play a central role in daily activities.

The integration of VR into the metaverse is critical for the efficient functioning of supply chains, particularly in both hardware (e.g., VR headsets and peripherals) and software (e.g., content updates and application distribution). Within VR technologies, SCM encompasses activities such as procuring high-quality components, precisely manufacturing VR devices, and their timely distribution to end-users. Effective management of these supply chains is crucial for maintaining the quality and performance of VR devices, essential for the metaverse (Lambert & Enz, 2017). The significance of SCM also extends to administering software and content distribution, ensuring users have uninterrupted access to the latest applications and updates necessary for an immersive metaverse experience. The integration of advanced logistics and real-time tracking systems significantly enhances supply chain efficiency, supporting the rapid deployment and scalability of VR technologies across multiple sectors, including gaming, healthcare, education, and military training (Ivanov et al., 2021). Moreover, the implementation of lean manufacturing principles and just-in-time inventory systems within SCM practices minimizes waste, reduces costs, and improves overall supply chain responsiveness, which is increasingly vital in a fast-paced digital landscape. In SCM risk management, VR platforms offer organizations significant potential for managing and mitigating risks. These platforms provide immersive training simulations that prepare employees to respond effectively to disruptions and emergencies. Organizations can simulate natural disasters, supply chain disruptions, or equipment failures to train their workforce in a controlled environment, allowing them to develop and practice response strategies without the risks associated with real-world scenarios. Additionally, VR facilitates remote collaboration among supply chain partners, allowing for real-time communication and problem-solving, crucial during crises. Through VR, teams can visualize and interact with data, enabling them to assess risks and make informed decisions more efficiently than traditional methods. This proactive approach helps businesses identify weaknesses, optimize response strategies, and enhance their resilience against potential disruptions. One of the primary challenges organizations face lies in selecting the appropriate VR platform based on their specific needs and operational requirements. The diversity of VR platforms available in the market poses a significant dilemma for decision-makers, as each platform offers unique features, functionalities, and user experiences. Selecting the right platform is critical, as the effectiveness of VR in SCM risk management is directly linked to how well the platform aligns with organizational objectives, employee capabilities, and the nature of the risks being managed.

To ensure that the chosen platform aligns with the organization's strategic objectives and operational requirements, several key human-centric attributes must be considered during the selection process. Human-centric design emphasizes creating systems and processes prioritizing users' needs, preferences, and capabilities. This approach is essential because it fosters user engagement, satisfaction, and productivity, ultimately leading to better outcomes for organizations. Attributes to consider include User Experience (UX), Accessibility, Collaboration Features, Customization Options, Realism and Immersion, and User Feedback Mechanisms. Each of these attributes plays a crucial role in ensuring that the VR system operates effectively and meets user needs. For example, UX focuses on creating an intuitive and user-friendly interface that enhances engagement and reduces the learning curve for users. Accessibility ensures that the platform accommodates a diverse range of users, including those with varying technical skills or disabilities, thereby broadening its usability and impact. Collaboration Features enable multiple users to interact within the virtual space, fostering teamwork and facilitating problem-solving through shared experiences. Customization Options allow organizations to tailor the VR platform to meet their specific operational needs and preferences, making it a more effective tool for their SCM processes. Additionally, Realism and Immersion are critical for creating engaging and authentic experiences that reflect real-world conditions, thereby enhancing the platform's utility for training, simulation, and operational planning. Finally, User Feedback Mechanisms are essential for continuous improvement, allowing organizations to gather insights from users to refine and enhance the system over time. Selecting the most appropriate VR platform for SCM risk management involves a complex evaluation of how these attributes interact within a broader system. This intricate process can be effectively addressed through the use of multiple criteria decision-making (MCDM) models, which allow organizations to manage the complexity of selecting the best VR platform. Various MCDM methods exist in the literature, including DEMATEL, ANP, WINGS, cognitive maps, and Bayesian Networks. Each of these methods offers different advantages for modeling interactions between criteria and assessing the relative importance of each attribute. They help organizations weigh the pros and cons of different platforms based on a multitude of factors, facilitating more informed decision-making.

A recently developed approach, the R.Graph method, proposed by Seiti et al. (2022), provides a causal mathematical framework specifically designed for risk analysis and management. The R.Graph method aims to address the limitations of existing causal models by estimating variability and risk factors within a network structure. It considers various scenarios in a causal chain of factors, making it particularly useful for analyzing indirect interactions between system components. This capability is especially beneficial in complex networks like supply chains, where numerous variables are interconnected. Unlike many existing models that primarily rely on quantitative data, the R.Graph method leverages qualitative data collected from expert opinions, rendering it applicable in situations where statistical data may be lacking (Seiti et al., 2022). The method is designed to be interpretable and explicable, enabling decision-makers to understand causal relationships and predict outcomes effectively. However, the successful application of the R.Graph method requires accurate estimation of input data, and it has not yet been fully extended into the Multiple Attribute Decision-Making (MADM) domain, presenting opportunities for further research and development.

This paper aims to develop a new MADM model that incorporates the R.Graph method to account for interactions between criteria. The proposed model will utilize qualitative input data and will involve creating two distinct algorithms designed to model these interactions. The objective is to apply this model to the human-centric selection of the best VR platform for managing risks in the food supply chain. This research will focus on identifying the required criteria, modeling the interactions between these criteria, and utilizing qualitative input data to offer a robust framework for selecting the most appropriate VR platform for SCM risk management in the metaverse. By bridging the gap between theoretical advancements in decision-making and practical applications, this study seeks to contribute to improved risk management and operational performance within SCM. Moreover, by leveraging advanced VR technologies and innovative decision-making models, this research aims to enhance the efficiency, scalability, and resilience of supply chains in the metaverse and beyond. In conclusion, the integration of human-centric principles in selecting VR technologies enhances operational effectiveness and ensures that the resulting systems meet the evolving needs of organizations, ultimately fostering innovation and growth in the digital economy.

Fig. 1. Various Applications of VR Technology in Supply Chain Management

The current manuscript is systematically organized into distinct sections. Section 2 presents applications of VR technologies in SCM. Additionally, it discusses recent studies on MADM models with interactions. Section 3 addresses the fundamentals of the classical R.Graph model. Section 4 introduces the proposed MADM models based on R.Graph for selecting the best VR technology. A pertinent case study is showcased in Section 5, illustrating the practical application of the proposed model within a tangible context. Finally, Section 6 summarizes the key findings of the research and provides insightful conclusions.

2. Literature Review

In this paper, Section 2.1 of the literature review explores the integration of VR technologies in supply chain operations. Subsection 2.2 discusses recent advancements in MADM models with interaction criteria.

2.1. Review of Integrating VR Technologies in Supply Chain Operations

Virtual reality (VR) technologies have become essential tools for modernizing supply chain management (SCM), enhancing training protocols, and optimizing operational efficiency. This literature review explores the transformative potential of VR in SCM alongside the associated challenges. Cobb et al. (1995) identified significant barriers to VR adoption in SCM, such as high initial costs, technical complexities, and resistance to change among personnel. Addressing these challenges necessitates strategic planning, substantial training investments, and fostering an innovative organizational culture. The immersive nature of VR allows trainees to engage with real-world logistics challenges in a risk-free environment, thereby enhancing skill acquisition and retention (Berta, 1999). The integration of VR with computer-aided design (CAD) tools has optimized design and visualization processes in SCM, reducing design cycle times and costs. This technological synergy empowers stakeholders to proactively identify potential inefficiencies, leading to streamlined supply chain operations. Ottosson (2002) emphasized that virtual prototyping through VR enables thorough testing and refinement of products and processes before physical implementation, which ultimately reduces errors and improves product quality. Moreover, VR-based simulations can enhance logistics and warehousing operations by addressing logistical challenges proactively. Teras et al. (2016) illustrated how VR technologies revolutionize training within SCM, creating immersive learning environments. The 'nDiVE' project exemplifies a significant advancement in logistics training by simulating complex scenarios that enhance learning efficacy. Guo et al. (2020) highlighted VR's critical role in industrial maintenance, encompassing maintenance needs identification, personnel training, and task visualization. This capability is vital in complex environments where timely maintenance ensures operational continuity. De Regt et al. (2020) suggested that future developments in VR within SCM will focus on enhancing realism and integrating emerging technologies like artificial intelligence (AI) and the Internet of Things (IoT), thus creating sophisticated solutions for navigating modern supply chain complexities. Bellalouna (2020) provided case studies illustrating VR's tangible benefits in sectors like automotive manufacturing, where it streamlines design, training, and operational planning. Despite these advantages, Akbari et al. (2022) noted that the adoption of VR in operations and supply chain management is still in its early stages, revealing a lack of theoretical frameworks to guide its application. This gap poses challenges for both academic research and practical implementation. Additionally, evidence from Du et al. (2022) demonstrates successful VR applications in risk management within the construction industry in China, validating its feasibility and potential impact. As VR technologies continue to evolve, their integration into SCM is expected to enhance operational efficiency, training effectiveness, and collaborative capabilities, ultimately positioning organizations to thrive in an increasingly complex global marketplace.

2.2 Recent Advances and Studies in the Application of Causal Models in MADM problems

The contemporary landscape of decision-making frameworks has witnessed a profound expansion in the development of advanced casual methodologies aimed at addressing both theoretical intricacies and technical challenges across various domains. Shamekhi Amiri et al. (2025) provided an incisive critique of the total engagement/prominence metric, identifying fundamental limitations in its capacity to capture both the vector and magnitude of influence across multiple attributes (Shamekhi Amiri et al., 2025). The authors advocated for the adoption of the total impact factor as a superior, more reliable evaluative tool, particularly within sophisticated decision-making contexts requiring granular analyses of influence dynamics Maden & Yücenur (2024). presented a rigorous evaluation of key attributes integral to the establishment of a sustainable metaverse, employing Fuzzy Cognitive Maps to mitigate the complexities of adverse causal linkages (Maden & Yücenur, 2024). Through an in-depth scenario analysis, their study accentuated the challenges of constructing a virtual ecosystem that is not only scalable and interoperable but also compliant with stringent regulatory frameworks. The research underscored the necessity of aligning virtual environments with real-world regulatory demands, revealing the nuanced interdependencies that define the sustainable development of metaverse infrastructures. In a parallel exploration of cognitive mapping methodologies, Vaz-Patto et al. (2024) introduced a groundbreaking integration of cognitive mapping, neutrosophic logic, and the Decision-Making Trial and Evaluation Laboratory (DEMATEL) method (Vaz-Patto et al., 2024). This composite framework was designed to systematically address the inherent subjectivity and multidimensional nature of evaluating urban Quality of Life. The authors proposed an analytical model that enables decision-makers to navigate uncertainties and identify critical determinants of urban well-being, significantly enhancing the robustness of decision-making processes within the domain of urban planning. In the renewable energy sector, Ebadi Torkayesh et al. (2024) proposed an advanced extension of the DEMATEL methodology, incorporating Type-2 Neutrosophic Numbers and K-means clustering to systematically analyze obstacles such as suboptimal renewable energy policies and technological constraints (Ebadi Torkayesh et al., 2024). The study's findings emphasized the disproportionate negative effects experienced by the road and maritime transport sectors compared to aviation and rail. This work highlighted the imperative for stronger policy frameworks and enhanced coordination within the renewable energy supply chain to alleviate sector-specific challenges and foster market resilience. Li et al. (2024) applied a sophisticated fuzzy-DEMATEL-ISM methodology to unravel the hierarchical structure of accident chains, with particular emphasis on explosions caused by interactions between molten aluminum and water (Li et al., 2024). The analysis identified equipment failure as the principal contributing factor, offering critical insights into improving safety protocols and preventive measures within industrial operations. These insights are essential for mitigating hazardous incidents and enhancing operational safety in high-risk industrial environments. Liu et al. (2024) advanced the discourse on preference heterogeneity by introducing a novel methodology that addresses preference variations at both individual and segment levels (Liu et al., 2024). Their approach, which leveraged probabilistic ranking, demonstrates an enhanced ability to manage sparse preference datasets and exceeds contemporary models in predictive accuracy. This advancement was particularly significant in contexts characterized by minimal heterogeneity, offering valuable contributions to the precision of targeting and pricing strategies in competitive environments. Tu et al. (2023) contributed to the decision-making literature by developing a multi-stage multi-criteria group decision-making framework, which incorporates a trust rejection threshold mechanism for optimizing expert weighting, alongside the DE-Shapley method for assessing hierarchical interactions among criteria (Tu et al., 2023). This model proficiently addressed the complexities of dynamic decision-making under uncertainty and has been validated in practical applications, such as the management of snowmelt flood scenarios. The framework's efficacy in real-world scenarios underscores its potential for broader adoption in complex, time-sensitive decision environments. Sonal & Ghosh (2022) proposed a comprehensive framework for resilience estimation in Active Distribution Networks, addressing challenges introduced by distributed energy resources and the demand for enhanced situational awareness (Sonal & Ghosh, 2022). Their hybrid MADM and Dynamic Bayesian Network framework evaluated the influence of non-deterministic resilience factors on operational performance, with a specific focus on the Load Not Served index. The study provided substantial contributions to the field of energy distribution by offering a robust approach to resilience assessment and operational optimization. Finally, Wu & Liao (2023) introduced a compensatory value function that captures the trade-offs between independent and interdependent criteria with greater precision than traditional additive models (Wu & Liao, 2023). Their approach demonstrated superior efficacy across multiple case studies, providing deeper insights into decision-makers' risk tolerance and behavior in multi-criteria decision environments. This method's ability to model complex decision-making processes offered substantial improvements over conventional value functions, thus enhancing the accuracy of decision analysis in high-stakes, multi-criteria contexts.

