TOWARDS A HIGH GENUS COSMIC MANIFOLD

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ABSTRACT

The universe is punctured with a high number of black holes that might stand for topological "holes" or "cavities". Starting from this observation, we discuss the possibility to think over the universe in terms of a manifold of very high genus (henceforward VHGM), rather than an isotropic and homogeneous manifold of genus zero. We argue about the feasibility and the likely physical consequences of a cosmic VHGM approach. In ΛCDM, a topological surface of very high genus might guarantee highly stable, topologically ordered structures protected against small perturbations, leading to fine-tuning and consistency of cosmic parameters. In non-standard cosmological models, a VHGM framework might help to elucidate spatial features like extra-dimensions, compactified universes, spacetime foam, multi-loops string interactions. Groups of almost infinite surface diffeomorphisms might also generate topological defects, impurities, passages and handles that connect different regions at either micro-, meso- or macroscopic physical scales, shedding light on the accelerated expansion of the Universe, on wormholes, on multiverse and bubble universes. VHGM might also contribute to the nonlocal interactions that are typical of quantum entanglement and to information and energy storage, with special attention to the quantum vacuum discrepancy and the holographic approaches. In conclusion, the theoretical possibility of a cosmic VHGM suggests a strong and methodologically manageable mathematical background subtending the mechanical, dynamical, energetic and informational features of the spacetime fabric in both ΛCDM and non-standard cosmological models.

Keywords: cosmology; astrophysics; ΛCDM; geometric topology; Planck results.

INTRODUCTION

The universe, isotropic and homogeneous at very large scales, can be regarded as a spatially flat, genus-zero four-dimensional manifold (de Bernardis et al., 2000). The assumption that the universe does not contain holes is implicitly taken for granted by the most influential accounts (Planck Collaboration 2016). Here we release a version that goes against the mainstream "genus-zero tenet". We argue that the cosmic manifold exhibits countless "handles" or "cavities" due to the occurrence of black holes that break its continuity. Very high genus manifolds (henceforward VHGMs) are mathematical objects pulled out when conventional, finite-genus systems' descriptions are inadequate. The higher the genus, the more complex the surface and/or the manifold. In mathematical jargon, the VHGM's groups of homeomorphisms, diffeomorphism and big mapping class groups, including the Riemannian ones, exhibit quantifiable algebraic and topological properties that affect one each other (Aramayona et al., 2020; Aougab et al., 2021; Mann and Rafi, 2023). The "holes" can be understood in a variety of topological ways, including regions, handles, punctures, loops, vortices, groups of symmetries, etc.

The question is: assuming our hypothesis is true and the universe can be described in terms of a high genus manifold, what are the physical outcomes? We will argue that the rich VHGM's topological features can be used to assess complex physical structures at different coarse-grained levels of observation, from the micro- to the meso- and the macro-scale. VHGMs can be used not just for the description of topological features, but also of spatial features (e.g., lines, perimeters, areas, volumes), images, spatial and temporal patterns, particle trajectories, vectors, tensors, functions, range of data, thermodynamic parameters, defects, impurities, signals (Tozzi et al., 2017). A list of feasible VHGM applications in physical and cosmic contexts is portrayed in **Table**. In each of the following chapters, our goal will be twofold:

1) To illustrate the feasible physical properties of a hypothetical manifold of very high, almost infinite genus.
2) To use VHGM to describe and/or assess both non-standard cosmological models and the standard, 6-parameter lambda cold dark matter model (henceforward ΛCDM), which continues to supply an excellent fit to the cosmic microwave background data at high and low redshift (Aghanim et al., 2020).

To sum up, we claim that the abstract topological manifolds of very high or infinite genus may have fascinating applications across various fields in physics, theoretical physics and astrophysics.

Spatial & geometric features	
	extra-dimensions
	compactified universes
	spacetime foam
	exotic cosmic topologies
	imaginary numbers in quantum mechanics
Highly stable structures	
	fine-tuning
	constant cosmic parameters
Impurities and defects, passages or handles	multiverse and bubble universes
	wormholes
	accelerated expansion of the Universe
Systems Dynamics interactions	
	strings interactions like splitting and joining
	multi-loops string interactions
Scale-free dynamics, chaotic dynamics	fractal-like distribution
Nonlocal interactions	
	quantum entanglement
Thermodynamic features	
	quantum vacuum discrepancy
	black hole thermodynamics
	holographic universe
Temporal dimensions	
	cosmic evolution

Table. Potential physical properties of topological objects with very high genus. The cosmic features that might be potentially described by high genus topological objects are displayed.

