

PSEUDO-DIFFERENTIAL CALCULATION

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Pseudo- Derivatives

Abstract: Pseudo derivatives. This is a kind of derivation that I apply to polynomials with several variables of degree n .

Such that the set of solutions of the system of pseudo derivatives of a polynomial P , belong to the set of solutions of the polynomial P .

With this very powerful mathematical tool, I manage to solve :

- an infinity of Diophantine equations
- The equations of higher degrees
- systems of linear and nonlinear equations with several variables

Finally, this calculation allows me to question the notion of $0/0$.

PSEUDO-DIFFERENTIAL CALCULATION

Definition: Let be a polynomial with two real or complex variables $P(x, y)$ The pseudo derivatives of $P(x, y)$, with respect to x and with respect to y are defined as follows:

$$\forall (x, y) \in \mathfrak{R}; \forall A_{ji} \in \mathfrak{R}; \forall P(x, y) \in \mathfrak{R}$$

$$P(x, y) = \sum_{j=0}^{n-1} \sum_{i=0}^{n-j} A_{ji} x^{n-j-i} y^i = 0 \dots \dots \dots (1)$$

The pseudo derivatives noted $p \frac{\partial P(x, y)}{\partial x}$; $p \frac{\partial P(x, y)}{\partial y}$ are

$$\left\{ \begin{array}{l} p \frac{\partial P(x, y)}{\partial x} = \frac{\sum_{j=0}^{n-1} \sum_{i=0}^{n-j-1} (n-i-j) A_{J(i+1)} x^{n-i-j-1} y^i}{n-j} = 0 \\ p \frac{\partial P(x, y)}{\partial y} = \frac{\sum_{j=0}^{n-1} \sum_{i=0}^{n-j-1} (i+1) A_{J(i+1)} x^{n-i-j-1} y^i}{n-j} = 0 \dots \dots \dots (2) \end{array} \right.$$

The zeros of system (2) are the same as those of equation (1)

Demonstration:

$$\forall (x, y) \in \mathfrak{R}; \forall A_{ji} \in \mathfrak{R}; \forall P(x, y) \in \mathfrak{R} : xp \frac{\partial P(x, y)}{\partial x} + yp \frac{\partial P(x, y)}{\partial y} = 0$$

This theorem is valid regardless of the number of variables.

Note 1: For n and m integers, the pseudo-derivatives of $(x^n y^m)$ with respect

to x is: $\frac{nx^{n-1}y^m}{n+m}$, and with respect to y is $\frac{mx^n y^{m-1}}{n+m}$

The pseudo-derivatives of dx^0 ; d real or complex is : $\frac{0x^{-1}}{x} = \frac{1}{x} \Rightarrow \frac{0}{0} = 1$

The pseudo-derivatives of $x^0 y^0$ with respect to x is:

$$\frac{0x^{-1}y^0}{0+0} = \frac{0x^{-1}y^0}{2*0} = \frac{1}{2x} \Rightarrow \frac{0}{2*0} = \frac{1}{2} \Rightarrow \frac{0}{0} = 1$$

With respect to y is :

$$\frac{0x^{-1}y^0}{0+0} = \frac{0x^0y^{-1}}{2*0} = \frac{1}{2y} \Rightarrow \frac{0}{2*0} = \frac{1}{2} \Rightarrow \frac{0}{0} = 1$$

The pseudo - derivative of a polynomial $P(x)$ with respect to x is: $\frac{P(x)}{x}$

That of $P(y)$ with respect to y is: $\frac{P(y)}{y}$

Note 2: The pseudo-derivatives of a real number (a) with respect to x is

$$\frac{a}{x}, \text{ and with respect to } y \text{ is } \frac{a}{y}$$

Example 1: let be the polynomial with two variables x, y following:

$$P(x, y) = x^2 - 2xy + 2y^2 + 2x + 3y = 0 \dots\dots(1)$$

$$\left\{ \begin{array}{l} p \frac{\partial P(x, y)}{\partial x} = x - y + 2 = 0 \\ p \frac{\partial P(x, y)}{\partial y} = -x + 2y + 3 = 0 \end{array} \right. \quad S = (-7; -5)$$

These solutions belong to the solutions of equation (1).