2.3. Research Gaps and Areas for Innovation

Despite the growing interest in integrating VR technologies within SCM, there is a significant gap in studies investigating the human-centric attributes essential for selecting VR platforms specifically tailored for risk management. Existing literature primarily emphasizes the technical capabilities and functionalities of VR systems, often overlooking critical factors such as user experience, accessibility, and collaborative features. These elements are vital as they greatly influence the effectiveness and adoption of VR technologies in real-world applications.

In the context of causal MADM models, many approaches primarily focus on direct impacts among criteria, often overlooking the complexities of indirect effects. Most models calculate only the weights of criteria based on direct relationships, rather than providing a holistic ranking of alternatives that accounts for the interdependencies between criteria. This limitation can obscure the full extent of how various attributes influence one another in decision-making scenarios. Additionally, the interpretation of impact matrices between criteria is often unclear. Some methods calculate precise values but may address only linear relationships, neglecting non-linear interactions or focusing solely on positive impacts while overlooking negative ones. Furthermore, methods like DEMATEL, which attempt to capture infinite impacts among criteria, fail to consider the influence of time, leading to potential misinterpretations of how criteria interact over specific periods. Moreover, many existing MADM models involve complex calculations that can hinder practical application and user comprehension. These challenges underscore the need for more intuitive and comprehensive models that effectively capture both direct and indirect, positive and negative, linear and non-linear impacts among criteria, while also accounting for time-sensitive dynamics. Such models would facilitate improved decision-making processes in dynamic environments like supply chain management. The novelties of the current study according to research gaps Are:

- **Focus on Human-Centric Attributes:** This study uniquely emphasizes the importance of human-centric attributes, such as user experience, accessibility, and collaboration features, in selecting VR platforms for risk management in SCM.
- **Novel MADM Model:** We propose a new multi-attribute decision-making model that incorporates the R.Graph method to effectively account for interactions between critical criteria, addressing a significant gap in existing methodologies.
- **Two Distinct Algorithms:** The study develops two algorithms: one that directly ranks alternatives based on attribute interactions and another that modifies weights to reflect these interactions, enhancing decision-making accuracy.
- **Integration of Direct and Indirect Impacts**: Our approach captures both direct and indirect positive, negative, and linear impacts among criteria, offering a more comprehensive understanding of how various attributes influence one another in decision-making processes.
- **Clarity in Impact Matrices:** We clarify the interpretation of impact matrices, specifying relationships and their effects over defined time periods to avoid misinterpretations prevalent in traditional methods.
- **Intuitive Calculations:** The study emphasizes intuitive and user-friendly calculations, overcoming the complexity often associated with existing MADM models, thus facilitating practical application and user understanding.

3. Preliminaries

This section explains the R. Graph causal method, a technique designed to analyze the causal relationships between variables and events. It is used to understand how changes in certain variables or the occurrence of specific events can affect other elements within a system. The method involves constructing a causal graph where variables and events are represented as nodes, and their causal interactions are illustrated by directed edges.

3.1. R. Graph casual method

The R.Graph technique comprises a fixed sequence of non-recursive causal elements that influence one another. Its objective is to analyze the variability within each element caused by fluctuations in other elements or distinct occurrences within a consistent temporal framework, based on the assumption that these occurrences are certain to happen. Suppose we have linear relationships between different factors in a static condition over a specific period of time. The key concepts are defined as follows:

1. **Variable:** A factor with intensity and quantity, denoted as V_i.
2. **Event:** A factor without intensity, usually indicated by 0 or 1, denoted as $E(j)$ that can influence variables or other events.

3. **Factor:** Any variable or event.
4. **Parent:** A factor that affects another factor.
5. **Arc:** A directional vector showing causality from cause to effect.

Definition 1: In the R. Graph method, risks or deviations are defined as the exact deviation of a parameter from its modified value, which can be computed using Eqs. (1) and (2).

$$R = \frac{|Changed\ Value - Initial\ Value|}{Initial\ Value} \tag{1}$$

where, if e_2 denotes the changed value and e_1 denotes the initial value, the risk (effect) value is calculated using Eq. (2).

$$R = \frac{|e_2 - e_1|}{e_1} \qquad R \geq 0 \tag{2}$$

In the R.Graph framework, "States" and "Effects" describe the relationships between variables and events, illustrating how one element influences another.

Various states depicting the effects of events and occurrences on one another are presented in Fig. 2a, while a specific type of R. Graph is illustrated in Fig. 2b.

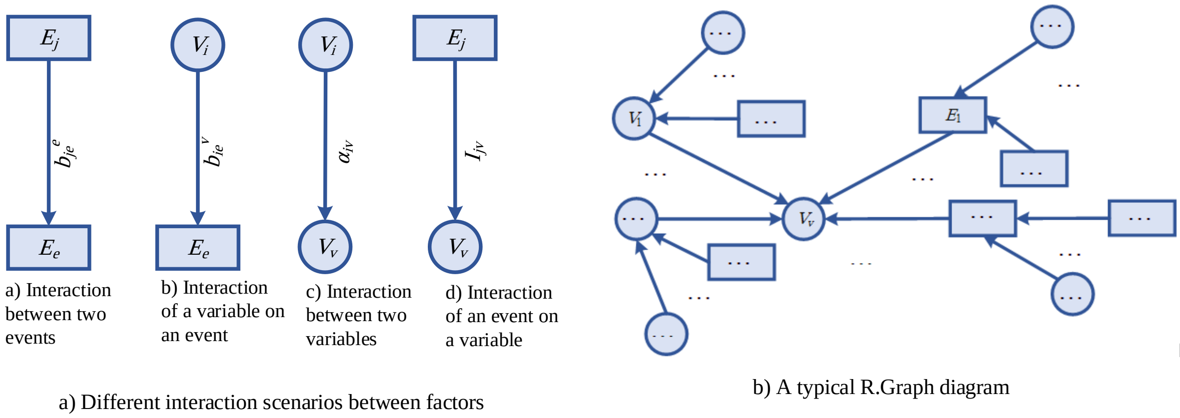


Fig. 2. The R.Graph visual concepts

Definition 2. In the R.Graph method, the way of influencing different factors is represented through the R.Graph matrix ($R^{R.Graph}$) as follows:

$$R_{R.Graph} = \begin{bmatrix} V-V & V-E \\ E-V & E-E \end{bmatrix}, \tag{3}$$

where the R.Graph matrix is composed of four distinct sub-matrices: $V-V$, $V-E$, $E-V$, and $E-E$. These sub-matrices are defined as follows:

$$V-V = \begin{array}{c} \\ \boldsymbol{V_1} \\ \boldsymbol{V_2} \\ \cdots \\ \boldsymbol{V_v} \\ \cdots \\ \boldsymbol{V_V} \end{array} \begin{array}{c} \begin{array}{cccccc} \boldsymbol{V_1} & \boldsymbol{V_2} & \dots & \boldsymbol{V_v} & \dots & \boldsymbol{V_V} \end{array} \\ \begin{bmatrix} 0 & \alpha_{12} & \dots & \alpha_{1v} & \dots & \alpha_{1V} \\ \alpha_{21} & 0 & \dots & \alpha_{2v} & \dots & \alpha_{2V} \\ \dots & \dots & 0 & \dots & \dots & \dots \\ \alpha_{v1} & \alpha_{v2} & \dots & 0 & \dots & \alpha_{vV} \\ \dots & 0 & \dots & \dots & 0 & \dots \\ \alpha_{V1} & \alpha_{V2} & 0 & \alpha_{Vv} & \dots & 0 \end{bmatrix} \end{array}, v = 1, \dots V, \forall i = 1, \dots, V, \alpha_{iv} \in \mathbb{R}, \tag{4}$$

$$V-E=\begin{array}{cc} & \begin{array}{cccccc} \boldsymbol{E_1} & \boldsymbol{E_2} & \dots & \boldsymbol{E_e} & \dots & \boldsymbol{E_E} \end{array} \\ \begin{array}{c} \boldsymbol{V_1} \\ \boldsymbol{V_2} \\ \dots \\ \boldsymbol{V_v} \\ \dots \\ \boldsymbol{V_V} \end{array} & \begin{bmatrix} b_{11}{}^{v} & b_{12}{}^{v} & \dots & b_{1e}{}^{v} & \dots & b_{1E}{}^{v} \\ b_{21}{}^{v} & b_{22}{}^{v} & \dots & b_{2e}{}^{v} & \dots & b_{2E}{}^{v} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_{v1}{}^{v} & b_{v2}{}^{v} & \dots & b_{ve}{}^{v} & \dots & b_{vE}{}^{v} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_{V1}{}^{v} & b_{V2}{}^{v} & \dots & b_{Ve}{}^{v} & \dots & b_{VE}{}^{v} \end{bmatrix} \end{array}, v=1,\dots V, e=1,\dots,E, \forall i=1,\dots,V,\ b_{ie}{}^{v}\epsilon\{0,1\}, \tag{5}$$

$$E-V=\begin{array}{cc} & \begin{array}{cccccc} \boldsymbol{V_1} & \boldsymbol{V_2} & \dots & \boldsymbol{V_v} & \dots & \boldsymbol{V_V} \end{array} \\ \begin{array}{c} \boldsymbol{E_1} \\ \boldsymbol{E_2} \\ \dots \\ \boldsymbol{E_e} \\ \dots \\ \boldsymbol{E_E} \end{array} & \begin{bmatrix} I_{11} & I_{12} & \dots & I_{1v} & \dots & I_{1V} \\ I_{21} & I_{22} & \dots & I_{2v} & \dots & I_{2V} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ I_{e1} & I_{e2} & \dots & I_{ev} & \dots & I_{eV} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ I_{E1} & I_{E2} & \dots & I_{Ev} & \dots & I_{EV} \end{bmatrix} \end{array}, v=1,\dots V, e=1,\dots,E, \forall j=1,\dots,E,\ I_{jv}\ \epsilon\mathbb{R}, \tag{6}$$

$$E-E=\begin{array}{cc} & \begin{array}{cccccc} \boldsymbol{E_1} & \boldsymbol{E_2} & \dots & \boldsymbol{E_e} & \dots & \boldsymbol{E_E} \end{array} \\ \begin{array}{c} \boldsymbol{E_1} \\ \boldsymbol{E_2} \\ \dots \\ \boldsymbol{E_e} \\ \dots \\ \boldsymbol{E_E} \end{array} & \begin{bmatrix} 0 & b_{12}{}^{e} & \dots & b_{1e}{}^{e} & \dots & b_{1E}{}^{e} \\ b_{21}{}^{e} & 0 & \dots & b_{2e}{}^{e} & \dots & b_{2E}{}^{e} \\ \dots & \dots & 0 & \dots & \dots & \dots \\ b_{e1}{}^{e} & b_{e2}{}^{e} & \dots & 0 & \dots & b_{eE}{}^{e} \\ \dots & \dots & \dots & \dots & 0 & \dots \\ b_{E1}{}^{e} & b_{E2}{}^{e} & 0 & b_{Ee}{}^{e} & \dots & 0 \end{bmatrix} \end{array}, e=1,\dots,E, \forall j=1,\dots,E,\ b_{je}{}^{e}\epsilon\{0,1\}. \tag{7}$$

The matrices represent, respectively, the influence of variable risks on other variables ($V-V$), the influence of variable risks on events ($V-E$), the influence of event risks on variables ($E-V$), and the influence of event risks on other events ($E-E$). Furthermore, in the above equation, α_{iv} represents the risk of variable v due to a 100% risk in variable i. As the R. Graph is non-cyclic, if α_{iv} takes a value, then $\alpha_{vi}=0$. Additionally, I_{jv} represents the risk to variable v caused by the occurrence of event j. In the above equations, $b_{je}{}^{e}$ denotes the likelihood of event e occurring due to event j, while $b_{ie}{}^{v}$ indicates the likelihood of event e occurring due to variable V_i. If $b_{je}{}^{e}$ and $b_{ie}{}^{v}$ take the value of one, it indicates the susceptibility of event e occurring due to event j and the variable i. If they are zero, it indicates non-susceptibility. Here, as the R. Graph is non-cyclic, if $b_{je}{}^{e}$=1 and $b_{ie}{}^{v}$ =1, we will have $b_{ej}{}^{e}$=0 and $b_{ei}{}^{v}$=0.

