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New methods based on the calculation of specific decimal fractions for decomposing an integer into a product of prime factors

Abstract

This article presents for the first time two methods for decomposing integers in products of prime factors which are based on the calculation of decimal fractions. Its originality lies in the fact that the divisors used are decimals and not prime divisors and in addition the decimal part is manipulated in such a way that two decimal digits are fixed and the others are variable. In the first method, the divisors are of type 2^n and which have a very interesting particularity which is that they always have two same digits at the end of their decimal parts (25 or 75). And it is this particularity which is exploited to develop these methods. The other method introduces a new notion that of the decomposition key which is a product of prime factors used to decompose all numbers having the same number of digits. It is similar to the first method because it also uses decimal fractions for the calculation and the denominator is the square root. This article paves the way for new applications in computer science.

Key words. Method. Factorization. Integers. Prime numbers. Decimal fraction. Decomposition.

1. Introduction

In mathematics, the fundamental theorem of arithmetic, also called the unique factorization theorem and prime factorization theorem, states that every integer greater than 1 can be represented uniquely as a product of prime numbers, up to the order of the factors ([https:// en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic](https://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic)). However, the problem is to be able to transform any number into a product of prime factors since a theorem must always be correct whatever the number. The solution that has always been imposed is the Euclidean division in series by prime factors called trial division (Zalak et Hajj-Boutros, 2011) [see 1]. Trial division is the most laborious but easiest to understand of the integer factorization algorithms. The essential idea behind trial division tests is to see if an integer n , the integer to be factored, can be divided by each prime number in turn that is less than the square root of n . It is for this reason that it has become necessary to develop new algorithms for decomposing natural numbers by taking advantage of the high computing speed of computers. Nowadays, these algorithms are capable of decomposing numbers containing several digits but are all limited by the value of the number to be factored, the larger it is, the more time and energy it requires (Bressoud, 2012; Harahap & Khairina, 2019) [see 2,3]. Among numbers, biprime numbers with two prime factors relatively closer in value are the most difficult to decompose. Hence their interest for cryptology. The problem of determining whether a given integer is prime is one of the better known and most easily understood problems of pure mathematics. This problem has caught the interest of mathematicians again and again for centuries. However, it was not until the 20th century that questions about primality testing and factoring were recognized as problems of practical importance, and a central part of applied mathematics. The advent of cryptographic systems that use large primes, such as RSA, was the main driving force for the development of fast and reliable methods for primality testing (Maurer, 1995) [see 4].

The defining property of a prime number p is that it is a positive integer $p \geq 2$ that is only divisible by 1 and p . Equivalently, p is prime if and only if p is a positive integer $p \geq 2$ that is not divisible by any integer m such that $2 \leq m < p$. A positive integer $n \geq 2$ which is not prime is called composite. A primality test is a mathematical procedure for determining whether a given number is a prime number (i.e. it has no divisor other than by 1 or itself). How to know if a number is a prime? To determine if a number is prime, it must now pass a primality test which confirms its status as a prime number. There exist several primeness tests to know if a number is a prime number, the oldest is the Sieve of Eratosthenes, and the most common are Miller–Rabin and Lucas-Lehmer tests (<https://www.dcode.fr/primality-test>). In number theory, the general number field sieve (GNFS) is the most efficient classical algorithm known for factoring integers larger than 10^{100} (Wikipedia).

There are broadly two categories of number primality tests: A deterministic primality test provides a certain answer about the primality of a number (it guarantees whether the number is prime or not). In contrast, a probabilistic test provides a probable answer, with a controlled margin of error, which means it can tell whether a number is probably prime or probably composite.

The Sieve of Eratosthenes is a simple and ancient algorithm used to find the prime numbers up to any given limit (Nicol, 1950; Harahap & Khairina, 2019) [see 5,3]. It is one of the most efficient ways to find small prime numbers. For a given upper limit n the algorithm works by iteratively marking the multiples of primes as composite, starting from 2. Once all multiples of 2 have been marked composite, the multiples of next prime, ie 3 are marked composite. This process continues until $p \leq n$ where p is a prime number (<https://brilliant.org/wiki/sieve-of-eratosthenes>).

2. The new proposals in this article

Let us suppose you have a biprime number $B_n = p \times q$ such that $p < q$ that you want to decompose and thus find its two prime factors p and q . You do not want to use a decomposition algorithm that automatically gives you the result but you want to proceed differently by doing your own calculation. First, let's agree on some rules. If you multiply B_n by another prime factor denoted r or a multiple of prime numbers s et t denoted $M = s \times t$ and you decompose it with your algorithm; you have three possible outcomes: $r \times p \times q$; $s \times t \times p \times q$ if your algorithm is able to decompose it, and $r \times B_n$ or $s \times t \times B_n$ if it is not able to do it. In the first case, you have not really decomposed the number B_n by your own calculation because you have just decomposed one of its multiples. In the second case, the algorithm reaches its limit and returns it to you undecomposed. However, a solution is available to you, which is to look for any numbers that have a common factor with B_n . Let us denote these numbers $M' = s' \times t' \times p$ and $M'' = s'' \times t'' \times q$ (p or q are the common prime factors between B_n and M' or M''). These numbers M' and M'' can be less than or greater than B_n , that does not matter. In case they are lesser than B_n , we would say that they are its submultiples and in the other that they are its supermultiples.

If you mark multiple by multiple of a prime number in an Erasthenes sieve, and if you choose a multiple in the middle, the submultiples will be those which precede it and the supermultiples those which follow it. A number to be decomposed therefore has a common factor with an infinite number of numbers which are its submultiples or supermultiples. This is the central idea of this article and it is also a truth of Erasthenes' sieve which has always gone unnoticed and which has not been exploited to its fair value to develop a factorization method. This article will then exploit this fact.

In fact, looking for a common factor between M' or M'' and B_n amounts to going up or down the Erasthenes' sieve, but if you want to do it manually by searching in this sieve for submultiples or supermultiples and knowing that B_n could be a very large number, you will quickly realize that the task is very difficult and very long or impossible to carry out. How then can you do it? This article gives you answers and alternative methods.

Now you understand that the Erasthenes' sieve is a fixed structure like a ladder that can only be ascended or descended step by step. You will therefore have to proceed differently and therefore you must carry out a precise and safe calculation. The question is the following: how can we find a submultiple or supermultiple of B_n which has one prime factor in common with it that we will use to divide it while being sure and certain that it is indeed one of its divisors?

In fact, finding a submultiple or supermultiple of a number is another way of breaking it down in the strict sense of mathematics. But be careful, the number sought must only have one factor p or q in common with B_n , never both because in the latter case and as said above we only multiply B_n by an integer and then the algorithm gives it back to us in the form of a product of prime factors, this is not decomposition. Decomposing a number means finding another number which has a single prime factor in common with it or finding an integer or decimal or even irrational divisor which gives a quotient which is a multiple of one of its prime factors. These are the main ideas of this article. The conventional idea is that a divisor is always an integer, this article ignores this classic concept and uses decimal (or irrational) divisors instead.

New calculation methods are thus necessary to which this article is all dedicated. As a result, this article will propose new methods for decomposing natural numbers. These methods use the calculation of decimal fractions between a denominator and a numerator in a continuous manner using calculators available on the web. An available decomposition algorithm is always necessary but will be used differently, only to find the common factor between B_n and one of its sub- or super-multiples and thus decompose it. This article is undoubtedly the first which uses classic mathematics and which decomposes a number by looking for submultiples which have a common factor with it.

All these new tests are deterministic because we only have two cases: 1) if the number is composite, you will at one time or another have an integer quotient between the numerator and the denominator; 2) if the number is prime, the calculation of fractions will run in an incessant loop giving only decimal or irrational numbers until the end. Examples of calculations will be given and the methods explained. Note that the calculation is mainly the calculation of fractions between a numerator and a denominator with a decimal part limited to few digits. The most important thing is to have a calculator capable of doing this calculation in series and continuously.

It is important to note now that the decomposition of a number is all the more difficult as the number is large with large prime factors and that if the known algorithms give a rapid answer it is only thanks to the power of computers. This simplicity is deceptive and breaking down a number could be an extremely difficult or an unachievable task if done manually. But the most important thing is to find an orderly and specific calculation method which leads to the result in a reproducible and programmable way. The calculation must also be done to take the shortest and least expensive path to decompose the number posed. These elements were taken into consideration in the design of the methods described in this article, which will now be described one after the other with illustrative examples.

3. Materials and Methods

This article is based on calculation in the first place which is obviously deduced from an equation which will be described and demonstrated. To calculate all decimal fractions a calculator was used available on the web (<https://calculatrices.app/calculatrice-de-grands-nombres>). Note that the latter calculates a decimal fraction but not continuously (We must write the decimal denominator). For very large numbers, the calculator (<https://www.123calculus.com/>) was also used. To factor the numbers, two sites were used: <https://www.dcode.fr/decomposition-nombres-premiers>; and <https://calculis.net/premier>. The prime numbers were obtained from the site http://compoasso.free.fr/primelistweb/page/prime/liste_online.php.

4. Results: New methods of integer decomposition

For a method to be valid and even conceivable, it must be based on an invariable and infinitely reproducible fact. Otherwise, the method will generate so many variations and it will be random and impractical. Mathematics reject exceptions and only works with axioms and theorems, but given the great complexity of prime numbers and the decomposition of natural numbers into prime factors, the development of algorithms and probabilistic methods were considered because they take advantage of the high speed of computer calculation and make it possible to verify the authenticity of the theorem of the decomposition of natural numbers at a very high level. In order to avoid probabilistic methods, we must therefore start with sure principles leaving no shadow of doubt.

4.1 Methods based on the series of divisions by 2^n

4.1A Principles

As a reminder, this study will be dedicated to biprime numbers because we know that they are the hardest to decompose, especially when their two prime factors are very large halfway between 0 and the square root (SR). But what will be described is valid for any odd number whatever its number of prime factors.

If you take an odd number > 1 and divide it by 2 you will have the remainder 1. Any odd number divided by 2 will therefore give a decimal extension 0.5. But if you divide it again by 2 (i.e. 4), you will only have two possible decimal parts 0.25 or 0.75. This is because the remainders of Euclidean divisions of an odd number by 4 is either 1 or 3 and in the first case it is 0.25 and the other it is 0.75. If you continue your divisions by 2^n (8, 16, 32, 64, 128, 256, and so on), you will always have a decimal extension which ends either with 25 or 75 (Table 1A-C).

