The cubic ellipsoid nuclear model: astrophysical phenomena

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Abstract

From the nuclear properties we infer in this study on astrophysical phenomena.
Under the assumption that a neutron star can be treated as a giant nucleus the TOV limit for neutron stars was roughly estimated with results in good agreement with the known data.
A hypothesis was raised that there is a minimum atomic size, beyond which the proton and electron are fused to create a neutron; this minimum size was estimated via the data of a white dwarf and was found to be in good agreement with the prediction of the model, that was achieved under the assumption of a constant tangential velocity of the nucleons in the nucleus and the electrons in the atom; in addition, based on the last assumption, the upper value of the rotation period for pulsars was estimated.

Content

Introduction

The purpose of this study is to find a connection between the cubic ellipsoid nuclear model and astrophysical phenomena.
We begin with some preparation steps and assumptions:

- the tangential velocity of the nucleons in the nucleus has a constant value and this possibly holds for electrons in the atom as well.
- there is a minimum volume that a bound electron can occupy; this volume determines the minimum size for an atom.
- there is a smallest size an atom can reach; beyond this size, the proton and electron are fused to create a neutron.
- a neutron star can be treated as a giant nucleus.
- the rotation period of a pulsar can be assessed through constant tangential velocity assumption.

The hypotheses of constant tangential velocity and minimum atomic size

We raise two hypotheses, that shall support our following research steps.

The constant tangential velocity of bound electrons and nucleons

Hypothesis: the tangential velocity of nucleons in a nucleus is constant; this might be the case also for bound electrons in an atom.
To calculate the tangential velocity, we analyze the nuclear rotation via its angular momentum:
$L \approx \hbar \cdot l \approx p \cdot r = m \cdot v \cdot r = m \cdot (\omega \cdot r) \cdot r = m \cdot \omega \cdot r^2.$
in our model we assume that the orbital radius of the nucleus grows by a constant value, r_0, while moving from one orbital to its next neighbor:
$r = l \cdot r_0$ with $l: \{0, 1, 2, 3\}$ for the orbitals $L: \{S, P, D, F\}$
we get: $L \approx \hbar \cdot l \approx m \cdot (\omega \cdot l \cdot r_0) \cdot l \cdot r_0$
which means: $v = \omega \cdot l \cdot r_0 = constant$
we define: $v_0 = \omega_0 \cdot r_0$ and $\omega_0 = \frac{\hbar}{m \cdot r_0^2}$ and so $\omega = \frac{\omega_0}{l}$ with:

- $r_0 = d_0 = 1.62 \cdot 10^{-15} m$: the distance between neighboring nucleons.
- $m_p = 1.67 \cdot 10^{-27} kg$: the nucleon mass (for a rough estimate, we consider the proton and neutron mass as equal).

we get:

- tangential velocity: $v_0 = \omega_0 \cdot d_0 \approx \frac{\hbar}{m_p \cdot d_0} = \frac{1.05 \cdot 10^{-34}}{1.67 \cdot 10^{-27} \cdot 1.62 \cdot 10^{-15}} \approx 3.8 \cdot 10^7 \ \frac{m}{s}$

the minimum volume of a bound electron

Hypothesis: the minimum volume occupied by a bound electron defines a lower limit for atomic volume; or the same statement in the opposite direction: the minimum size of an atom is determined by the minimum volume occupied by a bound electron.
We calculate it by comparing the angular momentum of the electron and the proton under the assumption of a constant tangential velocity of the bound particles:
$L_e \approx \hbar \approx L_p \quad \rightarrow \quad m_e \cdot r_e \approx m_p \cdot r_p \quad \rightarrow \quad r_e \approx \frac{m_p \cdot r_p}{m_e} \approx 1.5 \cdot 10^{-12}\, m$
which is in the range of its de-Broglie wavelength.
The meaning of this bound electron is assumed to be a minimum atomic size, before the proton and electron are fused to become a neutron; we assume that this is the case in the limit between white dwarfs and neutron stars.

Neutron star and the TOV limit

We want to assess the size and mass of a neutron star, and assume that the star is an expansion of the nuclear model, so we treat it as if it were a large nucleus.
A basic particle in the star is assumed to have the mass of a nucleon m_p and a basic cell size is derived from the distance between two neighboring nucleons in the star:
$d = 2 \cdot r = d_0 \approx 1.62 \cdot 10^{-15}\ m.$
A cubic bond is expected, so the volume of the basic cell and its density are:

- $V_{cell} = d^3 \approx 4.3 \cdot 10^{-45}\ m^3$
- $\rho = \frac{m_p}{V_{cell}} \approx 3.9 \cdot 10^{17}\ \frac{kg}{m^3}$ this is within expected range [1].

The gravitational pressure in the center of the star is $P = \frac{2 \cdot \pi \cdot G \cdot \rho^2 \cdot R^2}{3}$ [8] with R the star radius.
The force on the central nucleon in the star is about $F = P \cdot S$, with $S = 4 \cdot \pi \cdot r^2$ the nucleon surface. We get:

- $F = \frac{8 \cdot \pi^2 \cdot G \cdot \rho^2 \cdot R^2 \cdot r^2}{3}$ the force on the cell in the center of the star.
- $R = \sqrt{\frac{3 \cdot F}{8 \cdot \pi^2 \cdot G \cdot \rho^2 \cdot r^2}}$ the star radius.
- $M = \rho \cdot V = \frac{\rho \cdot 4 \cdot \pi \cdot R^3}{3}$ the star mass.

