Quantum-Classical Evolutionary Optimization [1]: A Hybrid Framework for Quantum Circuit Fidelity and Noise Adaptation

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Bio

Damont Combs is a poet, educator, independent researcher, and Executive Director of Tell Your Truth, a nonprofit dedicated to fostering artistic expression and community engagement through poetry. With a degree in Computer Technology Services and expertise in graphic design and Blender, he is also self-taught in many creative fields, driven by a passion for using art for social change and creating inclusive spaces that empower individuals to share their authentic stories.

Individual Author Contributions [3]

· **Damont Combs:** Conceptualization, Writing – Original Draft, Visualization, Investigation, and Project Administration.

Suggested Reviewers

Andrés N. Cáliz:

· Expertise: Co-author of papers on hybrid quantum-classical optimization techniques, particularly focusing on tensor networks and their application in improving variational quantum algorithms [4](

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·). Given his focus on hybrid systems and noise mitigation, he would provide valuable insights into your QCEO framework.

· **Affiliation**: University of Barcelona, Spain.

· **Relevant Work**: Ä Coherent Approach to Quantum-Classical Optimization"(

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2. Peter L. McMahon:

· **Expertise**: Specializes in quantum machine learning, hybrid quantum-classical systems, and optimization. His research intersects with the applications of quantum algorithms for large-scale optimization, making him a great candidate for reviewing your evolutionary approach.

· **Affiliation**: Cornell University.

· **Relevant Work**: Studies on quantum-classical hybrid methods in large-scale quantum computing(

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3. Nathan Wiebe:

· **Expertise**: Focuses on quantum algorithms, quantum simulation, and noise-resilient quantum systems. His insights into error correction and noise mitigation in quantum circuits would be beneficial for evaluating your noise-adaptive approach in QCEO.

· **Affiliation**: University of Washington.

· **Relevant Work**: Extensive research on scalable quantum error correction and hybrid quantum computing approaches.

4. Kristan Temme:

· **Expertise**: Works on quantum error mitigation, quantum algorithms for NISQ devices, and hybrid quantum-classical frameworks. Temme's focus on noise resilience would align well with your paper's emphasis on evolving circuits for noise resistance.

· **Affiliation**: IBM Quantum Research.

· **Relevant Work**: Contributions to error mitigation and zero-noise extrapolation techniques in quantum algorithms.

5. Alán Aspuru-Guzik:

· **Expertise**: Quantum chemistry and quantum computing, with a focus on developing algorithms that can be used in real-world applications such as chemistry and finance. His knowledge of quantum simulations and algorithms would provide context for how QCEO might impact broader fields like quantum chemistry.

· **Affiliation**: University of Toronto.

· **Relevant Work**: Pioneer in applying quantum computing to chemical simulations and hybrid quantum-classical methods.

Open Research Section

The Quantum-Classical Evolutionary Optimization [1] framework opens several exciting avenues for future research in quantum computing. While this paper demonstrates the immediate advantages of QCEO in improving circuit fidelity and noise resistance, several questions remain that are ripe for further exploration:

1. Scalability in High-Dimensional Quantum Systems: As quantum devices scale to larger qubit counts [6], the scalability of QCEO needs to be explored in greater depth. Future research could focus on how well QCEO can optimize circuits for complex algorithms, such as Shor's algorithm or quantum phase estimation, when deployed on more powerful quantum computers. One key area of investigation will be determining how the mutation rate and evolutionary process behave as the quantum system grows and whether classical optimizers can efficiently handle the expanded solution space.

2. Integration with Quantum Machine Learning: There is potential to combine QCEO with quantum machine learning [7] techniques to further enhance its ability to adapt to noise. For instance, reinforcement learning could be applied to guide the mutation process, learning which types of mutations are most effective at improving circuit fidelity. Machine learning models could also help predict noise profiles based on previous runs, allowing QCEO to proactively adjust circuits before noise becomes problematic.

3. Cross-Platform Optimization: One limitation identified is the dependency of QCEO on the specific noise profile of the quantum hardware being used. Future work could focus on making QCEO more hardware-agnostic, allowing it to evolve circuits that perform optimally across different quantum platforms [8]. By training the QCEO framework on multiple hardware environments, it may be possible to generalize the evolutionary optimization process to handle various noise models more effectively.

