Heuristic Challenges for a Single Model of Nature and the Supporting Mathematics

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Abstract

Over the last several centuries, science has discovered objects in the world along a continuum of scale. In one direction, we have found planets and stars, galaxies and galaxy clusters. In the other direction we have found cells and proteins, atoms and neutrinos. In order to locate and model this world, we use the 3 traditional directions of length, width and height. However inherent in all our measurements is the scale of what we are measuring – a continuum we do not directly see with our eyes. The author presents the hypothesis that we need to include this continuum in a complete model of nature and our world. A key reason we do not understand this direction as part of our world is that we do not know how to measure along this continuum because we lack the mathematical tools to do so. The author presents the mathematical conjecture that the appropriate tools require a numeric representational system with more power than our traditional decimal or positional based numerals. Such a system could provide a single value for complex numbers and has built into it more operations than addition/subtraction, multiplication/division, and exponentiation/logarithms (from whence the new system gets its additional power).
The author anticipates this to be the beginning of a much larger discussion, looking at a perspective of reality where objects at all levels of scale exist and interact together and considers some directions for defining more powerful mathematical tools than we have today.

Prelude

In mathematical circles, there is a well-known story 'Flatland: A Romance of Many Dimensions' by Edwin A. Abbott (Abbott 1884). It is a story with some interesting social discourse as well as the concept of two-dimensional beings (Flatlanders) moving around on a two-dimensional plane world. The protagonist is a square (called Square) who is shown a three-dimensional perspective by a three dimensional being called Sphere. Square has a revelation seeing three dimensions, but upon returning to the two-dimensional world is considered mentally unstable and put into jail.

We could enhance Square's two-dimensional world perspective by including organs and blood vessels, cells and proteins that the Flatlanders know are inside their bodies. When Sphere visits Square and pops him out of his plane, Square might see that all the people and objects of his world actually have a 'depth' to them filled with the other objects, like organs, cells, and proteins. So Square sees these other objects as existing both inside and at different levels of a third 'depth' dimension while Flatlanders believe these objects exist strictly in their two-dimensional plane world (since they do not know of a 'depth' dimension).

If we were like the Flatlanders, then instead of us actually living on a two-dimensional plane, we only believe we live on a two-dimensional plane, yet we really exist with three-dimensions. We can perceive objects in this other dimension with special tools, yet we believe everything that is 'below' us is 'inside' us. To shift to our perspective, we could say we only believe we live in a three-dimensional world, yet actually live in a four-dimensional spatial world. The organs, cells, proteins, and atoms exist at different locations along an additional (scale) dimension, even though we consider them strictly inside us. This could be the case without immediately changing anything we sense or measure about the world today.

Introduction

The size, or scale, of objects in our world is a commonplace part of our experience. We perceive objects smaller than ourselves, like a pin, and objects larger than ourselves, like a building. For centuries our perceptions did not go beyond what we can directly experience through our five senses - call this our 'Direct Sensory World'. The Greeks may have hypothesized atoms, but they could not perceive any such objects. In the last few hundred years, science and technology have shown us very small atomic particles and very large stellar objects. We cannot experience these objects directly with our five senses and need tools to perceive them. This world, expanded through technological tools, we might call our 'Indirect Sensory World'. For most anyone throughout history, these indirect objects could not be perceived and so this 'Indirect Sensory World' did not exist to humans until quite recently.

Today we know of and accept this Indirect Sensory World as a part of our reality. Further, we understand the scale of objects in this world to be along some sort of continuum – from the very small to our scale to the very large. This continuum has expanded several orders of magnitude just in the past century (see Figure 1). Along with this world comes what we consider to be 'space', so our 'Indirect Sensory Space' has expanded in this same time. A key concern of this paper involves our model of 'space' and whether it has been adequately adapted to account for this continuum of scale, introduced through our (relatively new) Indirect Sensory World.

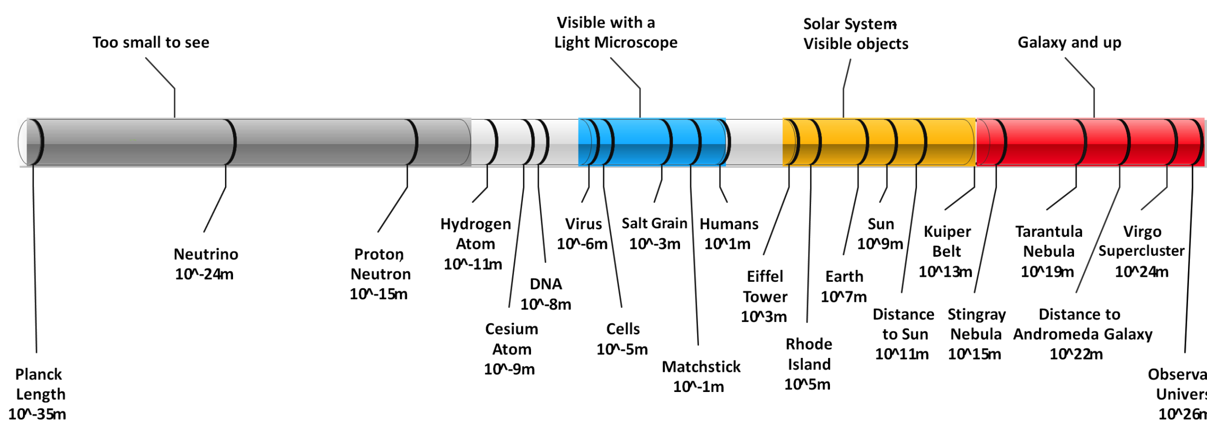

Figure 1: The Continuum of Scale

Our geometric model of space has not changed over several millennia. We have expanded the objects we find in space by many orders of scale, yet we still hold to a three-dimensional model of space devised more than two million years ago. We still consider the only possible continuums, the only dimensions, of physical space to be length, width, and height. This is a very human-centric perspective based upon out Direct Sensory World from our day-to-day experiences including only a small section of the continuum of scale. We continue to think our ‘original’ human-centric three dimensions are the only dimensions needed to uniquely locate an object in space and in which objects can travel. This paper proposes our few millennia old three-dimensional model of space is not sufficient to model the universe we see today encompassing Indirect Sensory Space. A better model is needed, in particular to encompass this continuum we find of scale.

In general, science has broken into levels of study at one or another level of scale to consider the objects found at that level. Thus, we have particle and atomic physics, molecular and protein chemistry, mitochondrial and cellular biology, bone and organ medicine, ecological studies, meteorological and atmospheric studies, solar system studies, star systems and galactic studies, and large-scale astrophysics. These are all important areas of scientific study; however, each level tends to have its own characteristics, objects, and interactions along with its own measurements and equations. This has made it difficult to unify all these levels, which could involve interactions between levels.