Tables 1: division by 2^n always generate 25 or 75 as digits in the decimal parts of the quotients.

Table 1A

	1/4	1/8	1/16	1/32
91	0.75	0.375	0.6875	0.84375
323	0.75	0.375	0.1875	0.09375
3397	0.25	0.625	0.3125	0.15625
10873	0.25	0.125	0.5625	0.78125
520187	0.75	0.375	0.6875	0.84375
1297603	0.75	0.375	0.1875	0.09375
5321531	0.75	0.375	0.6875	0.84375
20777459	0.75	0.375	0.1875	0.59375
152771243	0.75	0.375	0.6875	0,34375

Table 1B

	1/64	1/128	1/256	1/512
91	0.421875	0.7109375	0.35546875	0.177734375
323	0.046875	0.5234375	0.26171875	0.630859375
3397	0.078125	0.5390625	0.26953125	0.634765625
10873	0.890625	0.9453125	0.47265625	0.236328125
520187	0.921875	0.9609375	0.98046875	0.990234375
1297603	0.046875	0.5234375	0.76171875	0.380859375
5321531	0.921875	0.4609375	0.23046875	0.615234375
20777459	0.796875	0.8984375	0.94921875	0.974609375
152771243	0.671875	0.3359375	0.66796875	0.333984375

Table 1C

	1/1024	1/2048	1/4096	1/8192
91	0.0888671875	0.04443359375	0.022216796875	0.0111083984375
323	0.3154296875	0.15771484375	0.078857421875	0.0394287109375
3397	0.3173828125	0.65869140625	0.829345703125	0.4146728515625
10873	0.6181640625	0.30908203125	0.654541015625	0.3272705078125
520187	0.9951171875	0.99755859375	0.998779296875	0.998779296875
1297603	0.1904296875	0.59521484375	0.797607421875	0.3988037109375
5321531	0.8076171875	0.40380859375	0.201904296875	0.6009521484375
20777459	0.4873046875	0.24365234375	0.621826171875	0.3109130859375
152771243	0.6669921875	0.33349609375	0.666748046875	0.8333740234375

These are therefore decimal numbers but whose decimal part is all the longer as n of 2^n tends towards infinity. When 2^n is too high, we can say that the number obtained is between a decimal and irrational number since the digits after the decimal point are unpredictable and non-repeating, except that they always end with 25 or 75. Several examples of numbers of different values that have an increasing number of digits are shown in the following tables. We see that this fact remains true whatever the number of digits of the numbers tested (Table 2A-C). However, the most important fact to note is that for a given 2^n , we have the same number of digits before 25 or 75 in the resulting decimal part. For example, for $1/8$, we always have one digit before 25 or 75 (for example 0.375), for $1/16$ we have 2 digits (for example 0.6875), for $1/32$ we have 3 digits (for example 0.84375) and so on. The decimal part before 25 or 75 increases by one digit when going from 2^n to 2^{n+1} . The number of digits after the decimal separator = n being the exponent of 2^n .

Tables 2: division by 2^n still generate 25 or 75 as digits in the decimal parts of quotient obtained with numbers having increasing digits.

Table 2A

	1/4	1/8	1/16	1/32
4877863751	0.75	0.875	0.4375	0.21875
14048615549	0.25	0.625	0.8125	0.90625
329650448509	0.25	0.625	0.8125	0.90625
1657851033857	0.25	0.125	0.0625	0.03125
59708282945131	0.75	0.375	0.6875	0.34375
472250023232509	0.25	0.625	0.8125	0.90625
772405727553181	0.25	0.625	0.8125	0.90625
5422745703328963	0.75	0.375	0.1875	0.09375
69605379628229681	0.25	0.125	0.0625	0.53125

Table 2B

	1/64	1/128	1/256	1/512
4877863751	0.109375	0.5546875	0.27734375	0.638671875
14048615549	0.953125	0.9765625	0.48828125	0.244140625
329650448509	0.953125	0.9765625	0.48828125	0.244140625
1657851033857	0.015625	0.0078125	0.00390625	0.501953125
59708282945131	0.671875	0.8359375	0.41796875	0.208984375
472250023232509	0.953125	0.9765625	0.98828125	0.994140625
772405727553181	0.453125	0.2265625	0.61328125	0.306640625
5422745703328963	0.046875	0.5234375	0.76171875	0.380859375
69605379628229681	0.765625	0.3828125	0.19140625	0.095703125

Table 2C

	1/1024	1/2048	1/4096	1/8192
4877863751	0.8193359375	0.40966796875	0.704833984375	0.3524169921875
14048615549	0.1220703125	0.56103515625	0.780517578125	0.8902587890625
329650448509	0.1220703125	0.06103515625	0.530517578125	0.2652587890625
1657851033857	0.2509765625	0.12548828125	0.562744140625	0.7813720703125
59708282945131	0.2509765625	0.12548828125	0.562744140625	0.7813720703125
472250023232509	0.9970703125	0.49853515625	0.249267578125	0.1246337890625
772405727553181	0.6533203125	0.82666015625	0.413330078125	0.2066650390625
5422745703328963	0.1904296875	0.59521484375	0.797607421875	0.3988037109375
69605379628229681	0.0478515625	0.52392578125	0.261962890625	0.1309814453125

In tables 3A-B, there are two examples of 30- and 34-digit numbers produced by three prime factors of different values. Dividing them by 2^n in ascending order generates decimal parts which have the same kinds of decimal parts although with different digits before 25 (or 75). This further confirms that integers divided by 2^n always generate decimal parts ending with 25 or 75 and having the same number of digits, which depend on the magnitude of n in 2^n .

Tables 3: division by 2^n still generate 25 (or 75) as digits in the decimal parts of quotients obtained with two larger numbers.

Table 3A

	185592806269448167697065108937 = 91969 × 16588907 × 121647139715480539
$\frac{1}{4}$	46398201567362041924266277234.25
$\frac{1}{8}$	23199100783681020962133138617.125
$\frac{1}{16}$	11599550391840510481066569308.5625
$\frac{1}{32}$	5799775195920255240533284654.28125
$\frac{1}{64}$	2899887597960127620266642327.140625
$\frac{1}{128}$	1449943798980063810133321163.5703125
$\frac{1}{256}$	724971899490031905066660581.78515625
$\frac{1}{512}$	362485949745015952533330290.892578125
$\frac{1}{1024}$	181242974872507976266665145.4462890625
$\frac{1}{2048}$	90621487436253988133332572.72314453125
$\frac{1}{4096}$	45310743718126994066666286.361572265625
$\frac{1}{8192}$	22655371859063497033333143.1807861328125

Table 3B

	5643479997656899909896887654211013 = 17 × 113 × 2937782403777667834407541725253
$\frac{1}{4}$	1410869999414224977474221913552753.25
$\frac{1}{8}$	705434999707112488737110956776376.625
$\frac{1}{16}$	352717499853556244368555478388188.3125
$\frac{1}{32}$	176358749926778122184277739194094.15625
$\frac{1}{64}$	88179374963389061092138869597047.078125
$\frac{1}{128}$	44089687481694530546069434798523.5390625
$\frac{1}{256}$	22044843740847265273034717399261.76953125
$\frac{1}{512}$	11022421870423632636517358699630.884765625
$\frac{1}{1024}$	5511210935211816318258679349815.4423828125
$\frac{1}{2048}$	2755605467605908159129339674907.72119140625
$\frac{1}{4096}$	1377802733802954079564669837453.860595703125
$\frac{1}{8192}$	688901366901477039782334918726.9302978515625

4.1B First method based upon division by 2^n

Let a biprime number $B_n = p \times q$ therefore $B_n/4 = p/4 \times q$ or $p \times q/4$. Let $p < q$ and let us focus on the smallest factor p . So $B_n/4 : p/4 = q$ and it follows that $B_n : p/4 = 4 \times q$. We can generalize to any 2^n to the (power) n by setting the global equation: $B_n : p/2^n = 2^n \times q$. Let's first look at the case of 2^2 ($n = 2$), 2^3 ($n = 3$), 2^4 ($n = 4$), and 2^5 ($n = 5$). This is therefore the first method of decomposing natural numbers into prime factors in this article as described in Table 4. A calculator of decimal fractions is needed for the whole study. Put the number to decompose B_n as the numerator and the denominators as indicated. Then changes the denominators as indicated and lets the decimal quotients scroll until the desired quotient is obtained, i.e. an integer. The method requires a special calculator or a computer program where 25 or 75 digits are fixed in the decimal part. We only vary the values framed by the two vertical arrows including N for 2^2 fraction or X , XX and XXX for the others. This is also true for the whole study where it is indicated.

Table 4. First Method for the decomposition of integer into prime factors ($B_n = p \times q$). Note that two digits 75 or 25 of the decimal parts are fixed and only X, XX or XXX varies starting with 0, 00 or 000. N varies only in the case of 2^2 fraction. The digits that must change are framed by two vertical arrows in the whole study.

	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
Numerator /	$B_n /$	$B_n /$	$B_n /$	$B_n /$
Denominator	$\uparrow N \uparrow .75$	$N. \uparrow X \uparrow 75$	$N. \uparrow XX \uparrow 75$	$N. \uparrow XXX \uparrow 75$
Or				
Denominator	$\uparrow N \uparrow .25$	$N. \uparrow X \uparrow 25$	$N. \uparrow XX \uparrow 25$	$N. \uparrow XXX \uparrow 25$
The quotient must be an integer =	$2^2 \times q$	$2^3 \times q$	$2^4 \times q$	$2^5 \times q$

This first method based on the equation $B_n : p/2^n = 2^n \times q$ then gives us submultiples of the biprime number $B_n = p \times q$ including $4q, 8q, 16q$ and $32q$. This is a new method described for the first time in this paper which allows to factorize integers into products of prime factors. The only particularity is to be able to vary the denominator by fixing the two digits 75 or 25 of the decimal part while increasing X, XX or XXX starting from 0, 00, or 000. Here are the instructions or the calculation program in the box below to apply this method for any number to decompose. As soon as we have an integer as a result, we divide it by 2^n , and we have the prime factor of B_n .