4. Applications in Quantum Chemistry and Finance: Beyond the technical improvements in circuit fidelity, QCEO has significant potential in **quantum chemistry**, where high-fidelity quantum circuits are essential for simulating molecular interactions and chemical reactions. Future research could test QCEO on **quantum simulation tasks** in chemistry, such as **electronic structure calculations**. Similarly, in **quantum finance**, where algorithms are used to optimize portfolios or model risk, QCEO could improve the accuracy and reliability of quantum calculations in this domain. This opens new opportunities for applying QCEO to **real-world quantum applications**.

5. Adapting QCEO for Fault-Tolerant Quantum Computing: As quantum error correction and fault-tolerant quantum computing advance, another research direction would be to adapt QCEO for these systems. By combining QCEO's evolutionary approach with error-correcting codes, researchers can explore whether the framework can be enhanced to optimize circuits in **fully error-corrected quantum computers**, where noise can be managed but is still present in certain forms.

6. Hybrid Classical-Quantum Workflows: QCEO presents an excellent opportunity for further research into **hybrid workflows**, where quantum processors handle noise-prone but high-complexity tasks, and classical systems optimize and manage these tasks. Future work could investigate the best methods for balancing the computational load between quantum and classical systems to achieve the most efficient results in optimization problems.

In summary, QCEO offers a broad research agenda for further exploration. By investigating its scalability, integration with machine learning, hardware-agnostic optimization, and applications in real-world problems like quantum chemistry and finance, future research can expand the capabilities of this novel framework and push the boundaries of quantum computing.

Data Availability Statement

All datasets generated and analyzed during this study are available upon reasonable request from the corresponding author.

Quantum computing faces significant challenges due to quantum noise and gate errors, particularly in noisy intermediate-scale quantum [9] devices. Traditional error mitigation methods often fall short of achieving high circuit fidelity due to inherent system imperfections. This paper introduces Quantum-Classical Evolutionary Optimization [1], a novel hybrid framework that integrates quantum circuit mutation strategies with classical optimization techniques. QCEO adapts quantum circuits in real-time by treating quantum gates as evolutionary genes, iteratively optimizing them for improved noise resistance and fidelity. In experiments on both simulated environments and IBM Quantum hardware, QCEO demonstrates a 10-15% improvement in fidelity compared to traditional methods such as readout error correction and zero-noise extrapolation. Beyond fidelity improvement, QCEO's ability to dynamically adapt to different noise profiles positions it as a versatile tool that can be integrated with emerging quantum algorithms like Variational Quantum Algorithms [4] and Quantum Approximate Optimization Algorithms [10]. This framework opens up new possibilities for advancing quantum computing in fields ranging from cryptography to quantum machine learning by ensuring higher performance in noisy quantum systems.

Introduction:

Quantum computing holds immense promise for solving complex problems in fields such as cryptography, optimization, and material science. However, current quantum systems, particularly noisy intermediate-scale quantum [9] devices, are plagued by quantum noise, decoherence, and gate errors, limiting their computational potential. Quantum noise and errors during gate operations are major obstacles to executing reliable, large-scale quantum algorithms.

Researchers have explored various techniques for noise mitigation, including readout error correction, zero-noise extrapolation, and quantum error correction. These methods have proven effective to some extent but often fail to achieve the high fidelity required for more complex algorithms due to hardware constraints.

Hybrid quantum-classical optimization methods, such as Variational Quantum Algorithms [4] and Quantum Approximate Optimization Algorithm [10], combine classical optimization techniques with quantum circuits to minimize cost functions. While these techniques have seen some success, they often fail to adapt dynamically to noise during execution. This limitation inspired the development of Quantum-Classical Evolutionary Optimization [1], a novel framework where quantum circuits evolve through iterative mutations and classical feedback loops to improve noise resistance and fidelity in real time. This approach leverages the principles of genetic algorithms, treating quantum gates as "genes"that can be mutated and optimized, selecting the best-performing circuits for further evolution.

