Notes on Critical Phenomena and Primordial Cosmology

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Abstract

Critical phenomena describe continuous phase transitions characterized by power-law divergences, universality of scaling exponents and ergodicity breaking. Scaling exponents depend on the *dimension of the underlying spacetime* (d) and, in many cases, also on the *dimension of variables* defining criticality (D). Recent studies suggest that both dimensions (d and D) run the observation scale and, as a result, determine the rate of divergence near critical points. Building on these observations, the goal of this report is to close the gap between *critical behavior in continuous dimensions* and the anomalous findings of the James Webb Space Telescope (JWST).

Key words: criticality in continuous dimensions, multifractal scaling, primordial cosmology, James Webb Space Telescope.

The reader is cautioned upfront that these notes introduce a speculative proposal that needs to be scrutinized, challenged or developed through independent analysis

1. Introduction and Motivation

Recent data recorded by the JWST present a major challenge to our current understanding of primordial cosmology founded on the ΛCDM model. In particular, the unexpected abundance of supermassive galaxies and Black Holes and the discovery of stars whose age contradicts the Big Bang model, point to serious inconsistencies with ΛCDM cosmology. One of the main premises of ΛCDM is that large cosmic structures develop from small-scale Dark Matter (DM) filaments in a process called *hierarchical clustering*. The standard view is that density fluctuations during Universe expansion lead to the collapse/condensation of DM and the coalescence of progressively massive galaxies.

The goal of this provisional report is to highlight a largely unforeseen connection between *far-from-equilibrium critical behavior* and the JWST observations. To this end, we recall that critical phenomena describe continuous phase transitions characterized by power-law divergences, universality of scaling exponents and ergodicity breaking. Scaling exponents depend on the dimension of the underlying spacetime (d) and, in many cases, also on the dimension of variables defining criticality (D). Recent studies indicate that both dimensions run the observation scale and consequently determine the rate of divergence near critical points. Building on these observations, we consider three scenarios in which *critical phenomena in continuous dimensions* d and D impact the genesis of primordial Universe, namely,

1) *Percolation of the Cosmic Web from dimensional fluctuations*,

2) *Topological defects in primordial cosmology (Kibble-Zurek model)*,

3) *Primordial cosmology as Self-Organized Criticality (SOC).*

The brief report is organized as follows: sections 2.1 to 2.3 cover the three scenarios introduced above, an (incomplete) list of open questions is detailed in the last section.

2. Criticality in continuous dimensions and the early Universe

2.1 Percolation of the Cosmic Web from dimensional fluctuations

Building on references [1 - 3, 5 - 7], we explore in this section the genesis of the Cosmic Web from the percolation of dimensional fluctuations far above the Standard Model scale.

Since percolation has an intrinsic fractal structure at the critical point it can be cast using the terminology of continuous phase transitions/critical behavior.

All critical exponents are dependent on the redshift z, which assumes the role of *observation scale*. In particular [2],

a) Correlation length,

$$\xi(p,z) \propto \left|p - p_c\right|^{-\nu(z)} \tag{1}$$

b) Probability that a site belongs to the percolation infinite cluster,

$$P_\infty(p,z) \propto (p - p_c)^{\beta(z)} \tag{2}$$

where p_c stands for the critical percolation probability.

c) Average cluster size,

$$\chi(p,z) \propto \left|p - p_c\right|^{-\gamma(z)} \tag{3}$$

d) Characteristic cluster size,

$$s_\xi(p,z) \propto \left|p - p_c\right|^{-1/\sigma(z)} \tag{4}$$

e) The "mass" of an incipient infinite cluster denotes the *number of sites* of the percolating cluster in a window of size l and is given by,

$$M_\infty(\xi,l,z) \propto \begin{cases} l^D,\ l << \xi \\ l^D (l/\xi)^{d-D},\ l >> \xi \end{cases} \tag{5}$$