- First, put the decimal fraction with B_n as the numerator and as denominators the decimals shown below.
- Put $\uparrow N \uparrow .75$ or $\uparrow N \uparrow .25$ as denominators then increase N to divide B_n .
- Put $N. \uparrow X \uparrow 75$ or $N. \uparrow X \uparrow 25$ as denominators with $0 \leq X \leq 9$. Change X to divide B_n . N will increase from 1 to $+\infty$.
- Put $N. \uparrow XX \uparrow 75$ or $N. \uparrow XX \uparrow 25$ as denominators with $00 \leq XX \leq 99$. Change XX to divide B_n . N will increase from 1 to $+\infty$.
- Put $N. \uparrow XXX \uparrow 75$ or $N. \uparrow XXX \uparrow 25$ as denominators with $000 \leq XXX \leq 999$. Change XXX to divide B_n . N will increase from 1 to $+\infty$.

Factorize B_n once you get an integer as a result of the decimal fractions

To explain the method well, biprime numbers $B_n = p \times q$ whose prime factors we know are used and then we apply the method to fall back on their prime factors. We will give examples of numbers with an increasing number of digits (Tables 5A-D). All integers which are the quotients of the fractions in the tables are in the form of $2^n \times q$ corresponding to the box in which they are found.

Tables 5: Examples of the application of the first method based on series of divisions by 2^n ($n \leq 5$) to decompose randomly chosen biprime numbers (B_n). Quotients (Q) are highlighted.

Table 5A : The number 91 to decompose (SR = 9.5393920); $91 = p \times q$ with $p = 7$ and $q = 13$.

$B_n = p \times q$ ($q > p$)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
$t = p/2^n$	$7/4 = 1.75$	$7/8 = 0.875$	$7/16 = 0.4375$	$7/32 = 0.21875$
$Q = 91/t$	52	104	208	416
$t = q/2^n$	$13/4 = 3.25$	$13/8 = 1.625$	$13/16 = 0.8125$	$13/32 = 0.40625$
$Q = 91/t$	28	56	112	224
Decompose the integers (or the quotients Q) obtained and divide B_n by their prime factors > 2 . We will then find that $91 = 7 \times 13$.				

Table 5B : The number 152771243 to decompose (SR = 12360.0664642); p = 11383 and q = 13421

$B_n = p \times q$ ($q > p$)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
$t = p/2^n$	$11383/4 = 2845.75$	$11383/8 = 1422.875$	$11383/16 = 711.4375$	$11383/32 = 355.71875$
$Q = 152771243/t$	53684	107368	214736	429472
$t = q/2^n$	$13421/4 = 3355.25$	$13421/8 = 1677.625$	$13421/16 = 838.8125$	$13421/32 = 419.40625$
$Q = 152771243/t$	45532	91064	182128	364256
Decompose the integers (or the quotients Q) obtained and divide B_n by their prime factors > 2 . We will then find that $152771243 = 11383 \times 13421$				

Table 5C : The number 4877863751 to decompose (SR = 69841.7049548); p = 6247 and q = 780833.

$B_n = p \times q$ ($q > p$)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
$t = p/2^n$	$6247/4 = 1561.75$	$6247/8 = 780.875$	$6247/16 = 390.4375$	$6247/32 = 195.21875$
$Q = 4877863751/t$	3123332	6246664	12493328	24986656
$t = q/2^n$	$780833/4 = 195208.25$	$780833/8 = 97604.125$	$780833/16 = 48802.0625$	$780833/32 = 24401.03125$
$Q = 4877863751/t$	24988	5649976	99952	199904
Decompose the integers (or the quotients Q) obtained and divide B_n by their prime factors > 2 . We will then find that $4877863751 = 6247 \times 780833$				

Table 5D : The number 69605379628229681 to decompose (SR = 263828314.6825406); p = 854299 and q = 81476602019

$B_n = p \times q$ ($q > p$)	$t = 2^2$ Fraction	$t = 2^3$ Fraction	$t = 2^4$ Fraction	$t = 2^5$ Fraction
$t = p/2^n$	$854299/4 = 213574.75$	$854299/8 = 106787.375$	$854299/16 = 53393.6875$	$854299/32 = 26696.84375$
$Q = 69605379628229681/t$	325906408076	651812816152	1303625632304	2607251264608
$t = q/2^n$	$81476602019/4 = 20369150504.75$	$81476602019/8 = 10184575252.375$	$81476602019/16 = 5092287626.1875$	$81476602019/32 = 2546143813.09375$
$Q = 69605379628229681/t$	3417196	6834392	13668784	27337568
Decompose the integers (or the quotients Q) obtained and divide B_n by their prime factors > 2 . We will then find that $69605379628229681 = 854299 \times 81476602019$				

The method can be extended to all powers of 2 to infinity. As above, these results are summarized in the tables below: tables 6A-C for 2^n such n ranges from 6 to 13; and tables 7A-B for n ranging from 14 to 17.

Tables 6: Examples of the application of the first method based on series of divisions by 2^n ($5 < n \leq 13$) to decompose randomly chosen biprime numbers (Bn). Quotients (Q) are highlighted.

Table 6A : The number 4877863751 to decompose (SR = 69841.7049548); $p = 6247$ and $q = 780833$

$B_n = p \times q$ ($q > p$)	2^6 Fraction	2^7 Fraction	2^8 Fraction	2^9 Fraction
$t = p/2^n$	$6247/64 = 97.609375$	$6247/128 = 48.8046875$	$6247/256 = 24.40234375$	$6247/512 = 12.201171875$
$Q = 4877863751/t$	49973312	99946624	199893248	399786496
$t = q/2^n$	$780833/64 = 12200.515625$	$780833/128 = 6100.2578125$	$780833/256 = 3050.12890625$	$780833/512 = 1525.064453125$
$Q = 4877863751/t$	399808	799616	1599232	3198464
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2 . We will then find that $4877863751 = 6247 \times 780833$				

Table 6B : The number 69605379628229681 to decompose (SR = 263828314.6825406); $p = 854299$ and $q = 81476602019$

$B_n = p \times q$ ($q > p$)	2^6 Fraction	2^7 Fraction	2^8 Fraction	2^9 Fraction
$t = p/2^n$	$854299/64 = 13348.421875$	$854299/128 = 6674.2109375$	$854299/256 = 3337.10546875$	$854299/512 = 1668.552734375$
$Q = 69605379628229681/t$	5214502529216	10429005058432	20858010116864	20858010116864
$t = q/2^n$	$81476602019/64 = 1273071906.546875$	$81476602019/128 = 636535953.2734375$	$81476602019/256 = 318267976.63671875$	$81476602019/512 = 159133988.318359375$
$Q = 69605379628229681/t$	54675136	109350272	218700544	437401088
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2 . We will then find that $69605379628229681 = 854299 \times 81476602019$				

Table 6C : The number 4877863751 to decompose (SR = 69841.7049548); $p = 6247$ and $q = 780833$

$B_n = p \times q$ ($q > p$)	2^{10} Fraction	2^{11} Fraction	2^{12} Fraction	2^{13} Fraction
$t = p/2^n$	$6247/1024 = 6.1005859375$	$6247/2048 = 3.05029296875$	$6247/4096 = 1.525146484375$	$6247/8192 = 0.7625732421875$
$Q = 4877863751/t$	799572992	1599145984	3198291968	6396583936
$t = q/2^n$	$780833/1024 = 762.5322265625$	$780833/2048 = 381.26611328125$	$780833/4096 = 190.633056640625$	$780833/8192 = 95.3165283203125$
$Q = 4877863751/t$	6396928	12793856	25587712	51175424
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2 . We will then find that $4877863751 = 6247 \times 780833$				

Tables 7: Examples of the application of the first method based on series of divisions by 2^n ($14 < n \leq 17$) to decompose randomly chosen biprime numbers (Bn). Quotients (Q) are highlighted.

Table 7A : The number 4877863751 to decompose (SR = 69841.7049548); $p = 6247$ and $q = 780833$.

$B_n = p \times q$ ($q > p$)	2^{14} Fraction	2^{15} Fraction	2^{16} Fraction	2^{17} Fraction
$t = p/2^n$	$6247/16384 = 0.38128662109375$	$6247/32768 = 0.190643310546875$	$6247/65536 = 0.0953216552734375$	$6247/131072 = 0.04766082763671875$
$Q = 4877863751/t$	12793167872	25586335744	51172671488	102345342976
$t = q/2^n$	$780833/16384 = 47.65826416015625$	$780833/32768 = 23.829132080078125$	$780833/65536 = 11.9145660400390625$	$780833/131072 = 5.95728302001953125$
$Q = 4877863751/t$	102350848	204701696	409403392	818806784
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that $4877863751 = 6247 \times 780833$				

Table 7B : The number 5422745703328963 to decompose (SR = 73639294.5602343); $p = 549319$ and $q = 9871760677$

$B_n = p \times q$ ($q > p$)	2^{14} Fraction	2^{15} Fraction	2^{16} Fraction	2^{17} Fraction
$t = p/2^n$	$549319/16384 = 33.52777099609375$	$549319/32768 = 16.763885498046875$	$549319/65536 = 8.3819427490234375$	$549319/131072 = 4.19097137451171875$
$Q = 5422745703328963/t$	161738926931968	323477853863936	646955707727872	1293911415455744
$t = q/2^n$	$9871760677/16384 = 602524.45538330078125$	$9871760677/32768 = 301262.227691650390625$	$9871760677/65536 = 150631.1138458251953125$	$9871760677/131072 = 75315.55692291259765625$
$Q = 5422745703328963/t$	9000042496	18000084992	36000169984	72000339968
Decompose the integers (or the quotients Q) obtained and divide Bn by their prime factors > 2. We will then find that $5422745703328963 = 549319 \times 9871760677$				

Note that the more 2^n increases the more the decimal fraction obtained tends towards 0 which shows that this method can be very quick to decompose a natural number into products of prime factors. The most important thing is to have a continuous decimal fraction calculator where the denominator can be a decimal fraction. It is also possible to create a computer program or an algorithm by following the instructions indicated in Table 4 which directly gives the quotient Q and the prime factor of the number to be decomposed. Here are the instructions or the calculation program of the method in the box below (n from 4 to 7 for example). Make up the decimal fraction and put Bn as the numerator. Then follow what is below.