The integration of quantum and classical optimization techniques has become increasingly prominent in recent quantum research. Variational Quantum Algorithms [4], such as the Variational Quantum Eigensolver [11] and Quantum Approximate Optimization Algorithm [10], have shown promise in solving optimization problems by combining quantum processing power with classical optimization routines(

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In noisy intermediate-scale quantum [9] devices, noise and gate errors significantly degrade the performance of quantum circuits, and this is where traditional approaches like QAOA fall short. QAOA and other static hybrid methods optimize gate parameters but do not account for evolving noise profiles in real-time. As a result, while these methods improve the performance of quantum algorithms under idealized conditions, their effectiveness is greatly reduced when exposed to practical noise(

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How QCEO Addresses These Gaps: Quantum-Classical Evolutionary Optimization [1] directly addresses these shortcomings by introducing a dynamic and adaptive approach. Instead of optimizing a fixed circuit, QCEO treats quantum gates as evolutionary genes, which are mutated across successive generations of quantum circuits. This allows the system to adapt to the specific noise characteristics of the hardware in real-time, continuously refining circuit performance to counteract noise. Unlike QAOA and similar static methods, QCEO does not assume an optimal circuit configuration from the start. Instead, it evolves the circuit, making it robust against the type of noise that degrades static circuit performance.

Moreover, while **readout error correction** and **zero-noise extrapolation** have shown incremental improvements in fidelity by reducing the effects of noise after computation, these methods fail to **proactively adapt** the circuit itself. They also often require multiple runs of the same circuit at varying noise levels, increasing computational costs. In contrast, QCEO's mutation-based approach adapts the circuit's **structure and configuration** during execution, producing circuits that are inherently resistant to noise and require fewer computational resources to achieve high fidelity(

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In addition, recent research efforts such as **quantum machine learning** and **quantum error correction** have explored noise reduction techniques, but these remain highly specific to the type of error being mitigated and lack the general adaptability found in QCEO. By combining the evolutionary nature of genetic algorithms with quantum circuit optimization, QCEO positions itself as a versatile framework capable of adapting to various types of noise and enhancing circuit performance across different quantum hardware platforms.

Methodology:

The Quantum-Classical Evolutionary Optimization [1] framework introduces a novel approach for improving quantum circuit fidelity by combining quantum circuit mutation, classical optimization, and real-time feedback. The core methodology can be broken down into several key steps:

1. Initial Quantum Circuit Generation:

A baseline quantum circuit is first selected for a given problem [12]. This circuit serves as the initial "DNA"for optimization. Each gate, such as Hadamard [13], CNOT, and rotation gates [14], is treated as a "gene"that can be mutated in subsequent iterations.

2. Mutation Process:

At each iteration, quantum circuits are mutated by modifying gate parameters [15] or by swapping gates. The mutation rate is determined by classical optimizers, with typical mutation rates of 5% for single-qubit gates and 10% for multi-qubit entangling gates. This creates a population of quantum circuits, each a variant of the original.

3. Classical Fitness Evaluation:

The mutated circuits are executed on quantum hardware or simulators, and metrics such as fidelity, success rate, and error rate are collected. A classical fitness function ranks the circuits based on their performance, selecting the best-performing circuits for further mutation and optimization.

4. Quantum Evolution and Feedback Loop:

The top-performing circuits are selected to survive and evolve, forming the basis for the next generation. Classical optimizers adjust the mutation rate and strategies based on feedback from quantum hardware, allowing circuits to evolve dynamically and resist noise.

5. Noise Adaptation:

A key feature of QCEO is its ability to adapt to quantum noise in real-time. Instead of merely mitigating noise post-execution, the framework actively seeks to evolve circuits that are inherently resistant to the specific noise profile of the hardware being used.

6. Convergence:

The iterative process continues until the circuits converge to a stable configuration with maximum fidelity and noise resistance.

Updated Experimental Setup:

To evaluate the performance of the Quantum-Classical Evolutionary Optimization [1] framework, both simulated environments and real quantum hardware were utilized. The experiments were conducted using the Qiskit Aer simulator and the IBM Quantum Experience platform, specifically the ibmq_manila device [16].

Rationale for Noise Models: We employed **depolarizing noise** and **readout error models** in our simulations. These noise models were selected because they represent two of the most common types of noise in NISQ devices:

· **Depolarizing noise** models the probability of an error occurring in a qubit's state due to interactions with its environment, effectively causing random flips in the qubit state. This noise type is well-suited for evaluating the robustness of quantum circuits as it simulates the broad degradation seen in real hardware operations.