There have been attempts at including scale into certain scientific theories, with multiscale modeling a method to connect scale across levels of science. While multiscale modeling covers a range of scientific areas, there are also published articles attempting to include scale in many areas of science today, including biological processes (Berman 2018), (Sulpizi 2018), in geology and geophysics (Stewart 2022) and in complexity theory (Pavlos 2011) – as just a few examples. While these attempts tend to be specific to one area of science or another, we should consider a single model of nature that connects them all.

This paper is not about overturning theories of physics or mathematics. It is about stepping back and considering the unification of many areas of scientific discovery into a single model of what we know of nature (or ‘reality’, or ‘our universe’, or ‘our world’), then asking how this would impact our existing model of nature and the mathematical tools used by science. The real thesis of this paper is that engaging in this unification will indicate new directions of study and discovery in both science and mathematics. In a sense, this paper is as much philosophy and history as it is science and mathematics.

Philosophically, the author hopes that considering a single model of all nature will suggest new directions of study that will lead to a much larger discussion within both science and mathematics. Most of the particulars in either science or mathematics will require efforts considerably beyond those of the author. The complementary theses are:

1) Scientific Hypothesis: We should consider a single model of reality where objects at all levels of scale are included and interact together. This will involve identifying the scale of all objects to locate them within the model and require a four-dimensional spatial model.
2) Mathematical Conjecture: We will require the development and expansion of significantly more powerful mathematical tools that can adequately manage this model. In particular, we

will need the ability to measure across a non-linear scale dimension. This measurement ability will involve a more powerful numeric representational system to address combined linear and non-linear dimensional measurements.

A philosophical concern of this paper is to challenge the assumption that we already have the tools needed to understand nature. The assumption is broken into two concerns: 1) We believe our scientific tools (existing or that we can build) are theoretically capable of taking any measurement of nature; 2) We believe our mathematical tools are capable of representing any number that science requires for any measurement. Both of these assumptions will be addressed and found wanting.

1. Philosophical Overview of Discussion

Science is about developing theories of nature based upon evidence, in particular experimental evidence. Scientific theories use mathematical tools to measure, explain, and predict natural and human-made processes. Over the last few centuries, we have begun to perceive objects across many levels of scale. As noted above, we have come to understand that nature includes objects at many levels of scale and these objects are in motion at their levels. Scientific theories and disciplines tend to address one or another level of scale e.g., quantum physics, molecular chemistry, human medicine, planetary ecology, stellar systems, galaxy clusters. Some address multiple levels of scale. However, developing a single model of nature has to include all these levels operating together in some interconnected fashion.

In order to model our world today we use the three traditional directions of length, width and height, which are apparent to us directly at our scale. However inherent in all our measurements is the scale of the objects we are measuring, a continuum we do not directly see with our eyes. The many disciplines of science tend to focus on one or another level of objects along this scale continuum. The author's hypothesis is that to build a single model of nature we need to include this 'scale direction' into our scientific models of nature (see Figure 2).

Underlying science is mathematics, which needs to provide the quantitative means to account for this scale direction of nature. Using the mathematical tools of today, how can we compare and handle interactions at very different levels of scale, where the error term at the higher level is larger (sometimes by orders of magnitude) than the measurements at the smaller scale? The author conjectures that a key reason we do not include this direction as part of our model of nature is that we do not have the appropriate mathematical tools to handle measurements and activities that cross this scale continuum. The author introduces the conjecture that new mathematical tools will be required in order to adequately measure across and to manage the activities of this continuum. In particular we will need a new method of representing measurement values, which indicates we need a new method of representing numbers – a new (more powerful) numeric representational system.