-
- a) As denominators, Put $N.\uparrow XXXX\uparrow 75$ or $N.\uparrow XXXX\uparrow 25$ with $0000 \leq XXXX \leq 9999$. Change XXXX to divide BN. N will increase from 1 to $+\infty$.
 - b) As denominators, Put $N.\uparrow XXXXX\uparrow 75$ or $N.\uparrow XXXXX\uparrow 25$ with $00000 \leq XXXXX \leq 99999$. Change XXXXX to divide BN. N will increase from 1 to $+\infty$.
 - c) As denominators, Put $N.\uparrow XXXXXXX\uparrow 75$ or $N.\uparrow XXXXXXX\uparrow 25$ with $000000 \leq XXXXXXX \leq 999999$. Change XXXXXXX to divide BN. N will increase from 1 to $+\infty$.
 - d) As denominators, Put $N.\uparrow XXXXXXXX\uparrow 75$ or $N.\uparrow XXXXXXXX\uparrow 25$ with $0000000 \leq XXXXXXXX \leq 9999999$. Change XXXXXXXX to divide BN. N will increase from 1 to $+\infty$.
-

Here are the instructions or the calculation program of the method in the box below (n from 10 to 13) (see Table 6C). Make up the decimal fraction and put Bn as the numerator. Then follow what is below.

-
- a) As denominators, Put $N.\uparrowXXXXXXXXXX\uparrow75$ or $N.\uparrowXXXXXXXXXX\uparrow25$ with $0000000000 \leq XXXXXXXXXXXX \leq 9999999999$. Change XXXXXXXXXXXX to divide Bn. N will increase from 1 to $+\infty$.
- b) As denominators, Put $N.\uparrowXXXXXXXXXX\uparrow75$ or $N.\uparrowXXXXXXXXXX\uparrow25$ with $0000000000 \leq XXXXXXXXXXXX \leq 9999999999$. Change XXXXXXXXXXXX to divide Bn. N will increase from 1 to $+\infty$.
- c) As denominators, Put $N.\uparrowXXXXXXXXXX\uparrow75$ or $N.\uparrowXXXXXXXXXX\uparrow25$ with $0000000000 \leq XXXXXXXXXXXX \leq 9999999999$. Change XXXXXXXXXXXX to divide Bn. N will increase from 1 to $+\infty$.
- d) As denominators, Put $N.\uparrowXXXXXXXXXX\uparrow75$ or $N.\uparrowXXXXXXXXXX\uparrow25$ with $000000000000 \leq XXXXXXXXXXXX \leq 999999999999$. Change XXXXXXXXXXXX to divide Bn. N will increase from 1 to $+\infty$.
-

We can continue for increasing values of n of the powers of 2^n to infinity but this study is limited to what has been shown (n = 17).

Conclusion: This method is used to find submultiples of the number to be decomposed or, in other words, to go up against the grain of Eratosthenes's sieve. The goal is to find by specific calculations a submultiple which has a prime factor of the number to be decomposed. This is achieved by using powers of 2 to divide the number to be decomposed. The number is finally factorized according to the equation initially posed $B_n : p/2^n = 2^n \times q$. The instructions given show how to apply the method for numbers with unknown prime factors.

4.1C Variant of the first method described by standardizing the denominators by multiplication with powers of 10

Since the method uses decimal divisors it is possible to modify it and make it uniform, i.e. operating in the same way whatever the decimal length of the divisors. Below are the instructions to follow and this time we will just use a few divisors (up to 2^5) without covering them all because the method will always apply in the same way. Make up the decimal fraction and put Bn as the numerator. Then follow what is below. Here are the instruction to follow to apply this new method. Change N for $\frac{1}{4}$ and change NX, NXX, or NXXX for $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$.

-
- a) Divide B_n by N.25 and N.75 \rightarrow N.25 or N.75 (1/4)
- b) If the divisor has one digit in the decimal part before 25 or 75 digits such N.X25 ou N.X75 multiply by 10 \rightarrow $\uparrow NX\uparrow.25$ or $\uparrow NX\uparrow.75$ as denominators (1/8)
- c) If the divisor has two digits in the decimal part before 25 or 75 digits such N.XX25 ou N.XX75 multiply by 100 \rightarrow $\uparrow NXX\uparrow.25$ or $\uparrow NXX\uparrow.75$ as denominators (1/16)
- If the divisor has three digits in the decimal part before 25 or 75 digits such N.XXX25 ou N.XXX75 multiply by 1000 \rightarrow $\uparrow NXXX\uparrow.25$ or $\uparrow NXXX\uparrow.75$ as denominators (1/32)
-

Therefore, all denominators of the decimal fractions have the same form N.75 or N.25. It is as if you divided Bn by the decimals N.75 and N.25 except that this time you will recover at the same time the quotients $\frac{1}{8}$ and $\frac{1}{16}$ and $\frac{1}{32}$ during the same one calculation because these will be decimals which we will make integers by multiplying them by 10^m . To explain it differently, you only divide Bn by 4 but this time you do not limit yourself to taking only the integer quotients but also any decimals with 1, 2 or 3 digits after the decimal point which you multiply by 10^m in order to be able to decompose them and find the q factor of Bn. You could recover all the decimal quotients having one digit, two, three, four, five and by increasing n in $2n$ (or the exponent n) and so on to find all the submultiples of the number to be decomposed. This method does not offer a single solution for factoring a number as in Trial division, but an infinity.

Representative examples are shown in Tables 8A-D. Indeed, with this method three quotients obtained are decimals with one, two and three digits after the decimal separator following the initial equation $B_n : p/2^n = 2^n \times q$ and so its two terms are multiplied by 10^m ($1 \leq m \leq 3$ for $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$) to have integer quotients such $(B_n : p/2^n) \times 10^m = (2^n \times q) \times 10^m$. We therefore multiply the obtained quotient by 10^m so as to have a natural number as a quotient $Q' = p \times 2^n$ whose decomposition will give the factor of the number Bn.

Tables 8: Examples of the application of the first method with variations aimed at standardizing the calculation. Note $1 \leq m \leq 3$ in 10^m .

Table 8A : The number 91 to decompose (SR = 9.5393920)

$B_n = p \times q$ ($q > p$)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
$t = p/2^n$	$7/4 = 1.75$	$7/8 = 0.875$	$7/16 = 0.4375$	$7/32 = 0.21875$
N.75	1.75	8.75	43.75	218.75
$Q = 91/t$	52	10.4	2.08	0.416
Q' (integer) = $Q \times 10^m$	52	104	208	416
$t = q/2^n$	$13/4 = 3.25$	$13/8 = 1.625$	$13/16 = 0.8125$	$13/32 = 0.40625$
N.25	3.25	16.25	81.25	406.25
$Q = 91/t$	28	5.6	1.12	0.224
Q' (integer) = $Q \times 10^m$	28	56	112	224
Decompose the integers (or the quotients Q') obtained and divide B_n by their prime factors > 2 . We will then find that $91 = 7 \times 13$				

Table 8B : The number 152771243 to decompose (SR = 12360.0664642)

$B_n = p \times q$ ($q > p$)	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
$t = p/2^n$	$11383/4 = 2845.75$	$11383/8 = 1422.875$	$11383/16 = 711.4375$	$11383/32 = 355.71875$
N.75	2845.75	14228.75	71143.75	355718.75
$Q = 152771243/t$	53684	10736.8	2147.36	429.472
Q' (integer) = $Q \times 10^m$	53684	107368	214736	429472
$t = q/2^n$	$13421/4 = 3355.25$	$13421/8 = 1677.625$	$13421/16 = 838.8125$	$13421/32 = 419.40625$
N.25	3355.25	16776.25	83881.25	419406.25
$Q = 152771243/t$	45532	9106.4	1821.28	364.256
Q' (integer) = $Q \times 10^m$	45532	91064	182128	364256
Decompose the integers (or the quotients Q') obtained and divide B_n by their prime factors > 2 . We will then find that $152771243 = 11383 \times 13421$				

Table 8C : The number 4877863751 to decompose (SR = 69841.7049548)

$B_n = p \times q (q > p)$	2^2 Fraction	2^3 Fraction	2^4 Fraction	2^5 Fraction
$t = p/2^n$	$6247/4 = 1561.75$	$6247/8 = 780.875$	$6247/16 = 390.4375$	$6247/32 = 195.21875$
N.75	1561.75	7808.75	39043.75	195218.75
$Q = 4877863751/t$	3123332	624666.4	124933.28	24986.656
$Q' \text{ (integer)} = Q \times 10^m$	3123332	6246664	12493328	24986656
$t = q/2^n$	$780833/4 = 195208.25$	$780833/8 = 97604.125$	$780833/16 = 48802.0625$	$780833/32 = 24401.03125$
N.25	195208.25	976041.25	4880206.25	24401031.25
$Q = 4877863751/t$	24988	4997.6	999.52	199.904
$Q' \text{ (integer)} = Q \times 10^m$	24988	5649976	99952	199904
Decompose the integers (or the quotients Q') obtained and divide B_n by their prime factors > 2 . We will then find that $4877863751 = 6247 \times 780833$				

Table 8D : The number 69605379628229681 to decompose (SR = 263828314.6825406)

$B_n = p \times q (q > p)$	$t = 2^2$ Fraction	$t = 2^3$ Fraction	$t = 2^4$ Fraction	$t = 2^5$ Fraction
$t = p/2^n$	$854299/4 = 213574.75$	$854299/8 = 106787.375$	$854299/16 = 53393.6875$	$854299/32 = 26696.84375$
N.75	213574.75	1067873.75	5339368.75	26696843.75
$Q = 69605379628229681/t$	325906408076	65181281615.2	13036256323.04	2607251264.608
$Q' \text{ (integer)} = Q \times 10^m$	325906408076	651812816152	1303625632304	2607251264608
$t = q/2^n$	$81476602019/4 = 20369150504.75$	$81476602019/8 = 10184575252.375$	$81476602019/16 = 5092287626.1875$	$81476602019/32 = 2546143813.09375$
N.75	20369150504.75	101845752523.75	509228762618.75	2546143813093.75
$Q = 69605379628229681/t$	3417196	683439.2	136687.84	27337.568
$Q' \text{ (integer)} = Q \times 10^m$	3417196	6834392	13668784	27337568
Decompose the integers (or the quotients Q') obtained and divide B_n by their prime factors > 2 . We will then find that $69605379628229681 = 854299 \times 81476602019$				

The other option is to completely remove the decimal point and continue the same divisions process. So we must then multiply B_n by a power of 10 to avoid having decimals as a result. We choose a power of 10 (10^m) high enough ($m \geq 5$ for 2^5 and much more when increasing n of 2^n) to be divisible by all the divisors obtained and give an integer number. Even if 10^m is too high, this has no impact because the most important thing is to recover the prime factor which divides B_n . It follows that the quotient is also a multiple of 5 and 2 and of the prime factor which divides B_n . Make up the decimal fraction and put B_n as the numerator. Then follow what is below. The instructions are summarized below and representative examples are shown in Tables 9A-C.