SPATIAL & GEOMETRIC FEATURES

The holes may represent systems with a highly flexible or "deformable" geometry characterized by a high number of possible shapes or sizes. In differential topology and in geometric group theory, but also in physics, VHGM may play crucial roles in building exotic, differentiable structures with unusual spatial properties, in which the high dimensional, smooth manifold structure differs from the canonical ones. Topological holes can be used to assess infinite-dimensional moduli spaces in which the parameters describing the system's geometry can vary in an almost infinitely complex way. High genus surfaces can be also equipped with more rigid structures consisting of translation constructions (Hooper 2013). Since the holes may represent not just natural numbers, but also integers, rationals, irrationals and imaginary numbers (Tozzi et al., 2017), they can be used to assess the the complex plane's mathematics subtending quantum dynamics.

In higher-dimensional physical theories, the geometry of the extra dimensions can be highly nontrivial, with bulky effects on the large-scale structure on the universe (Bars and Terning, 2009; Watts et al., 2017). In string theory, the universe is often modeled with extra dimensions that can be compactified in complicated ways through Calabi-Yau manifolds (Douglas 2015). These compactified universes may include "multi-connected" spaces with repeating regions that can be assimilated to the concept of VHGM.

Surfaces of almost infinite genus could arise in scenarios where fluctuations become extreme. Particularly in case of higher-dimensional spaces or complex boundary conditions, the number of holes, handles and path integrals increase without bounds. In the context of quantum gravity and in more exotic scenarios like certain models of loop quantum gravity and string theory, the concept of the universe as a smooth manifold is challenged at the Planck scale's high energy regimes, where the spacetime is thought to dynamically behave like a "foam" with large fluctuations in geometry and topology (Davis 1994; Carlip 2023).

The homogeneous and isotropic large-scale structure described by the cosmological principle in ΛCDM is usually modeled as a 3-dimensional spatial manifold with relatively simple topologies involving either flat, convex or concave spaces (Boylan-Kolchin 2023). Yet, there is room for other hypothetical spacetime structures where space is not simply connected but rather is characterized by a high number of topological features and a large variety of shapes. Infinite-genus manifolds embrace different structures that could theoretically match the shape of the universe, including:

1) The infinite Loch Ness monster surface, equipped with an orientable surface of infinite genus that displays only one end (Valdez 2009; Arredondo and Ramírez Maluendas, 2017). Its metrizable topological surface provides an example of chaotic groups of countable products' homeomorphisms (Zhukova and Korotkov, 2022).
2) The two ends surface termed Jacob's ladder surface (Ghys 1995).
3) The groups with no planar ends and with self-similar end spaces (Aougab et al., 2021).
4) The Weierstrass' one-ended, periodic minimal surfaces (Thayer 2012).
5) The Veech groups of tame translation surfaces, where every end has genus (Ramírez Maluendas and Valdez 2017).
6) The blooming Cantor trees with infinite number of handles and wild translation surfaces (Randecker 2018).
7) Parabolic or infinite translation surfaces with wild singularities. In this case, there is a failure of the classical Gauss-Bonnet formula for translation surfaces, which relates the cone angle of the singularities (geometry) to the genus of the surface (topology) (Randecker 2018).

It should be noted that non-trivial spatial topologies of the universe may give rise to potentially measurable signatures in the cosmic microwave background. To date, machine learning approaches have been used to classify viable topologies of the microwave background, at least in the test case of small toroidal universes (Tamosiunas et al., 2024).

HIGHLY STABLE, TOPOLOGICALLY ORDERED STRUCTURES

The holes may represent a highly stable, topologically ordered phase of matter where the large number of topological features protects against small perturbations, as in the case of topological quantum computing. Surfaces of almost infinite genus can be also associated with low-dimensional topology, infinite cyclic knot group and knot complements, i.e., the space left after removing a knot from a three-dimensional space (Conway and Powell, 2023).