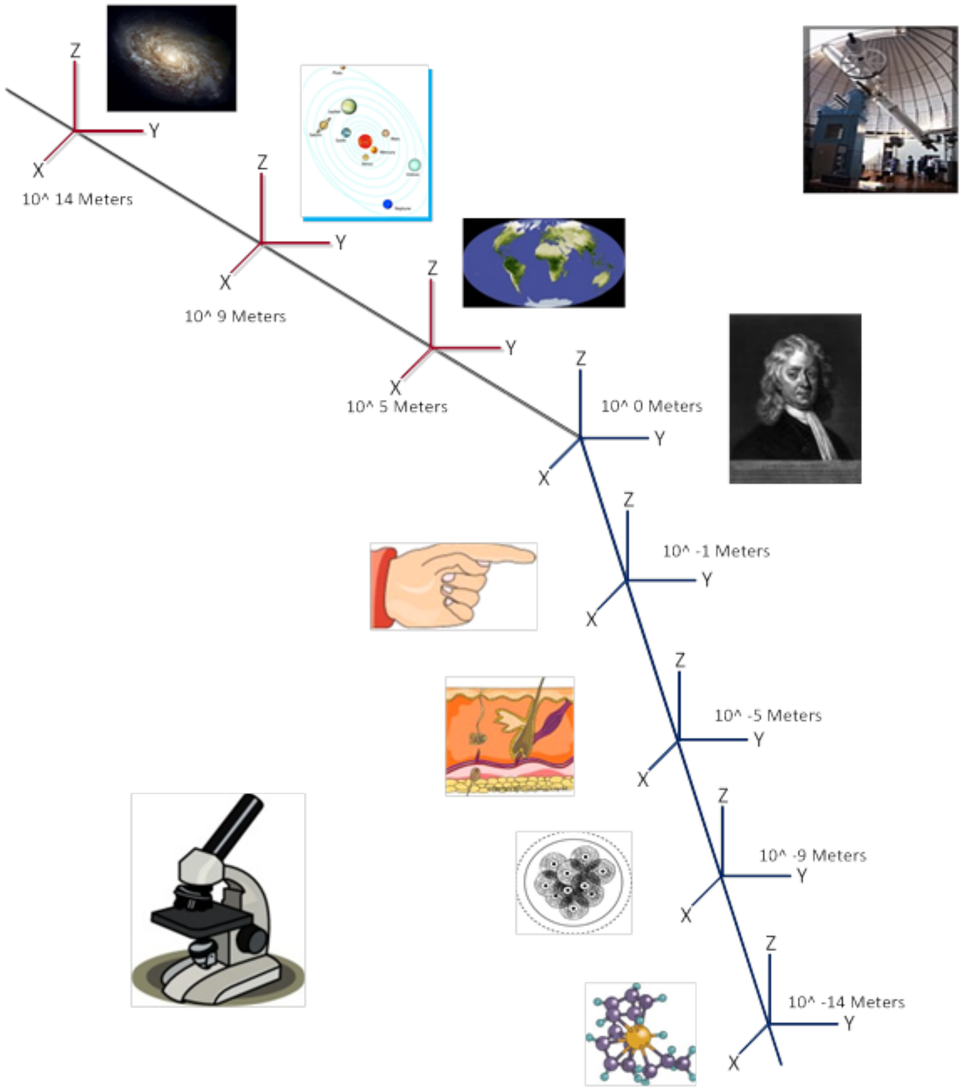

Figure 2: The Hypothesis of Scale Space

2. Heuristic Challenges for A Single Scientific Model of Nature

If we are to model all of nature, then we need to include all the many levels science has discovered into a single model. The first step is to dispel the notion that we are limited to one or another level, as suggested by the article *Why Physics Says You Can Never Actually Touch Anything* (Trosper 2014). Consider the observation if we touch our finger to a pane of glass, the direct evidence is of our finger touching the glass. If we perceive the action with a magnifying glass, we would see specific ridges of our skin touching the less than smooth surface of the glass. If we perceive the action with a microscope, we would see cells touching the rough surface of the glass. We can continue indirect observations using different magnifying tools down to the protein and molecular scale levels. We could setup multiple observational tools to observe different scale levels during the same action and we would gather the observational evidence that the action occurs at all these levels together, not one or the other. If science is about observations, then we should reject the single level perspective and agree that nature operates as a cohesive whole, not at only one or another level. This allows us to include objects and actions at all levels in our new model as our observations indicate.

Note that science tends to have many different models of nature, depending upon which level we are concerned with. There is the 'classical' model of what we see around us. We have medical models of our bodies, organs, and cells. Then there are the models of proteins and macromolecules that are built up from models of molecules and atoms, which build upon models of smaller particles. Multi-scale modeling attempts to bridge adjacent models, and there are efforts to explain natural phenomenon at multiple levels, in particular using emergence and complexity theories. However, to model nature we should be attempting to describe all levels using a single model that can explain the actions and observations at any and all levels.

If we are to efficiently model this one action of touching the pane of glass at all levels, we will need to be able to specify the actions at every level and then combine them across levels. Each level exists at a certain scale, with differing objects at different scales. Since we model each level as actions in a three-dimensional space, combining them would most effectively require a four-dimensional model, identifying the scale level as one of the locators in the modeled space. This heuristic model would provide a means of locating a pen on a table, an atom of the pen, and a star in a different galaxy all in one model. It would not be limited by individual models at each scale, nor require squashing all levels down to a single (three-dimensional) level. Rather it would require these individual models to be integrated together across scale. Such a wholistic model could allow for actions between levels, both upward in scale as well as downward in scale.

Adding scale as a dimension is a change in perspective of nature that does not (immediately) change our observations to date. Measurements at each level remain unchanged, so all current measurements and equations at one scale or another would remain intact. Interpretations of those measurements and equations might involve a cross-scale review. One example might be the equations of gravity interpreted as a four dimensional 'space-time'. The equations of gravity would not change, only the interpretation of what the equations represent would need to be reviewed. The above discussion about touching our finger to glass suggests modeling the many levels of nature using four spatial dimensions is more direct and potentially better than our traditional three-dimensional space that still needs to account for objects at different scales. The implication is that 'space-time' as four dimensional might not be a good model, perspective, or interpretation. The hypothesis introduced is to expand space to four dimensions and then we could tack time onto these four spatial dimensions – conceivably modeled as a five dimensional 'space-time'.

This heuristic model could provide the means to account for actions on the molecular level to affect actions on the protein or mitochondrial levels. Since it provides a means of connecting activities at different levels, its usefulness in medicine could be powerful, directly connecting actions on the chemical and protein level to those on the cellular and even organ levels – potentially without depending upon statistical inferences (given new mathematical tools). It also provides the capability for actions on a larger level, say our human scale, to affect actions on smaller scales, such as humans building the Large Hadron Collider (LHC) to guide sub-atomic particles into collisions with each other.

a. Short Critique of Nature as Three Dimensional

If we only lived in three dimensions, then we should expect higher precision of measurements to produce more accurate information about the objects we are taking measurements on at the original scale. This would be the expectation of a mathematical space of three dimensions – more precise measurements of objects result in more accurate models. However, nature does not always mesh nicely with mathematics, since adding more and more 'precision' to measurements at one scale shifts us into another scale with different objects, say from our bodily organs to cells or to molecules. More importantly, the objects we can measure at one level, say our body at our level, are not the same objects we can measure at other levels, say the cellular level. Further, the volume of our body does not change at different levels of scale since our body at our level and at the cellular or molecular levels has essentially the same physical three-dimensional volume.

There is a statement in physics that two objects cannot occupy the same physical space. If we believe we only exist in three-dimensional space, why are there multiple levels inhabited by different objects somehow at the same three-dimensional location? How can these different objects exist in the same physical three-dimensional space? Maybe the explanation is that they do not occupy the same locations, rather they take up different three dimensional 'surfaces' in four-dimensional space. If we use a four-dimensional model, then this all makes sense.

Consider a three-dimensional cube with 5cm sides that surrounds the joint of your knee at our level. Now build that 5cm cube surrounding your knee at the cellular level. Should we consider this the same cube, somehow encapsulating different objects? Maybe it is a better solution to consider them to be different three-dimensional cubes, at different levels of scale. Say we do consider them to be different cubes encompassing different objects. Now connect the vertices of each cube. The resulting figure would constitute a four-dimensional hypercube, with a three-dimensional 'surface-cube' at each end and an enclosed space through the intervening scales including the objects at these intermediate scales, like bone and blood vessels and nerves. We have just visualized a four-dimensional hypercube or tesseract. Also note that projecting this tesseract of two connected cubes at different levels of scale onto three dimensions could resemble a cube-within-a-cube as depicted by multiple sources (see Figure 3). This should be another clue that our world may be better modeled in four dimensions since we actually observe different objects at different scales. It also means we need to identify the scale of the objects we measure in order to properly locate an object in our world - constituting a fourth location axis.

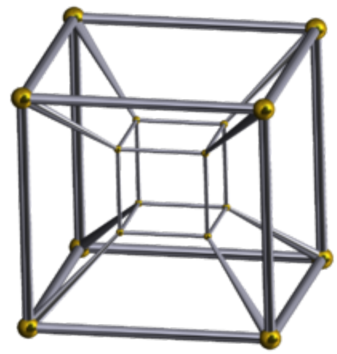

Figure 3: Tesseract from Wikipedia

b. Scale as a Requirement of Position

To understand how basic this concept of scale is, consider specifying a point in space. We need a reference system for this, usually thought of as a three-dimensional set of axes. One axis exists for each dimension and locating a point in space requires one measurement made along each axis. Our current paradigm is that three axes (& three measurements) are all we need to locate a point in space (see Figure 4)

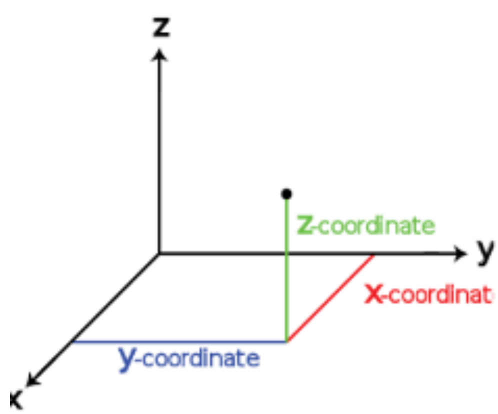

Figure 4: Three-dimensional Cartesian coordinate system with the x-axis pointing towards the observer

However, reality is not comprised of geometric space. For scientific work, we do not need to identify an infinitesimally small point in some geometric space. We need to identify the position or location of a real object in space, through measuring distances along some agreed upon set of practical geometric axes.