The relationship between critical exponents of (1), (2), (4) and (5) is encoded in

$$\Delta(z) = d(z) - D(z) = \frac{\beta(z)}{\nu(z)} \tag{6}$$

with

$$D(z) = \frac{1}{\nu(z)\sigma(z)} \tag{7}$$

Here, ξ is the correlation length (1) and z is the redshift. By (5) and (6), we write,

$$M_{\infty}(\xi, l, z) \propto l^{D(z)} (l/\xi)^{\Delta(z)} \tag{8}$$

It is apparent from (8) that there is an upsurge in percolating mass when the rate of exponents with the redshift satisfies

$$\boxed{\frac{d\Delta}{dz} > 0; \quad \frac{dD}{dz} > 0} \tag{9}$$

The boost in (8) leads to oversized cosmic objects, an upsurge in the overall gravitational pull, as well as in the likelihood of gravitational collapse and the formation of massive Black Holes.

It is instructive to note that the boost in the overall gravitational attraction is on par with the effect of Dark Matter/Cantor Dust on the galactic rotation curves [3, 5, 7].

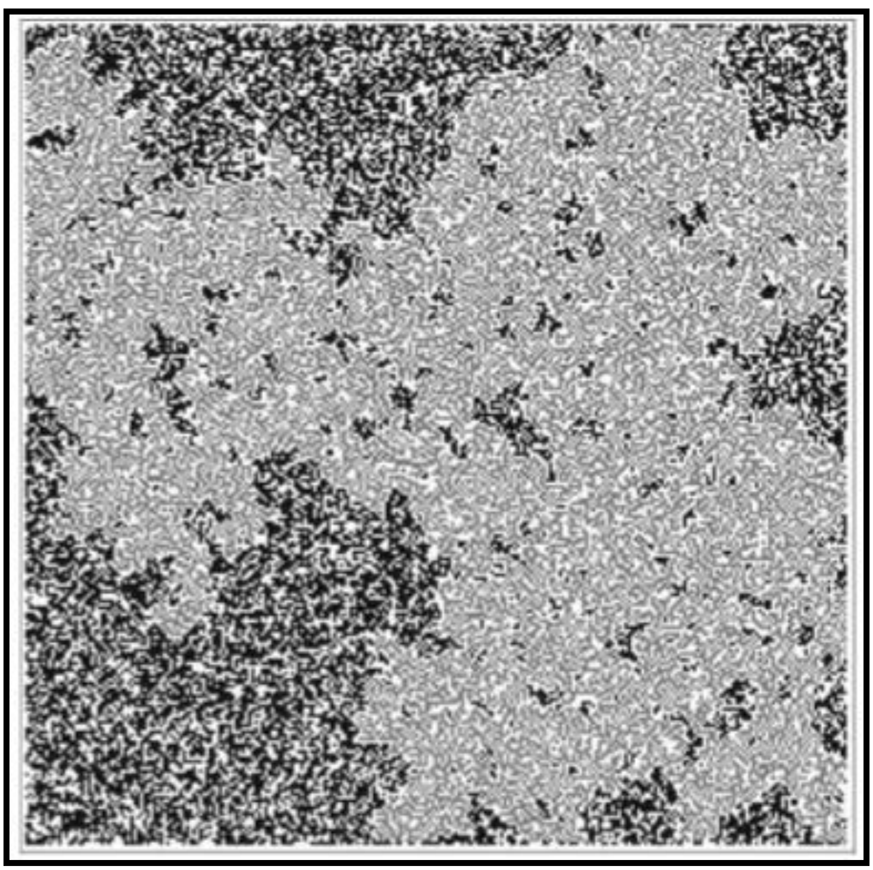

Fig. 1: Percolation cluster as replica of Cantor Dust [3]

2.2 Topological defects in primordial cosmology

In the contest of a spacetime endowed with continuous dimensions, topological defects described by the Kibble-Zurek mechanism (KZM) arise from the continuous distribution of dimensional gradients forming *domain walls*.