In this metatheoretic cosmic context, the holes may represent a highly stable, topologically ordered structure of the universe where the large number of topological features protect against small perturbations. This suggests that a VHGM approach to the universe might contribute to solve the fine-tuning problem, constraining the cosmic parameters to remain constant within tight limits (Landsman 2016).

IMPURITIES AND DEFECTS, "PASSAGES" AND "HANDLES"

The holes may represent the systems' impurities that affect the conductive or magnetic properties, especially in certain models of spin liquids or quantum Hall states. To provide an example, colloidal platinum nanocrystals produced from the same synthesis batch may display intertwined topological defects that include size variations, lattice distortions, structural degeneracies, strain (Kim et al., 2020). These tiny differences in structure may generate non-trivial topologies that lead to systems dynamics' modifications and energetic frustration, unexpectedly influencing the system's physical properties (Kim et al., 2020).

In chaotic inflationary models characterized by a process of eternal inflation, the universe is thought to spawn an almost infinite number of "bubble universes" or "pocket universes" that become isolated from one another but are potentially connected through an almost infinite number of complex topological structures (Fialko et al, 2015). These connections could manifest through the presence of bridges between different regions. In the VHGM context, the hypothetical wormholes can be thought of as handles that connect different regions of either spacetime or multiverse. It has been conjectured that billions of microscopic wormholes could be responsible for the current accelerated expansion of the universe, where the visible horizon is 10^{29}, expanding at a speed of 74,3±2,1 km/sec per megaparsec. In a VHGM framework, the effective cosmological constant depends on the Gauss-Bonnet coupling and the wormhole density, attaining an effective dark energy sector of topological origin (Tsilioukas et al, 2024)

SYSTEMS DYNAMIC INTERACTIONS

The holes may represent the number of possible configurations involving different physical quantities. High topological complexity with an unbounded number of degrees of freedom describes physical systems with an enormous number of interaction pathways, including particles, fields, vortices, connections, charge configurations, excitations, phase transitions, chaotic bifurcations, etc.

The holes may represent loops, cycles, fluxes and vortices hinting at conserved quantities in dynamical systems. This approach has remarkable methodological and operational implications. For instance, when assessing the variance in observables' measurement during scattering events, the classical regime characterized by negligible uncertainty may emerge because of an infinite set of relationships among multiloop amplitudes in a momentum-transfer expansion (Cristofoli et al., 2021). VHGM could represent a high number of quantum loops or topological excitations, corresponding to physical systems that have complex boundary conditions or nontrivial vacuum states, leading to topological fluctuations of the spacetime at the quantum level.

In string theory, high genus surfaces can be used to describe different classes of interactions. For instance, the worldsheet of a propagating string can be modeled as a two-dimensional surface where strings interactions such as splitting and joining correspond to different topologies (Demulder et al., 2023). In perturbative string theory, the worldsheet surface's almost infinite genus is related to the number of loops in the string interactions, characterized by the vanishing capacity of the ideal boundary (Davis 2005). When the higher-order corrections to string interactions involve more loops, the surface's genus tends to increase. It is noteworthy that, in a cosmic VHGM context, the countable series of the above-mentioned infinite genus Loch Ness monster's loops may stand for string interactions.

SCALE-FREE DYNAMICS, CHAOTIC BEHAVIOUR

The holes may represent scale-free dynamics, namely (spatial) fractal-like distribution, (temporal) power laws, $1/f\alpha$ behavior, self-similarity, scale-invariant structure. A surface with fractal structure displays the same topological features repeating at each scale and generating almost infinite complexity at all scales (Hernández et al., 2017). Also, the holes may represent chaotic behavior, where sensitivity to initial conditions, density of closed orbits of homeomorphism groups and their countable products are correlated with high genus topological features (Zhukova and Korotkov, 2022).

If the universe has a fractal structure either at small or large scales as suggested by a few authors (see, e.g., Dickau 2009), the cosmic distribution of matter may not be homogeneous, but instead follow a fractal-like distribution, generating an almost infinitely complex, self-similar pattern that might involve surfaces of very high genus. This is also the case of the Everett III's formulation of quantum mechanics (Everett III 1957), where the number of the branching trees of all the possible outcomes is almost infinite and can therefore be described by a VHGM mathematical approach.