A common example of specifying a location in space is to use the lay-out of a city giving the intersection of two streets and the floor above street-level: say the corner of South and 34th streets and on the second floor. As the streets intersect at a corner, we are provided two practical axes along the ground. The third axis is the floor above the ground. So, we have South Street, 34th Street, and the second floor as the three measurements. Such a system is useful for specifying a room or the location of a person, all objects of essentially the same scale. However, this is insufficient if we are specifying the position of a molecule that is part of the surface of a pen sitting on a table in this second-floor room at the corner of South and 34th streets (see Figure 5). We believe that three dimensions are sufficient to define a theoretic point, and we could identify a point corresponding to the molecule in three-dimensional space. However, to locate an object in reality, the scale of our three-dimensional reference system is also required to identify such a location, especially if we are to locate it relative to our scale, as with the street and floor system.

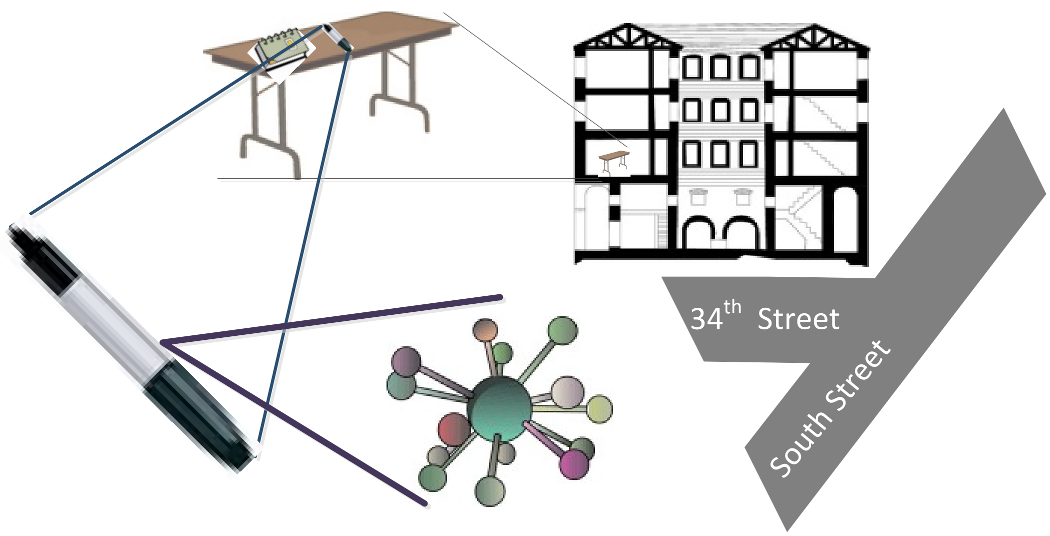

Figure 5: Table in room on 2nd floor of 34th and South Street. Molecule of pen on Table

This is what is meant by 'the fourth dimension is scale': We require the scale of our measuring sticks in order to specify an object in the universe (specifying a theoretic point is insufficient). This means we require four measurements and four axes, which also means the appropriate geometric model of our 'normal' space is four dimensional. This additional dimension is not hidden or beyond what we already can 'see' with technology. It is simply not accounted for in a three-dimensional model of space. The author notes that since most of our experiences do not extend far from the scale of our bodies, we don't generally need to account for this dimension in our normal day to day activities.

Let us re-consider the figure of specifying the position of a molecule that is part of the surface of a pen sitting on a table in this second-floor room at the corner of South and 34th streets (see Figure 5). In this figure, showing the position of the molecule is a series of expansions, or magnifications, starting with the streets, building, and figure of the table. These expansions or magnifications occur along the continuum we call scale. Using this physical continuum of reality, we can build out and position the objects we find in these successive expansions or magnifications. And the location of objects along this continuum is a necessary aspect to define the relative location of objects to each other. Placing these expansions along an axis, it might look something like Figure 6, and this model of reality could be better than our current 3-dimensional one.

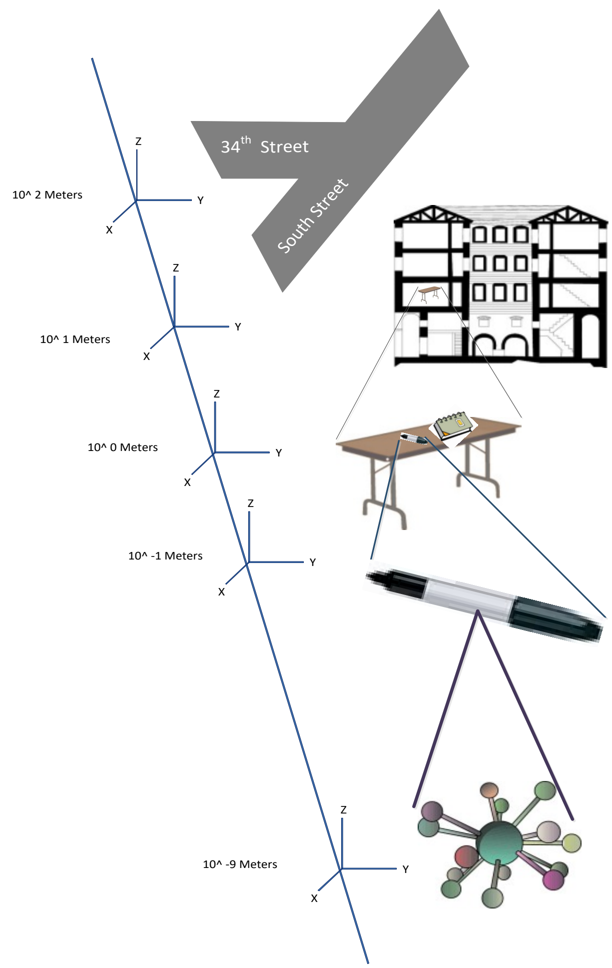

Figure 6: Molecule of pen on Table in room on 2nd floor of 34th and South Street – Scale View

To re-cap: Given that locating an object of nature requires us to specify the scale of the object and we can provide this along an axis we can also represent (the scale continuum), we then have actually described an additional axis of physical space, which our current models do not adequately account for. Expanding our physical model of space to include scale can locate objects at any different scale and also suggests there is a 4-dimensional relative distance between them.

c. Accuracy versus Scale

The previous discussion implies a change in what we consider to be 'close-by'. To say a molecule of our finger is simply tiny presumes that molecule is 'close by.' In fact, don't we consider this molecule to be 'part of' ourselves? So, it is not just 'close-by' but exists as part of one of our cells, which is a part of us and so the distance should be zero. In a strict 3-dimensional space world, we consider this tiny molecule to be just a more 'accurate' measurement of a part of ourselves.

