Relationships of the Fine-structure Constant to 2π and e

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Abstract

This paper is a brief review of our previous papers concerning 2π-e formula, formulas of the fine-structure constant, formulas of the speed of light in atomic units and formulas of the anomalous magnetic moments of electron, muron and tauon. In this paper, these formulas are mainly elucidated from points of view of their relationships with 2π and e so as to answer the physicist Feynman's question whether the fine-structure constant was related to π and e. In addition, we also give some new formulas of the fine structure constant and the speed of light in atomic units, especially a formula of a half of the speed of light in atomic units incorporating three factors of 141, 173 and 157, which are related to values of square root of 2, square root of 3 and π/2 in atomic units respectively.

Keywords: the fine-structure constants, 2π and e.

摘要

本文是对我们以前文章中所述的 2π-e 公式、精细结构常数公式、原子单位制中的光速公式以及电子、缪子和陶子的反常磁矩公式的一个简洁的综述，着重从精细结构常数与 2π 和 e 的关系进行阐明，以回答物理学家 Feynman 提出的问题：精细结构常数与 π 和 e 有关吗？另外，我们给出了一些新的精细结构常数和原子单位制中的光速公式，特别是一个原子单位制中的半光速公式，其含有 141、173 和 157 三个因子，它们分别与原子单位制中根号 2、根号 3 和 π/2 的取值有关。

关键词：精细结构常数公式，2π 和 e。

1. 精细结构常数简介以及 Feynman 等物理学家的相关评论

精细结构常数是物理学的百年之谜，由物理学家 Sommerfeld 于 1916 年引入，其有如下三种定义。

$$\alpha = \frac{\lambda_e}{2\pi a_0}, \quad \alpha = \frac{2\pi r_e}{\lambda_e}, \quad \alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

$$\alpha = \frac{1}{137.035999\cdots} \approx \frac{1}{137.036} \approx \frac{1}{137}$$

以下为几位物理学家对精细结构常数的评论。

Pauli: *When I die, my first question to the devil will be: What is the meaning of the fine structure constant?*

Dirac: The origin of the number is "*the most fundamental unsolved problem of physics*"

Wheeler: *Physicists love this number not just because it is dimensionless, but also because it is a combination of three fundamental constants of nature. Why do these constants come together to make the particular number 1/137.036 and not some other number?*

Feynman: *It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from:* ***is it related to pi or perhaps to the base of natural logarithms?*** *Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!*[1]

本文对我们以前的相关文章进行简洁的综述，以回答 Feynman 的疑问，即精细结构常数与 π 和 e 的关系。另外，我们还给出了一个新的原子单位制中的半光速公式，其中同时出现 141、173 和 157 因子，它们分别与原子单位制中根号 2、根号 3 和 π/2 的取值有关。

2. 2π-e 公式与自然群理论

我们于 2013 年推导出了 2π-e 公式及其相关公式，并认为它们是一种自然群的形式，这些公式于 2020 年公开[2]。

$2\pi - e$公式：

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$(2\pi)_{Chen-k} = \left(\frac{e}{e^{\gamma_{c,k}}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

注意：2π是一个平方数

相关公式：

$$1 = 4\gamma_c + \frac{4\gamma_1}{1(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \cdots$$

$$= |B_1| \frac{2\pi}{4} + \sum_{n=1}^{\infty} \frac{|B_{2n}|}{(2n)!} \left(\frac{2\pi}{4}\right)^{2n}$$

$$2N + 1 \sim -|B_1| + \sum_{n=1}^{N} \frac{|B_{2n}|(2\pi)^{2n}}{(2n)!}$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

其中：$\gamma_c = \lim_{N\to\infty}\left(\int_1^{N+1} \log(x)\,dx - \sum_{n=1}^{N} \log(n) - \frac{\log(N+1)}{2}\right)$

$$\gamma_s = \lim_{N\to\infty}\left(\sum_{n=1}^{N} \frac{1}{n^s} - \int_1^{N+1} \frac{1}{x^s}\,dx\right) - \frac{1}{2},\ s \in \mathrm{N}$$