NONLOCAL INTERACTIONS

The holes may represent nonlocal interactions. In this case, information is spread out across the entire area of a high genus surface, with no single "local" description being sufficient. Non-local covers of infinite-type surfaces have different possible homeomorphism types (Biringer et al., 2024) so that complex space connectivity can be used to assess physical nonlocal interactions, especially in the cases of quantum entanglement and topological phases of matter.

If spacetime itself was modeled as a surface of very high genus, this could imply that distant regions of the universe are connected in a highly intricate and nonlocal manner. Nonlocality could manifest through the presence of particles connected by topologically nontrivial paths. This is the case with quantum entanglement.

A Riemann surface with very high genus can be thought of as having an indefinite number of handles or punctures corresponding to the moduli spaces of highly intricate geometric structures. This means that the holes may represent "passages" that can be visualized as "handles" connecting different elements at either the micro-, meso- or macroscopic physical scales.

THERMODYNAMIC FEATURES

The holes may represent thermodynamic parameters like energy. In physical contexts, energetic changes do not depend anymore on thermodynamic parameters, but rather on topological features like affine connections, projections, homotopies and continuous functions (Tozzi and Papo, 2017). Yet, random surfaces of infinite genus may describe the large-N thermodynamic limits that can be reached in statistical mechanics as well as in string theory.

The holes may represent information storage. A surface of almost infinite genus could store an almost infinite amount of topological information, particularly in the thermodynamic limit where the number of the components (e.g., particles, strings or signals) goes to infinity, their behavior becoming indefinitely complex (La 1991). This explains why certain theoretical models of quantum memory or holographic systems are described as supplied with vast or even infinite information capacity.

In conformal field theory and topological quantum field theory, surfaces of almost infinite genus can be used to model systems where the number of topological interactions or degrees of freedom becomes unbounded. The fact that an almost infinite entropy might correspond to an almost infinite number of possible cosmic configurations could be relevant in situations where the number of accessible microstates is extremely large, as is the case of black hole thermodynamics or cosmical systems undergoing phase transitions. Also, a VHGM-like approach with the energy endowed inside the topological holes might help to solve the scale discrepancy between the observed vacuum energy of just 10^{-9} J/m^3 and the theoretical quantum vacuum energy of 10^{133} j/m^3 (Henke 2017).

Black hole's exponential complexity is a matter of fact in the holographic dictionary. When coping with isolated, single-sided, non-evaporating black holes, reconstruction of interior outgoing modes is always exponentially complex (Engelhardt et al., 2021). The "quantum extremal surface" appears to encode the amount of holographic entanglement entropy that has radiated away from the black hole, evolving over the black hole's lifetime exactly as expected if information escapes (Engelhardt and Wall, 2014). New results show that an infinite number of black holes configurations are conceivable in dimensions higher than three, leading to a full range of possible topologies for the horizon as well as the domain of outer communication (Khuri and Rainone, 2022).

TEMPORAL DIMENSIONS

The holes may represent not just "spatial" dimensions, but also other kinds of dimensions such as the temporal ones. This means that a high genus topological structure can be made not just of space, but also of time, portraying the temporal evolution of the universe. Certain approaches to quantum gravity suggest that the topology of the universe could evolve over time, acquiring more and more topological complexity, possibly approaching an almost infinite genus in some future state. Therefore, the evolutionary processes of the universe might be studied via VHGM methodologies.

Summarizing, the abstract mathematical framework described by VHGM can be fruitfully used to investigate physical systems that are characterized by a very high amount of geometric, mechanical, dynamical, energetic, informational interactions and/or configurations. Some applications of VHGM are highly theoretical but are crucial for studying the limits in applications like quantum Hall effects and topological insulators. In our paper, we narrowed our list of feasible applications, focusing on the issue of a VHGM-like physical universe in both non-standard cosmological models and ΛCDM.

CONCLUSIONS

The universe regarded as a topological manifold of high genus is a speculative idea. Direct experimental evidence is lacking and none of the widely accepted cosmological models posit the universe as having a very high genus. Nevertheless, several theoretical backgrounds in modern physics implicitly allow for such a possibility, especially when considering the relationship between complex topological features and the spacetime structure.

