On Quantum Tangential Spacetime as a finite Hilbert-space

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Abstract:

Nature knows no zero. On a fundamental level there is a minimal length. If there has to be a sort of quantum-spacetime it can not include fourvectors but must be described in tensor-form from first principles. Every coordinate, even in tangential spacetime without gravity-force has constant components in other dimensional directions, which can‘t be set to zero but must be defined over Planck-lenghts. In this case tangential spacetime has to be rewritten in a corrected form. This rewriting will be done. Maybe, from this calculation there can later derived a consistent version of quantum gravity, which leads to finite physical states in this theory. In $T_Q(M)$, there is ergo constructed a Frobenius-scalarproduct of matrices and therefore a finite Hilbert-space. A Hilbert space is a real or complex vector space H with a dot product $\langle .,. \rangle$, which is complete with respect to the norm induced by the dot product, i.e. in which every Cauchy sequence converges. A Hilbert space is therefore a complete pre-Hilbert space. Therefore the matrix space K (m × n) of the real or complex matrices with the Frobenius -scalar product is a Hilbertspace. Instead of classical continuos spacetime, there is the transition of $O(3,1) \rightarrow (S(2,1)_{PL} x \mathbb{Z})^4$ to a fourdimensional spacetime-lattice.

Key-words:

Quantum-lineelement; quantum tangential spacetime; finite Hilbert-space; Planck-length; metron; fundamental Ricci-scalar: Planck-Minkowski-lattice.

1. Introduction:

Citation: M.v.Laue: “Strictly speaking, the world lines of two bodies cannot intersect, but can only pass close to each other.” (“*Die Weltlinien zweier Körper können sich genaugenommen, nicht schneiden, sondern nur nahe aneinander vorübergehen.”).*[1.]

Planck-length is defined as the smallest physical lenght-size and from this principle must be derived a consistent description of spacetime.[2.] The first trying is to formulate this principle in tangential spacetime to see what consequences there are and if this description can be done in a consistent form without contradictions or if it has to be denied. From this first priciples every spacetime-coordinate is coupled over constant coefficients to each other coordinate in sizes of Planck-length. This change of paradigm means, that there are no fourvectors any longer but every coordinate must be described over a line-vector, so that the whole fourvectors mutates into a tensorlike matrix even in tangential spacetime. (Example given: There is no sphere in real fourdimensional spacetime O(3,1) but only a minimal polyeder in subspace O(3) with a finite, maximal number of edges, points and corners which can evolve in O(3,1). „Real“spheres belong to affine Hausdorff-pointspaces in mathematics, where the minimal distance between two events can be set to zero. See also the comment of Richard Feynman on the theorem of Banach-Tarski). There must be a group or even a mathematical form, where as well as the summing but also the difference leads to a minimal neutral element of addition of the spacetime greater than zero. In full consequence from this idea there are no points in spacetime (and also no point-particles or pointlike events), which means also there are no point-coordinates. Every pointlike coordinate has to be developed into a linevector and so every fourvector must be a matrix of a fourtensor with some constant coefficients even in tangential spacetime, which can be possibly later developed to consistent quantum gravity descriptions. Even, if symmetric tensors are assumed, where the 64 components can reduced to 40 components, there mustn‘t be zeros in the known 24 other components but minimally constant numbers which lead to constant fundamental length of r_{PL} . In general fact this could possibly lead to an eightdimensional theory, which can in this special case but folded to a well known fourdimensional description. The usual restriction to rectangular coordinates in SRT-described tangential spacetime is used. [3.]

2.Methods/Calculations:

The coordinate- transition, which has to be made, is:

$$\begin{aligned} x &\rightarrow (x/r_{PL}/r_{PL}/r_{PL}) \\ y &\rightarrow (r_{PL}/y/r_{PL}/r_{PL}) \\ z &\rightarrow (r_{PL}/r_{PL}/z/r_{PL}) \\ ct &\rightarrow (r_{PL}/r_{PL}/r_{PL}/ct)\,. \end{aligned} \qquad \text{(1a-d.)}$$