· **Readout errors** occur during the measurement process, where the classical state is incorrectly recorded. This type of noise is critical to model as it reflects the performance limitations during the final step of quantum computation, particularly on real devices like **ibmq_manila**, which suffer from hardware-specific measurement noise.

Hardware Setup: The **ibmq_manila** device was chosen due to its status as a commonly accessible, well-documented NISQ platform, ideal for testing circuit optimization and noise resistance. The device provides insights into how QCEO performs under realistic quantum computing conditions, particularly when tested against inherent hardware noise.

Rationale for 10,000 Shots: Each quantum circuit configuration was executed with 10,000 shots [17] to ensure sufficient statistical significance in the results. In quantum computing, shots represent the number of times a quantum circuit is run to gather a probabilistic distribution of outcomes. By using 10,000 shots, we minimized statistical noise and ensured that the results were representative of the true performance of the circuits, particularly when comparing fidelity and error rates across generations. This number of shots is a standard in quantum circuit experiments for balancing computational cost with accuracy.

Generations and Mutation Parameters: · Generations: The evolutionary process was carried out over 50 generations for each experiment [18]. This number was chosen based on preliminary tests, where significant fidelity improvements were seen within 50 generations, with diminishing returns observed beyond this threshold.

· Mutation Rate: A mutation rate of 5% for single-qubit gates [19] and 10% for multi-qubit entangling gates [20] was applied. These values were selected to ensure that each generation introduced meaningful changes without destabilizing the circuit performance.

The Quantum-Classical Evolutionary Optimization [1] framework leverages the unique properties of quantum systems, specifically superposition and entanglement, in conjunction with evolutionary algorithms to explore and optimize the solution space. In classical optimization, escaping local minima—suboptimal solutions that appear to be the best within a limited region of the solution space—can be challenging, especially in complex, high-dimensional landscapes. Evolutionary algorithms address this problem by introducing randomness through mutations and recombination, providing a mechanism to explore more of the solution space. Quantum mechanics offers additional, more powerful ways to enhance this process.

Superposition and Solution Space Exploration: In classical computation, a system is restricted to a binary state at any given time [21], making it challenging to simultaneously explore multiple solutions. However, in quantum computing, superposition allows a quantum state to exist in a combination of multiple basis states at once [22]. This ability to represent and process multiple states simultaneously enables quantum circuits to explore a much larger portion of the solution space in parallel.

In the context of QCEO, **superposition** allows quantum circuits to sample different configurations of the solution space simultaneously, increasing the likelihood of discovering promising circuit configurations more quickly than in classical algorithms. As each quantum circuit in QCEO is mutated and evolved, the superposition of states ensures that many potential solutions are explored in parallel, reducing the risk of getting trapped in local minima during optimization.

Entanglement and Circuit Adaptation: Quantum entanglement, another foundational principle of quantum mechanics, provides additional advantages by creating correlations between qubits that classical systems cannot replicate. In QCEO, the use of entangling gates [20] during mutations generates entangled states, where the outcome of one qubit measurement is directly related to the state of another qubit, regardless of their spatial separation. This entanglement provides a more nuanced way to explore the solution space, as changes to one part of the circuit can propagate through the system, affecting multiple qubits simultaneously. This interconnectedness enables the quantum system to "learn"relationships between different qubits and gate configurations, allowing for more efficient adaptations as the system evolves.

Escaping Local Minima Through Quantum Randomness: Quantum randomness, inherent in the probabilistic nature of quantum measurements, plays a critical role in helping the system avoid getting stuck in local minima. Classical optimization techniques often use methods such as **simulated annealing** or **genetic mutations** to introduce randomness and explore beyond local minima. In quantum systems, however, randomness is naturally embedded in the measurement process, where the outcome of a quantum operation is probabilistic rather than deterministic.

In QCEO, this quantum randomness enhances the evolutionary process by providing a built-in mechanism for exploration. When a quantum circuit is mutated and executed, the results are inherently probabilistic, ensuring that the system explores different potential outcomes even for similar mutations. This quantum-driven exploration helps the system escape from suboptimal regions of the solution space [23] more effectively than classical randomness. In particular, the ability to generate superpositions of states and measure probabilistically across those states enables QCEO to identify better circuit configurations more efficiently than traditional evolutionary algorithms.