The correlation length and relaxation time of KZM are given by [4]

$$\xi(\varepsilon)=\frac{\xi_0}{|\varepsilon|^{\nu}} \tag{10}$$

$$\tau(\varepsilon)=\frac{\tau_0}{|\varepsilon|^{z\nu}} \tag{11}$$

where ν, z denote the critical and dynamic exponents, respectively. Here, the dimensional deviation can be further mapped to a reduced distance parameter λ,

$$\varepsilon=1-\frac{\lambda}{\lambda_c} \tag{12}$$

such that $\varepsilon = 0$ when $\lambda = \lambda_c$. Under the linear quench assumption, the parameter λ evolves according to,

$$\lambda(t) = \lambda_c[1 - \varepsilon(t)] \tag{13}$$

in which the time-dependent dimensional deviation is measured relative to the quench time τ_Q as in,

$$\varepsilon(t) = \frac{t}{\tau_Q} \tag{14}$$

Fig. 2 shows the schematic diagram of KZM, where $t \in [-\tau_Q, \tau_Q]$ with the critical point being reached at $t = 0$. The dynamics of the KZ phase transition remains adiabatic outside the critical region $[-\hat{t}, \hat{t}]$ and enters the "frozen" stage once $t \in [-\hat{t}, \hat{t}]$. In this stage KZM is no longer adiabatic and $\tau(\varepsilon) \to \infty$. As a result, $\hat{t}$ plays the role of a "freeze out" time and marks the *crossover* boundary between the adiabatic and the critical regime of KZM.

The density of topological defects in the KZM assumes the form,

$$n = \frac{1}{\xi_0^{d-D}} \left(\frac{\tau_0}{\tau_Q}\right)^{\kappa} \tag{15}$$

Here, d and D are the dimensions of the underlying space and of the defects, respectively, and

$$\kappa = (d - D) \frac{\nu}{1 + z\nu} \tag{16}$$

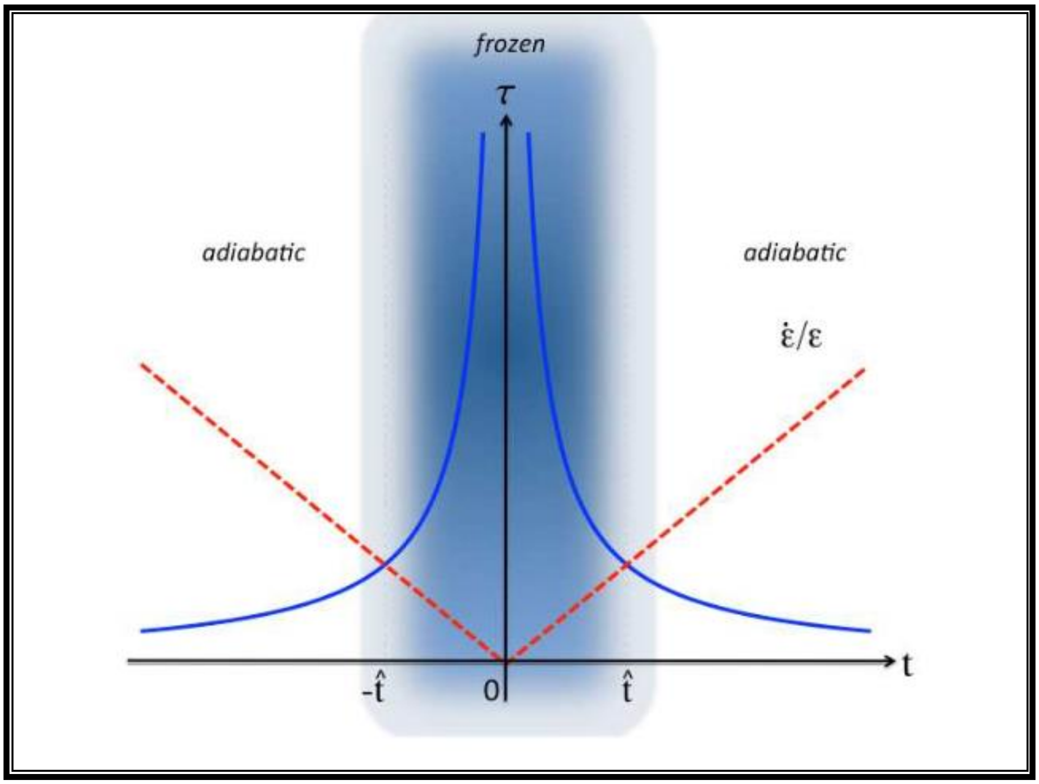

Fig 2: Schematic diagram of the KZM [4].