我们认为元素是一种以 2π-e 公式为代表的自然群，由此构建了新的环形元素周期表，并对传统的方框形元素周期表进行了改进[3]。

3. 手性理论的四条原理与原子单位制

我们经过多年的思考构筑了如下手性理论的四条原理[4, 5]。

原理一 (Principle 1)：

手性(Chirality) $= \pm 2\pi$

一双手可抽象为反时针或顺时针的圆；$\pm 2\pi$代表右手和左手。

原理二 (Principle 2)：

圆应该分为420°，手性与840°相对应。

这是亚原子世界的一条定律

圆 $= 2\pi = 420^\circ$

手性(Chirality) $= \pm 2\pi = \pm 420^\circ = 840^\circ$

右撇子和左撇子两双手 $= \pm 840^\circ$

$840^\circ = 1(2\ 4\ 8)(3\ 5\ 7)$，$\pm 840^\circ = \pm 1(2\ 4\ 8)(3\ 5\ 7)$

亚原子世界例如原子核处于手性空间，适用840°。

其中最稳定数为$56=8\times7$，元素的自然终点为112号Cn^*。

原理三 (Principle 3)：

$$2\pi-e\text{公式}:\ 2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$$

$$(2\pi)_{Chen-k}=(\frac{e}{e^{\gamma_{c,k}}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

原理四 (Principle 4)：

在亚原子世界，适用的数轴是百分度的自然数数轴，

无理数例如$\sqrt{2}$ $\sqrt{3}$和π表现为141/100 173/100和314/100，

且有$141+173=314$，并与元素核素对应，例如：

$^{157}_{64}\mathrm{Yb}_{93}$ $^{173}_{70}\mathrm{Yb}_{103}$ $^{257}_{100}\mathrm{Fm}^*_{157}$ $^{285}_{112}Cn^*_{173}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$

我们对 Hartree 原子单位制进行了如下的改进[6]。

Hartree Atomic Units (au):

$$\hbar_{au}=e_{au}=a_{0/au}=m_{e/au}=1$$

$$\hbar_{au}=\frac{h_{au}}{2\pi}=1,\ h_{au}=2\pi$$

Hartree-Chen Atomic Units (still abreviated as au):

$$\hbar_{au}=e_{au}=a_{0/au}=m_{e/au}=1$$

$$\hbar_{au}=\frac{h_{au}}{(2\pi)_{au}}=1,\ \ h_{au}=(2\pi)_{au}=\frac{4\times157}{100}=6.28,\ (2\pi)_{au}=420^\circ$$

In the subatomic world, $\sqrt{2}$, $\sqrt{3}$ and π express as rational numbers

$$(\sqrt{2})_{au}=\frac{141}{100}=1.41,\quad(\sqrt{3})_{au}=\frac{173}{100}=1.73$$

$$(\pi)_{au}=\frac{2\times157}{100}=3.14,\quad(2\pi)_{au}=\frac{4\times157}{100}=6.28$$

$$(\sqrt{2})_{au}+(\sqrt{3})_{au}=(\pi)_{au},\ 1.41+1.73=3.14,\ 141+173=2\times157$$

$$(\frac{\sqrt{2}}{2})_{au}+(\frac{\sqrt{3}}{2})_{au}=(\frac{\pi}{2})_{au},\quad(\sin\frac{\pi}{4})_{au}+(\sin\frac{\pi}{3})_{au}=(\frac{\pi}{2})_{au}$$

$$\frac{141}{2}+\frac{173}{2}=157\text{ 或 }157=\frac{141+173}{2}$$

$(\sin\frac{\pi}{3})_{au}=(\sin70^\circ)_{au}=\frac{173}{2\times100}$与元素核素$^{173}_{70}Yb_{103}$相对应。

4. 基于 112 号元素和 2π-e 公式的精细结构常数公式

物理学家 Feynman 根据精细结构常数的定义，认为最大的类氢原子（即只有一个电子但核电荷可以增加的原子）其核电荷数不能超过 137，否则基态电

子的等效速度将超过光速。所以他认为最大的元素是 137 号，被非正式称为 Feynmanium（Fy）。我们由此通过以下步骤推导出精细结构常数公式[2, 7-9]。

H原子基态电子等效线速度与光速之比：

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{v_e}{c} \approx \frac{1}{137.036}$$

Feynman注意到类H原子（Z个质子但仍然只有1 个电子的原子）基态：

$$Z\alpha = \frac{v}{c} = \frac{Ze^2}{4\pi\varepsilon_0\hbar c} = \frac{Zv_e}{c},$$

由于电子速度v不能超过光速c，所以：$Z_{\max-\text{ideal}} = 137 \approx \dfrac{1}{\alpha}$，即：$\alpha = \dfrac{1}{Z_{\max-\text{ideal}}}$

这是一个粗略的精细结构常数公式，是我们比Feynman多走的第一步。

我们根据原子核的手性模型[4]、核素周期表[4]和新的元素周期表[3]认为 112 号元素 Cn*为元素的自然终点。试问这两个终点之间是什么关系，由此将 112 变换为 137 即得到合理和精确的精细结构常数公式[2]。

$$\alpha = F(Z\,NA) = f(Z\,NA)\frac{1}{Z_{\max-\text{real}}} \approx \frac{1}{Z_{\max-\text{ideal}}}$$ (ZNA代表质子、中子和总核子数)