Definition 3. Consider a set of $\acute{V}$ variables and $\acute{E}$ events that influence a specific variable V_v, where $i=1,\dots,\acute{V}, j=1,\dots,\acute{E}$. If the objective is to analyze the rate of change (risk) of the variable V_v with respect to all these factors, under the assumption that all factors are independent, the following expression holds:

$$R(V_v)=R\big(V_v|Par(V_v)\big)=\sum_{i=1}^{\acute{V}}\alpha_{iv}R(V_i)+\sum_{j=1}^{\acute{E}}I_{jv} \tag{8}$$

Here, $Par(V_v)$ denotes all the parent variables of V_v, $R(V_i)$ represents the risk or impact associated with the i variable, and $R\big(V_v|Par(V_v)\big)$ refers to the impact (or risk) resulting from alterations or occurrences in the parents of V_v.

Definition 4. The risk associated with variable V_v according to the desired event V_i, can be defined in the following manner:

$$\begin{cases} R(V_v|V_i)=\alpha_{iv}R(V_i)+\sum_{l=1}^{L}\alpha_{lv}R(V_l|V_i)+\ \sum_{k=1}^{K}I_{kv} & V_i\epsilon Par(V_v) \\ R(V_v|V_i)=\sum_{l=1}^{L}\alpha_{lv}R(V_l|V_i)+\ \sum_{k=1}^{K}I_{kv} & V_i\notin Par(V_v) \end{cases} \tag{9}$$

where V_l denotes the variables that are either directly or indirectly influenced by V_i;V_i represents the impact of events on V_v , which are the parent variables of V_v, and V_i influences their occurrence.

Definition 5. The impact of the variable V_v can be defined according to the desired event E_j, i.e., $R(V_v|E_j)$ as follows:

$$\begin{cases} R(V_v|E_j) = I_{jv} + \sum_{k=1}^{K} I_{kv} + \sum_{l=1}^{L} \alpha_{lv} R(V_l|E_j) & E_j \epsilon Par(V_v) \\ R(V_v|E_j) = \sum_{k=1}^{K} I_{kv} + \sum_{l=1}^{L} \alpha_{lv} R(V_l|E_j) & E_j \notin Par(V_v) \end{cases} \tag{10}$$

In this relation, I_{kv} represents the influence of events on V_v, which acts as the parent of V_v, and also demonstrates how E_j impacts their occurrences. Additionally, V_l refers to variables that are directly or indirectly influenced by E_j.

4. Proposed Qualitative R.Graph Model for Decision Making with Interactive Criteria

In this section, we propose a MADM approach based on the R.Graph model for selecting the best AR technology for supply chain risk management. The method addresses the complexities of evaluating multiple interconnected criteria related to risk mitigation. By utilizing the R.Graph framework, it models interactions among these criteria, offering a view of their dependencies for a systematic and transparent evaluation. To support this, we present two algorithms: the first directly ranks alternatives without recalculating weights, while the second provides an approximate method for comparing results by calculating weights based on impact.

Algorithm 1: The steps of the proposed first interactive MADM R.Graph algorithm are outlined as follows:

Step 1: *Development of Decision and Interaction Matrices Along with Criteria Weight Assignment*

The R.Graph method facilitates MADM by modeling the influences among criteria through either intensifying or diminishing effects. This method assumes acyclic influences, meaning no criterion affects itself, similar to the ANP. Unlike DEMATEL, R.Graph posits that effects occur only once within a specific timeframe. Each event is treated as an option, and each criterion as a variable (Fig. 3a), with direct or indirect influences being either positive or negative. The relationships between criteria are linear, represented in an influence matrix ($C-C$ matrix), which shows changes in one criterion due to a 100% change in another. The decision matrix ($E-V$ matrix) captures the impact of options on criteria, while the $C-C$ matrix accounts for interdependencies. Decision-makers assign subjective weights to each criterion based on its importance, adjusting cost criteria by multiplying those columns by negative one. This framework evaluates and selects the most suitable options by considering direct impacts and interdependencies among criteria. To implement this, decision-makers define influential attributes and their relationships, deriving causal relationships from experts or methods like Interpretive Structural Modeling (ISM). Custom causal diagrams may also be developed to explore attribute relationships. This approach ensures all relevant factors are considered, leading to informed and effective decision-making.

$$c_1 \quad c_1 \quad \cdots \quad c_n \tag{11}$$

$$X = \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \text{ where } \begin{cases} x_{ij} \geq 0 \text{ for beneficial attributes} \\ x_{ij} \leq 0 \text{ for non} - \text{beneficial attributes} \end{cases}$$

$$C - C = \begin{array}{c} \\ \boldsymbol{C_1} \\ \boldsymbol{C_2} \\ \dots \\ \boldsymbol{C_j} \\ \dots \\ \boldsymbol{C_n} \end{array} \begin{array}{c} \begin{array}{cccccc} \boldsymbol{C_1} & \boldsymbol{C_2} & \dots & \boldsymbol{C_j} & \dots & \boldsymbol{C_n} \end{array} \\ \left[\begin{Vmatrix} 0 & \alpha_{12} & \dots & \alpha_{1v} & \dots & \alpha_{1n} \\ \alpha_{21} & 0 & \dots & \alpha_{2v} & \dots & \alpha_{2n} \\ \dots & \dots & 0 & \dots & \dots & \dots \\ \alpha_{j1} & \alpha_{j2} & \dots & 0 & \dots & \alpha_{jn} \\ \dots & 0 & \dots & \dots & 0 & \dots \\ \alpha_{n1} & \alpha_{n2} & 0 & \alpha_{nj} & \dots & 0 \end{Vmatrix}\right] \end{array} \tag{12}$$

$$W = [w_{js}]_{1 \times n} \tag{13}$$

In this context, X represents the decision matrix evaluating mmm alternatives across n criteria. $C - C$ denotes the interaction matrix between the criteria, while α_{jk} indicates the effect of the j-th attribute on the k-th criterion, assuming a complete 100% change in attribute j. W is the subjective weight matrix, with w_{js} denoting the subjective weight assigned to the j-th attribute. Both the $C - C$ matrix and the decision matrix can be deterministic (using exact values) or qualitative, particularly when defined by experts. This paper employs a qualitative model to better align with MADM applications.

Step 2: *Calculating the Influenced Decision Matrix*

In this step, we apply the R.Graph concept to determine how the influenced matrix is formed based on criteria interactions. For example, in the initial decision matrix, let Alternative 1 have a value of 0.2 for Attribute 1 and 0.3 for Attribute 2. If Attribute 1 positively affects Attribute 2, causing a 100% change in Attribute 1 to influence Attribute 2, the new value for Attribute 2 would be $0.3 + (0.2 \times 0.3)$. This can be calculated using $R(C_2|A_1)$ in the R.Graph. The influenced matrix, which includes the initial matrix plus these influences, can be derived using the following relation:

$$X_i{}^I = [R(C_j|A_i)]_{m \times n} \tag{14}$$

where

$$R(C_j|A_i) = x_{ij} + \sum_{l=1}^{n} |\alpha_{lj}| \times R(C_l|A_i) \tag{15}$$

In this context, $X_i{}^I$ is the influenced matrix, α_{lj} represents the influence of criterion l on criterion j, and $\sum_{l=1}^{n} |\alpha_{lj}| \times R(C_l|A_i)$ denotes the total effects on criterion C_j resulting from the impacts of other criteria.

Step 3: *Calculating the Normalized Influenced Decision Matrix*

Now, since the influence matrix from Step 2 is a new matrix with potential values exceeding 1, it should be normalized. We can apply logistic normalization (Eq. (17)) to transform the values between 0 and 1. It is important to note that this normalization does not lead to rank reversal.

$$X_i{}^{IN} = \left[R(C_j|A_i)^N\right]_{m \times n} \tag{16}$$

$$\text{where } R(C_j|A_i)^N = \frac{1}{1 + e^{-R(C_j|A_i)}} \tag{17}$$

${X_i}^{IN}$represents the normalized influenced matrix, while $R\left(C_j\middle|A_i\right)^N$denotes the normalized evaluation of alternative A_i with respect to the j-th criterion.

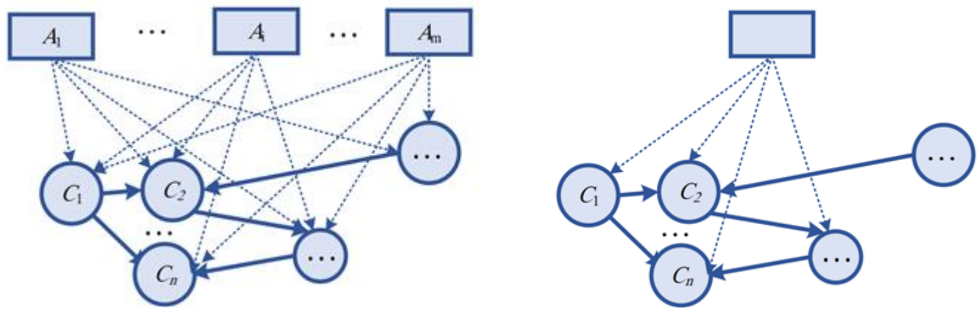
***Step 4:** Calculating the Weighted Sum and Ranking*

Now, to calculate the final ranking of each alternative, we compute the weighted sum for each alternative using the normalized matrix from Step 3 and the subjective weights from Step 1. The alternatives are then ranked in ascending order based on the following relation.

$$Score(A_i) = \sum_{j=1}^{m} w_{js} R\left(C_j\middle|A_i\right)^N \tag{18}$$

where the $Score(A_i)$ is the taotal performance of alternative i.

In the next section, we will discuss Algorithm 2, which is used for weight determination.



a) Corresponding R.Graph of Algorithm 1 b) Corresponding R.Graph of Algorithm 2

Fig.3. Different structures of the two algorithms corresponding to the R.Graph method.

Algorithm 2: This algorithm calculates the weights of each attribute based on interactions and subjective weights, similar to existing methods such as DEMATEL, ANP, and others. It is important to note that this is an approximate method and does not fully utilize the original concepts of the R.Graph method.