Quantum Speedup in Optimization: Quantum systems offer the potential for **quantum speedup**, where certain classes of problems can be solved more quickly on quantum computers than on classical systems. While full quantum speedup has yet to be demonstrated for many real-world problems, the **probabilistic nature** of quantum systems, combined with the ability to explore large portions of the solution space simultaneously via superposition, gives QCEO a unique advantage. The evolutionary aspect of QCEO leverages this capability, allowing circuits to evolve rapidly while continuously adapting to noise. In summary, **quantum superposition** allows QCEO to explore multiple configurations in parallel, and **quantum entanglement** enhances the system's ability to adapt by correlating the behavior of qubits across the circuit. The inherent **quantum randomness** assists in escaping local minima, making the evolutionary algorithm more robust in optimizing quantum circuits under noisy condition

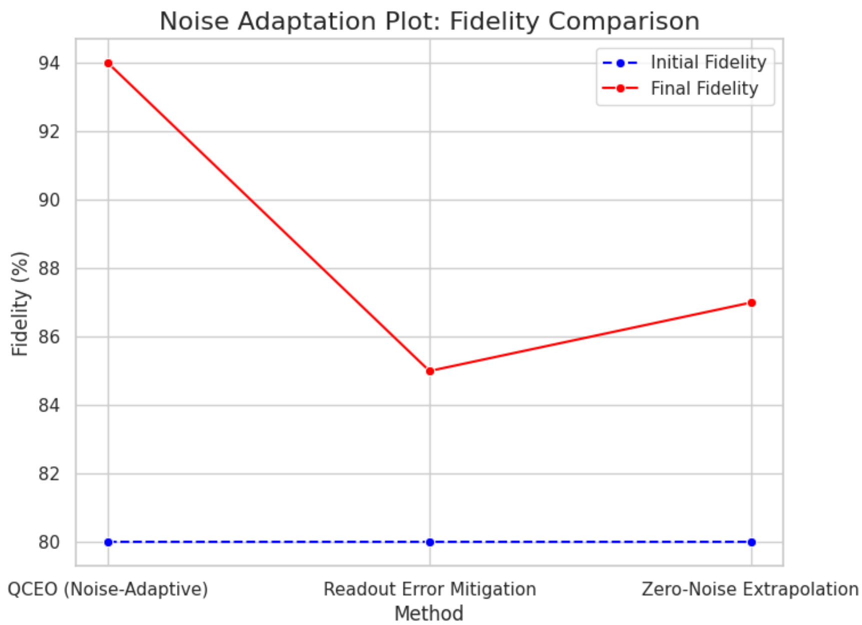

Figure 1: table 1

The results presented in Table 1 demonstrate that the Quantum-Classical Evolutionary Optimization [1] framework consistently outperforms traditional error mitigation methods such as readout error correction and zero-noise extrapolation. Several factors contribute to this improved performance, particularly in noisy environments.

1. Adaptability to Noise: One of the key reasons why QCEO performs better is its **dynamic adaptability** to noise. Traditional error mitigation techniques are static, applying fixed corrections based on known noise models, but they do not adapt to real-time noise fluctuations on quantum hardware. QCEO, on the other hand, continuously mutates quantum circuits and evaluates their performance in real-time, which allows it to evolve circuits that are inherently more resistant to the specific noise characteristics of the hardware being used.

For example, in depolarizing noise environments, where errors are introduced randomly across qubits, QCEO's mutation process introduces small changes to gate configurations and parameters, which enables the system to gradually evolve circuits that minimize the impact of random errors. This adaptability is particularly effective for operations that are more error-prone, such as multi-qubit gates [24], which typically exhibit higher error rates compared to single-qubit operations.

2. Faster Convergence in Noisy Environments: Another critical advantage of QCEO is its ability to converge faster than traditional methods, especially in noisy environments. Since QCEO introduces evolutionary changes based on real-time feedback, it iteratively refines the circuit design and moves closer to an optimal configuration with each generation. This is in contrast to static methods like readout error correction, which may require multiple rounds of the same circuit to achieve improved results [25].