Example 2: let be the polynomial with three variables x, y, z following:

$$Q(x, y, z) = x^2 + y^2 - z^2 + 4xy - 2xz + 4yz + 2x - y - 2z = 0 \dots\dots(2)$$

$$\left\{ \begin{array}{l} p \frac{\partial Q(x, y, z)}{\partial x} = x + 2y - z + 2 = 0 \\ p \frac{\partial Q(x, y, z)}{\partial y} = 2x + y + 2z - 1 = 0 \\ p \frac{\partial Q(x, y, z)}{\partial z} = -x + 2y - z - 2 = 0 \end{array} \right.$$

$S = (-2; 1; 2)$ These solutions belong to the solutions of equation (2).

Example 3: in this example we will make the 0/0 appear

$$P(x, y) = x^2 - 4xy + 4y^2 + 5x - 3y + dx^0 - dy^0$$

$$\left\{ \begin{array}{l} p \frac{\partial P(x, y)}{\partial x} = x - 2y + 5 + \frac{d}{x} = 0 \\ p \frac{\partial P(x, y)}{\partial y} = -2x + 4y - 3 - \frac{d}{y} = 0 \end{array} \right. \quad \text{see (a)}$$

Important note 3: the zero can be written as being : 1-1; or 2-2

Or d-d, or $x^0 - y^0$ or, $dx^0 - dy^0$, d real or complex number.

The pseudo derivative of : $dx^0 = \frac{0x^{-1}}{0} = \frac{1}{x} \Rightarrow \frac{0}{0} = 1$

let's take $d = \alpha xy$, α real or complex number

$$\left\{ \begin{array}{l} p \frac{\partial P(x, y)}{\partial x} = x - 2y - 5 + \frac{\alpha xy}{x} = x - 2y - 5 + \alpha y = 0 \\ p \frac{\partial P(x, y)}{\partial y} = -2x + 4y - 3 - \frac{\alpha xy}{y} = -2x + 4y - 3 - \alpha x = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} p \frac{\partial P(x, y)}{\partial x} = x + (\alpha - 2)y - 5 = 0 \\ p \frac{\partial P(x, y)}{\partial y} = -(\alpha + 2)x + 4y - 3 = 0 \end{array} \right.$$

let's take $\alpha = 1$

$$\left\{ \begin{array}{l} p \frac{\partial P(x, y)}{\partial x} = x - y + 5 = 0 \\ p \frac{\partial P(x, y)}{\partial y} = -3x + 4y - 3 = 0 \end{array} \right. \quad S = (-17; -12)$$

Example 4: $P(x, y) = 11y^3 + 2xy - 15x + 7y - 23 = 0$

We take the following variable change : $\begin{array}{l} x = X + k \\ y = Y + \lambda \end{array}$ k, λ real or complex

$$P(X,Y) = 11Y^3 + 33\lambda Y^2 + 33\lambda^2 Y + 2XY + 2\lambda X + 11\lambda^3 + 2KY + 2K\lambda - 15X - 15K + 7Y + 7\lambda - 23 = 0$$

Let's take : $11\lambda^3 + 7\lambda - 23 + (2\lambda - 15)K = 0 \Rightarrow \lambda = 7; K = 3799$

We replace the values of k, λ in $P(x, y)$

$$P(X,Y) = 11Y^3 + 231Y^2 + 5425Y + 2XY - X = 0$$

Let's calculate the pseudo derivatives:

$$\left[\begin{array}{l} p \frac{\partial P(X,Y)}{\partial X} = Y - 1 \\ p \frac{\partial P(X,Y)}{\partial Y} = 11Y^2 + 231Y + 9222 + X = 0 \end{array} \right. \quad X = -9464; Y = 1$$

Let's calculate the values of $x; y$

$$x = X + K \Rightarrow x = X + 3799 \Rightarrow x = -9464 + 3799 = -5665$$

$$y = Y + \lambda \Rightarrow y = Y + 7 \Rightarrow y = 1 + 7 = 8$$

$$S = (-5665; 8)$$