***Step 1:** Determination of Appropriate Inputs*

In this proposed algorithm, instead of using a decision matrix and analyzing its impact on the results, it is assumed that there is a single event (option) that affects all criteria equally (Fig. 3b). Consequently, the $E - V$ matrix, analogous to the R.Graph method, is represented as a $1 \times n$ vector with all elements equal to one. Furthermore, the interaction matrix between criteria, accounting for both positive and negative influences, is defined accordingly. Additionally, the qualitative weight matrix is derived using Eq. (13):

$$C - C = \begin{array}{c} \\ \boldsymbol{C_1} \\ \boldsymbol{C_2} \\ \cdots \\ \boldsymbol{C_j} \\ \cdots \\ \boldsymbol{C_n} \end{array} \begin{array}{c} \begin{array}{cccccc} \boldsymbol{C_1} & \boldsymbol{C_2} & \dots & \boldsymbol{C_j} & \dots & \boldsymbol{C_n} \end{array} \\ \begin{bmatrix} 0 & \alpha_{12} & \dots & \alpha_{1v} & \dots & \alpha_{1n} \\ \alpha_{21} & 0 & \dots & \alpha_{2v} & \dots & \alpha_{2n} \\ \dots & \dots & 0 & \dots & \dots & \dots \\ \alpha_{j1} & \alpha_{j2} & \dots & 0 & \dots & \alpha_{jn} \\ \dots & 0 & \dots & \dots & 0 & \dots \\ \alpha_{n1} & \alpha_{n2} & 0 & \alpha_{nj} & \dots & 0 \end{bmatrix} \end{array} \tag{19}$$

***Step 2:** Determination the influence matrix*

In this step, we determine the total effect of each attribute on the others, considering all interactions. The influence of an alternative on criterion j is denoted by $R(C_j|C_i)$, which leads to the construction of the influence matrix between criteria, represented as an $n \times n$ matrix, as follows:

$$I = [R(C_j|C_i)]_{n\times n} \tag{20}$$

where I is influence matrix and can be formed basen R.Graph concepts using Eq. (21).

$$R(C_j|C_i) = \alpha_{iv}R(C_i) + \sum_{l=1}^{n} \alpha_{lv}R(C_l|C_i) \tag{21}$$

where the term $\alpha_{iv}R(C_i)$ represents the direct effect of attribute i on j, and $\sum_{l=1}^{n} \alpha_{lv}R(C_l|C_i)$ captures the indirect effects of i on j based on the influence of other attributes that also affect j. Using the R.Graph method and considering the previously mentioned $E-V$ matrix, we have:

$$R(C_j) = 1 + R\left(C_j|Par(C_j)\right) = 1 + \sum_{i=1}^{n} \alpha_{ij}R(C_i) \tag{22}$$

$R(C_j)$ is percentage change in attribute j.

Step 3: *Obtaining the Normalized Influence Decision Matrix*

Now, similar to Step 3 of Algorithm 1, logistic normalization Eq. (24), should be applied to normalize the matrix as follows:

$$I^N = \left[R(C_j|C_i)^N\right]_{m\times n} \tag{23}$$

where $R(C_j|C_i)^N = \frac{1}{1+e^{-R(C_j|C_i)}}$ (24)

I^N is the normalized influence matrix, and $R(C_j|A_i)^N$ is the normalized impact.

Step 4: *Calculating the combined weight of each attribute*

In the final step, the combined weight of each attribute (C_j), taking into account the subjective factors, is calculated using Eq. (25).

$$weight\ (C_j) = \frac{\sum_{i=1}^{n} w_{is}R(C_j|C_i)^N}{\sum_{j=1}^{n}\sum_{i=1}^{n} w_{is}R(C_j|C_i)^N} \tag{25}$$

The obtained weights can now be applied to any MADM method to rank the alternatives. In this paper, we multiply the weights with the decision matrix in Eq. (11). The proposed methods are detailed in Algorithms 1 and 2 (Appendix) and visually represented in Fig. 4.

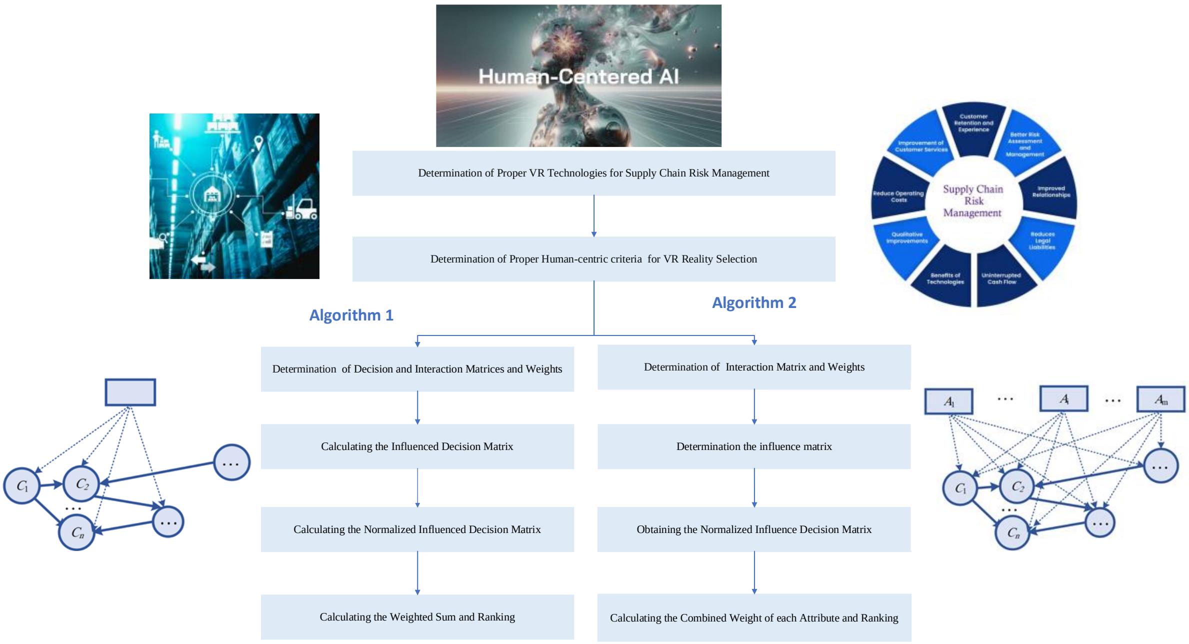


Fig.4. Overview of proposed algorithms for causal MADM in VR selection

5. Case Study

The Food production sector (Fig. 5) faces significant risks at every stage, from sourcing to delivery. Challenges such as supplier delays, natural disasters, and regulatory issues can lead to ingredient shortages and increased costs. Additionally, the preparation phase risks equipment malfunctions and contamination, while transportation is impacted by traffic and fuel price fluctuations. Effective supply chain risk management is vital for minimizing losses and ensuring customer satisfaction. Strategies like supplier diversification and safety stock maintenance enhance visibility. Innovations such as VR technologies significantly improve risk management by simulating scenarios for early threat detection and proactive mitigation, supporting business continuity and resilience in a global marketplace. Effective risk management in supply chain operations is crucial for ensuring business continuity and customer satisfaction. It enables systematic identification, evaluation, and mitigation of risks, minimizing disruptions and controlling costs. In a globalized marketplace, robust risk management practices are essential for maintaining a competitive advantage. Strategies to address supply chain risks include supplier diversification, safety stock maintenance, and leveraging advanced technologies for enhanced visibility and forecasting. While traditional methods like risk assessment matrices remain effective, advanced technologies, particularly VR technologies in the Metaverse, offer innovative solutions for enhancing supply chain resilience. VR can simulate various supply chain scenarios, providing insights into potential risks and facilitating proactive management.

In this section, we aim to identify the most effective VR technology model for managing supply chain risks within the Metaverse by utilizing the the proposed MADM algorithm bsed on R.Graph method. We evaluate several VR technology models, including Bentley Synchro XR, VisualLive, Augmentir, Unity Reflect, SimLab VR, Revizto, Fuzor, and InsiteVR. Each model offers distinct advantages relevant to supply chain risk management. A summary of their key characteristics is provided in the Table 1.

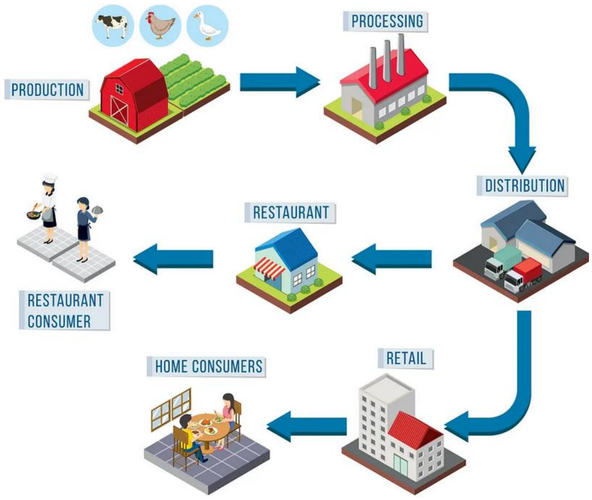


Fig. 5. A simplified food industries supply chain system

Table 1. Descriptions of VR technology models for supply chain risk management

Alternative	Descriptions
SYNCHRO™	Bentley Synchro XR is a mixed reality platform tailored for construction and infrastructure project management. It integrates 4D (3D + time) BIM models with project schedules, allowing users to visualize and interact with construction progress in immersive environments, enhancing decision-making and project execution.
VISUAL LIVE	VisualLive provides AR solutions primarily for architecture, engineering, and construction industries. It enables the overlay of 3D BIM models onto physical environments using AR headsets or mobile devices, allowing real-time visualization and interaction with digital twins for improved coordination and issue resolution on-site.
	Augmentir is an artificial intelligence (AI)-driven AR platform designed for industrial and manufacturing applications. It supports operational tasks by providing contextualized AR guidance and digital work instructions, improving operational efficiency, maintenance, and training via AI insights.
unity Reflect	Unity Reflect is a real-time 3D visualization platform that enables the synchronization of BIM data from Revit and other design tools into a real-time interactive 3D environment. Its primary function is to enhance design review, coordination, and decision-making through dynamic rendering and immersive simulations.
	SimLab VR is a VR software platform that transforms CAD models into fully immersive virtual experiences. It is widely used for creating interactive simulations, virtual walkthroughs, and presentations in various industries, facilitating enhanced communication and spatial understanding in design and engineering contexts.
revizto	Revizto is a real-time issue tracking and BIM collaboration platform used in architecture, engineering, and construction. It consolidates 3D models, 2D drawings, and project data into a unified interface, enabling project teams to track and resolve design issues collaboratively, improving project coordination and reducing rework.
Fuzor	Fuzor is a comprehensive virtual design and construction tool that integrates with BIM software to provide advanced 4D simulations, clash detection, and design coordination features. It supports project visualization, planning, and scheduling, enhancing the accuracy of construction workflows and project management.
InsiteVR	InsiteVR is a VR platform designed for immersive design and coordination reviews. It allows project teams to conduct collaborative virtual meetings, explore 3D models in a VR environment, and improve communication and decision-making processes during the design and construction phases.

It is worth mentioning that, in this case study, the relevant data for evaluating the decision matrix and interaction matrix was gathered through an iterative process involving experts highly qualified in the fields of the metaverse and VR. These experts possess a comprehensive understanding of VR technologies and their practical applications, particularly within supply chain and food delivery systems. Their deep familiarity with both the technical aspects of VR and the operational demands of food delivery systems ensures that the evaluation process is well-grounded, facilitating accurate and effective decision-making in selecting the most appropriate VR models for mitigating supply chain risks in the metaverse. Table 2 delineates the criteria and sub-criteria, including human-centric ones, alongside their respective weights, which are essential for evaluating the effectiveness of VR technology models in mitigating supply chain risks within the metaverse. These criteria are carefully selected to cover various aspects of performance, reliability, and overall impact. By assigning specific weights to each criterion, the methodology enables prioritization of the most critical factors for effective risk management through VR models. This structured approach ensures a comprehensive assessment, allowing stakeholders to identify the strengths and weaknesses of different models in real-world applications.

The decision matrix for this research, which outlines the performance of various alternatives based on the identified criteria, is presented in Table 3. It serves as a comprehensive tool for comparing and evaluating different VR technology models in terms of effectiveness, efficiency, adaptability, impact, and human-centricity in managing supply chain risks within the metaverse. The evaluation employed a set of linguistic variables and their corresponding crisp values (e.g., VL: 0.07 – Very Low, L: 0.25 – Low, ML: 0.45 – Moderately Low, M: 0.5 – Medium, MH: 0.65 – Moderately High, H: 0.75 – High, VH: 0.93 – Very High) to provide qualitative insights into each tool’s performance. These linguistic variables were then converted into crisp values, leading to the deterministic evaluation presented in Table 4.

