Relativistic Relation Between Linear Speed and Angular Speed

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Abstract

The Newtonian formula for the relationship between linear motion and rotational motion is not suitable for application in the theory of relativity. Therefore, the relativistic relationship between linear speed and angular speed must be derived. This can be done by taking advantage of the rules of summing speeds in special relativity.

Introduction

Relativity is now considered one of the most established, convincing, and coherent theories. However, some issues, especially related to rotational motion, still raise many questions and doubts. Although many scientific papers [1] have been written on this topic, most physics references, even those specializing in relativity, still avoid the topic of rotational motion in relativity, and I think the reason for this is that there is not yet an agreement or a standard way to address this topic.

Newtonian and Relativistic formulas

There is no doubt that the most important relationship in rotational motion is the relationship between angular velocity and linear velocity because it is the basic link between the two types of motion. It is known in Newtonian physics that the relationship is a simple proportional relationship: linear speed equals angular speed multiplied by the radius. The very important question that I want to answer here is: Is this formula relativistic? If not, what is the relativistic form of this relationship?

Given that the theory of relativity has brought about major changes, not only in the relationships between all the physical quantities associated with motion, but also in their definitions, verifying whether a relationship is relativistic or not is justified in itself and does not need additional justification. However, the traditional relationship between angular speed and linear speed in particular has something that calls us to verify their validity in relativistic issues because it leads to a major contradiction with one of the basics of relativity, which is the principle that all speeds must not exceed the speed of light.

In the context of this paper, the reader should not be concerned with the distinction between velocity and speed, because this research does not include any comparison between speeds that change in their directions.

If we have a reference system rotating around itself with respect to another system with angular speed, then the points far enough away from the center that are stationary in the second system will be moving with respect to first system at speeds greater than the speed of light!

Some scientists [2] have tried to get rid of this problem by introducing practical obstacles that prevent physical bodies of large dimensions from rotating around themselves due to their fine structure. This way of dealing with the issue is incorrect because relativity has purely kinematic relationships that have nothing to do with the structure of matter. The movement of the reference systems themselves as reference systems, regardless of whether they are fixed in material frameworks or not, should not lead to contradictions in their results. Nothing in relativity prevents that one reference system rotates around itself in relation to another reference system.

Others [3] deal with the issue with a cunning escape by simply saying: Well, rotational motion is an accelerated motion, and accelerated motion is somehow related to non-Euclidean geometry, and non-Euclidean geometry may not lead us to the conclusions that led us to this contradiction. It is sufficient to demonstrate the error of this statement to know that in the case of studying rotation in regions far from the center, the relative motion between the two systems is very close to uniform motion, so the issue can be addressed with special relativity neglecting the acceleration.

Some hasty writers have very bold tendencies that try to destroy what is known and agreed upon or build something new to solve this contradiction, such as claiming that there are exceptions to the rule of not exceeding the speed of light or the existence of a rule that determines the amount of angular velocity, and it is generally very easy to get rid of any contradiction by getting rid of the existing theory or adding new principles to it, but this method is incorrect unless the existing theory is truly unable to resolve the contradiction.

We have a clear and simple way to solve this problem, which is to review the formula that links linear velocity and angular velocity and write it in a form that complies with the rules of relativity, and the required formula can be easily deduced from the rule of summing velocities in special relativity.

Suppose the speed v_1 is added to the speed v_2, according to special relativity, the result will be:

$$v_1 \text{ added to } v_2 = \frac{v_1+v_2}{1+\frac{v_1 v_2}{C^2}} \qquad (1)$$

In the same way, in a reference system that rotates around a specific center with angular speed ω, if the linear speed at a point at a distance R from the center of rotation is $v(R)$ then the linear speed at the neighboring point $R + dR$ will increase by an amount of $\omega\, dR$, and by applying the law of addition of velocities, then:

$$v(R + dR) = \frac{v(R)+\omega\, dR}{1+\frac{v(R)\omega\, dR}{c^2}} \qquad (2)$$

From this, it follows that the linear speed will not be distributed linearly along the distance from the center, but rather it will be distributed in the form of a curve that converges to the speed of light and coincides with the linear Newtonian formula at small speeds as shown in fig (1)

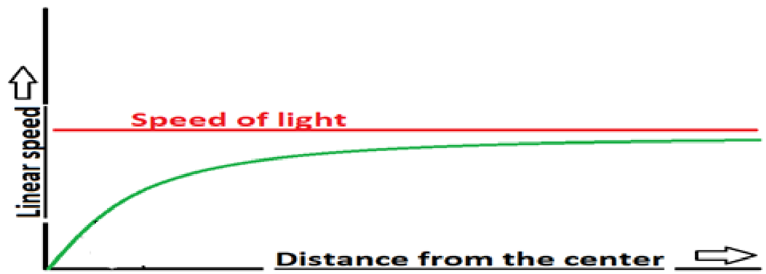

fig (1)

The relativistic formula for the relationship between linear speed and angular speed deduced from the law of summation of velocities is:

$$v(R) = \frac{\omega R}{\sqrt{1+\frac{\omega^2 R^2}{c^2}}} \qquad (3)$$

This is the relationship between linear velocity and angular velocity, which is based in its derivation on the principles of relativity and therefore does not contradict it in results.

It is truly useful and surprising to compare this behaviour in motion with cases in which we try to accelerate objects with a constant acceleration [4]. In that case, we find that the value of the resulting speed does not increase linearly with time, but rather increases in a curve that converges to the speed of light. We must point out here the symmetry and similarity between acceleration and angular velocity, not only in this circumference, but also in the units of measurement. If we make the units of measurement for distance and time the same thing, which is something that special relativity encourages, then the unit of measurement for acceleration will match the unit for measuring angular speed.

References

[1] See: Relativistic Rotation: A Comparison of Theories by Robert D. Klauber.

[2] See more discussion by Don Koks, 2017. Rotating Coordinates in Relativity.

[3] Ibid

[4] See: Berkeley Physics Course, Vol. 1. Page 354.