- a) Calculate $B_n' = B_n \times 10^m$ ($m \geq 5$ for up to 2^5)
b) Divide B_n' by changing N only and without changing 75 or 25 digits such that $\uparrow N \uparrow 75$ or $\uparrow N \uparrow 25$ with N increasing from 1 to $+\infty$. Note that the decimal separator is removed.
c) When you get an integer as the quotient, Decompose it and get prime factors of B_n' . You also get prime factors by dividing the denominators that give integer quotients by $2 \times 5n$.

Tables 9: Examples of the application of the first method with other variations aimed at standardizing the calculation. This involves eliminating the decimal point. Note that we multiply B_n by 10^5 in all cases.

Table 9A : The number 91 to decompose (SR = 9.5393920). Note that we multiply $B_n = 91$ by 10^5 .

7	7/4	7/8	7/16	7/32
Divisors N75	175	0875	04375	021875
$Q' = 9\ 100\ 000/N75$	52000	10400	2080	416
13	13/4	13/8	13/16	13/32
Divisors N25	325	1625	08125	040625
$Q' = 9\ 100\ 000/N25$	28000	5600	1\ 120	224
Decompose the integers (or the quotients Q') obtained and divide B_n by their prime factors > 5 . We will then find that $91 = 7 \times 13$				

Table 9B : The number 152771243 to decompose (SR = 12360.0664642). Note that we multiply $B_n = 152771243$ by 10^5 .

11383	f/4	f/8	f/16	f/32
Divisor N75	284575	1422875	7114375	35571875
$Q = 15277124300000/N75$	53684000	10736800	2147360	429472
13421	f/4	f/8	f/16	f/32
Divisor N25	335525	1677625	8388125	41940625
$Q' = 15277124300000/N25$	45532000	9106400	1821280	364256
Decompose the integers (or the quotients Q') obtained and divide B_n by their prime factors > 5 . We will then find that $152771243 = 11383 \times 13421$				

Table 9C : The number 69605379628229681 to decompose (SR = 263828314.6825406). Note that we multiply Bn = 69605379628229681 by 10^5 .

854299	$854299/4 = 213574.75$	$854299/8 = 106787.375$	$854299/16 = 53393.6875$	$854299/32 = 26696.84375$
Divisor N75	21357475	106787375	533936875	2669684375
Q' = 6960537962822968100 000/N75	32590640807600000	6518128161520000	1303625632304000	260725126460800
81476602019	$81476602019/4 = 20369150504.75$	$81476602019/8 = 10184575252.375$	$81476602019/16 = 5092287626.1875$	$81476602019/32 = 2546143813.09375$
Divisor N75	2036915050475	10184575252375	50922876261875	254614381309375
Q' = 6960537962822968100 000/N75	3417196000	683439200	136687840	683439200
Decompose the integers (or the quotients Q') obtained and divide Bn by their prime factors > 5. We will then find that $69605379628229681 = 854299 \times 81476602019$				

Overall conclusion on the methods based on divisions by 2^n

First this method is set with a calculator of decimal fractions that works continuously. We put Bn as the numerator and the denominators are defined in the tables and in the instructions cited above. These first methods also require specific calculators where it is necessary to fix two decimal digits after the decimal point (or without the decimal separator) which are 75 or 25 and only vary those which are before (framed by arrows) as if 25 and 75 are separated from the rest of the number which continues to grow as if they were not present. The other option is that you can have several calculators for each fraction based on 2^n because as soon as an integer quotient appears, all the calculations on all the calculators combined automatically stop. Here the approach differs from that of serial divisions because the calculation consists of determining decimal fraction after decimal fraction by increasing the values of the divisors as indicated in the instructions. The calculation is therefore continuous but the path is shorter than that of Euclidean divisions in series and the number of operations can be drastically reduced in these new methods because in the end we are only counting, i.e. increasing the value of the divisor until you find the one which gives a quotient in the form of an integer. This method is robust, and applicable to any number, even those with two digits like 91. It can help in the decomposition of very large numbers by calculating the quotients with several fractions whose denominator is the desired factor divided by powers of 2. It therefore applies without limits.

4.1D Can we use other decimal divisors?

If we were able to use the 2^n divisors it is because they form a repeating pattern and therefore predictable, however this is not the case for all other numbers. We can always calculate submultiples or supermultiples of the number $Bn = p \times q$, that is to say multiples of p or q.

Let $r = p/n$. Then $Bn : r = pq : r = pq : p/n = pq \times n/p = nq$. However, very few numbers give recognizable fractions and among them are 2, 3, 6 in addition to powers of 10.

Table 10 : Decimal divisors that can work in a decomposition method. The table shows 2, 3, 6 as good divisors but not 7 that vary from a prime number to another. The number $B_n = 5601385979857$ is used as an exemple but the data are true for all composite odd numbers. F means prime factor either p or q of the chosen numbers B_n .

$B_n = 5601385979857$ (p x q)	F:2	$B_n/f:2$
99793 (p)	49896.5	$112260098 = 2 \times 56130049$
56130049 (q)	28065024.5	$199586 = 2 \times 99793$
$B_n = 5601385979857$ (p x q)	F:3**	$B_n/f:3$
99793 (p)	33264.333333	$168390147 = 3 \times 56130049$
56130049 (q)	18710016.333333	$299379 = 3 \times 99793$
$B_n = 5601385979857$ (p x q)	F: 6**	$B_n/f:6$
99793 (p)	16632.16666666 666666666667	$336780293 = 2 \times 3 \times 56130049$
56130049 (q)	9355008.166666 6666666666667	$598758 = 2 \times 3 \times 99793$
$B_n = 5601385979857$ (p x q)	F:7	$B_n/f:7$
99793 (p)	14256.14285714 285714285714	$392910343 = 7 \times 56130049$
56130049 (q)	8018578.428571 42857142857143	$698551 = 7 \times 99793$

** An odd number divided by 3 can also give a decimal extension = 0.666..While divided by 6 can also give another decimal extension of 0.83333...This is true for all odd numbers.

Table 10 shows us the other decimal extensions obtained with other denominators. All prime numbers divided by 2 give a decimal extension 0.5 for example. Odd numbers divided by 3 give two extensions either 0,3333...or 0.66666... The table shows a prime number p divided by 3 giving a decimal extension of 0.33333. But prime number like 597566339 divided by 3 gives instead an extension of 0.66666...Prime numbers divided by 6 either gives an extension of 0.1666666... like in the table or 0.8333333 with another prime numbers like 597566339. Although prime numbers divided by 11 give decimal fractions with a repeating pattern, it varies from one number to another. For example $99793/11 = 9072.0909090909...$ while another prime number $597566273/11 = 54324206.6363636363...$ Therefore, we cannot make a method with 11 because of variable results. Prime numbers divided by 7 give several decimal extensions either 0.428571428571...; 0,14285714285714285714...; 0.28571428571428571... or 0.71428571428571428571.... Even if we can tinker with 7, this will complicate the method and therefore 7 is not a good candidate nor is 11 nor 6^n like 36. We can only develop a calculation method with 2, 3 or 6. We have seen the case of 2^n but this does not work with the powers of 3 or 6. Here are the instructions in the case where one wants to decompose a number using decimal extensions of 2, 3 and 6.

In a similar way this method is based on the calculation of decimal fractions:

-
- Put B_n to decompose into numerator ($B_n = p \times q$ such that $q > p$)
 - Put the denominator by fixing the decimal part and only change N. Therefore, $\uparrow N \uparrow .5$ (we will detect the submultiple $2 \times q$), $\uparrow N \uparrow .3333$ or $\uparrow N \uparrow .6666$ (we will detect $3 \times q$), $\uparrow N \uparrow .1666...$ or $\uparrow N \uparrow .83333...$ (we will detect $6 \times q$). The decimal part in bold must remain unchanged despite N changes and therefore you must have a calculator capable of carrying out this type of specific calculation as metioned above with 2^n .
 - If the quotient is an integer or very close** then decompose B_n by its larger prime factor.

** When we divide BN by a decimal number having .3333... or .6666... or similar decimal parts we could obtain a quotient close to an integer having the form $N.999999999...XXX$ or $N.00000000...XXX...$, in the first case take $N + 1$ and in the second case N.

By contrast, it is entirely possible to develop a reliable and extended method with 10 and its powers which is as robust and relevant as that which we saw with the 2^n subdivisions.

