Resonance Phenomena May Be Interpreted by DBZC: $(f(x)/x)(x=0) := f'(0)$

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September 24, 2024

Abstract: In this note, we would like to show the simple result that resonance phenomena may be interpreted by DBZC: $(f(x)/x)(x = 0) := f'(0)$ by a typical simple example.

Key Words: Division by zero, division by zero calculus, ordinary differential equation, resonance phenomena, singularity.

2010 Mathematics Subject Classification: 30A10, 30H10, 30H20, 30C40.

1 Results

In this note, we would like to show the simple result that resonance phenomena may be interpreted by DBZC: $(f(x)/x)(x = 0) := f'(0)$ by a typical simple example.

For the equation

$$y''(t) + k^2 y(t) = \sin \omega t \quad (k, \omega > 0, k \neq \omega)$$

satisfying the initial conditions

$$y(0) = 0$$

and

$$y'(0) = 1,$$

we have the solution

$$y(t) = \frac{k^2 - \omega^2 - \omega}{k(k^2 - \omega^2)} \sin kt + \frac{\sin \omega t}{k^2 - \omega^2}.$$

By the division by zero calculus

$$\frac{f(x)}{(x-a)^n}|_{x=a} := \frac{f^{(n)}(a)}{n!}$$

(see the basic references), for $\omega = k$, we have, directly

$$y(t) = \frac{\sin kt}{k} + \frac{\sin kt}{2k^2} - \frac{t \cos kt}{2k}.$$

Note that this solution satisfies all the requested conditions.

Of course, for $k = \omega$, we obtain the same corresponding solution.

Acknowledgement

The author is gathering examples on the division by zero and division by zero calculus, and this was listed with No. 1318 on 21th, September, 2024.

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