Let us consider what it means to have distance between one point and another. From a strictly geometric perspective, there only has to be space between the two points. But in our world, traveling between two points, say New York and Los Angeles, means things change as we travel. We don't just see 'space' we see lots of other objects between the two points. In a three-dimensional world, these objects would all also be three-dimensional objects (cars, houses, buildings, pastures, fields, etc.). If there is no distance between two objects, there should not be any other objects 'between' them.

There are some photographers who synthesize a dense image from individual pictures at high magnification (see image at http://www.docbert.org/MP/). We can look at the entire scene and then zoom in on one part again and again. This would seem to be what we should expect in a 3-dimensional universe where 'zooming in' only changes the accuracy of our view. In these images, the details are enhanced, but the overall perspective of what we see 'zoomed out' isn't different than what we see 'zoomed in'. The objects in the picture do not change as we change the accuracy (or resolution) of our view.

Taking the 'touching the pane of glass action' with ourselves and a molecule of our finger, is there no change between them? If we start at our level and consider what would occur if we 'travelled' to the level of the molecule, what would we see? We would see the ridges of our fingerprint, and the bumps in our skin. The cells that make up our skin would come into view, followed by the one cell of which the molecule is a part (see Figure 7). We would see the internal parts of a cell and then a particular protein of which the molecule is a part.

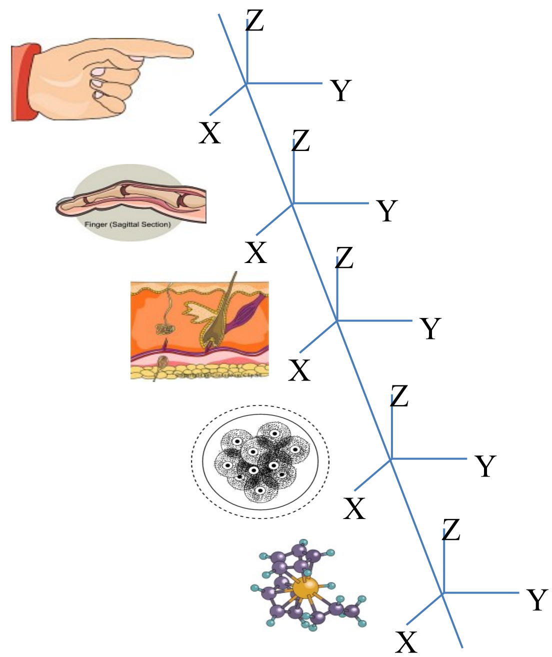

Figure 7: Finger at Different Scales

This does not mimic the magnification of the high-resolution photographs, where we see the same objects in more and more detail. It also does not appear like we are looking at the same object with no distance between us and the molecule. This does appear like we are traveling across some distance, where the view changes. It is hard to rectify the concept of a three-dimensional world that considers a molecule of our finger as simply needing more ‘accurate’ measurements compared to our finger – when we have this sense of travel, of changing perspectives and different objects, in the ‘shift of accuracy’ from our finger to that molecule of our finger.

The concept of scale as a fourth dimension overcomes this problem, since this conception requires we travel a distance to get from our finger (at our scale) to that molecule ‘in’ our finger. You and I still consist of all these objects and levels, just with an additional distance included by the different scales. The expectation of different objects and views as we travel this fourth-dimensional distance is inherent in this conception – and is at odds with a strict three-dimensional conception.

The four-dimensional conception implies there is a distance between us, at our level, and that molecule, at a different level. So that molecule is not ‘zero’ distance from us at our level. It can still be a part of us, just at a scale distance from the level we see ourselves in. This indicates we are four-dimensional beings comprising a hypervolume of space.

d. Perceiving the Dimension of Scale

There are two classic videos of traveling across scale: *Cosmic Zoom* (National Film Board of Canada 1968) and *Powers of Ten* (Eames 1977) both inspired by Kees Boeke's 1957 book *Cosmic View: The Universe in 40 Jumps*. More recently is a 2010 book that looks at traversing the levels of scale by Gott, J. Richard and Vanderbei, Robert J. *Sizing Up the Universe: The Cosmos in Perspective* and a worthy interactive web site by the Huang brothers http://htwins.net/scale/. These all provide a perspective of traveling upward and downward in scale, seeing different objects come into view at different levels of scale. They also give a feeling about how it would appear to travel through different levels of scale. Two things to note in all these videos (and books):

1) As we progress up or down, we see different objects (especially noticeable when progressing down). This is characteristic of travel in our normal three dimensions – we see different objects as we travel. As the preceding discussions indicate, this strongly implies a four-dimensional model of nature.

2) From a standard unit of length perspective, travel up or down in scale involves traveling in 'Powers of Ten'. One unit upward would be an increase of 10 of our 'standard units' and two units upward would be an increase of 100 of our 'standard units'. This means a linear movement in the scale direction involves a power (or exponential) change in the lengths we measure moving across scale. What this translates into is a linear movement in scale will appear to us as an exponential movement according to our standard units of length.

e. Why we have not included Scale today

First, as noted in the introduction, there have been recent attempts at including scale into certain scientific theories, with multiscale modeling being a key area. From Wikipedia on Multiscale modeling: "In physics and chemistry, multiscale modeling is aimed at the calculation of material properties or system behavior on one level using information or models from different levels. On each level, particular approaches are used for the description of a system. The following levels are usually distinguished: level of quantum mechanical models (information about electrons is included), level of molecular dynamics models (information about individual atoms is included), coarse-grained models (information about atoms and/or groups of atoms is included), mesoscale or nano-level (information about large groups of atoms and/or molecule positions is included), level of continuum models, level of device models. Each level addresses a phenomenon over a specific window of length and time." There are also journals on multiscale modeling as well special issues within specific scientific areas (SIAM-MM&S), (BBRC 2018).