-
- Put the number BN to be decomposed into the numerator of the decimal fraction
 - As a denominator, Put $N.\uparrow X \uparrow$ ($0 \leq X \leq 9$) to detect $10 \times q$. Or $N.\uparrow XX \uparrow$ ($00 \leq XX \leq 99$) to detect $100 \times q$. $N.\uparrow XXX \uparrow$ to detect $1000 \times q$ ($000 \leq XXX \leq 999$). $N.\uparrow XXX...Xn \uparrow$ ($000...0n \leq XXX...Xn \leq 999...9n$) to detect $10^n \times q$. Change X; XX; XXX;...and then N will increase from 1 to $+\infty$
 - Divide the quotient by 10^n et get the prime factor of B_n .
-

Here is an example $B_n = 77633670588622783 = 9865069 \times 7869551707$. If we divide B_n by a decimal number like 986506.9 (in the form of $N.\uparrow X\uparrow$) we will get 10×7869551707 . If we divide it by 98650.69 ((in the form of $N.\uparrow XX\uparrow$) we will get $10^2 \times 7869551707$. If we divide it by 9865.069 (in the form of $N.\uparrow XXX\uparrow$) we will get $10^3 \times 7869551707$. If we divide it by 986.5069 (in the form of $N.\uparrow XXXXX\uparrow$) we will get $10^6 \times 7869551707$. Finally by a decimal divisor like 0.9865069 (in the form of $N.\uparrow XXXXXXX\uparrow$) we will get $10^7 \times 7869551707$. Even with 0.0009865069 (in the form of $N.\uparrow XXXXXXXXX\uparrow$) we will get $10^{10} \times 7869551707$ and so on.

Let us recall to better understand the basis of the method. In reality we have no idea about the prime factor of a number that we want to decompose, we will just put the decimal fraction $B_n/N.X$ or $B_n/N.XX$, or $B_n/N.XXX$ and so on. We are just going to vary X from 0 to 9; XX from 00 to 99, and XXX from 000 to 999 and so on. N will increase and increase till we get that decimal divisor which gives a quotient that is an integer ending up with one or many zeros as digit units and which is $= 10^n \times q$. Then we can decompose our number B_n by dividing it with q .

4.1E Case of very large numbers

The larger the number, the more we can increase the exponent n of the denominator 2^n . If we divide an odd number by 2^n , it will have a decimal part of n digits that end in 25 or 75. Here are four examples of numbers divided by 2^n ($n = 20; 40; 60$ and 80). The quotients are shown in the table below.

$$A = 4478426278314973908054947361209202363226832797223$$

$$B = 4371670289392423861780657263209609734863176577149611$$

$$C = 296656153839032545132756393640972499493454450571947475882071924710747657058731$$

$$D =$$

$$1223895571497399066025333324562904297975123047381083842035962765384958406174755360357788075768515853039701361219$$

Number	$/2^{20}$	$/2^{40}$	$/2^{60}$	$/2^{80}$
A	4270960119547819049887 606965264513362147171.7 8079891204833984375	1113126537800234533301 8941569779638466344504 6260775037993513502932 2699125418471667542497 085167738853.7061245744 7259657783433794975280 76171875	3884415600212213621387 133539669.3333333561638 7630481044501706833216 3034938275814056396484 375	3704467392170156117808 469.333333333333551062 3579484028341013749328 9027155185522133251652 12154388427734375
B	4169149674789832937031 4190513702485417014852 30.58854198455810546875	3976010966100533425361 079264993904630376.3248 7353185842948732897639 27459716796875	2573081971787784815008 5588200487425906055759 4735267856226178.174927 9339138482365287319275 6374010059516876935958 8623046875	3616160907859496933598 238015.2871925296279413 2466284138853309648438 2209356844839476252673 19381237030029296875
C	2829133547201466990783 2755436036348294587559 7545573688394615101538 417489.1078290939331054 6875	2698072001649348250182 4145732914303106868324 0457128227610221005953. 2332316473284663516096 770763397216796875	2573081971787784815008 5588200487425906055759 4735267856226178.174927 9339138482365287319275 6374010059516876935958 8623046875	2453882190501961531647 2614479529786974006423 4481113296.724489378860 4106081468930518454814 5650873240963818489035 4750677943229675292968 75
D	1167197772500418725991 5669675473254184485655 2827938446239086414850 7078558134799347265041 423576846936263753.2818 0217742919921875	1113126537800234533301 8941569779638466344504 6260775037993513502932 2699125418471667542497 085167738853.7061245744 7259657783433794975280 76171875	1061560190010294469167 6084108142507997841362 5965857542031777861530 5613637369605701010224 423568.4765374242063278 8905771048959536528855 0965487957000732421875	1012382688532156438033 6841686384685514298784 8249299566299226628809 5105779046636296282028. 6021885648130647719649 2105265664062057999472 7101791340828640386462 21160888671875

Here is an example where we see that if we divide a number by 2^{20} , we can factor it after a quick number of operations. In the example below where A is the numerator, we must reach the denominator $75307.34239673614501953125$ to obtain an integer quotient whose decomposition gives the largest factor of the number to be decomposed (A).

$$296656153839032545132756393640972499493454450571947475882071924710747657058731 (A) = 78965471861$$

$$(B) \times 3756783146451975902703159225296742756247967386181329259467691040671 (C)$$

$$78965471861(B) / 2^{20} = 75307.34239673614501953125$$

$$296656153839032545132756393640972499493454450571947475882071924710747657058731 (A) /$$

$$75307.34239673614501953125 =$$

$$3939272644574027084152867887824757332375468649932473509575593600662634496 (D) = 220 \times$$

$$3756783146451975902703159225296742756247967386181329259467691040671$$

4.2 The second Method based on the use of a decomposition key

4.2A- Decomposition Key (Kd)

A decomposition key is the product of prime numbers ($p_1 \times p_2 \times p_3 \times p_4 \times p_5 \times \dots \times p_n$) whose values are in ascending order ($p_n > p_5 > p_4 > p_3 > p_2 > p_1$) which helps with decomposition of all numbers having a given number of digits. So there is one key to decomposing two-digit, another for three-digit, another for 4-digit numbers, and so on. The more digits the number has, the more the key will encompass more prime factors and will have a much greater value. Not only is it a product of prime factors, but some are put into power ($p_1^n \times p_2^m \times p_3 \times p_4 \times p_5 \times \dots \times p_n$) depending on the number of digits of the numbers. This key is calculated by analyzing the value of each number having a given number of digits. This article is limited to Kd specific for numbers with two or three digits. Two-digit numbers are denoted NX and three-digit numbers are NXX. But since the decimal numbers such like N.X or N.XX can be converted into fractions between two integers with 10^n as the denominator such that $NX/10$ or $NXX/100$, whether the number is NX or N.X or is NXX or NX.X or N.XX has no importance for the calculation that we will develop subsequently. Since the key will considerably inflate the value of the number B_n , it is necessary to have very large number calculators or specialized computer programs. See the box below where Kds used in this article are presented. To put it another way, Kd2 will factor all two-digit numbers NX such as $1 \leq NX \leq 99$ and Kd3 for three-digit numbers NXX such as $100 \leq NXX \leq 999$.

$$*Kd2 = 2671979643323542381608979200 = 2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47.$$

$$*Kd2' \text{ (Kd2 extended)} = Kd2 \times 53 \times 59 = 8355280344672717027291277958400$$

$$*Kd2'' \text{ (Kd2 extended)} = Kd2 \times 53 = 163565233866050646890193661728000 = 2^8 \times 3^3 \times 5^3 \times 7^4 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53$$

$$**Kd3 = 2^9 \times 3^6 \times 5^4 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23^2 \times 29^2 \times 31^2 \times 37 \times 41 \times 43 \times 47 \times 53 \times 59 \times 61 \times 67 \times 71 \times 73 \times 79 \times 83 \times 89 \times 97 \times 101 \times 103 \times 107 \times 109 \times 113 \times 127 \times 131 \times 137 \times 139 \times 149 \times 151 \times 163 \times 167 \times 173 \times 179 \times 181 \times 191 \times 193 \times 197 \times 199 \times 211 \times 223 \times 227 \times 229 \times 233 \times 239 \times 241 \times 251 \times 257 \times 263 \times 269 \times 271 \times 277 \times 281 \times 283 \times 293 \times 307 \times 311$$

* Used in this study. ** Not used but replaced by extended Kd2 including Kd2' and Kd2''.

4.2B Principles

Let $B_n = p \times q$. Let's call Sri the integer part of its square root (SR), ignoring the decimal part. We calculate a decimal fraction between the number $B_n \times Kd$ to be decomposed and Sri such that $B_n \times Kd / Sri$. Let $Q = (B_n \times Kd) / Sri$. We will from the start decide the calculation such that Q will be in the form of N.X or N.XX. In the first case we use a Kd2 and in the other a Kd3. For this method as for the first ones we need a calculator for decimal fractions in series, that is to say in a continuous and automatic way.

The principle of this method is based on the fact that if we vary the value of the SR_i by increasing or reducing it continuously, we will go through decimal values which can be used by the decomposition key Kd to factorize the number B_n . If $B_n = p \times q$ such that $p < q$, at one time or another we will come across a value of SR_i that allows B_n decomposition. For instance, $Sri = N.X \times p$; $Sri = N.XX \times p$ or $Sri = N.XXX \times p$ and so on. If we divide $B_n \times Kd$ by these values of SR_i , we can obtain a number whose decomposition will give the largest factor of the number B_n (q). Let's see an example with Kd2, that is to say a two-digit number like N.X. For example, $Q = B_n \times Kd2 / SR_i$ and suppose that one value we reach is $Sri = N.X \times p$ therefore $Q = B_n \times Kd2 / N.X \times p = B_n \times Kd2 \times 10 / NX \times p$. Let $Kd2a = Kd2 \times 10$ and $B_n = p \times q$. Hence $p \times q \times Kd2a / NX \times p = q \times Kd2a / NX$. Let $NX = p_1 \times p_2$ and knowing that $Kd2a = p_1 \times p_2 \times p_3 \times p_4 \times p_5 \times p_6 \times \dots \times p_n \times 10$ then $Q = q \times Kd2a' / NX$ with $Kd2a' = p_1 \times p_2 \times p_3 \times p_4 \times p_5 \times p_6 \times \dots \times p_n \times 10$.

Indeed, Kd2 contains all the prime factors which factor any two-digit number, here we have assumed that $NX = p_1 \times p_2$ to simplify but whatever the prime numbers which are factors of NX, they will be included in Kd2, and therefore by dividing Kd2 by NX, we eliminate the prime factors of NX. Since the chosen value of Sri corresponds to $NX \times p$, by dividing $B_n \times Kd / Sri$, we have $B_n \times Kd / p \times NX$ and given that $B_n = p \times q$, we then eliminate p from the numerator and denominator, and then we finally have the remainder of $Kd \times q$ (the largest factor of B_n).