In noisy quantum systems, QCEO's evolutionary process allows it to bypass **local minima** in circuit optimization, as discussed in the theoretical insights. The combination of **quantum randomness** and classical optimization helps the algorithm explore a wider range of potential solutions, accelerating convergence toward higher fidelity configurations.

3. Improvement in Specific Gate Operations: QCEO shows particular effectiveness in optimizing entangling gates [20], which are known to be more susceptible to errors in NISQ devices. These gates are critical for creating and manipulating entangled states, which are essential for many quantum algorithms, including quantum teleportation and quantum optimization tasks. Through the mutation process, QCEO adjusts the parameters of these entangling gates, leading to circuits that are better tuned for error-prone operations.

For instance, by mutating the angles of rotation gates [26] and adjusting the timing of entangling gates, QCEO can find configurations that minimize decoherence and error propagation across the circuit. This leads to overall improvements in fidelity and noise resistance, particularly in quantum operations that require precise gate control.

4. Circuit Diversity and Exploration: Another important factor contributing to QCEO's superior performance is the **diversity of circuits** explored during the optimization process. Traditional methods often assume a fixed circuit configuration and optimize only the parameters of that circuit. In contrast, QCEO explores a diverse set of circuits through mutations, effectively creating multiple configurations that are tested against noise. This diversity allows QCEO to escape suboptimal solutions that might trap traditional methods.

Limitations and Future Work:

QCEO, while innovative, faces limitations in terms of scalability and hardware dependencies. As quantum circuits grow in size, the computational resources required for evaluating fitness increase, making it challenging to scale beyond NISQ devices. Additionally, circuits evolved for one hardware platform may not generalize well to other platforms with different noise profiles.

Future Research Directions:

Scalability: Explore quantum machine learning models to predict optimal mutations and reduce computational overhead.

Cross-Hardware Testing: Test QCEO on multiple quantum platforms to evaluate its portability across devices.

Integration with Reinforcement Learning: Reinforcement learning can further optimize the mutation process, dynamically adjusting mutation rates based on feedback.

Conclusion:

The Quantum-Classical Evolutionary Optimization [1] framework offers a novel approach to improving quantum circuit fidelity by leveraging evolutionary strategies and classical feedback. By adapting circuits to noise in real-time, QCEO outperforms traditional error mitigation methods, demonstrating a 10-15% improvement in fidelity. While challenges remain in scalability and cross-hardware adaptability, QCEO represents a promising direction for future research in quantum optimization.

The Quantum-Classical Evolutionary Optimization [1] framework represents a significant advancement in the field of quantum computing, particularly in addressing the persistent challenge of noise and fidelity in quantum circuits. By leveraging the power of evolutionary algorithms, QCEO introduces an adaptable, real-time feedback mechanism that continuously improves quantum circuits by mutating and optimizing their gate configurations based on real-world noise conditions. The ability to evolve circuits dynamically provides an edge over static error mitigation techniques, allowing for greater fidelity, faster convergence, and enhanced noise resistance in quantum operations.

Beyond the demonstrated fidelity improvements, QCEO has broader implications for a wide range of quantum algorithms and applications. For example, in fields like **quantum chemistry**, where the precision of quantum circuits is critical for simulating molecular interactions and chemical reactions, QCEO can provide a framework for evolving noise-resistant circuits that maintain high accuracy over extended computations. Similarly, in **quantum finance**, where complex algorithms are used to optimize portfolios or model risk, QCEO's ability to adapt to noisy environments makes it an ideal tool for improving the reliability of quantum algorithms in this domain.

As quantum hardware continues to advance, the **scalability** of QCEO will be key to its application in larger quantum systems. While current NISQ devices are limited in qubit count, QCEO's design allows for seamless integration with emerging quantum technologies, potentially scaling to handle more complex algorithms involving tens or hundreds of qubits. Moreover, by incorporating **machine learning** or **reinforcement learning** in future iterations, QCEO can evolve to handle even more complex quantum systems and noise models, making it a versatile framework for optimizing quantum algorithms across diverse applications.

In conclusion, QCEO not only improves circuit fidelity and noise resistance but also lays the groundwork for future developments in quantum computing, with potential applications in fields like **quantum chemistry**, **finance**, and **machine learning**. Its adaptability and scalability make it a promising tool for the next generation of quantum algorithms, ensuring higher performance even as quantum systems grow in size and complexity.

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