Based on data from [7] we assume that the maximum force a nucleon can bear is about:

- $F_{max} \approx [3.0, 4.0] \cdot 10^4\ N$ maximum tolerable force in the star center.

this means:

- $R \approx [1.3, 1.5] \cdot 10^4\ m\ ,\quad M \approx [3.6, 5.6] \cdot 10^{30}\ kg$

and with $M_{sun} = 2 \cdot 10^{30}\ kg$ we get: $\frac{M}{M_{sun}} \approx [1.8, 2.8]$

which delivers a rough estimation to the maximum star mass, before it collapses to become a black hole; this is in the range of the Tolman-Oppenheimer-Volkoff limit [5].
We summarize the process:

- from the assumption of constant cubic arrangement, we get the density ρ.
- assuming the maximum force a nucleon can bear is F_{max} we get the star radius R through the gravitational pressure P in the center of the star.
- via R we get the star volume V.
- the star mass M is calculated using the star volume and density.

Remark: the main assumptions here are that the neutron star is a kind of large nucleus with a constant density and a rough estimate of the maximum force a nucleus can bear.

White dwarf and the lower limit of atomic size

The discussion of white dwarfs is done in a somewhat similar way to that of neutron stars above.
Hypothesis: there is a maximum pressure that atoms inside a white dwarf can bear, beyond this they collapse, and their electron and proton are fused to form a neutron.
Via the constant tangential velocity assumption, we make a rough estimate of the minimum basic cell radius for an atom to be in the range of $r_e \approx r_p \cdot \frac{m_p}{m_e} \approx 1.5 \cdot 10^{-12}\ m$.
Another assumption is that the star was created by light elements, so each basic cell consists of one electron, one proton and one neutron [11] meaning the atomic mass is about $A = 2$.
The star consists of atoms, so unlike the cubic arrangement in the neutron star, here the basic cell volume is assumed to be a sphere and not a cube.
Assuming a typical white dwarf radius and mass:

- $R \approx 1 \cdot 10^7\ m$ [11]
- $M \approx 0.5 \cdot M_{sun} = 1.0 \cdot 10^{30}\ kg$ [11]

We get:

- $V = \frac{4 \cdot \pi \cdot R^3}{3} \approx 4.2 \cdot 10^{21}\ m^3$ white dwarf volume.
- $\rho = \frac{M}{V} \approx 2.4 \cdot 10^8\ \frac{kg}{m^3}$ white dwarf density.
- $V_{cell} = \frac{m}{\rho} = \frac{A \cdot m_p}{\rho} \approx 1.4 \cdot 10^{-35}\ m^3$ white dwarf basic cell volume.
- $r_{cell} = \sqrt[3]{\frac{3 \cdot V_{cell}}{4 \cdot \pi}} \approx 1.5 \cdot 10^{-12}\ m$ white dwarf basic cell radius.

We see that the result for r_{cell} is in the range of r_e.

This strengthens the hypothesis, that there is a minimum atom size, beyond which the electron and proton are fused to form a neutron.

Pulsar - the rotation period

To analyze pulsars, we assume also here that a neutron star acts somewhat as a giant nucleus and as such maintains some of the nuclear properties.
We discuss it now in the light of the model. The elements formed in a star before undergoing supernova are assumed to be mainly up to the fourth row of the periodic table.

The upper limit of the rotation period
In order to calculate the pulsar angular velocity, we use the assumption made above regarding the constant tangential velocity, in the ideal situation in which all nuclei lie parallel to each other and so the superposition leads to a maximal angular and tangential velocity.

Data of a pulsar [2]:

- radius: $R \approx 1.5 \cdot 10^4\ m$
- the minimum rotation period: $T \approx 10^{-3}\ s$

We use the tangential velocity found above:

- $v = \omega \cdot R \approx 3.8 \cdot 10^7\ \frac{m}{s}$

and get a rough estimation for the angular velocity:

- $\omega = \frac{v}{R} \approx \frac{4 \cdot 10^7}{1.5 \cdot 10^4} \approx 2.5 \cdot 10^3\ s^{-1}$

this implies a rotation period of:

- $T = \frac{2 \cdot \pi}{\omega} \approx 0.0025\ s \approx 3\ ms$

which is within expected range [2].

The longer periods, or shorter angular velocities, are assumed to be due to a less parallel arrangement of the nuclei (or a smaller relative part with parallel nuclei) or of older pulsars that slowed down with time.

Discussion of the results and conclusion

The calculations of this article deliver only very rough estimations, yet they show how the model assumptions ease the calculations and create visualization of the concepts in astrophysics.
We claim the following:

- a constant tangential velocity of the nucleons in the nucleus and possibly also of the electrons in the atom, with a value of $v \approx 3.8 \cdot 10^7 \; \frac{m}{s}$.
- the smallest size a bound electron can occupy is about $r_e \approx \frac{m_p \cdot r_p}{m_e} \approx 1.5 \cdot 10^{-12} \; m$; beyond this size, the proton and electron are fused to create a neutron.
- a neutron star can be treated as a giant nucleus.
- using the model and an assessment for the maximum force tolerable by the nucleon, the TOV limit can be estimated.
- the minimum rotation period of a pulsar is assessed to be about $T \approx 3 \; ms$.

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