Table 2. Detailed Definitions of Criteria for Weighting VR Technology Models in Supply Chain Risk Management Evaluation

Criteria	Wi	Sub-Criteria	wi	Definition	Human-Centric Relevance
Effectiveness	0.20	Risk Identification	0.399	Ability of the system to identify and anticipate potential risks within the supply chain by using advanced monitoring features.	Helps users proactively recognize threats early, enhancing decision-making.
		Mitigation Capabilities	0.437	The system's ability to reduce the impact of identified risks, allowing for prompt intervention and risk reduction strategies.	Gives users confidence that the system can manage and lessen risks effectively.
		Monitoring and Alerting	0.084	The system's capability to continuously monitor supply chain activities and send real-time alerts on potential issues or risks.	Improves user awareness by providing timely notifications, fostering quick actions.
		Accuracy	0.080	The precision with which the system detects risks, issues, and performance metrics, minimizing errors and false positives.	Ensures that users can trust the information and act with certainty.
Efficiency	0.20	Response Time	0.409	The speed at which the system responds to emerging risks, problems, or user inputs in real-time, reducing delays in reaction.	Essential for maintaining smooth operations in high-pressure situations.
		Resource Utilization	0.249	Efficiency in the use of available resources (computing power, energy, time, etc.), minimizing waste and ensuring optimal performance.	Ensures that users experience high performance without unnecessary resource drain.
		Cost-Effectiveness	0.210	The balance between the system's cost and the value it provides, ensuring the user gets maximum benefit for the lowest possible cost.	Provides users with a cost-efficient solution without sacrificing quality.
		Operational Efficiency	0.134	Ability to optimize workflows, reduce delays, and maintain high performance levels with minimal errors and disruptions.	Improves productivity by supporting seamless operations, boosting user output.
Adaptability	0.20	Scalability	0.235	Ability to scale the system up or down depending on the size or complexity of operations, accommodating growth or shifts in demand.	Allows the system to grow alongside user needs without needing a complete overhaul.
		Flexibility	0.223	Capacity to adjust to new requirements, user preferences, or external changes with minimal reconfiguration or interruption.	Directly human-centric, enabling users to adapt the system for various scenarios.
		Compatibility	0.209	Integration with existing technologies, software, and infrastructure to work seamlessly within the current IT ecosystem.	Eases the transition for users by avoiding compatibility issues with current tools.
		User Training and Support	0.333	Availability of user-friendly training resources and prompt support services, ensuring users are well-equipped to use the system.	Ensures that users feel competent and supported, enhancing satisfaction.
Impact	0.20	Operational Continuity	0.314	Ability to maintain consistent and uninterrupted operations without breakdowns or downtime, especially in critical situations.	Vital for ensuring smooth, continuous operations, preventing user frustration.
		Safety and Compliance	0.277	Adherence to industry-specific safety regulations and standards, ensuring compliance and reducing risk to users and stakeholders.	Protects users from safety risks and ensures compliance with legal frameworks.
		Reputation Management	0.067	The ability to build and maintain trust with stakeholders by ensuring reliability, quality, and accountability in operations.	Reflects user trust and satisfaction, helping build long-term relationships.
		Financial Stability	0.342	Ensures the system's financial sustainability and continued development, guaranteeing longevity and reducing risks of obsolescence.	Users can trust the system to remain viable and supported in the long term.
Human-Centric	0.2	User Experience (UX)	0.352	How easy, intuitive, and enjoyable the system is for users to navigate and interact with, minimizing frustration and maximizing efficiency.	Central to human-centric design, ensuring smooth and positive user interactions.
		Accessibility	0.263	How accessible the system is to a wide range of users, including those with different levels of technical expertise or disabilities.	Ensures inclusivity, making the system easy for users regardless of their skill levels.
		Collaboration Features	0.118	Tools and features that allow multiple users to collaborate, share information, and make joint decisions within the VR environment.	Facilitates teamwork and cooperation, critical for collaborative supply chain operations.
		Customization Options	0.030	Ability for users to tailor the VR environment, interface, and workflows to meet specific needs or preferences.	Increases user satisfaction by allowing personalized experiences.
		Realism and Immersion	0.198	How immersive and realistic the virtual environment feels, providing a high level of human engagement and sensory experience.	Enhances user engagement and focus by creating a believable, engaging experience.
		User Feedback Mechanisms	0.039	Availability of channels for users to provide feedback, report issues, and suggest improvements, fostering system improvement.	Empowers users by making them part of the improvement process.

Table 3. Decision Matrix for Evaluating VR Technology Models in Supply Chain Risk Management

Criteria	Sub-Criteria	Cost/Benefit	Bentley Synchro XR	VisualLive	Augmentir	Unity Reflect	SimLab VR	Revizto	Fuzor	InsiteVR
Effectiveness	Risk Identification (C_1)	Benefit	VH	M	H	L	H	H	VL	H
	Mitigation Capabilities (C_2)	Benefit	H	M	VH	L	MH	H	M	VL
	Monitoring and Alerting (C_3)	Benefit	VH	M	H	VL	MH	VH	ML	H
	Accuracy (C_4)	Benefit	H	M	VH	VL	L	MH	L	M
Efficiency	Response Time (C_5)	Benefit	H	VL	VH	MH	M	H	L	ML
	Resource Utilization (C_6)	Benefit	VH	L	VH	VL	H	M	ML	L
	Cost-Effectiveness (C_7)	Benefit	H	M	H	L	MH	M	ML	M
	Operational Efficiency (C_8)	Benefit	H	M	VH	VL	H	H	ML	MH
Adaptability	Scalability (C_9)	Benefit	H	M	H	L	MH	H	M	VL
	Flexibility (C_{10})	Benefit	VH	M	H	L	H	VH	ML	H
	Compatibility (C_{11})	Benefit	H	VL	H	L	M	H	ML	VH
	User Training and Support (C_{12})	Benefit	H	M	VH	L	MH	H	VL	H
Impact	Operational Continuity (C_{13})	Benefit	H	M	H	L	H	H	ML	MH
	Safety and Compliance (C_{14})	Benefit	VH	M	M	VL	MH	H	M	H
	Reputation Management (C_{15})	Benefit	H	M	VH	L	H	H	VL	H
	Financial Stability (C_{16})	Benefit	H	L	H	VL	M	MH	VL	L
Human-Centric	User Experience (UX) (C_{17})	Benefit	H	VH	H	M	MH	H	M	H
	Accessibility (C_{18})	Benefit	M	MH	H	H	H	MH	ML	MH
	Collaboration Features (C_{19})	Benefit	H	H	VH	H	MH	H	M	H
	Customization Options (C_{20})	Benefit	M	MH	H	M	ML	H	MH	MH
	Realism and Immersion (C_{21})	Benefit	VH	H	MH	H	MH	MH	M	H
	User Feedback Mechanisms (C_{22})	Benefit	H	H	M	H	M	VH	L	MH

Very High (VH), High (H), Moderately High (MH), Medium (M), Moderately Low (ML), Low (L) and Very Low (VL)

Table 4. Quantified Decision Matrix for Evaluating VR Technology Models in Supply Chain Risk Management

Criteria	Sub-Criteria	Cost/Benefit	Bentley Synchro XR	VisualLive	Augmentir	Unity Reflect	SimLab VR	Revizto	Fuzor	InsiteVR
Effectiveness	Risk Identification (C_1)	Benefit	0.93	0.5	0.75	0.25	0.75	0.75	0.07	0.75
	Mitigation Capabilities (C_2)	Benefit	0.75	0.5	0.93	0.25	0.65	0.75	0.5	0.07
	Monitoring and Alerting (C_3)	Benefit	0.93	0.5	0.75	0.07	0.65	0.93	0.45	0.75
	Accuracy (C_4)	Benefit	0.75	0.5	0.93	0.07	0.25	0.65	0.25	0.5
Efficiency	Response Time (C_5)	Benefit	0.75	0.07	0.93	0.65	0.5	0.75	0.25	0.45
	Resource Utilization (C_6)	Benefit	0.93	0.25	0.93	0.07	0.75	0.5	0.45	0.25
	Cost-Effectiveness (C_7)	Benefit	0.75	0.5	0.75	0.25	0.65	0.5	0.45	0.5
	Operational Efficiency (C_8)	Benefit	0.75	0.5	0.93	0.07	0.75	0.75	0.45	0.65
Adaptability	Scalability (C_9)	Benefit	0.75	0.5	0.75	0.25	0.65	0.75	0.5	0.07
	Flexibility (C_{10})	Benefit	0.93	0.5	0.75	0.25	0.75	0.93	0.45	0.75
	Compatibility (C_{11})	Benefit	0.75	0.07	0.75	0.25	0.5	0.75	0.45	0.93
	User Training and Support (C_{12})	Benefit	0.75	0.5	0.93	0.25	0.65	0.75	0.07	0.75
Impact	Operational Continuity (C_{13})	Benefit	0.75	0.5	0.75	0.25	0.75	0.75	0.45	0.65
	Safety and Compliance (C_{14})	Benefit	0.93	0.5	0.5	0.07	0.65	0.75	0.5	0.75
	Reputation Management (C_{15})	Benefit	0.75	0.5	0.93	0.25	0.75	0.75	0.07	0.75
	Financial Stability (C_{16})	Benefit	0.75	0.25	0.75	0.07	0.5	0.65	0.07	0.25

Criteria	Sub-Criteria	Cost/Benefit	Bentley Synchro XR	VisualLive	Augmentir	Unity Reflect	SimLab VR	Revizto	Fuzor	InsiteVR
Human-Centric	User Experience (UX) (C_{17})	Benefit	0.75	0.93	0.75	0.5	0.65	0.75	0.5	0.75
	Accessibility (C_{18})	Benefit	0.5	0.65	0.75	0.75	0.75	0.65	0.45	0.65
	Collaboration Features (C_{19})	Benefit	0.75	0.75	0.93	0.75	0.65	0.75	0.5	0.75
	Customization Options (C_{20})	Benefit	0.5	0.65	0.75	0.5	0.45	0.75	0.65	0.65
	Realism and Immersion (C_{21})	Benefit	0.93	0.75	0.65	0.75	0.65	0.65	0.5	0.75
	User Feedback Mechanisms (C_{22})	Benefit	0.75	0.75	0.5	0.75	0.5	0.93	0.25	0.65

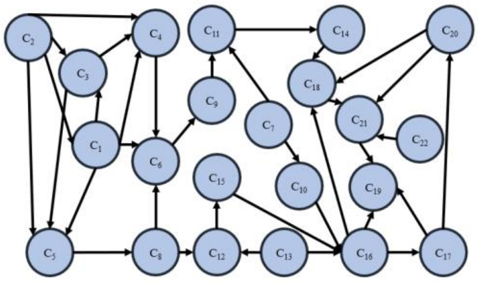


Fig. 6. Chain of factors and their interactions in supply chain risk management using the R.Graph method

In the next phase, experts defined the causal relationships between different criteria through an interactive process, with the corresponding R.Graph figure illustrating the relationships among attributes depicted in Fig. 6. These visual aids clarify both the magnitude and direction of the interdependencies, helping to identify the most critical factors in supply chain risk management. To further elucidate the complex interrelations among the criteria, we conducted a meticulously designed questionnaire targeting domain experts. The primary objective was to evaluate the impacts and interdependencies among various criteria by soliciting qualitative percentage relationships between different attributes. For example, experts were asked the following question: "For Criteria 1 (Risk Identification), what is the percentage change in Criteria 'Monitoring and Alerting' (C3), 'Accuracy' (C4), 'Response Time' (C5), and 'Resource Utilization' (C6) if Criteria 1 changes by 100%?" The results are provided in Table 5, offering a qualitative assessment, while Table 6 quantitatively examines the relationships among criteria using the corresponding crisp values for the aforementioned linguistic terms. This comprehensive analytical process reveals synergistic effects, where improvements in areas such as the accuracy of demand volatility forecasts lead to significant enhancements in other domains, including cost efficiency and expedited decision-making. This interconnected view is essential for developing holistic strategies that enhance the overall effectiveness and resilience of supply chains.