例证：${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{222}_{86}Ac^{*}_{136}$ ${}^{223,224}_{87}Fr^{*}_{136,137}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

根据原子核的手性模型、核素周期表和新的元素周期表：$Z_{\max-\text{real}} = 112$

根据精细结构常数的定义：$\alpha = \dfrac{\lambda_e}{2\pi a_0}$，$\alpha = \dfrac{2\pi r_e}{\lambda_e}$

我们认为有两个精细结构常数，即：$\alpha_1 = \dfrac{\lambda_e}{2\pi a_0}$，$\alpha_2 = \dfrac{2\pi r_e}{\lambda_e}$

引入2π因子，再代入$2\pi-e$公式和考虑2π是平方数，
由此构建如下两个精细结构常数公式：

$$\alpha_1 = \frac{36}{7(2\pi)_{Chen-112}}\frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

注意：α_1公式中两个112的巧合，36和2π都为平方数。

与以下元素核素相对应：

${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{285}_{112}Cn^{*}_{173}$

$$\alpha_2 = \frac{13(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot 3\cdot 29}} = 1/137.035999111818$$

注意：100和2π都为平方数。

与以下元素核素相对应：

$$ {}^{56}_{26}Fe_{30} \quad {}^{63,65}_{29}Cu_{34,36} \quad {}^{100}_{44}Ru_{56} \quad {}^{112}_{48}Cd_{64} \quad {}^{140,142}_{58}Ce_{82,84} \quad {}^{136,137,138}_{56}Ba_{80,81,82} \quad {}^{157}_{64}Gd_{93} \quad {}^{169}_{69}Tm_{100} $$

$$ {}^{188}_{76}Os_{112} \quad {}^{223,224}_{87}Fr^{*}_{136,137} \quad {}^{257}_{100}Fm^{*}_{157} \quad {}^{278}_{109}Mt^{*}_{169} \quad {}^{285}_{112}Cn^{*}_{173} \quad {}^{2\cdot157}_{126}Ch^{ie}_{188} \quad {}^{2\cdot173}_{137}Fy^{ie}_{209} \quad {}^{426}_{169}Ch^{ie}_{257} $$

5. 基于 137²≈112×168 和(137/2)²≈56×83 的原子单位制中的光速公式

我们根据以上精细结构常数公式推导出以下原子单位制中的光速公式[2,7]。

$$ c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}} $$
$$ = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot141} - \frac{1}{14\cdot112(2\cdot173+1)})} $$
$$ = 137.035999074626 $$

最近（2024/9/4）我们将以上公式进行如下修改，并认为新的公式更准确。

$$ c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}} $$
$$ = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot141} - \frac{1}{14\cdot112(2\cdot173+1) + \frac{7}{24}})} $$
$$ = 137.035999074627 $$

我们最近（2024/8/10, 9/4）得到如下一个新的原子单位制中的半光速公式。

$$ \frac{c_{au}}{2} = \sqrt{56(83 + \frac{157}{4\cdot47} - (\frac{1}{8\cdot141} + \frac{1}{56^2(2\cdot173+1) + \frac{7}{12}}))} $$
$$ = 137.035999074627 / 2 $$

注意：$3\times47 = 141$，$4\times47 = 188$，$7\times12 = 84$，$7\times24 = 168$。

141、173和157与$\sqrt{2}$、$\sqrt{3}$和2π有关，且有$141 + 173 = 2\times157$。

56为原子核中的最稳定数，56主要代表56号元素Ba。

83号元素Bi为稳定元素的终点和放射性元素的起点。

6. 基于 137=56+81 的精细结构常数公式和原子单位制中的光速公式

我们以前构建了以下精细结构常数和原子单位制中的光速公式[7]。

${}^{137}_{56}Ba_{81}$：137=56+81

$$ 1/\alpha_1 = 56 + 81 + \frac{1}{28 - \frac{13(112\cdot11-1)}{3\cdot5(112\cdot43+1)}} = 137.035999037435 $$

$$ 1/\alpha_2 = 56 + 81 + \frac{1}{28 - \frac{2(16\cdot27-1)}{3(16\cdot81+1)}} = 137.035999111818 $$

$$c_{au}=56+81+\cfrac{1}{28-\cfrac{5(4\cdot 3\cdot 7\cdot 17-1)}{2\cdot 5(4\cdot 5\cdot 7\cdot 23+1)+1}}=137.035999074626$$