So there are scientific movements to include scale into theories today. These have, however, been more about taking information from specific levels of scale and attempting to bridge nearby levels. Consider zooming in on a map of the globe down to a house and car level as with a number of mapping apps these days. Mapping information at specific resolutions are extrapolated between levels so as to provide a relatively smooth experience of zooming into an area. There are large data issues about attempting to actually hold the data at many close resolutions so we would see an accurate zooming experience. There is also the problem of scaling out, then back in, using smoothing algorithms, since there is a loss of information when zooming out (the edges of a lake get smoothed into curves) and then zooming back in (the smoothing does not return to the accurate picture of the original perspective). Put this way, there appear to be mathematical issues with moving across scale, since we currently cannot build an algorithm that holds the resolution, the accuracy, of information when moving across multiple levels of scale.

There is another consideration for challenges to including scale in our model of nature: There is a definite difference in perceiving upward in scale versus perceiving downward in scale. First, we use different tools to look upward into space as opposed to downward inside objects. We use a telescope to look outward into space, even across distances on earth. This is distinct from using a microscope to look downward into the cells and crystalline structures of objects around us. A continuum might be thought of as a direction that smoothly flows across the entire continuum, which would appear to need the same tool to perceive any direction of the continuum. However, this is not what we encounter. We need different tools to perceive in different directions of the scale continuum. This would seem to discredit scale as a smooth continuum of nature.

What is missing in our concept of a continuum are the objects found along this continuum. There is no actual continuum, there are just the objects we find in nature that can be mapped onto a scale continuum. Now, four dimensional objects would have three-dimensional surfaces to them – an upper and a lower surface, as well as ‘side’ surfaces (that spans the object in scale). Think back to the image of a tesseract around your knee. There is an upper surface to this object, which would correspond to our level of scale. In a four dimensional model of nature, we – and all objects in our Direct Sensory World, should have a three-dimensional surface to them. It is this three dimensional surface that makes looking upward in scale different than looking downward in scale. To look downward, we have to penetrate the surface of the object, which very possibly would require a different tool to perceive. So the differences in upward and downward and the need for different tools would be explained through us experiencing a three-dimensional surface, or boundary, of four dimensional scale-space.

As a potentially more important item, while the hypothesis of including scale as a spatial dimension might seem a simple expansion of space to four dimensions, there is a key difference between our traditional 3 dimensions and that of scale. The measurements in our traditional dimensions are all equivalent – 1 unit (e.g., one centimeter) in any direction equates to the same unit in any other dimension. These are linearly equivalent related dimensions. This is not the case with scale, which has a different measurement or metric. As noted in 2) of the previous section, relative to the units we use in our traditional dimensions, scale has an exponential measurement.

This introduces an additional challenge as such a model has three equivalent linear dimensions plus one that is exponential in extent (when related to the other three). This last is a challenge for mathematics rather than science. Traditional geometry assumes all dimensions of a space have the same units with the distance between any two points in space defined by linear dimension measurements. However, to model nature, we may need a geometry that does not have the same distance measurement, or metric, in all spatial dimensions. Since the position of objects uses distance measurements and the location of objects is key to science, we will need to address distance measurements across scale. For this we will need to consider the mathematics we use for measurements in science which bridges into the mathematical conjecture.

3. Heuristic Challenge for Measurements and Mathematics

a. Do We have the most Appropriate Tools to Measure Nature?

There is an intriguing interplay between numbers and their representations. We have numbers and we have representations of numbers (or 'numerals'). It is simple to state that the representation of a number is not the same as that number. However, the two are intimately connected and we could argue that only by some representation of a number (even if verbally or by show of fingers) could there be the concept of number in the first place. Early humans had marks on a stick or on a papyrus pad. Romans invented Roman numerals, and Greeks used a version of fractions for ratios. All of these involved representations for numbers in order for them to be utilized and communicated. Note that this discussion is not about the type of abstract numbers we refer to – such as Integers, Rationals, Reals, or Complex. It is about how these numbers are represented and manipulated as values and measurements. We could state that numerals are critical to the development and application of numbers.

Consider that current science would not be possible without the decimal (and positional) numeric system we use to represent numbers today (Tobias 1954). The author submits that a system such as Roman numerals is completely inadequate for the measurements of current science. Fractions are, likewise, not up to the task of capturing measurements and providing the arithmetic for equations defining scientific laws today. Measurements on the quantum scale would not be possible using fractions or other limited numeric representational systems. A strong statement could be: Without our current method of representing number values – particularly for measurements – we would not have the science of today.

The decimal numeric system became the defining means of representing numbers and measurements less than a thousand years ago. Its use predates the explosion of science in the last 400 years[1], lending support to the idea that current science needs such a representational system to manage the measurements of today. We cannot manipulate nor measure nor calculate quantities without numeral systems. And, as implied above, the power of science has a significant dependency upon the power of the numeric systems used (e.g., decimals vs Roman numerals). This provides a view into how deeply science depends upon our numeral systems – it is at a very foundational level.

If defining the position of any object is only a question of accuracy in a 3-dimensional world, then we should have no trouble measuring the distance between any two objects in nature. But how can we measure the distance between objects at very different scales - say from a pen on the table to a molecule of the table? There is a large difference in the accuracy, or error term, of a measurement, say in meters for the position of the pen and in nanometers (meters * 10^{-9} or .000000001 m) for that of a molecule. In order to compare measurements made at vastly different levels of scale we need to be consistent and use the same units and scale for the measurements. However, using units at our scale provides an accuracy - or rather an error term – that is many times larger than the measurement of the molecule (e.g., 0.436 m ± .001 m for the pen and .00000003.1 m ± .0000000001 for the molecule). The level of accuracy (or error term) on the measurements at the different scales do not match up and are therefore not compatible. While some might say this is 'just how measurements work', the author suggests this shows the inadequacy of our current system of measuring values to address differences in the scale of objects we find in our world.

For typical theories that apply to one level of scale, the scale location can be held constant, and we can consider only the traditional three-dimensions of linear space. This is okay for scale-limited models as the scale value does not change (significantly) for any objects under observation. However, this constancy of scale would seem more like an assumption of the model to limit the scale it applies to. Actions at different scales could impact the experimental apparatus and even tools used for observation, altering the results of an experiment. As an example, consider the magnifying glass and microscopes to view our finger touching the glass pane if we are on the surface of the sun or near a black hole. Our current models need to account for the location of the apparatus as an external condition that has to be addressed separately, rather than an integral part of the model.

If scale is a continuum of reality, then we should be able to measure it and produce equations that demonstrate actions and influences that cross multiple levels of scale. We might think we just need to identify a 'length' to scale, to which we apply our traditional numeric values and mathematical tools. We might attempt to measure the distance between the surface of the moon and a molecule near us. Since the accuracy of measurements are vastly different at these two levels, we generally reduce the problem to one at the same level of scale (say, that of the molecule). This produces other problems, as the surface of the moon, at the molecular level, is no longer really a surface, but has to be defined by many moving particles. As noted previously, this is a consequence of our attempts to be more accurate introducing different objects involved rather than resulting in more accuracy. Why should we have to collapse measurements across levels of scale down to one level of scale in order to measure something apparently as simple as distance? If this is strictly a three-dimensional spatial world, we should be able to measure the direct distance between any two objects, regardless of their scale. However, if scale is a continuum of space that we must account for, then we must address distance across scale as well as our traditional directions and we should make sure we have the appropriate tools.