Table 5. Matrix of Factors and their Impact in Supply Chain Risk Management

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22
C1	0	0	VH	MH	M	ML	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C2	L	0	M	L	H	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C3	0	0	0	H	M	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C4	0	0	0	0	0	L	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C5	0	0	0	0	0	0	0	H	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C6	0	0	0	0	0	0	0	0	H	0	0	0	0	0	0	0	0	0	0	0	0	0
C7	0	0	0	0	0	0	0	0	0	L	M	0	0	0	0	0	0	0	0	0	0	0
C8	0	0	0	0	0	M	0	0	0	0	0	M	0	0	0	0	0	0	0	0	0	0
C9	0	0	0	0	0	0	0	0	0	0	L	0	0	0	0	0	0	0	0	0	0	0
C10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0	0	0	0	0	0
C11	0	0	0	0	0	0	0	0	0	0	0	0	0	L	0	0	0	0	0	0	0	0
C12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0	0	0	0	0	0	0
C13	0	0	0	0	0	0	0	0	0	0	0	M	0	0	0	M	0	0	0	0	0	0
C14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	L	0	0	0	0
C15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0	0	0	0	0	0
C16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	L	M	0	0	0
C17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	L	M	0	0
C18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0
C19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0	0	M	0
C21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0	0	0
C22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	M	0

Very High (VH), High (H), Moderately High (MH), Medium (M), Moderately Low (ML), Low (L) and Very Low (VL)

Table 6. Quantified Matrix of Factors and their Impact in Supply Chain Risk Management

	C_{1}	C_{2}	C_{3}	C_{4}	C_{5}	C_{6}	C_{7}	C_{8}	C_{9}	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}	C_{21}	C_{22}
C_{1}	0	0	0.93	0.65	0.5	0.45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{2}	0.25	0	0.5	0.25	0.75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{3}	0	0	0	0.75	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{4}	0	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{5}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{6}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{7}	0	0	0	0	0	0	0	0	0	0.25	0.5	0	0	0	0	0	0	0	0	0	0	0
C_{8}	0	0	0	0	0	0.5	0	0	0	0	0	0.5	0	0	0	0	0	0	0	0	0	0
C_{9}	0	0	0	0	0	0	0	0	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0

C_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0	0
C_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0	0	0	0	0	0	0	0
C_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0	0	0
C_{13}	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0.5	0	0	0	0	0	0
C_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3	0	0	0	0
C_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0	0	0	0
C_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0.3	0.5	0	0	0
C_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.3	0.5		0
C_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0
C_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0.5	0
C_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0	0	0
C_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.5	0

Table 7. Ranking Results of Algorithms 1 and 2 Compared with Other MADM Approaches

	Proposed Model				**TOPSIS**				**MACBETH**				**WASPAS**				**TODIM**				**MABAC**				**MARCOS**			
Method	**Algorithm 1**		**Algorithm 2**		**Dematel**		**Combined**		**Dematel**		**Combined**		**Dematel**		**Combined**		**Dematel**		**Combined**		**Dematel**		**Combined**		**Dematel**		**Combined**	
	Utility	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**	**Utility**	**Rank**
A1					0.826	1	0.827	1	82.522	1	82.677	1	0.889	1	0.889	1	1.000	1	1.000	1	0.286	1	0.287	1	0.856	1	0.857	1
A2					0.481	6	0.484	6	51.425	6	51.521	6	0.575	6	0.574	6	0.492	6	0.491	6	-0.025	6	-0.024	6	0.586	6	0.586	6
A3					0.782	2	0.790	2	81.805	2	82.145	2	0.871	2	0.874	2	0.948	2	0.946	2	0.279	2	0.282	2	0.843	2	0.846	2
A4					0.255	8	0.255	8	24.111	7	23.765	8	0.341	8	0.337	8	0.000	8	0.000	8	-0.298	7	-0.302	8	0.371	8	0.367	8
A5					0.694	3	0.697	3	72.634	3	72.805	3	0.786	3	0.786	3	0.821	3	0.821	3	0.187	3	0.189	3	0.760	3	0.761	3
A6					0.660	4	0.662	4	65.752	4	65.989	4	0.750	4	0.750	4	0.750	4	0.750	4	0.119	4	0.120	4	0.728	4	0.729	4
A7					0.320	7	0.318	7	23.846	8	23.872	7	0.397	7	0.392	7	0.058	7	0.056	7	-0.300	8	-0.301	7	0.410	7	0.407	7
A8					0.552	5	0.548	5	57.799	5	57.635	5	0.636	5	0.632	5	0.611	5	0.613	5	0.039	5	0.037	5	0.646	5	0.644	5

51. Step-by-Step Analysis of the Outputs Obtained by the Algorithm 1 and Algorithm 2 Methods

First, we utilized Algorithm 1 to calculate the rankings of the alternatives. Since all attributes are of the beneficial type, there is no need to apply a negative sign to any non-beneficial attributes. Using this algorithm, we captured both the direct and indirect effects of one alternative on other criteria. Next, in the first step, we calculated the influence matrix using Eqs. (14) and (15), and the resulting matrix is presented in Table A-1 (Appendix) . Following this, in the next step, the influence matrix was normalized using logistic normalization (Eqs. (16) and (17)) to ensure that all values fall between 0 and 1, facilitating better comparisons across alternatives. Finally, in Step 4, we computed the weighted sum by multiplying the normalized matrix by the subjective weights assigned to each criterion; the results are presented in Table A-2 (Appendix). This weighted matrix was then used to determine the overall ranking of the alternatives, enabling a comprehensive comparison of VR models for managing supply chain risk. The final ranking results from Algorithm 1 are presented in Table 7.

In Algorithm 2, we calculate the weights of each attribute by considering their interactions and subjective weights. In the first step, we define the appropriate inputs. Instead of utilizing a full decision matrix, we assume that a single event affects all criteria equally, simplifying the process. In Step 2, we calculate the total influence of each attribute on the others, resulting in a 22×22 influence matrix (I) using Eqs. (20-22). The results from this step are presented in Table A-3 (Appendix), which outlines the interactions and the computed values of the influence matrix. In the next step, we normalize the influence matrix using logistic normalization (Eqs. (23) and (24)) to ensure that all values fall within the range of 0 and 1, allowing for consistent comparisons across attributes. Finally, we determine the combined weight of each attribute by considering both the subjective weights and the normalized influence matrix using Eq. (25), as depicted in Table #. These weights can then be multiplied by the decision matrix to effectively rank the alternatives. The results from this step are summarized in Table 7, displaying the combined weights of the attributes used for ranking.

5.2. Validation of the Results

MADMtechniques aim to provide reliable and consistent results; however, their rankings can fluctuate due to factors such as variations in criterion weights, changes in alternatives, subjective judgments, and criteria selection. This section validates the case study results through various methods. Section 5.2.1 compares case study rankings with established MADM methods to evaluate correlations. Section 5.2.2 assesses ranking stability in response to input weight changes, while Section 5.2.3 conducts rank reversal analysis to explore these dynamics.

5.2.1. Ranking Results of other Methods

This section evaluates the results of the proposed algorithms against various alternative ranking methodologies to assess their concordance. To this end, we first calculated the new weights of the criteria by considering the interactions and subjective weights using the DEMATEL method, with the obtained weights presented in Table 3#. Subsequently, we employed several common ranking techniques, including MABAC, WASPAS, TODIM, MACBETH, MARCOS, and TOPSIS, incorporating the DEMATEL weights. The results are summarized in Table 7, where discrepancies are highlighted in blue. We then compared these findings with the results obtained from Algorithms 1 and 2, as well as the DEMATEL method, also shown in Table 7.

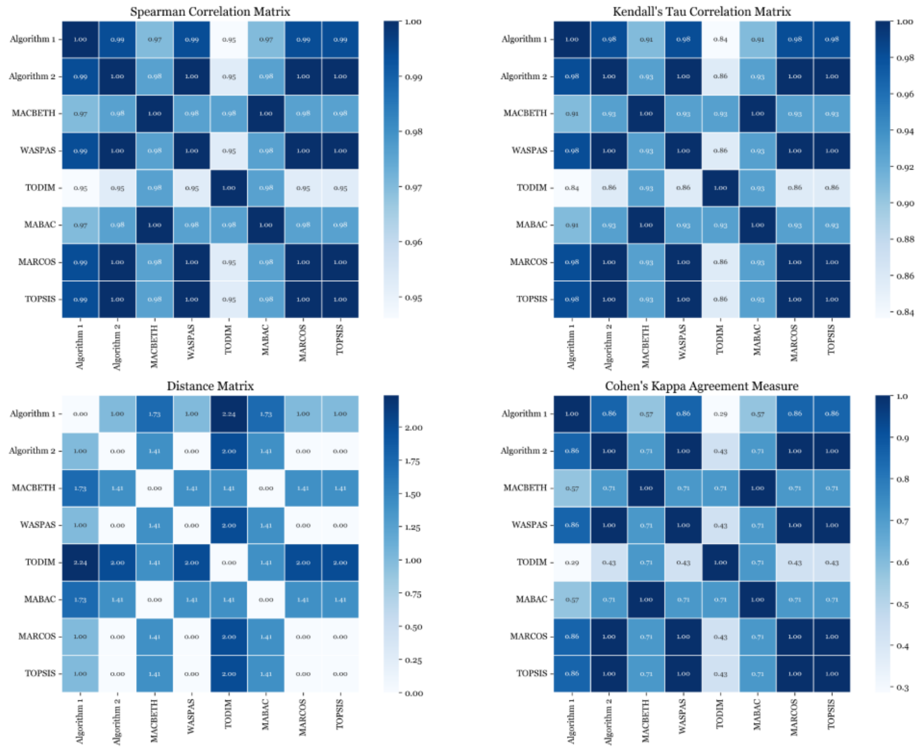


Fig. 7. Rank Correlation Measures with Different Methods

Spearman's correlation coefficient analysis, in conjunction with three sophisticated statistical methodologies, was employed to evaluate the coherence and divergence among multiple MADM techniques. These comprehensive results are encapsulated in Fig. 7, which amalgamates four distinct analytical outputs into a singular, cohesive illustration. The Kendall's Tau Correlation Matrix examines the ordinal interrelations across methods, elucidating the degree of concordance in their ranking outcomes. The Distance Matrix computes the Euclidean separation between rankings, providing nuanced insights into the dissimilarities and methodological divergence. Moreover, Cohen's Kappa Agreement Measure assesses the extent of alignment between method pairs, accounting for the likelihood of fortuitous agreement. Together with Spearman's correlation, these advanced analyses furnish a rigorous and granular understanding of the consistency and variability in ranking outcomes across methods, as presented in Fig. 7.

The comparative analysis shows that our proposed models perform consistently well across all evaluation measures, including the Spearman and Kendall's Tau Correlation matrices, the Distance matrix, and Cohen's Kappa Agreement measure. The proposed models exhibit strong positive correlations with established methods like MACBETH, WASPAS, and VIKOR, indicating high alignment in ranking results. The low distance values further demonstrate minimal differences in rankings compared to other approaches, while the high agreement in Cohen's Kappa confirms the stability and reliability of the proposed models. Overall, our models provide robust performance, comparable to traditional MADM methods, with no risk of rank reversal. The reason the results are close across the various methods is due to the high number of attributes (22), combined with the fact that only a few attributes significantly influence each other. This leads to similar weight distributions, which is why the weights obtained from the DEMATEL method closely align with those derived from Algorithm 2. Additionally, the slight discrepancies between the final rankings produced by our proposed Algorithms 1 and 2 can be attributed to the nature of Algorithm 1, which is designed to calculate the rankings directly without first determining the weights. This direct ranking approach results in minor variations when compared to Algorithm 2, which computes the weights before ranking the alternatives.

5.2.2. Sensitivity Analysis

The objective of conducting a sensitivity analysis within a MADM algorithm is to assess how changes in predetermined conditions affect the resulting rankings. This analysis is crucial for evaluating the robustness and reliability of the MADM outcomes. Notably, variations in criteria weights significantly impact final rankings. In this section, we examine result stability against intentional perturbations in the input weight matrix. Given the importance of subjective weights for decision-makers, an error margin is introduced to these values to evaluate its effect on alternative performance. To assess sensitivity to fluctuations in weights w_S^j, a percentage change, denoted as ererer, is applied, resulting in adjusted weights w_j^{ms}, calculated as follows:

$$w_j^{ms} = (1 - \frac{er}{100})\, w_S^j \tag{26}$$

And other modified attributes weight due to the change in w_S^j is calculated as:

$$w_i^s = \frac{(1-\left(1-\frac{er}{100}\right)w_j^s)w_i^s}{1-w_j^s} \qquad \forall\, i = 1, \dots m,\; j = 1, \dots, n \tag{27}$$

For analyzing the sensitivity, we consider 13 different values for er, i.e.,

$er \in \{-100\%, -77\%, -55\%, -33\%, -11\%, 11\%, 33\%, 55\%, 77\%, 100\%\}$

The overall performance of all alternatives is calculated for each percentage error value along with the corresponding adjusted weights, and the resulting data points are illustrated in a scatter plot, as shown in Fig. 8. This visual representation provides a clear depiction of the relationship between performance and the degree of error in weight modifications, allowing for a more detailed analysis of how variations in weight assignments impact the overall evaluation of alternatives.

The radar charts in Fig. 8 offer a visual analysis of weight sensitivity across various attributes under different weight fluctuation scenarios. Each chart focuses on a specific attribute, illustrating how the weights assigned to it change across ten scenarios, ranging from -100% to +100% adjustments. This analysis is crucial for evaluating the robustness of the decision-making process. Most attributes display a consistent trend, indicating that the overall framework is resilient to weight variations. However, attributes such as 1, 5, 10, and 21 exhibit significant fluctuations, suggesting a greater impact on the final decision. Additionally, attributes 2, 3, 7, 14, 15, and 18 show some changes. Since the initial subjective weights of these criteria are relatively close to each other, the fluctuations in these attributes reflect changes in importance due to interactions between criteria and the nature of the decision matrix itself.

