# Multiple of a Sixth Power is a Sum of Two Cubes


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We arrive at the below mentioned Identity:

$$\boldsymbol{a^3 + b^3 = w(c)^6}$$

Above has parametric solution as:

$$a = (x^3 - 3x^2y + y^3)(x^3 + 3x^2y - 6xy^2 + y^3)$$

$$b = 3xy(x^2 - y^2)(x - 2y)(2x - y)$$

$$c = (x^2 - xy + y^2)$$

$$w = (x^6 + 6\,x^5y - 30\,x^4y^2 + 20\,x^3y^3 + 15\,x^2y^4 - 12\,x\,y^5 + y^6)$$

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**Method:**

$a^3 + b^3 = w(c)^6$ -------- (1)

$$We\ take\ (a + b) = w$$

$$Hence\ we\ have, \qquad (a + b)(a^2 - ab + b^2) = w[c]^6$$

$$Hence, (a^2 - ab + b^2) = [c]^6$$

$$We\ take, \quad (a,b) = (p^2-q^2),(2pq-q^2)$$

$$Hence, \ (a^2-ab+b^2) = (p^2-pq+q^2)^2 --- (2)$$

$$Hence\ from\ (2), \qquad (p^2-pq+q^2) = [c]^3 \ --- (3)$$

$$We\ take\ c = (x^2-xy+y^2)$$

$$We\ factor, (x^2-xy+y^2) = (x+yu)(x+yu^2) \ where\ u^3 = 1$$

$$And, \qquad (p^2-pq+q^2) = (p+qu)(p+qu^2) \qquad where\ u^3 = 1$$

$$Hence\ from\ (3)\ we\ have:$$

$$[c]^3 = (x^2-xy+y^2)^3 = [(x+yu)(x+yu^2)]^3 = (p^2-pq+q^2)$$

Hence, $(p+qu)(p+qu^2) = [(x+yu)(x+yu^2)]^3$

Equating real & imaginary parts we get:

$$p = x^3-3xy^2+y^3, \qquad q = 3x^2y-3xy^2$$

$$Since, \qquad c = (x^2-xy+y^2) \quad \& \quad w = (a+b)$$

$$And, \qquad (a,b) = (p^2-q^2),(2pq-q^2)$$

We substitute values of (p,q) & we get:

$$a = (x^3-3x^2y+y^3)(x^3+3x^2y-6xy^2+y^3)$$

$$b = 3xy(x^2-y^2)(x-2y)(2x-y)$$

$$c = (x^2-xy+y^2)$$

$$w = (x^6+ 6x^5y - 30x^4y^2 + 20x^3y^3 + 15x^2y^4 - 12xy^5 + y^6)$$

$$For\ (x,y) = (4,1)$$

$$(1513)^3 + (2520)^3 = (4033)(13)^6$$

(For references see below)

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