我们最近（2024/9/4）对以上公式修改，得到如下原子单位制中的光速公式。

$$c_{au}=56+81+\cfrac{1}{28-\cfrac{1}{4}+\cfrac{1}{2\cdot 17}-\cfrac{1}{64\cdot 17-\cfrac{49}{50}}}$$

$$=56+81+\cfrac{1}{28-\cfrac{5(2\cdot 3\cdot 29(2\cdot 3\cdot 157-1)-1)}{4\cdot 81\cdot 11\cdot 17\cdot 61}}$$

$$=56+81+\cfrac{1}{28-\cfrac{5(2\cdot 3\cdot 29(4\cdot 5\cdot 47+1)-1)}{4\cdot 81\cdot 11\cdot 17\cdot 61}}$$

$$=56+81+\cfrac{1}{28-(\cfrac{28\cdot 13}{27\cdot 61}+\cfrac{14}{27\cdot 17\cdot 61}+\cfrac{1}{4\cdot 81\cdot 11\cdot 17\cdot 61})}$$

$$=137.035999074627$$

$$\frac{c_{au}}{2}=\frac{56+81}{2}+\cfrac{1}{56-\cfrac{1}{2}+\cfrac{1}{17}-\cfrac{1}{32\cdot 17-\cfrac{49}{100}}}$$

$$=\frac{56+81}{2}+\cfrac{1}{56-\cfrac{5(2\cdot 3\cdot 29(2\cdot 3\cdot 157-1)-1)}{2\cdot 81\cdot 11\cdot 17\cdot 61}}$$

$$=\frac{56+81}{2}+\cfrac{1}{56-\cfrac{5(2\cdot 3\cdot 29(4\cdot 5\cdot 47+1)-1)}{2\cdot 81\cdot 11\cdot 17\cdot 61}}$$

$$=\frac{56+81}{2}+\cfrac{1}{56-(\cfrac{56\cdot 26}{54\cdot 61}+\cfrac{56}{54\cdot 17\cdot 61}+\cfrac{1}{3\cdot 54\cdot 11\cdot 17\cdot 61})}$$

$$=137.035999074627/2$$

Note: $81=54\times 3/2$ or $54=81\times 2/3$

Relationships with nuclides:

$^{54,56,58}_{26}Fe_{28,30,32}$ $^{60,61,62}_{28}Ni_{32,33,34}$ $^{76-78,80}_{34}Se_{42-44,46}$ $^{96,98}_{42}Mo_{54,56}$ $^{97,98,99}_{43}Tc^{*}_{54,55,56}$

$^{100}_{44}Ru_{56}$ $^{107,109}_{47}Ag_{60,62}$ $^{118,119,120}_{50}Sn_{68,69,70}$ $^{124,126,128-132,134,136}_{54}Xe_{70,72,74-78,80,82}$

$^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{145}_{61}Pm^{*}_{84}$ $^{157}_{64}Gd_{93}$ $^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$

$^{188}_{76}Os_{112}$ $^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}Po^{*}_{125}$ $^{223,224}_{87}Fm^{*}_{136,137}$ $^{235}_{92}U^{*}_{143}$ $^{257}_{100}Fm^{*}_{157}$

$^{273,2\cdot 137}_{107}Bh^{*}_{166,167}$ $^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{344,436,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{400}_{157}Ch^{ie}_{243}$

我们最近（2024/8/31-9/4）得到如下新的精细结构常数和原子单位制中的半光速公式，是以前得到的类似公式的改进[7]。

$$1/\alpha_1 = 56+81+\frac{103}{27\cdot 107}+\frac{1}{27\cdot 107}+\frac{1}{2\cdot 27\cdot 7(2\cdot 3\cdot 5\cdot 11\cdot 19-1)}$$

$$= 56+81+\frac{103}{27\cdot 107}+\frac{1}{27\cdot 107}+\frac{1}{2\cdot 27\cdot 7(4(32\cdot 49-1)+1)}$$

$$= 56+81+\frac{103}{27\cdot 107}+\frac{1}{27\cdot 107}+\frac{1}{7(7\cdot 137(32\cdot 11+1)-1)}$$

$$=137.035999037435$$

$$1/\alpha_2 = 56+81+\frac{103}{27\cdot 107}+\frac{1}{27\cdot 107}+\frac{1}{5\cdot 17\cdot 137\cdot 173}$$

$$=137.035999111818$$

$$c_{au} = 56+81+\frac{103}{27\cdot 107}+\frac{1}{27\cdot 107}+\frac{1}{8\cdot 137(6(4\cdot 83-1)+1)}$$