It is possible a three-dimensional space model is the only hammer we have, so that everything looks like a three-dimensional nail. We have discovered three dimensional molecules or atoms inside of us. Since we believe we only need three-dimensional measuring tools, we will think we only need to measure three dimensional objects in three dimensional ways. Our models will preclude us from thinking there is a scale to measure. However, our models would show anomalies, since we think we are measuring three-dimensional space. The objects we are measuring may not be on (or 'in') the same three-dimensional space we believe is our world.

It could be that anomalies in our current scientific models are a consequence, not just of an incorrect model, but of incorrect tools to build a better model. Consider if the mathematical hammer we have is our decimal (or positional) numeric system, which can handle Real numbers that fit on a continuum (the Real number line). We will see all continuums as Real ones that can be entirely handled by decimal (or positional) numeric values. If we come across something more than this, such as complex numbers, we couch such a different tool in terms of the hammer we know – Real numbers that have decimal values. In expanding our concept of space, we may also need to expand our measuring tools at a basic mathematical level.

If our scientific hammer is three-dimensional space and our mathematical hammer is the decimal numeric system, what might occur if the mathematical hammer affects, even causes, the scientific hammer? If we are unable to represent certain mathematical values, then we might also be unable to represent certain measurements in nature and therefore could be missing aspects of nature we endeavor to study. Part of the mathematical conjecture in this paper is that a more powerful numeric system could provide more advanced scientific theories than we have today – we need new hammers in both disciplines.

Once we locate objects using four space location values, we can start to consider how an equation changes when scale is altered. However, most current mathematics presumes all dimensions have the same metric, which appears not to be the case for scale. This wrinkle introduces a measurement challenge as such a model needs three equivalent linear dimensions plus one that is exponential in extent. This last is a challenge for measurements in mathematics as the metric for geometric distance is traditionally defined linearly (e.g., square root of $[x^2 + y^2 + z^2]$). The author's conjecture is that this situation indicates a need for additional mathematics. Traditional distance measurements assume all dimensions of a space can be measured using equivalent units. However, to model nature, we may need a method that does not have the same distance measurement, or metric, in all spatial dimensions.

If we are unable to adequately compare and relate measurements at significantly different scale levels, then there are limits to our current measurement tools. If we are unable to properly measure across scale with our current tools and we agree that differences in scale constitute distances in scale-space, then we have identified an aspect of our new four-dimensional model for which we do not have the mathematical tools to measure. At a time when the reigning philosophy is to only consider what we can measure, we have hit that uncomfortable situation that might be stated as "we don't know what we cannot measure" – and we have now identified aspects of nature we cannot measure.

If we require a different 'measuring stick' for distances across scale, we may need to enhance our current measuring tools to address both our traditional three linear dimensions and measurements across non-linear scales. This would suggest different measuring tools that can account for – that can measure – the linear and non-linear distances. Such tools could have a direct impact upon what and how science measures the universe.

b. Finding Adequate Measurement Tools – Representational Numeric Systems

Our simple whole numbers can be defined using a basic unit (one) and the operations of addition and subtraction. This is how we represent the Integers, including negative whole numbers. Examples include: 1 + 1 = 2, 4 + 1 = 5, and 2 – 3 = (-1). Fractions are defined using integers plus the reversing operations of multiplication and division. This is how we can represent the Rational numbers as ratios, which include the Integers. Examples include: ½, 2/3, 4/4, 47/35, -5/7, 13/93762. Positional base numerals, like decimals, add the reversing operations of exponents and logarithms in order to represent Real numbers, which include the Rationals. An example of how we build up a decimal is: 5x10^2 + 1x10^1 + 2x10^0 + 3x10^-1 + 6x10^-2= 512.36. Using these three pairs of reversing operations, we have a representational system that can theoretically represent all Real numbers, although infinite decimals present a practical limitation. Note that in all three cases we have defined a specific value for each Real number that can be used as a quantity or measurement and in calculations.

While simple numerals, primarily for small counting numbers, have unique representations for each number, fractions are not the same as there can be an infinite number of ratios, and therefore of numerals, to represent the same Rational number (e.g., 1/3, 2/6, 3/9...). Transitioning to decimals, we find that these numerals may not be as precise as fractions, since the fraction 1/3 is exact while the decimal equivalent is 0.333… - a representation that can only be an approximation of the same fractional value. Further, adding two fractions and two equivalent decimals, we get 1/3 + 2/3 = 3/3 = 1. However, using decimals, we have 0.333… + 0.666… =

0.999… = 1.00(?) So we have to make 1.000… = 0.999… for the arithmetic to work. Even with decimals, we have the situation of two numeric representations for the same number value.

Practical measurements are not theoretical number values without an accuracy or error term. Also, manipulation of measurements must conform to operations that are manageable by humans or human inventions (such as computers). Although theoretically possible, it is not practically possible to multiply by π when π is only accurately represented by an infinite non-repeating decimal expansion. This is an example of the important distinction between a number system (such as Integers, Rationals, or Reals) and a numeral system (such as Roman numerals, fractions, or decimals). Limitations of numeric representations are not a consequence of the accuracy of any measurement using the value. We can specify the accuracy of, say, 1/3 = 0.333333… to any degree we wish or need. However, the value will never be exactly the number it is supposed to represent. In general, we can use the accuracy of our measurements to define the accuracy of the numbers we need to calculate with. An example would be determining the circumference of a circle, where the calculation is 2πr, with r = the radius of the circle. If r = 2.51cm ± 0.005cm, then we can use 2.00 x 3.14 x 2.51cm = 15.76cm ± 0.005cm to get the circumference, even though π = 3.1415926898… Note that, in determining the value for the circumference, we end up with a single numeric value.

c. Finding Adequate Measurement Tools – Numeric Representations for Complex Numbers

It is important to understand that representations of numbers have characteristics and limitations that are not true for the numbers they represent. This aspect maybe especially important if we are unable to adequately represent a number as a single measurement value, such as a complex number. By not accounting for the limits of our mathematical tools, our models maybe missing measurements our tools cannot account for. This would be the situation with how we currently represent Complex numbers. We are unable to represent a complex number as a single value that could be used as a measurement, so we manage with what we know and represent a complex value using two Real numbers: x + iy. However, this means our models cannot account for complex values that require a value for ‘i’.