Any Q value that is divisible in this way by Kd2 allows B_n decomposition. This also applies to the largest factor q if we increase SR_i value. A certain number of operations is required before obtaining a desired value with a very short decimal part such like N.X or N.XX; N.XXX; or N.XXXX depending on the Kd we want to use. In all cases, we will have the closest one when the progress of the continuous calculation of decimal fractions reaches it.

The speed of the method depends on the distance between Sri and the values like Sri x N.X (in the case of Kd2) or Sri x N.XX (in the case of Kd3) (or even Sri x N.XXX in the case of kd4 and so on). Note that these values of Sri could be obtained by reducing and/or increasing Sri and this is why two calculators must be operational in parallel.

4.2C- Representative examples of the calculation

In order to explain more this method, we will give various examples. We will see a first example before establishing the instructions to follow for this method.

Here, a Kd2 adapted and readjusted to the chosen examples is used to save space and make the explanations more plausible and be able to carry out the calculations because if Kd3 or Kd4 are used, the limits of the calculators available on the web or conventional will be exceeded. The Kd3 in the box above gives you an idea of the complex construction of the decomposition keys which must be the product of all the prime factors and powers of prime numbers capable of forming a three-digit number (up to 999) either combined together into products or in powers. Examples of the use of Kd to factor numbers are listed in tables 11.

Tables 11 : Decomposition by multiplication of the number Bn to factorize by a decomposition key and dividing the product by the integer part of SR (Sri). SF: small factor. LF : large factor. SR : square root. Sri = integer part of SR. Kd2 (decomposition key for two-digit odd numbers). Kd2' or Kd2'' the key decomposition for two-digits numbers extended by its multiplication with more prime factors.

Table 11A

SF	LF	Bn	SRi	SRi/SF	2.5SF	2.5SF - SR
14459876343 7	<u>895514715619</u>	129490320518 144110022503	35984763514 3	2.4885941386 3370574904	361496908592 .5	1649273449.3 131689930371 0662683

2671979643323542381608979200 (Kd2) x 129490320518144110022503 = 345995500431921880816230698761118569805858858937600
345995500431921880816230698761118569805858858937600 : 361496908592.5 = 957118836172255643149703593577954449920 = 2⁹ × 3² × 5 × 7³ × 11² × 13² × 17² × 19² × 23 × 29 × 31 × 37 × 41 × 43 × 47 × 895514715619

SF	LF	Bn	SRi	SRi/SF	6.5SF	6.5SF - SR
9471240377	<u>389714237467</u>	369107722138 9216605059	60754236242	6.4146018708 9495089055	61563062450. 5	808826208.5

2671979643323542381608979200 (Kd2) x 3691077221389216605059 = 986248319748721086334613933548235322254545772800
986248319748721086334613933548235322254545772800 : 61563062450.5 = 160201309111566301048923146690502105600 = 2⁹ × 3² × 5² × 7³ × 11² × 13 × 17² × 19² × 23 × 29 × 31 × 37 × 41 × 43 × 47 × 389714237467

Table 11B

SF	LF	Bn	SRi	SRi/SF	1.32SF	SR - 1.32SF
39871425701 3	<u>698754303007</u>	278603302758 072676738091	52782885745 1	1.3238273981 1029692832	526302819257 .16	1526038193.8 4

2671979643323542381608979200 (Kd2) x 278603302758072676738091 = 744422353532275922257173111915367167756492266707200
744422353532275922257173111915367167756492266707200 : 526302819257.16 = 1414437328272298725648035663718757920000 = 2⁸ × 3 × 5⁴ × 7³ × 11 × 13² × 17² × 19² × 23 × 29 × 31 × 37 × 41 × 43 × 47 × 698754303007

SF	LF	Bn	SRi	SRi/SF	4608.35 x SF	(4608.35 x SF) - SR
22823697877 120277	<u>484703753292</u> <u>603814487663</u>	11062732025 056632153858 950364786583 642651	10517952284 0981894967	4608.3471402 07266808538 7	105179588112 027228512.95	1526038193.8 4

2671979643323542381608979200 (Kd2) x 53 x 59 = 8355280344672717027291277958400 (Kd'2)

141614921096147746225275897600 x 11062732025056632153858950364786583642651 =
92432227447337082323592188819134365189249446133336982453719218443718400

92432227447337082323592188819134365189249446133336982453719218443718400 : 105179588112027228512.95 =
878803854497767400436190504899792571840663105152000 = 2¹⁰ × 3² × 5³ × 7³ × 11² × 13² × 17² × 19² × 23 × 29 × 31 × 41 × 43 × 59 ×
484703753292603814487663

Table 11C

SF	LF	Bn	SRi	(SRi x 10 ⁻⁹)/SF	9.45SF - (SRi x 10 ⁻⁹)	9.45SF
10780251859 59431232990 7551187	<u>956478064792</u> <u>755281357337</u> <u>812662039047</u> <u>944195630644</u> <u>07</u>	103110744366 432692787769 570948735978 666103180950 244877199079 356660902693 0301109	10154346082 66001010577 60046910822 5957770	9.4193959611 646247622	329919246566 150459866311 8063	101873380073 166251517626 3587171.5

163565233866050646890193661728000 (Kd2'') = 2⁸ × 3³ × 5³ × 7⁴ × 11³ × 13² × 17² × 19² × 23 × 29 × 31 × 37 × 41 × 43 × 47 × 53

163565233866050646890193661728000 x
1031107443664326927877695709487359786661031809502448771990793566609026930301109 =
168653330163981276345273748179618754268957139381396147326143518369510602293588945526304179190121658329256
352000 : 1018733800731662515176263587171.5 =
165551913603782604293375443373105817974577638062084096212007067318352254337728000 = 2⁹ × 5³ × 7³ × 11³ × 13² ×
17² × 19² × 23 × 29 × 31 × 37 × 41 × 43 × 47 × 53 × 95647806479275528135733781266203904794419563064407

In the first example in Table 11A, we have the number $B_n = 129490320518144110022503 = 144598763437 (p) \times 895514715619 (q)$. As explained above, we start from numbers with known factors to explain how the method works. The SR of this $B_n = 359847635143.18683100696289337317$. We will consider $S_{ri} = 359847635143$ and ignoring the decimal part. But the question that interests us in the first place is the following: how far is the S_{ri} from a multiple of the smallest factor of B_n , i.e. p ($B_n = p \times q$ such $q > p$)? p is called SF (small factor) in this section. In truth we don't know, but we will decide to set the square root at a predictive value, that is to say we will, for example, look for the case where the $S_{ri}/SF = N.X$ (like $N.5$ thus we look at the two-digits at the end of the quotient digits). We therefore need a key K_{d2} which decomposes all the two-digit numbers NX . We know that the $S_{ri} > SF$, but we are primarily interested in a ratio of type $N.X$. In our example (Table 11A), we have $S_{ri}/SF = 2.48859413863370574904$. We retain 2.48 and we will then look for $SF \times 2.5$ which is one ratio closest to 2.48. The calculation shows that $SF \times 2.5 = 361496908592.5$. It takes 1649273449 operations or decimal fractions before reaching this value from the S_{ri} .

The $K_{d2} = 2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47$. Therefore, $B_n \times K_{d2} = 345995500431921880816230698761118569805858858937600$ and so $345995500431921880816230698761118569805858858937600 : 361496908592.5 = 2^9 \times 3^2 \times 5 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times \underline{895514715619}$. We therefore have the largest factor (LF or q) of B_n and we can break it down.

Indeed, the detailed calculation is as follows:

$345995500431921880816230698761118569805858858937600$: 361496908592.5 =
 $345995500431921880816230698761118569805858858937600$: $(3614969085925 : 10)$ =
 $(345995500431921880816230698761118569805858858937600 \times 10) : 3614969085925$. Because 3614969085925 must be a multiple of SF (which is $= 144598763437$) and indeed $3614969085925 = 25 \times 144598763437$. Therefore, we have:

$(345995500431921880816230698761118569805858858937600 \times 10) : 3614969085925 = 2^9 \times 3^2 \times 5^3 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times \underline{144598763437} \times 895514715619 : (5^2 \times \underline{144598763437}) = 2^9 \times 3^2 \times 5 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 895514715619$.

Note that in this example we have the ratio $25/10 = 2.5$ and therefore the two-digit number $25 = 5^2$ and we have $10 = 2 \times 5$. The method works just as well with all decimals generating natural quotients like 2.1 or 2.2, and even 2.01, 2.001 except that in the last two cases we must have K_{d3} and K_{d4} . Here the K_{d2} has been optimized for the example and we must always have a K_{d2} capable of decomposing any two-digit number. In this example we have set the value $2.5 \times SF$ closest to the S_{ri}/SF ratio, but in reality we ignore the factor of the number to be decomposed and therefore we will just put one digit or two or more after the decimal point of the decimal fraction and wait to have an integer as the quotient. Another key element is how to reduce the number of operations to get the desired value? The only solution is to use higher K_d and thus increase the number of decimals. We have as said above $S_{ri}/SF = 2.48859413863370574904$ (Table 11A, first example):

- if we use 2.49 instead of 2.5 we will have less operations 203285815 (compared to 1649273449 with 2.5). However $249 = 3 \times 83$ and we have to use K_{d3} .
- If we use 2.4885 instead of 2.5 we will have 13612330 operations that are much less than 1649273449 with 2.5. However $24885 = 3^2 \times 5 \times 7 \times 79$ meaning that we have to use K_{d5} .
- If we use 2.48859 instead of 2.5 we will have 598441 operations that are much lesser than 1649273449 with 2.5 or 13612330 with 2.4885. Nevertheless, $248859 = 3^3 \times 13 \times 709$. Hence a need for a K_{d6} .
- If we use 2.4885941386 instead of 2.5 the number of operations decreases drastically because we only have 4 operations to go through from the S_{ri} to decompose the number. However, $24885941386 = 2 \times 12442970693$ and thus we need a K_{d11} .
- If we use two k_d for a same calculation with S_{ri} , we will have much less operation with the higher one because this signifies the use of more decimals after the decimal separator.
- In the example cited above in Table 11A, 2.5 is the closest values to S_{ri} by using K_{d2} . For each key used, we have one closest value. Requiring the least number of operations. Note that K_{d2} factors all numbers between 1 and 99. If we set NX as being any two-digit number, thus any product $S_{ri} \times NX$ such that $1 \leq NX \leq 99$ can be factorized by K_{d2} , we therefore have 99 chances of decomposing B_n using K_{d2} .