$$= 56+81+\frac{103}{27\cdot 107}+\frac{1}{27\cdot 107}+\frac{1}{8\cdot 137(4\cdot 7\cdot (70+1)-1)}$$

$$= 137.035999074627$$

$$\frac{1}{2\alpha_1} = \frac{56+81}{2}+\frac{103}{54\cdot 107}+\frac{1}{54\cdot 107}+\frac{1}{2\cdot 7(7\cdot 137(32\cdot 11+1)-1)}$$

$$=137.035999037435/2$$

$$\frac{1}{2\alpha_2} = \frac{56+81}{2}+\frac{103}{54\cdot 107}+\frac{1}{54\cdot 107}+\frac{1}{2\cdot 5\cdot 17\cdot 137\cdot 173}$$

$$=137.035999111818/2$$

$$\frac{c_{au}}{2} = \frac{56+81}{2}+\frac{103}{54\cdot 107}+\frac{1}{54\cdot 107}+\frac{1}{16\cdot 137(6(4\cdot 83-1)+1)}$$

$$= \frac{56+81}{2}+\frac{103}{54\cdot 107}+\frac{1}{54\cdot 107}+\frac{1}{16\cdot 137(4\cdot 7\cdot (70+1)-1)}$$

$$= 137.035999074627/2$$

注：$2\cdot 27 = 2\cdot 81/3 = 54$，$11\cdot 19 = 209$，$2\cdot 83 = 166$

Relationships with nuclides:

$^{27}_{13}Al_{14}$ $^{36,38,40}_{18}Ar_{18,20,22}$ $^{49,50}_{22}Ti_{27,28}$ $^{50,51}_{23}V_{27,28}$ $^{54,56,58}_{26}Fe_{28,30,32}$ $^{59}_{27}Co_{32}$ $^{68}_{30}Zn_{38}$

$^{76-78,80}_{34}Se_{42-44,46}$ $^{82,83,84}_{36}Kr_{46,47,48}$ $^{90,94}_{40}Zr_{50,54}$ $^{96,98}_{42}Mo_{54,56}$ $^{97,98,99}_{43}Tc^{*}_{54,55,56}$

$^{100}_{44}Ru_{56}$ $^{103}_{45}Rh_{58}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{118,119,120}_{50}Sn_{68,69,70}$ $^{126,128}_{52}Te_{74,76}$

$^{124,126,128-132,134,136}_{54}Xe_{70,72,74-78,80,82}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{157}_{64}Gd_{93}$

$^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{179,180}_{72}Hf_{107,108}$ $^{188}_{76}Os_{112}$ $^{200}_{80}Hg_{120}$

$^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}Po^{*}_{125}$ $^{257}_{100}Fm^{*}_{157}$ $^{264,165}_{103}Lr^{*}_{161,162}$ $^{273,2\cdot 137}_{107}Bh^{*}_{166,167}$

$^{276}_{108}Hs^{ie}_{168}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{400}_{157}Ch^{ie}_{243}$ $^{438}_{173}Ch^{ie}_{265}$

7. 电子、缪子和陶子的反常磁矩公式

根据以上精细结构常数公式，我们推导出如下电子、缪子和陶子的反常磁矩公式。特别是我们 2021/6/13 和 2023/8/10 分别都计算出缪子的反常磁矩为 0.00116592057，被 Fermi 国际合作组（Fermilab Collaboration）于 2023/8/10 公开的最新测量值 0.00116592057(25)完美证实[10, 11]。

$$a_e=\frac{\alpha_2\gamma_1}{(2\pi)_{Chen-109}}=\frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}}\frac{1+\frac{1}{3\cdot47\cdot73\cdot137}}{112-\frac{1}{64\cdot3\cdot29}}=0.00115965218058$$

$$a_\mu=\frac{\alpha_2\gamma_1\gamma_2}{(2\pi)_{Chen-109}}=\frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}}\frac{(1+\frac{1}{3\cdot47\cdot73\cdot137})(1+\frac{1}{5\cdot37})}{112-\frac{1}{64\cdot3\cdot29}}$$

$$=0.00116592057\quad(2021/6/13,\ 2023/3/10)$$

$$a_\tau=\frac{\alpha_2\gamma_1\gamma_2\gamma_3}{(2\pi)_{Chen-109}}=\frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}}\frac{(1+\frac{1}{3\cdot47\cdot73\cdot137})(1+\frac{1}{5\cdot37})(1+\frac{1}{103})}{112-\frac{1}{64\cdot3\cdot29}}$$

$$=0.00117724019$$

Fermilab measurement: $a_\mu=0.00116592057(25)\quad(2023/8/10)$

Relationships with nuclides:

$^{83}_{36}Kr_{47}$ $^{103}_{45}Rh_{58}$ $^{107,109}_{47}Ag_{60,62}$ $^{140,142}_{58}Ce_{82,84}$ $^{137}_{56}Ba_{81}$ $^{146}_{61}Pm^{*}_{85}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$

$^{5\cdot37}_{75}\mathrm{Re}_{112}$ $^{4\cdot47}_{76}Os_{112}$ $^{209}_{83}Bi^{*}_{126}$ $^{223,224}_{87}Bi^{*}_{136,137}$ $^{8\cdot29}_{90}Th^{*}_{142}$ $^{238}_{92}U^{*}_{146}$ $^{257}_{100}Fm^{*}_{157}$ $^{265}_{103}Lr^{*}_{162}$

$^{278}_{109}Mt^{*}_{169}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$ $^{314}_{126}Ch^{ie}_{4\cdot47}$ $^{357}_{141}Ch^{ie}_{216}$ $^{370}_{146}Ch^{ie}_{224}$ $^{426}_{169}Ch^{ie}_{257}$ $^{6\cdot73}_{173}Ch^{ie}_{265}$

8. 2π-e 公式与元素的直接对应关系

根据以上精细结构常数公式以及电子、缪子和陶子的反常磁矩公式，我们得出以下 2π-e 公式与元素的直接对应关系。

$$(2\pi)_{Chen-112}\leftrightarrow {}^{285}_{112}Cn^{*}_{173}$$

$$\left.\begin{array}{l}(2\pi)_{Chen-109}\\(2\pi)_{Chen-278}\end{array}\right\}\leftrightarrow {}^{278}_{109}Mt^{*}_{169}$$

特别地，112 号元素 Cn*和 109 号元素 Mt*又分别为我们构筑的新的元素周期表中 7sp 周期中的 2π 族和π族元素[3]，是元素的两个自然终点。

9. 精细结构常数与元素的关系图示

精细结构常数是元素的函数，具体说精细结构常数公式中的因子与一些特

殊元素的核素的核子数相关，可图示如下（**Fig. 1**）。

$$2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$$

$$(2\pi)_{Chen-k}=(\frac{e}{e^{\gamma_{c,k}}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

$$\alpha_1=\frac{36}{7(2\pi)_{Chen-112}}\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

$$\alpha_2=\frac{13(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot 3\cdot 29}}=1/137.035999111818$$

$$c_{au}=\frac{c}{v_e}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$=\sqrt{112(168-\frac{1}{3}+\frac{1}{4\cdot 141}-\frac{1}{14\cdot 112(2\cdot 173+1)+\frac{7}{24}})}=137.035999074626$$

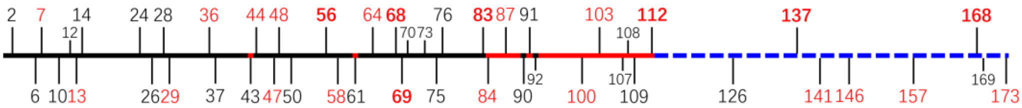
$$\frac{c_{au}}{2}=\sqrt{56(83+\frac{157}{4\cdot 47}-(\frac{1}{8\cdot 141}+\frac{1}{56^2(2\cdot 173+1)+\frac{7}{12}}))}=137.035999074627/2$$

$$a_e=\frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}}\frac{1+\frac{1}{3\cdot 47\cdot 73\cdot 137}}{112-\frac{1}{64\cdot 3\cdot 29}}=0.00115965218058$$

$$a_\mu=\frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}}\frac{(1+\frac{1}{3\cdot 47\cdot 73\cdot 137})(1+\frac{1}{5\cdot 37})}{112-\frac{1}{64\cdot 3\cdot 29}}=0.00116592057$$

$$a_\tau=\frac{13(2\pi)_{Chen-278}}{100(2\pi)_{Chen-109}}\frac{(1+\frac{1}{3\cdot 47\cdot 73\cdot 137})(1+\frac{1}{5\cdot 37})(1+\frac{1}{103})}{112-\frac{1}{64\cdot 3\cdot 29}}=0.00117724019$$

$$3\times 47=141,\ 4\times 47=188,\ 6.28=\frac{4\times 157}{100},\ 141+173=314,\ \frac{141}{2}+\frac{173}{2}=157$$



black line: stable elements (with primordial nuclides);
red line: radioactive elements; blue line: ideal extended elements