The number of defects N follows by multiplying (15) with the *effective space volume* $V = L^{d-D}$, which yields,

$$N = (\frac{L}{\xi_0})^{d-D} (\frac{\tau_0}{\tau_Q})^{\kappa} \tag{17a}$$

On account of (6) and by demanding that all scaling exponents in (17a) depend on the redshift z leads to

$$\boxed{N(z) = (\frac{L}{\xi_0})^{\Delta(z)} (\frac{\tau_0}{\tau_Q})^{\kappa(z)}} \tag{17b}$$

(17b) indicates that the number of topological defects undergoes a boost upon demanding $d\Delta(z)/dz > 0;\ d\kappa(z)/dz > 0$.

2.3 Primordial cosmology as Self-Organized Criticality (SOC)

The size distribution of fractal volumes (V) and fractal areas (A) are given by the following scaling relationships [10]

$$N(V, z) \propto V^{-\omega_V(z)} \tag{18}$$

$$\omega_V(z) = 1 + d_V(z)/D_V(z) \tag{19}$$

$$N(A,z) \propto A^{-\omega_A(z)} \tag{20}$$

$$\omega_A(z) = 1 + d_A(z)/D_A(z) \tag{21}$$

The condition for a boost in the volume and area distributions of *fractal avalanches* amounts to

$$\boxed{\frac{d\omega_{V,A}(z)}{dz} < 1} \tag{22}$$

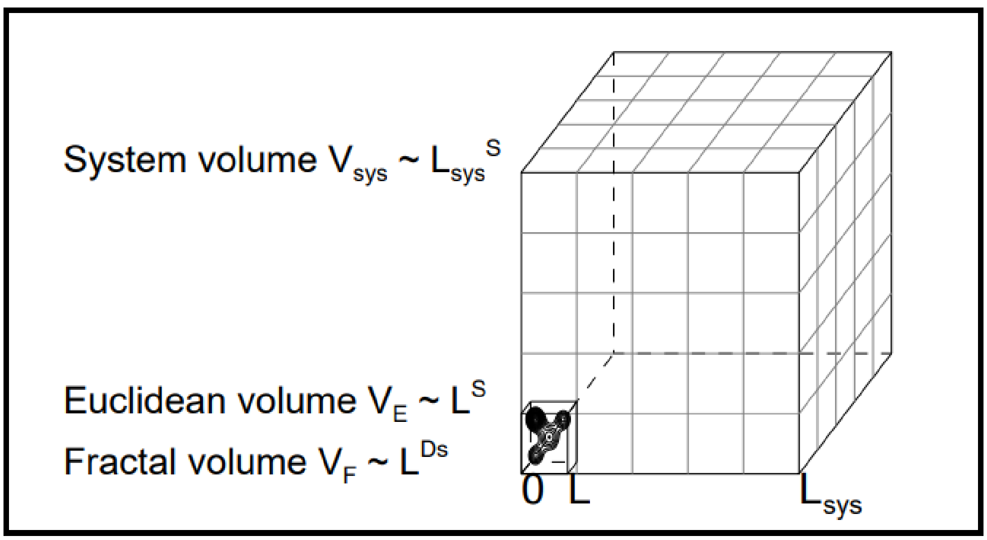

Fig. 3 Geometry of the avalanche and system volumes [10]

We close this section by noting that further developments of the topics discussed here are likely to follow references [2 – 3, 8 - 10].

4. Open questions

a) How strong is the hypothesis that criticality in continuous dimensions can do better in explaining primordial cosmology than current models (in which turbulence, gravitation, thermodynamics play a collective role)?

b) How is the limited optical resolution of distant clusters dealt with when building data analysis based on our approach? It is apparent that careful calibration of window sizes may likely be required in both percolation and SOC studies of primordial cosmology.

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