Fig. 8. Impact of weight sensitivity

5.2.3. Rank Reversal Analysis

The rank reversal phenomenon is explored by simulating the deliberate deletion of alternatives in two different sequences: from best to worst and from worst to best (Jiang et al., 2024). These scenarios are illustrated in Fig. 10, where variations in ranking results are shown as alternatives are deleted in both sequences. It is assumed that when the alternatives are ranked from best to worst and the best alternative is removed, the second-best should take its place (Su et al., 2023). However, as observed in Fig. 10, deleting the best alternative (A1) leads to an interchange in ranking between the second-best (A3) and the third-best alternatives, while the other alternatives exhibit a smoother decline in their positions. Similarly, when deleting from worst to best, dynamic changes in rankings occur, but the pattern of movement differs depending on the sequence of deletion (Faramondi et al., 2023).

The results indicate that the proposed model is free from the rank reversal problem when alternatives are removed, whether from best to worst or worst to best. This is because the model does not consider the relationships between alternatives, eliminating the need for normalization of the decision matrix.

Additionally, since logistic normalization is applied, which inherently does not lead to rank reversal, the method consistently maintains ranking stability. Consequently, the approach is robust and ensures that rank reversal is avoided, supporting reliable decision-making.

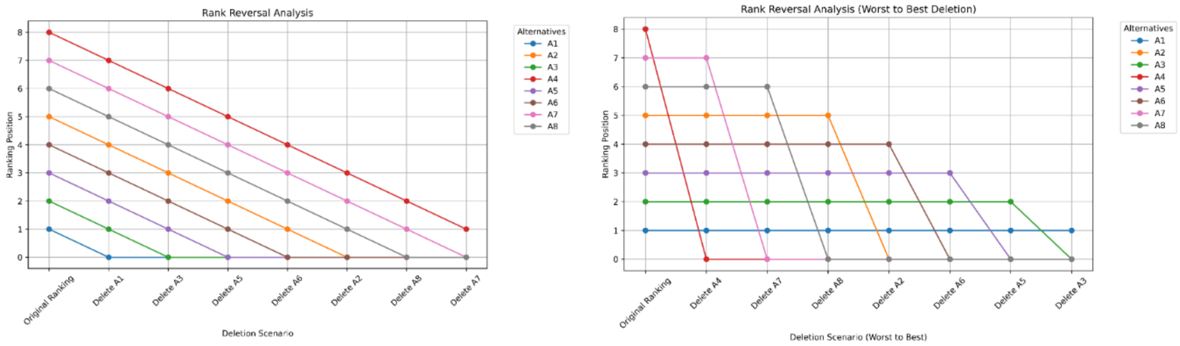


Fig. 9. Variations in ranking results by deleting the alternatives

5.3. Discussion and Managerial Insights

In this study, we propose two causal MADM models based on R.Graph models, designed to address decision-making problems characterized by interactive criteria. These models are specifically tailored for scenarios where direct relationships exist between criteria and alternatives. We consider specific interaction scenarios within defined time periods to select the most effective human-centric VR technology in the context of supply chain risk management. Our proposed models differ from existing frameworks, such as Bayesian networks, by requiring fewer calculations and simplifying data inputs. Furthermore, the models are capable of taking dynamic inputs and user feedback to continuously update information, ensuring that the decision-making process remains relevant and responsive to changing conditions. They accommodate both quantitative and qualitative data, allowing historical data to be quantified effectively. This enables us to investigate various scenarios involving interactions within a single loop over a specified period, focusing on both positive and negative impacts rather than mere correlations. These frameworks prove to be practical for strategic decision-making within the MADM field, especially when decision-makers aim to explore the interactions among critical criteria. One notable advantage of our first model is that it does not necessitate the calculation of attribute weights, thereby preventing information loss. Furthermore, our models effectively eliminate the rank reversal problem since no direct relationships exist between alternatives. This versatility allows the model to be applied to various decision-making challenges in MADM, such as resource allocation, supplier selection, and project prioritization, among others. By leveraging these frameworks, organizations can enhance their decision-making processes and improve outcomes in complex, interactive environments.

The proposed model enhances decision-making by enabling managers to evaluate various VR platforms based on human-centric attributes. This targeted analysis ensures that selected technologies align with organizational needs and user preferences, leading to more effective implementations. Additionally, the model's adaptability across diverse industrial applications allows managers to tailor VR solutions to specific operational contexts. This flexibility enhances the overall effectiveness of supply chain management strategies. A human-centric design approach prioritizes user needs, leading to higher engagement and satisfaction. Managers can create immersive training environments and collaborative platforms that resonate with employees, boosting productivity. Establishing user feedback mechanisms allows for continuous refinement of VR systems. By incorporating real-world insights, organizations can enhance the long-term effectiveness and sustainability of their VR applications in supply chain management. Employing VR technologies for immersive training and simulation helps managers prepare their teams for potential disruptions. This proactive approach to risk management fosters resilience within the supply chain, ensuring organizations are well-equipped to navigate challenges. Finally, the integration of VR technologies with advanced decision-making models encourages innovation. Managers can leverage these tools to explore new possibilities, streamline processes, and enhance overall operational efficiency.

Incorporating dynamic modeling techniques empowers managers to respond swiftly to changes in supply chain dynamics. By leveraging real-time data, organizations can proactively adjust their strategies, minimizing risks associated with market fluctuations. Utilizing the R.Graph framework and E-E matrices enables comprehensive assessment of interactions between alternatives. This deeper analysis aids in making informed decisions regarding platform selection, ultimately enhancing collaboration among supply chain partners. The key capabilities and characteristics of our model are:

- **Human-Centric Attributes Assessment:** Evaluates Virtual Reality (VR) platforms based on critical human-centric factors such as user engagement, satisfaction, usability, and adaptability to individual workflows, ensuring alignment with user needs in the supply chain.
- **Dynamic Weight Adjustment:** Employs a mechanism for continuously estimating and updating the weights of criteria in response to evolving organizational requirements and market dynamics, facilitating ongoing relevance in decision-making.
- **Integration of R.Graph Methodology:** Utilizes the R.Graph method to analyze the complex interactions among criteria, enhancing the depth and nuance of decision-making processes.
- **Interaction Analysis:** Assesses the interactions between alternatives, allowing for a comprehensive evaluation of relationships among various VR platforms, which helps in identifying synergies and potential conflicts.
- **Real-Time Data Integration:** Incorporates dynamic modeling techniques to reflect real-time changes in supply chain dynamics, ensuring decisions are based on the most current data available.
- **User Feedback Mechanisms:** Integrates ongoing user feedback to continuously refine the model, ensuring long-term effectiveness and sustainability of VR systems within supply chain management.
- **Multi-Industry Applicability:** Designed to be flexible and adaptable across various industries, allowing customization to specific operational contexts and enhancing overall effectiveness.

- **Support for Immersive Training Environments:** Facilitates the creation of immersive training and simulation environments, equipping teams to effectively manage potential supply chain disruptions.
- **Enhanced Collaboration Framework:** Promotes improved collaboration among supply chain partners through selected VR platforms, enabling real-time communication and problem-solving critical for effective risk management.

6. Conclusion

In conclusion, this study highlights the critical role of Virtual Reality (VR) technologies in enhancing Supply Chain Management (SCM) through advanced risk management practices. A central focus of our research was the integration of human-centric networks, essential for developing and implementing AI technologies. By prioritizing users' needs, preferences, and capabilities, human-centric design ensures that technology aligns with workflows, fostering engagement and productivity. We developed two frameworks: the first directly calculates the ranks of VR platforms based on human-centric attributes, while the second updates the weights of criteria for dynamic adjustments to evolving organizational needs. The first algorithm closely aligns with the R.Graph method, distinguishing it from existing approaches that may overlook the interactions among criteria. The selected VR platforms, Bentley Synchro XR and Augmentir, demonstrate the transformative potential of immersive technologies in SCM. Managers can leverage these platforms to create immersive training environments, enhance collaboration among supply chain partners, and visualize complex data for better decision-making. By facilitating real-time communication and problem-solving, these VR technologies enable organizations to effectively manage risks and adapt to disruptions.

Future studies should focus on validating the proposed model across diverse industrial applications to assess its adaptability and effectiveness in various operational contexts. Incorporating data-driven methodologies will enhance the model's predictive capabilities, enabling organizations to respond more adeptly to market fluctuations. Additionally, research should investigate the incorporation of interactions between alternatives within the R.Graph framework, facilitating a more comprehensive analysis of the relationships among options. Moreover, exploring dynamic modeling techniques that reflect real-time changes in supply chain dynamics will significantly enrich the decision-making process. Lastly, investigating user feedback mechanisms for ongoing refinement will be essential in ensuring the long-term effectiveness and sustainability of VR systems in supply chain management. Ultimately, the intersection of VR technologies, SCM, and advanced decision-making models presents significant opportunities for fostering resilience and innovation in supply chains within the rapidly evolving digital landscape.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration Regarding the Use of Generative AI and AI-Assisted Technologies in the Writing Process

In the course of preparing this manuscript, the authors utilized OpenAI's ChatGPT tool to assist in editing and drafting certain sections of the paper. Following the use of this service, the authors thoroughly reviewed and revised the content as necessary, assuming full responsibility for the final publication.

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Appendix

Algorithm 1:

Input: Decision matrix (X), Interaction matrix ($C - C$) and subjective weight matrix (W)
Output: Ranked Alternatives

Step 1: *Development of Decision and Interaction Matrices Along with Criteria Weight Assignment*

$$X = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_1 & \cdots & c_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \end{array} \text{ where } \begin{cases} x_{ij} \geq 0 \text{ for beneficial attributes} \\ x_{ij} \leq 0 \text{ for non} - \text{beneficial attributes} \end{cases}$$

$$C - C = \begin{array}{c} \\ C_1 \\ C_2 \\ \cdots \\ C_j \\ \cdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & \dots & C_j & \dots & C_n \end{array} \\ \left[\begin{bmatrix} 0 & \alpha_{12} & \dots & \alpha_{1v} & \dots & \alpha_{1n} \\ \alpha_{21} & 0 & \dots & \alpha_{2v} & \dots & \alpha_{2n} \\ \dots & \dots & 0 & \dots & \dots & \dots \\ \alpha_{j1} & \alpha_{j2} & \dots & 0 & \dots & \alpha_{jn} \\ \dots & 0 & \dots & \dots & 0 & \dots \\ \alpha_{n1} & \alpha_{n2} & 0 & \alpha_{nj} & \dots & 0 \end{bmatrix}\right] \end{array} \quad \& W = [w_{js}]_{1\times n}$$

Step 2: *Calculating the Influenced Decision Matrix*

$X_i{}^I = [R(C_j|A_i)]_{m\times n}$ where: $R(C_j|A_i)^N = \frac{1}{1+e^{-R(C_j|A_i)}}$

Step 3: *Calculating the Normalized Influenced Decision Matrix*

$X_i{}^{IN} = \left[R(C_j|A_i)^N\right]_{m\times n}$ where: $R(C_j|A_i)^N = \frac{1}{1+e^{-R(C_j|A_i)}}$

Step 4: *Calculating the Weighted Sum and Ranking*

$$Score\ (A_i) = \sum_{j=1}^{m} w_{js} R(C_j|A_i)^N$$

Algorithm 2:

Input: Decision matrix (X), Interaction matrix ($C - C$) and subjective weight matrix (W)
Output: Ranked Alternatives

Step 1: *Determination of Appropriate Inputs*

$$X = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_1 & \cdots & c_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{pmatrix} \end{array} \text{where } \begin{cases} x_{ij} \geq 0 \text{ for beneficial attributes} \\ x_{ij} \leq 0 \text{ for non} - \text{beneficial attributes} \end{cases}$$

$$C - C = \begin{array}{c} \\ \boldsymbol{C_1} \\ \boldsymbol{C_2} \\ \cdots \\ \boldsymbol{C_j} \\ \cdots \\ \boldsymbol{C_n} \end{array} \begin{array}{c} \begin{array}{cccccc} \boldsymbol{C_1} & \boldsymbol{C_2} & \dots & \boldsymbol{C_j} & \dots & \boldsymbol{C_n} \end{array} \\ \begin{bmatrix} 0 & \alpha_{12} & \dots & \alpha_{1v} & \dots & \alpha_{1n} \\ \alpha_{21} & 0 & \dots & \alpha_{2v} & \dots & \alpha_{2n} \\ \dots & \dots & 0 & \dots & \dots & \dots \\ \alpha_{j1} & \alpha_{j2} & \dots & 0 & \dots & \alpha_{jn} \\ \dots & 0 & \dots & \dots & 0 & \dots \\ \alpha_{n1} & \alpha_{n2} & 0 & \alpha_{nj} & \dots & 0 \end{bmatrix} \end{array} \quad \& W = [w_{js}]_{1\times n}$$

Step 2: *Determination the influence matrix*

$I = [R(C_j|C_i)]_{n\times n}$ Where: $R(C_j|C_i) = \alpha_{iv} R(C_i) + \sum_{l=1}^{n} \alpha_{lv} R(C_l|C_i)$ & $R(C_j) = 1 + R\left(C_j|Par(C_j)\right) = 1 + \sum_{i=1}^{n} \alpha_{ij} R(C_i)$

Step 3: *Obtaining the Normalized Influence Decision Matrix*

$I^N = \left[R(C_j|C_i)^N\right]_{m\times n}$
Where: $R(C_j|C_i)^N = \frac{1}{1+e^{-R(C_j|C_i)}}$

Step 4: Calculating the combined weight of each attribute

$$weight\ (C_j) = \frac{\sum_{i=1}^{n} w_{is} R(C_j|C_i)^N}{\sum_{j=1}^{n} \sum_{i=1}^{n} w_{is} R(C_j|C_i)^N}$$

Questionnaire

Instructions: For each question, please provide the quliative percentage change using scales based on the scenario described. The questionnaire aims to assess the impacts and relationships between criteria. For all questions related to Criteria, assume a 100% percentage change in the value of the criteria.