Fig. 1. Relationships of Formulas of the Fine-Structure Constant with Elements

Relationships with nuclides:

$${}^{14}_{7}N_{7}\quad {}^{27}_{13}Al_{14}\quad {}^{28,29}_{14}Si_{14,15}\quad {}^{54,56,58}_{26}Fe_{28,30,32}\quad {}^{58,60,61,62}_{28}Ni_{30,32,33,34}\quad {}^{63,65}_{29}Cu_{34,36}\quad {}^{64,66,68}_{30}Zn_{34,36,38}$$

$${}^{69,71}_{31}Ga_{38,40}\quad {}^{70,72}_{32}Ge_{38,40}\quad {}^{82,83,84,86}_{36}Kr_{46,47,48,50}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{98}_{42}Mo_{56}\quad {}^{99}_{43}Tc^{*}_{56}\quad {}^{100}_{44}Ru_{56}\quad {}^{103}_{45}Rh_{58}$$

$${}^{107,109}_{47}Ag_{60,62}\quad {}^{112}_{48}Cd_{64}\quad {}^{118,119,120}_{50}Sn_{68,69,70}\quad {}^{136,137,138}_{56}Ba_{80,81,82}\quad {}^{141}_{56}Ba^{*}_{85}\quad {}^{140,142}_{58}Ce_{82,84}\quad {}^{141}_{59}\mathrm{Pr}_{82}$$

$${}^{157}_{64}Gd_{93}\quad {}^{166,167,168}_{68}Er_{98,99,100}\quad {}^{169}_{69}Tm_{100}\quad {}^{173}_{70}Yb_{103}\quad {}^{185,187}_{75}\mathrm{Re}_{110,112}\quad {}^{188}_{76}Os_{112}\quad {}^{208}_{82}Pb_{126}\quad {}^{209}_{83}Bi^{*}_{126}$$

$${}^{209}_{84}Po^{*}_{125}\quad {}^{210}_{85}Po^{*}_{125}\quad {}^{222}_{86}Rn^{*}_{136}\quad {}^{223,224}_{87}Fr^{*}_{136,137}\quad {}^{226}_{88}Ra^{*}_{138}\quad {}^{227}_{89}Ac^{*}_{138}\quad {}^{232}_{90}Th^{*}_{142}\quad {}^{231}_{91}Pa^{*}_{140}\quad {}^{257}_{100}Fm^{*}_{157}$$

$${}^{265}_{103}Lr^{*}_{162}\quad {}^{273,2\cdot137}_{107}Bh^{*}_{166,167}\quad {}^{276}_{108}Hs^{*}_{168}\quad {}^{278}_{109}Mt^{*}_{169}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{2\cdot157}_{126}Ch^{ie}_{188}\quad {}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$$

$${}^{355}_{141}Ch^{ie}_{214}\quad {}^{370}_{146}Ch^{ie}_{224}\quad {}^{400}_{157}Ch^{ie}_{243}\quad {}^{420}_{168}Ch^{ie}_{252}\quad {}^{426}_{169}Ch^{ie}_{257}\quad {}^{438}_{173}Ch^{ie}_{265}$$

10. 精细结构常数与 2π 和 e 的关系

我们看到以上精细结构公式、原子单位制中的光速公式以及电子、缪子和陶子的反常磁矩公式中含有 2π-e 公式、与 840 度相关的 56 和 112 以及与 2π 相关的 157 等因子，而且这些公式中的因子往往与我们构建的核素周期表[4]中的稳定核素和新的元素周期表[3]中的 2π 族或 π 族元素相对应，这样我们就解答了 Feynman 的疑问，即精细结构常数与 π 和 e 有关吗？

11. 补充：e 的新的展开式及其与精细结构常数的关系

我们还曾经推导出 e 的新的展开式，其中出现了 112、136 和 137 等因子，显示出其与精细结构常数的相关性[7]。

$$e=1+\sum_{k=1}^{\infty}\frac{1}{k!}\,,\quad \frac{1}{e}=1+\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k!}$$

$$e=2\sum_{k=2}^{\infty}\frac{\sum\limits_{l=1}^{k-1}l}{k!}\,,\quad \frac{1}{e}=2\sum_{k=2}^{\infty}\frac{(-1)^{k-2}\sum\limits_{l=1}^{k-1}l}{k!}$$

$$e=\frac{24}{11}\sum_{k=3}^{\infty}\frac{\sum\limits_{l=2}^{k-1}l\sum\limits_{m=1}^{l-1}m}{k!}\,,\quad \frac{1}{e}=\frac{24}{5}\sum_{k=3}^{\infty}\frac{(-1)^{k-3}\sum\limits_{l=2}^{k-1}l\sum\limits_{m=1}^{l-1}m}{k!}$$