- For comparison, and if we pose NXX as being any three-digit number, Kd3 will offer more chances because if we have Sri x NXX such that $100 \leq NXX \leq 999$, we have 899 chances of factoring Bn. To express it differently Bn x Kd2/SRi x NX offers 99 possible factorizations and Bn x Kd3/SRi x NXX offers 899. This is why this method is robust and safe and this is above all the major advantage offered by the use of Kd. Unlike the method of division by series which has only one solution, this method offers several factorization solutions and therefore accelerates the decomposition process.

Here are the instructions to follow for this method

- Calculate SR of Bn to factorize and take only the integer part by ignoring the decimal one (SRi). Let's name it Sri. Choose your Kd. Then calculate Bn x Kd and put it as a numerator in the calculator of decimal fractions. Put Sri as the denominator.
- Let us therefore set the decimal fraction Bn x K/SRi = N.X or N.XX (in this article we only consider one or two digits after the decimal point and the corresponding Kd2 and Kd3). You might use two calculators : one to increase SRi (Bn x K/ \uparrow SRi \uparrow = N.X or N.XX) and the other to decrease it (Bn x K/ \downarrow SRi \downarrow = N.X or N.XX). If one gives the desired quotient, all the calculations stop all at once.
- Start the calculation by reducing and/or increasing Sri which must then be set as N.0 or N.00. For example if we start from 359847635143 from the example cited above, we then start with 359847635142.9 or 359847635142.99. We let it scroll towards 0 or to higher values.
- Stop the calculation as soon as you obtain an integer, decompose it and take the prime factor p or q in order to break down the number Bn.

The second example of the Table 11A is the number $Bn = 3691077221389216605059 = 9471240377 \times 389714237467$ has for SRi = 60754236242 and the ratio SRi/SF = 6.41460187089495089055 and therefore the closest number is SF x 6.5. And we therefore have the ratio 65/10 which will impose itself and therefore $65 = 5 \times 13$ and $10 = 2 \times 5$ and thus the Kd2 defined above is able to decompose it. It takes 808826208 decimal fractions to go from SRi = 60754236242.0 up to SRi x 6.5 = 61563062450.5.

In table 11B, we will use the same Kd2 for the decomposition of a three-digit number. As said above the use of a Kd3 is avoided to save space and not to interrupt the text with a large gap, but in reality we have to use a Kd3 for a three-digit number but the methods works the same anyway. Indeed, a Kd2 can work for some three-digit numbers which are the products of smaller prime factors and in this study it was extended to be able to provide explanatory demonstrations.

In table 11B, we have the number $278603302758072676738091 = 398714257013 \times 698754303007$ and whose SRi = 527828857451. The SRi/SF ratio = 1.32382739811029692832. Here we set the ratio Bn/SRi = N.XX. You have to count or scroll through 1526038193 decimal fractions on calculators before arriving at $1.32SF = 526302819257.16$. We therefore have the ratio 132/100 and then $132 = 2^2 \times 3 \times 11$ and $100 = 2^2 \times 5^2$. We will use to simplify the Kd2 cited above because it works. $Kd2 = 2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47$ and after calculation as shown in table 11B, we arrive at the decomposition which gives the LF (the largest factor of the number Bn) as follows = $744422353532275922257173111915367167756492266707200 : 526302819257.16 = 1414437328272298725648035663718757920000 = 2^8 \times 3 \times 5^4 \times 7^3 \times 11 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 698754303007$.

In the second example of Table 11B, we use a more extended Kd2 noted Kd2' (meaning that we multiply the initial Kd2 with two more prime factors to enlarge it). $Kd2' = Kd2 \times 53 \times 59 = 2^8 \times 3^2 \times 5^2 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53 \times 59 = 8355280344672717027291277958400$. We see that we have the ratio 295/100 in this example with $295 = 5 \times 59$ et $100 = 2^2 \times 5^2$. The number is broken down as explained in the table 11B with the quotient = $878803854497767400436190504899792571840663105152000 = 2^{10} \times 3^2 \times 5^3 \times 7^3 \times 11^2 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 41 \times 43 \times 59 \times 484703753292603814487663$.

In the example described in Table 11C we are dealing with an SR which is too large compared to the prime factor SF. This case is to be mentioned in giant numbers and especially when the two prime factors are large with a large gap between them and also far from each other from the SR. In this case, it will be necessary to provide a certain number of calculators which operate in parallel but interrelated so that if one displays an integer, the calculation stops on all of them simultaneously. We multiply the SR with 10^{-n} depending on its value. The higher Kd, the fewer operations are required. But if we configure several interconnected calculators, we could work with Kd2 and Kd3.

The number Bn in Table 11C that we want to decompose consists of 79 digits. You have to count 3299192465661504598663118063 decimal fractions to get to $9.45 \times SF = 1018733800731662515176263587171.5$. Here we use a Kd2" extended (meaning that we multiply the initial Kd2 with more prime factors to enlarge it). $Kd2" = 163565233866050646890193661728000 = 2^8 \times 3^3 \times 5^3 \times 7^4 \times 11^3 \times 13^2 \times 17^2 \times 19^2 \times 23 \times 29 \times 31 \times 37 \times 41 \times 43 \times 47 \times 53$. We then have the ratio $945/100$ with $945 = 3^3 \times 5 \times 7$ and $100 = 2^2 \times 5^2$. The breakdown of the number is explicit in table 11C.

Conclusion on the method

The method which uses the decomposition key is a new method which can be promising. It can be used for numbers to decompose by starting with their square roots. It works with all numbers. It is necessary to have calculators for very large numbers.

5. Discussion

This article provides original material and proposes new methods for decomposing numbers. It draws its originality from 1) proposing decimal divisors instead of natural integer divisors as in the case of serial divisions known as Trial division algorithm; 2) it also proposes a calculation trick which consists of fixing two decimal digits by varying the rest of the digits of the number which makes it possible to accelerate the decomposition of the odd number into its prime factors; 3) it varies the methods which proves that they are robust since they are flexible and determinate or semi-deterministic; 4) it takes advantage of the decimal parts of decimal or irrational numbers to decompose numbers. 5) it proposes a new notion that of decomposition key (Kd) which is only the product of consecutive prime numbers and their powers (only certain ones) which decomposes all numbers having a fixed number of digits. Here the article uses those specific to two- and three-digit numbers. The Erasthenes's sieve is ineffective when it comes to carrying out an indeterminate number of steps to decompose a number going multiple by multiple, the method described in this article in fact makes it possible to exploit this sieve by a careful calculation of decimal fractions and go up or down to find the submultiples or supermultiples of a number to be decomposed. This sieve gives us all the numbers having a common factor. A number to be decomposed then has an infinity of numbers having a common factor with it. In this article, those having a common fact $<$ the number to be decomposed are called submultiples; and those $>$ are called supermultiples.

The methods described in this article do not only allow the factorization of biprime numbers or any multiples of prime factors, they make it possible to find a whole set of numbers having a common factor with the number to be decomposed, unlike the unique solution offered by trial division where we only have one prime factor that works; or by comparison to other methods such as that of Fermat based on the subtraction between two perfect squares. Here, the decomposition of the number has limitless solutions or outcomes and therefore we can now argue that these methods described in this paper have the particularity of offering several solutions (even unlimited) and therefore of considerably increasing the chances and the speed of decomposition of an odd number whose prime factors we do not know.

The method described here based on the divisions of the number to be decomposed $B_n = p \times q$ by 2^n or 10^n which gives submultiples (or symmetrically supermultiples of B_n) should not be confused with the classic methods of searching for common factors between two numbers, whether manual or using a specific calculator. Let us recall here that a submultiple is of the form $n \times p$ or $n' \times q$ which are both $< B_n$, and a supermultiple is $m \times p$ or $m' \times q$ that both are $> B_n$. Firstly, we are assumed not to know the p and q factors of B_n and secondly the common factor calculator will not be of any help since the p and q factors of B_n are unknown. We cannot list the factors of B_n neither since it only has two which are unknown. The method described here is above all a method of decomposition and factorization like any other algorithm. The decomposition method of this paper has its own specificity and cannot be deduced from any known algorithms which are more complex and based on more advanced mathematical notions. The relative simplicity of this article's methods is an additional advantage because it can be used by a very wide range of audiences.

By using decimal divisors and playing with the location of digits and decimal commas or by allowing the square root to unfold towards 0 or infinity, this study shows that it is possible to decompose numbers with an efficiency that is not negligible compared to existing algorithms whether it is The trial division or others such as Pollard's. These are all limited by the length of the number and the time of calculation or analysis (Mosca & Verschoor, 2022) [see 6]. Unless there is an unintentional error, this type of calculation detailed in this article has not been reported before. The calculation methods are all based on the calculation of decimal fractions with the number to be decomposed into the numerator and the denominator is a decimal divider concocted to result in a sub- or super-multiple of the number to be decomposed which contains one of its prime factors.

The calculator best suited to the methods described here would be that which allows continuous calculation of decimal fractions with decimal numbers as denominators. It is also possible to perform that calculation with specific program on a computer. By analogy, the methods described here would amount to going back through Erasthenes's sieve upstream or downstream to find multiples having a common factor with the number to be decomposed. The idea of decomposing a number by looking for limitless numbers that have a common factor with it is a new idea of this paper.

The methods described are also limited by the length of the operations like those known, but are very promising since they offer possibilities to shorten the paths and save time required for the decomposition of a number. Dramatically increasing the exponent n of 2^n or 10^n which are used to fix the decimal part of the denominator results in decimal fractions tending towards 0 meaning that the decomposition of a giant number could be done quickly.

These new methods described in this article and the concepts they convey might lead to new algorithms or programs to decompose an integer into its prime factors. More importantly, this article attests that the decomposition of a biprime number or multiple of prime factors, does not have only one solution (finding the right prime divisor) but can have an infinity of solutions. This idea has always been present in Erasthenes' sieve if we follow multiple by multiple, but it was necessary to develop a calculation method that puts it into practice, and that is what this article was dedicated to.

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