1. For Criteria 1 (Risk Identification); what is the percentage change in the Criteria “Monitoring and Alerting” (C3), “Accuracy” (C4), “Response Time” (C5), and “Resource Utilization” (C6) if Criteria 1 changes?

   a) Monitoring and Alerting (C3):
   b) Accuracy (C4):
   c) Response Time (C5):
   d) Resource Utilization (C6):

2. For Criteria 2 (Mitigation Capabilities); what is the percentage change in the Criteria “Risk Identification” (C1), “Monitoring and Alerting” (C3), “Accuracy” (C4), and “Response Time” (C5) if Criteria 2 changes?

   e) Risk Identification (C1):
   f) Monitoring and Alerting (C3):
   g) Accuracy (C4):
   h) Response Time (C5):

3. For Criteria 3 (Monitoring and Alerting); what is the percentage change in the Criteria “Accuracy” (C4) and “Response Time” (C5) if Criteria 3 changes?

   i) Accuracy (C4):
   j) Response Time (C5):

4. For Criteria 4 (Accuracy); what is the percentage change in the Criteria “Resource Utilization” (C6) if Criteria 4 changes?

   k) Resource Utilization (C6):

5. For Criteria 5 (Response Time); what is the percentage change in the Criteria “Operational Efficiency” (C8) if Criteria 5 changes?

   l) Operational Efficiency (C8):

6. For Criteria 6 (Resource Utilization); what is the percentage change in the Criteria “Scalability” (C9) if Criteria 6 changes?

   m) Scalability (C9):

7. For Criteria 7 (Cost-Effectiveness); what is the percentage change in the Criteria “Flexibility” (C10) and “Compatibility” (C11) if Criteria 7 changes?

   n) Flexibility (C10):
   o) Compatibility (C11):

8. For Criteria 8 (Operational Efficiency); what is the percentage change in the Criteria “Resource Utilization” (C6) and “User Training and Support” (C12) if Criteria 8 changes?

p) Resource Utilization (C6)

q) User Training and Support (C12):

9. For Criteria 9 (Scalability); what is the percentage change in the Criteria “Compatibility” (C11) if Criteria 9 changes?

r) Compatibility (C11):

10. For Criteria 10 (Flexibility); what is the percentage change in the Criteria “Financial Stability” (C16) if Criteria 10 changes?

s) Financial Stability (C16)

11. For Criteria 11 (Compatibility); what is the percentage change in the Criteria “Safety and Compliance” (C15) if Criteria 11 changes?

t) Safety and Compliance (C14):

12. For Criteria 12 (User Training and Support); what is the percentage change in the Criteria “Reputation Management” (C15) if Criteria 12 changes?

u) Reputation Management (C15)

13. For Criteria 13 (Operational Continuity); what is the percentage change in the Criteria “User Training and Support” (C12) and “Financial Stability” (C16) if Criteria 13 changes?

v) User Training and Support (C12):

w) Financial Stability (C16)

14. For Criteria 14 (Safety and Compliance); what is the percentage change in the Criteria “Accessibility” (C18) if Criteria 14 changes?

x) Accessibility (C18)

15. For Criteria 15 (Reputation Management); what is the percentage change in the Criteria “Financial Stability” (C16) if Criteria 15 changes?

y) Financial Stability (C16)

16. For Criteria 16 (Financial Stability); what is the percentage change in the Criteria “User Experience (UX)” (C17), “Accessibility” (C18), and “Collaboration Features” (C19) if Criteria 16 changes?

z) User Experience (C17):

aa) Accessibility (C18):

bb) Collaboration Features (C19):

17. For Criteria 17 (User Experience); what is the percentage change in the Criteria “Collaboration Features” (C19) and “Customization Options” (C20) if Criteria 17 changes?

cc) Collaboration Features (C19):

dd) Customization Options (C20):

18. For Criteria 18 (Accessibility); what is the percentage change in the Criteria "Realism and Immersion" (C21) if Criteria 18 changes?

    ee) Realism and Immersion (C21):

19. For Criteria 20 (Customization Options); what is the percentage change in the Criteria "Accessibility" (C18) and "Realism and Immersion" (C21) if Criteria 20 changes?

    ff) Accessibility (C18):

    gg) Realism and Immersion (C21):

20. For Criteria 21 (Realism and Immersion); what is the percentage change in the Criteria "Collaboration Features" (C19) if Criteria 21 changes?

    hh) Collaboration Features (C19):

21. For Criteria 22 (User Feedback Mechanisms); what is the percentage change in the Criteria "Realism and Immersion" (C21) if Criteria 21 changes?

    ii) Realism and Immersion (C21):

Table A-1. The influenced matrix of Algorithm 1

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22
A1	1.118	0.750	2.344	3.422	3.043	3.805	0.750	3.033	3.603	1.118	2.026	2.641	0.750	1.436	2.071	2.719	2.110	2.391	4.313	1.705	3.353	0.750
A2	0.913	0.650	1.674	2.661	1.851	2.345	0.500	2.038	2.259	0.625	0.885	1.769	0.500	0.721	1.384	1.505	1.682	1.952	3.346	1.491	2.847	0.750
A3	0.983	0.930	2.129	3.398	3.183	3.880	0.750	3.317	3.660	0.938	2.040	2.684	0.750	1.010	2.272	2.730	2.115	2.589	4.298	1.807	2.948	0.500
A4	0.313	0.250	0.486	0.700	1.237	0.884	0.250	0.997	0.913	0.313	0.603	0.874	0.250	0.221	0.687	0.695	0.847	1.516	2.519	1.074	2.420	0.750
A5	0.913	0.650	1.824	2.623	2.356	3.075	0.750	2.517	2.956	0.938	1.864	2.283	0.750	1.116	1.892	2.540	1.920	2.609	4.062	1.890	3.124	0.450
A6	1.118	0.750	1.914	3.000	3.008	3.131	0.500	2.756	2.998	0.575	1.750	2.453	0.650	1.187	1.677	2.101	1.800	2.172	3.701	1.400	2.901	0.930
A7	0.195	0.500	0.881	1.163	1.163	1.490	0.450	1.322	1.617	0.563	1.079	0.956	0.450	0.770	0.548	0.850	0.925	1.411	2.100	1.113	1.887	0.250
A8	0.518	0.070	1.266	1.804	1.394	1.782	0.500	1.696	1.406	0.875	1.532	1.923	0.650	1.133	1.711	2.118	1.809	2.240	3.748	1.555	2.972	0.650

Table A-2. Weighted normalized influenced matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22
A1	0.040	0.044	0.008	0.008	0.041	0.025	0.021	0.013	0.024	0.022	0.021	0.033	0.031	0.028	0.007	0.034	0.035	0.026	0.012	0.003	0.020	0.004
A2	0.060	0.059	0.015	0.015	0.078	0.049	0.029	0.026	0.046	0.034	0.037	0.062	0.043	0.045	0.012	0.064	0.063	0.048	0.023	0.005	0.038	0.005
A3	0.057	0.057	0.014	0.015	0.071	0.045	0.026	0.024	0.043	0.029	0.030	0.057	0.039	0.037	0.011	0.056	0.059	0.046	0.023	0.005	0.037	0.005
A4	0.058	0.063	0.015	0.015	0.079	0.049	0.029	0.026	0.046	0.032	0.037	0.062	0.043	0.041	0.012	0.064	0.063	0.049	0.023	0.005	0.038	0.005
A5	0.046	0.049	0.010	0.011	0.063	0.035	0.024	0.020	0.034	0.026	0.027	0.047	0.035	0.031	0.009	0.046	0.049	0.043	0.022	0.004	0.036	0.005
A6	0.057	0.057	0.014	0.015	0.075	0.048	0.029	0.025	0.045	0.032	0.036	0.060	0.043	0.042	0.012	0.063	0.061	0.049	0.023	0.005	0.038	0.005
A7	0.060	0.059	0.015	0.015	0.078	0.048	0.026	0.025	0.045	0.029	0.036	0.061	0.041	0.042	0.011	0.061	0.060	0.047	0.023	0.005	0.038	0.006
A8	0.044	0.054	0.012	0.012	0.062	0.041	0.026	0.021	0.039	0.028	0.031	0.048	0.038	0.038	0.008	0.048	0.050	0.042	0.021	0.005	0.034	0.004

Table A-3. The influence matrix of Algorithm 2

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22
C1	0	0	1.163	1.684	1.206	1.436	0	0.905	1.077	0	0.269	0.452	0	0.067	0.226	0.113	0.057	0.059	0.093	0.028	0.044	0
C2	0.250	0	0.733	0.962	1.241	0.818	0	0.931	0.614	0	0.153	0.465	0	0.038	0.233	0.116	0.058	0.053	0.093	0.029	0.041	0

C3	0	0	0	1.997	1.331	0.998	0	0.998	0.749	0	0.187	0.499	0	0.047	0.250	0.125	0.062	0.059	0.100	0.031	0.045	0
C4	0	0	0	0	0	1.015	0	0	0.761	0	0.190	0	0	0.048	0	0	0	0.012	0.003	0	0.006	0
C5	0	0	0	0	0	1.390	0	2.780	1.042	0	0.261	1.390	0	0.065	0.695	0.347	0.174	0.147	0.276	0.087	0.117	0
C6	0	0	0	0	0	0	0	0	3.350	0	0.838	0	0	0.209	0	0	0	0.052	0.013	0	0.026	0
C7	0	0	0	0	0	0	0	0	0	0.250	0.500	0	0	0.125	0	0.125	0.063	0.078	0.105	0.031	0.055	0
C8	0	0	0	0	0	1.890	0	0	1.417	0	0.354	1.890	0	0.089	0.945	0.472	0.236	0.199	0.375	0.118	0.159	0
C9	0	0	0	0	0	0	0	0	0	0	1.088	0	0	0.272	0	0	0	0.068	0.017	0	0.034	0
C10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.625	0.313	0.234	0.488	0.156	0.195	0
C11	0	0	0	0	0	0	0	0	0	0	0	0	0	0.647	0	0	0	0.162	0.040	0	0.081	0
C12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.695	0.847	0.424	0.318	0.662	0.212	0.265	0
C13	0	0	0	0	0	0	0	0	0	0	0	0.500	0	0	0.250	0.625	0.313	0.234	0.488	0.156	0.195	0
C14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.412	0.103	0	0.206	0
C15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.347	0.674	0.505	1.053	0.337	0.421	0
C16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.736	1.302	2.713	0.868	1.085	0
C17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.684	1.197	1.368	1.026	0
C18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.866	0	1.732	0
C19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.184	0.888	0	1.776	0
C21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2.208	0	0	0
C22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.250	0	0.500	0