This is a transition, where every spacetime coordinate has a constant Planck-Length component in the other three dimension-directions. In this case, every pointlike coordinate becomes a line-vector. If there is only the contemplation of two parallel moving inertial systems with constant velocity v in x,ct -direction, than the components of y,z has also to be reduced to Planck-length r_{PL} . Ergo, every coordinate point-component must be translated into a line-vector, so the whole fourvector for a spacetime-event becomes a tensorlike four-matrix even in $T_Q(M)$, not only in case of gravity. Then there is a transition of the classical form of linelement-product too:

$$s^2=\eta_{i,k}\cdot x^i\cdot x^k\cdot\rightarrow s^2=\hat{\eta}_{i,k}\cdot A^{i,k}:A^{i,k} \qquad \text{(2.)}$$

where $A^{i,k}:A^{i,k}$ means the classical Frobenius scalarproduct of two real matrices. With this product there is defined a norm and the matrices build a finite Hilbert-space with all needed conditions for this space. The definition of the assigned matrices are with [4.]:

with

$$\hat{\eta}_{1,1}=n\cdot K$$
$$\hat{\eta}_{i,k}=m\,;i\neq k \qquad m,n\in\mathbb{N} \qquad (3a\text{-}d.)$$
$$\hat{\eta}_{i,k}=n\,;i,k=2\,;3\,;$$
$$\hat{\eta}_{4,4}=n\cdot(1-K).$$

and

$$A^{1,1}=x\,;$$
$$A^{i,k}=r_{PL},i\neq k\neq 1\,;4. \qquad (4a\text{-}c.)$$
$$A^{4,4}=c\cdot t$$

for a movement of two inertial systems I_1 and I_2 parallel to x-axis with $v=const$. (Definition of K see below (7e.)) .

With Frobenius scalar product of two matrices, [4.],[5.] this leads to the correct quantized lineelement of tangential spacetime $T_Q(M)$ in following form. This linelement includes 64 elements instead of 4 by classical Minkowski-tangential spacetime without quantization. Even if tensors are assumed as symmetric, they mustn‘t account any zeros but normed Planck-sizes as minimal values.

This conditions leads to the whole, complete and correct quantum-line-element of $T_Q(M)$ - tangential-spacetime metric of:

$$s^2=n\cdot K\cdot x^2+50\cdot m\cdot r^2_{PL}+3\cdot n\cdot K\cdot r^2_{PL}+3\cdot m\cdot c\cdot t\cdot r_{PL}+3\cdot m\cdot x\cdot r_{PL} +3\cdot n\cdot(1-K)\cdot r^2_{PL}+n\cdot c^2\cdot t^2\cdot(1-K). \qquad (5a.)$$

This lineelement shall be called Minkowski-quantum-spacetime (MQST) or Minkowski-Planck-lattice (MPL).

For the bounding conditions of $m=0\,;n=1;K\rightarrow 1\,;1-K\rightarrow -1$ this quantum-lineelelement gives back the classical Minkowski-lineelement in two dimensions with constant velocity of the two inertialsystems parallel to x-axis with constant velocity v of:

$$s^2=x^2-c^2\cdot t^2 \qquad (5b.)$$

with its well known $\eta_{i,k}$ of metric tensor for no curvature in flat spacetime.

So the correct transition and connection from the here described elementary quantum case to classical theory of unquantized SRT to this quantized system is given. The metrical tensor for QSRT of $T_Q(M)$ doesn‘t need to be written as an eightdimensional system with its sixtyfour components or doesn‘t need to be reduced to six dimensions because of a possible symmetry with reducing the components of metrical tensor to thirtysix. But the whole system can be described in only four dimensions like classical SRT/GRT.

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For this elementary system of parallel IS the description of coordinate-transformations can be made in only four dimensions. If this folded description also can be made for whole movement in all dimensions $x, y, z, c\cdot t$ or in full gravity-description is well known.

The local fundamental Lorentz-Planck-transformations for the quantum-lattice in two dimensions in $T_Q(M)$ come in following form:

$$\begin{aligned} x' &= C\cdot(x+B\cdot c\cdot t)+2\cdot n\cdot r_{PL} \\ y' &= n\cdot x+3\cdot n\cdot r_{PL} \\ z' &= n\cdot x+3\cdot n\cdot r_{PL} \\ ct' &= C\cdot(B\cdot x+ct)+2\cdot n\cdot r_{PL} \end{aligned} \tag{6a-d.}$$

with following abbrevations:

$$A := \frac{D\cdot E\cdot c^2\cdot t^2}{\tau}$$

$$B := \sqrt{1-A\cdot\frac{v^2}{c^2}}$$

$$C := \frac{A^2}{D}\cdot\left(1+\frac{F}{E\cdot c^2\cdot t^2}\right)$$

$$D := n\cdot K$$
$$E := n\cdot(1-K)$$
$$F := 50\cdot m\cdot r^2_{PL}+3\cdot n\cdot K\, r^2_{PL}+3\cdot m\cdot r_{PL}\cdot c\cdot t+3\cdot m\cdot r_{PL}\cdot x+3\cdot n\cdot(1-K)\cdot r^2_{PL}$$