While mathematically possible, a very high genus universe must face serious challenges. Observational evidence and the current understanding of cosmology suggest that, if such complexity exists, it may be hidden beyond the reach of current experiments. An objection may be raised concerning the cosmic microwave background (henceforward CMB), which describes a large-scale cosmic manifold that is isotropic and homogeneous, providing strong constraints on the possible topologies of the universe (Linker et al, 2024). The answer to this objection is that CMB was emitted from the surface of last scattering about 379,000 years after the Big Bang, while the most recent candidates for the oldest-known black holes date back to 400 million years after the Big Bang (Maiolino et al., 2024). This means that the primeval, continuous, uniform, isotropic and homogeneous manifold carved by CMB preceded the onset of singularities like the first black holes. Starting from an isotropic and homogeneous universe after the Big Bang, the production of many generations of black holes might have pierced the spatiotemporal continuum, progressively leading to a universe equipped with higher and higher genus. Moreover, the introduction of very high topological complexity into a physical model might raise issues of stability and energy. Almost infinite genus might imply an almost infinite amount of energy or complexity in spacetime, which could be physically problematic. This issue could be solved by considering that this huge amount of energy could be hidden inside the cosmic holes.

It could be speculated that a cosmic VHGM approach could help to elucidate the mystery of the physical nature of the dark sector of the universe. However, when comparing the topological properties of the large-scale dark matter distribution in various cosmological models, the ΛCDM and warm dark matter models produce nearly identical genus curves, indicating no topological differences in structure formation (Watts et al., 2017). In turn, the quintessence model produces significant differences in the strength and redshift evolution of non-Gaussian modes associated with higher cluster and lower void abundances (Watts et al., 2017).

To oppose to these drawbacks, there is one advantage to using a VHGM approach, namely the possibility of using the tools at our disposal such as the Teichmüller's infinite-dimensional spaces, that allow not just the mathematical classification of the elements of big mapping class groups, but also the assessment of countless physical systems including geodesic currents, foliations, laminations (Saric 2018).

It would be obvious to think that black holes are too small to affect the macrocosm, but it is not the case. Supermassive black holes can deeply influence the macroscopic structure of the spacetime, regulating cool gas accretion in massive galaxies (Wang et al., 2024). Still, micrometer-scale waves simulating black hole ringdown signatures are able to modify the background velocity field via intricate wave–vortex interactions (Švančara et al., 2024). Further, simulated postmerger gravitational-wave signals generated by binary black hole coalescence produce deep changes in nonlinear spacetime dynamics (Mitman et al., 2024). Galactic overdensities around ancient supermassive black holes support the idea that the most distant black holes grow within huge dark matter halos (Mignoli et al., 2020), generating gigantic high-power jets that represent the largest galaxy-made structures (Oei et al., 2024). Information change between black holes and spacetime is another clue testifying the effects of the black holes on cosmic structures (Chesler et al., 2019).

We showed that very high genus manifolds could be theoretically acknowledged also in ΛCDM (Aghanim et al., 2020). A VHGM-like approach could explain why the ΛCDM universe is highly connected and displays a very high number of quantum loops and topological excitations. Another theoretical source of cosmic "holes" could be represented by the occurrence of cosmic voids, i.e., vast pockets that are on average less dense than the universe. Making up together at least 80% of the universe's volume, such "bubbles" can span hundreds of millions of light-years (Contarini et al., 2022). Here, contrary to the usual efforts to evaluate how the expansion of the universe would affect the number and density of voids, we emphasize the possible effects of the voids on the cosmic manifold.

The holes may represent also biological features. Apart from the VHGM applications in physics, aspects of biological complexity like large interaction networks (e.g., indefinitely scaling neural networks, protein folding, metabolic pathways, cell signaling networks, ecosystems), and spatial or temporal self-similar structures (e.g., vascular systems, evolutionary processes), can be approximated to intricate, high genus topological structures in which the complexity could dramatically increase.

In sum, while there is no direct evidence to suggest that the universe has a topological manifold of infinite genus, certain theoretical frameworks, especially quantum gravity and string theory, open the door to such a possibility. Surfaces of infinite genus could model exotic, fractal, chaotic structures of spacetime, especially at very small or very large scales. But there is also room for a VHGM-like interpretation of the standard ΛCDM model.

DECLARATIONS

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