We should note that science uses complex numbers for many calculations and equations. Our numeral system for representing complex numbers theoretically appears to produce unique values for all complex numbers. Yet there are always two parts to any complex value; a Real part that we can use decimals to represent, and an ‘imaginary’ part that we cannot resolve into a useable numeric value. Complex numbers are represented as two numeric values: “x + iy”, where x and y are known numeric values, however i = sqrt(-1) and is not representable as a numeric value. We use this 2-part 'work-around' for representing complex values as we have been unable to identify a value for ‘i’. However, this is not a full representation of a complex value precisely because it includes the undefined term. This is not a question of accuracy, as with π, since we do not have a numeric value at any level of accuracy for i.

From a practical standpoint, we use complex numbers for all sorts of calculations, but because we cannot resolve the imaginary part into an actual value, we need to ignore at least the undefined term, if not the entire imaginary part, when we use complex values for quantities or measurements. Distance is understood as a single valued measurement (with appropriate units). Since we cannot represent a complex number as a single value, we are unable to provide a single value that could be used as a complex distance measure. If it could measure across four-dimensional scale, then we would have a solution to our inability to measure across scale.

Ignoring the entire imaginary part allows many theoretic calculations involving measurements to produce different complex values yet result in the same real quantity (5 + yi equates to the Real value 5, regardless of what 'y' or 'yi' are). This is a logical problem for physical theories, since calculations in a theory could produce different complex values, yet the calculations for the theory would have to reduce to something we can measure and represent as a single value. Even if we account for 'y', we still cannot account for 'i' and thus cannot identify what a single complex value looks like, let alone what it could measure. We are left with two separate parts of a complex number that we can only evaluate as two parts. Five and π are understood as single values and can be used in measurements and calculations as a single value. Not so with complex numbers – as we represent them today. If the currently accepted scientific philosophy is that all we can know of the physical world is through measurements, and we realize that our numeral systems are not capable of entirely specifying all practical quantities, then we have a direction to look into before any scientific theories can be considered complete.

On the theoretic mathematics side, we have become used to understanding complex numbers as 2-dimensional numbers. This situation appears to have 'gelled' into the idea that this is a property of the complex numbers. However, as noted previously, our numeric representations have aspects and limitations that are not true of the abstract numbers they represent. This might very likely be the case for the numeric methods we use to represent complex numbers. We only know how to represent them as 2-part numerals that involve an 'always unknown' value. We are unable to resolve the imaginary part into an actual numeral value and so we leave it apart – unresolved.

What if we could find a means of fully representing a value for that pesky 'i'? This value certainly does not fall into the mathematical notations of today. So maybe mathematics needs to take a new step here. Maybe the more than 1000-year-old numerals we use today are not sufficient to represent 'modern' complex values. The 'yi' symbols used to define 'imaginary' values could be consolidated with the 'real' part of a complex number and be reduced to a single value. This could simplify many equations made complicated due to 2-part complex values.

d. Possible Directions for a Complex Numeral System

To gain a perspective on how to include the non-linear scale dimension into a single model of the universe, we could consider a simple model that collapses all three 'standard' dimensions onto a single dimension and then models scale along a separate axis. The collapsed dimension holds all linear dimensions, while the separate dimension is the non-linear dimension. We could model this using complex numbers and the complex plane as our simple model. This model might suggest that we are already making use of mathematics that differentiates our standard dimensions and measurements with a dimension that acts differently.

If we start to consider how to define a complex numeric system, we might extrapolate the pattern identified previously of adding reversing operations into the definition of a complex numeral. Maybe we need a fourth pair of reversing operations in order to fully represent complex numbers as single values without any unknown placeholder. A possibility would be integration and differentiation added into the definition of a complex numeral value. In order to represent negative square roots, we might need to define an undefined area of mathematics – that of negative bases. This might indicate a bit of theoretical work is required – maybe even a little inventing.

As Donald Knuth worked on more than 60 years ago (Knuth 1960), maybe we need to develop – to invent – numerals using negative bases, which can represent negative square roots. Euler's great equation (e^(πi) + 1 = 0) might provide a clue to how to construct complex numerals, using 'e' as a base. When used for integration and differentiation, base 'e' allows for continuous integration or differentiation and does not 'bottom out' as typical bases do (e.g. Derivative of x^2 = 2x and derivative of 2x = c, bottoming out – while derivative of e^(2x) = 2e^(2x) and so the exponent does not decrease). A complex numeral might involve the positional placement of integrated and/or differentiated 'digits' in some similar way as exponential digits are used for decimal and positional base numerals. Maybe an extension of Taylor series, that involves infinite derivatives, for e^x or ln(x) could be a direction for research.

The capability of incorporating integration and/or differentiation into numerals suggests integration and differentiation operations should be simplified. Consider that modeling upward in scale generally involves integration, suggesting such a numeral system could 'take on' the difficulties of the scale continuum. Differential equations permeate many areas of science and so might also benefit using such a numeric system.

This representational discovery or invention could open up a new universe of possibilities for mathematics. It might also alter the interpretation of physical equations that 'toss out' the imaginary value for quantities and measurements given that we can only use real numeral values. Now we could have a value or measure that included the imaginary part. Now a complex value could be handled in its complete form, possibly opening up measurements not possible before (e.g., across scale). There would be complex measurements, not real measurements + imaginary placeholders – potentially identifying measurements we cannot make today.

Conclusion

As we move toward digitally modeling entire bodies in the universe, we will find the need for locating objects in scale, in addition to locating objects in three-dimensional space. This will require a four-dimensional scientific model of objects in space, which will require some distance measure and units that cross non-linear scale. Since the distance metric of scale is different than our traditional three-dimensional space, we may need to develop new values of measurement and potentially new mathematics to understand the characteristics of such a space.

The ability to measure across non-linear scale becomes an imperative for a four-dimensional scale space model of nature. This leads to the need to advance the underlying means of representing numbers, so that such distance measurements can be adequately quantified. Continuing the pattern of developing numeral systems to represent values for Integers, Rations, and Reals, the author recommends developing a numeral system that can adequately represent Complex numbers. As with the other numeral systems, this includes representing a complex value as a single value rather than the current method used today.

There is an assumption that complex values should be able to measure across scale. Supporting this assumption is the pattern suggesting integration and differentiation appear to be the next reversing operations to add to a numeral system along with the use of integration and differentiation in science and in multi-scale modeling today. In addition, the author notes that decimals use three reversing operations to represent the Real numbers, useful for three-dimensional work, and the next numeral system to represent Complex numbers would use four reversing operations to represent four-dimensional work.

It is very possible that such a numeral system may not be representable using traditional paper and pencil methods – requiring the use of computers.

It is not a huge step to consider systems beyond a complex numeral system - beyond where we do not quite see yet. So, we may still be in the early stages of understanding the extend of what mathematics can provide and science can utilize. Where mathematics needs to go could be well outside the 'standard model' of current mathematics (with only a Real line continuum) - and there might be tremendous dividends for science as well.

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