$$K := \frac{\hbar^2}{c^2\cdot m^2_0}\cdot(\alpha\cdot R+\beta\cdot\Lambda);\ \alpha,\beta\in R.\ Mostly\ choice\ for\ both:1$$

$$\tau := \frac{1}{r^2_{PL}} := R_{Fund} \tag{7a-f.}$$

With choice for alpha and beta of: $\alpha=1;\beta=4$ the bracket can be substituted by T , the Laue-scalar of Einstein-tensor, which can‘t be zero in a quantum form of a tangential spacetime, because it is:

$$\Lambda = \frac{\chi\cdot\mathrm{T}-R}{4} \quad \text{with} \quad \mathrm{T}=\mathrm{T}^k_k=g^{i,k}\mathrm{T}_{i,k} \tag{7g,h.}$$

and $T \neq 0$ but containing constant coefficients means the description of quantum state of spacetime-field without a matter-caused gravity $T_{i,k}$ -tensor.
This means:

$$K := \frac{\hbar^2}{c^2 \cdot m^2{}_0} \cdot T \tag{7i.}$$

τ however is called a metron (after Diplom-Physicist Burkhard Heim). This is the smallest length-square of nature and can be identified with fundamental Ricci-scalar.

The whole quantum-metric of spacetime for gravity in two dimensions now can be formulated over:

$$\begin{aligned} \Delta s^2 = g_{1,1} \cdot \Delta x^2 + \Delta x \cdot r_{PL} \cdot [g_{2,1} + g_{3,1} + g_{4,1}] + r^2{}_{PL} \cdot \sum_{k=2}^{4} [g_{1,k} + g_{2,k} + g_{3,k} + g_{4,k}] \\ + 2 \cdot r^2{}_{PL} \sum_{k=1}^{4} [g_{1,k} + g_{2,k} + g_{3,k} + g_{4,k}] \\ + r^2{}_{PL} \cdot \sum_{k=1}^{3} [g_{1,k} + g_{2,k} + g_{3,k} + g_{4,k}] + r_{PL} \cdot c \cdot \Delta t \cdot [g_{1,4} + g_{2,4} + g_{3,4}] + g_{4,4} \cdot c^2 \cdot \Delta t^2 \end{aligned} \tag{8a.}$$

These are 64 components in form of: $(1/3/12/32/12/3/1)$ .For a quasi-symmetrical tensor this form reduces to 40 real working, functioning components of $(1/0+3 \cdot c./9+3 \cdot c./20+12 \cdot c./6+6 \cdot c./3/1)$, where c. means a non-zero constant n . So this system is no real symmetric tensor with zeros in it but called half-symmetric or quasi-symmetric. There has namely to be distinguished between physical zeros, which don‘t exist in nature and mathematical zeros, which can be set very well in calculations. In fully four dimensions the coordinates of y and z appear additionally in this quantum-description of spacetime, but don‘t change the line-element very much in its groundstructure. They only change some terms. In contradiction to classical theory of infinitesimal length-difference d here the notation of a finite difference Δ is used because of the lattice of metron-structure of quantum spacetime in microscopic states.

For a quasisymmetric tensor this lineelement reduces to:

$$\begin{aligned} \Delta s^2 = g_{1,1} \cdot \Delta x^2 + 3 \cdot n \cdot \Delta x + r^2{}_{PL} \cdot [\sum_{k=2}^{4} (g_{1,k} + g_{2,k}) + g_{3,3} + g_{3,4} + g_{4,4} + 3 \cdot n] \\ + r^2{}_{PL} \cdot [2 \cdot \sum_{k=1}^{4} (g_{1,k}) + 2 \cdot (g_{2,2} + g_{2.3} + g_{2,4} + g_{3,3} + g_{3,4} + g_{4,4}) + 12 \cdot n] \\ + r^2{}_{PL} \cdot [\sum_{k=1}^{3} (g_{1,k}) + g_{2,2} + g_{2,3} + g_{3,3} + 6 \cdot n] + r_{PL} \cdot c \cdot \Delta t \cdot (g_{1,4} + g_{2,4} + g_{3,4}) + g_{4,4} \cdot c^2 \cdot \Delta t^2 \end{aligned} \tag{8b.}$$

These are 40 function-coefficients of metrical quantum tensor and 24 constant quantum coefficients.