$$e^{2}=\frac{12}{7}\sum_{k=3}^{\infty}\frac{2^{k-3}\sum\limits_{l=2}^{k-1}l\sum\limits_{m=1}^{l-1}m}{k!}\,,\quad \frac{1}{e^{2}}=12\sum_{k=3}^{\infty}\frac{(-2)^{k-3}\sum\limits_{l=2}^{k-1}l\sum\limits_{m=1}^{l-1}m}{k!}$$

$$e=\frac{16}{7}\sum_{k=4}^{\infty}\frac{\sum\limits_{l_1=3}^{k-1}l_1\sum\limits_{l_2=2}^{l_1-1}l_2\sum\limits_{m=1}^{l_2-1}m}{k!}\,,\quad \frac{1}{e}=\frac{16\cdot3}{5}\sum_{k=4}^{\infty}\frac{(-1)^{k-4}\sum\limits_{l_1=3}^{k-1}l_1\sum\limits_{l_2=2}^{l_1-1}l_2\sum\limits_{m=1}^{l_2-1}m}{k!}$$

$$e=\frac{2^{7}\cdot3^{2}\cdot5}{2\cdot9\cdot136-1}\sum_{k=5}^{\infty}\frac{\sum\limits_{l_1=4}^{k-1}l_1\sum\limits_{l_2=3}^{l_1-1}l_2\sum\limits_{l_3=2}^{l_2-1}l_3\sum\limits_{m=1}^{l_3-1}m}{k!}\,,\quad \frac{1}{e}=\frac{2^{7}\cdot3^{2}\cdot5}{3\cdot112+1}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum\limits_{l_1=4}^{k-1}l_1\sum\limits_{l_2=3}^{l_1-1}l_2\sum\limits_{l_3=2}^{l_2-1}l_3\sum\limits_{m=1}^{l_3-1}m}{k!}$$

$$e = \frac{2^8 \cdot 3^2}{7 \cdot 137} \sum_{k=6}^{\infty} \frac{\sum_{l_1=5}^{k-1} l_1 \sum_{l_2=4}^{l_1-1} l_2 \sum_{l_3=3}^{l_2-1} l_3 \sum_{l_4=2}^{l_3-1} l_4 \sum_{m=1}^{l_4-1} m}{k!}$$

$$\frac{1}{e} = \frac{2^8 \cdot 3 \cdot 5}{137} \sum_{k=6}^{\infty} \frac{(-1)^{k-6} \sum_{l_1=5}^{k-1} l_1 \sum_{l_2=4}^{l_1-1} l_2 \sum_{l_3=3}^{l_2-1} l_3 \sum_{l_4=2}^{l_3-1} l_4 \sum_{m=1}^{l_4-1} m}{k!}$$

Note: $16 \cdot 3 = 48$, $16 \cdot 5 = 80$, $16 \cdot 7 = 2 \cdot 56 = 112$

Releationships with nuclides:

$${}^{20,22}_{10}Ne_{10,12} \quad {}^{23}_{11}Na_{12} \quad {}^{24,26}_{12}Mg_{12,14} \quad {}^{40,42,44}_{20}Ca_{20,22,24} \quad {}^{52}_{24}Cr_{28} \quad {}^{56}_{26}Fe_{50} \quad {}^{82,83,84}_{36}Kr_{46,47,48} \quad {}^{100}_{44}Ru_{56}$$

$${}^{112}_{48}Cd_{64} \quad {}^{118,119,120}_{50}Sn_{68,69,70} \quad {}^{126}_{52}Te_{74} \quad {}^{136,137,138}_{56}Ba_{80,81,82} \quad {}^{157}_{64}Gd_{93} \quad {}^{168}_{68}Er_{100} \quad {}^{169}_{69}Tm_{100} \quad {}^{173}_{70}Yb_{103}$$

$${}^{185,187}_{75}\mathrm{Re}_{110,112} \quad {}^{188}_{76}Os_{112} \quad {}^{200}_{80}Hg_{120} \quad {}^{208}_{82}Pb_{126} \quad {}^{209}_{83}Bi_{126} \quad {}^{209}_{84}Po^{*}_{125} \quad {}^{210}_{85}At^{*}_{125} \quad {}^{222}_{86}Rn^{*}_{136}$$

$${}^{223,224}_{87}Fr^{*}_{136,137} \quad {}^{257}_{100}Fm^{*}_{157} \quad {}^{276}_{108}Hs^{*}_{168} \quad {}^{285}_{112}Cn^{*}_{173} \quad {}^{300}_{120}Ch^{ie}_{180} \quad {}^{2\cdot 157}_{126}Ch^{ie}_{188} \quad {}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$$

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