For special case of tangential spacetime there is, see above in (5a.):

$$g_{1,1}=K\,;g_{4,4}=1-K\,;all\ other:g_{i,k}=m\,,n;m\,,n\in\mathbb{N}\,. \qquad (9.)$$

In full quantum gravity, the $g_{i,k}$ will change from simple constants to full functionals.

Suggestion for projection-operators:

The operator-system may be like the product of two complex values, which includes the states and operators in a mixed form like:

$$(A+Bi)\cdot(a+bi)=Aa+Abi+Bia-Bb=Aa-Bb+Abi+Bia\quad , \qquad (10a.)$$

where for a physical, gravitational system the real components containing the A,a‘s represent the forms of classical states, the B,b‘s stand for the quantum states and the imaginary terms are the projection operators from the first to the last, like:

$$\begin{array}{l} Aa\rightarrow Abi\rightarrow -Bb \\ Aa\rightarrow Bia\rightarrow -Bb \end{array} \qquad (10b)$$

or

$$\begin{pmatrix} Aa\rightarrow & Abi \\ \downarrow & \downarrow \\ Bia\rightarrow & -Bb \end{pmatrix} \qquad (10c.)$$

One of the operator-components may include $\hbar$.

This projector-system must eventually contain ten operators, every one for every component of metrical tensor $g_{i,k}$ resp. Ricci-tensor $R_{i,k}$. The operators may be entangled. They have to be summation operators no differential-operators because differentiation and integration aren‘t defined on lattice of quantum spacetime.
So may be, twenty components of metric tensor could be interpreted as projection operators, which couple over each other (ten to ten) the ten classical metrical components to the ten quantum metric functions of spacetime. Because of nonlinearity of gravity ten of these operators couple to classical metric tensor components and project ***down***, coupling to the ten others, which project ***up*** and coupled to the other ten projectors, and then couple to quantum-spacetime states. So the projection between classical and quantum states could be described over a form of formal spin-formalism.

3.Conclusion:

A quantum Minkowski-tangential spacetime can be constructed on a Planck-lattice, which for now can assumed to be static, not dynamic. But dynamic can be included. Gravity is not really included yet because there has to be defined an operator, which projects classical $g_{i,k}$ into quantum states. This operator can‘t be the Schroedinger-operator for ordinary quantum-mechanics - nor the Dirac-operator for quantum-electrodynamics. Eventually ten different difference-operators, which depends on these ten components has to be defined for every part of fundamental-tensor according to the key-lock-principle. Reason for this is the non-linearity of gravity-equations.

Anyway it can seen, that the quantized metric lineelement must have 64 components instead of four, like in classical Minkowski-spacetime. Even, if some of them are set to constant coefficients, there can't be a zero but it must be build a minmal, positive size of a constant. Possibly, an advanced quantum gravity theory can be build with a lineelement of 64 components.This doesn't include the other three known physical interactions like $U(1) x\ SU(2) x\ SU(3)$ like in superstring- or M- theory. If this ansatz here is constructive and consistent for a quantum gravity theory , must be seen, because the mathematical descriptions for such a theory must be examined to see, if it is possible to quantize their formations of concepts in a finite form.

4.Summary:

There is no choice to touch anything. Always there is a Planck-distance between two microobjects or four-events. This distance must be seen as a minimal size of a distance. In full consequence there are no point-events in light-cone of SRT at the center but only minimal volumes. Real spacetime is no affine Hausdorff-space with a defined zero-distance but a lattice of Planck-planes or volumes, possibly dynamic.[7.] Described is a consequent form of tangential spacetime lineelement $T_Q(M)$ in this model, which leads basically to 64 components of spacetime. Some of them can safely assumed as small and constant but not set to zero. This model has to be advanced to a gravity description over well defined projection-operators.

5.Discussion:

The question is, if this model can be developed to a full gravity representation without any contradictions or if the used paradigma has to be rejected because the way of its explanation as an extension of classical SRT towards a form of QSRT will not lead to any consistent description of a quantum gravity-theory of GRT. [6.] If the underlying tensor is assumed as symmetric, the number of terms reduced from 64 to 36 components but the rest of 28 terms are not zero but different and must set to be constant, which generates no real symmetric tensor. These facts must be taken into account in the further description.

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7.Verification:

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September 2024