A problem of Seiyō Sampō revisited with $1/0 = 0$

*Hiroshi Okumura, **Saburou Saitoh, ***Jun Ozone
*Maebashi Gunma 371-0123, Japan
e-mail: hokmr@yandex.com
**Institute of Reproducing Kernels,
Kawauchi-cho, 5-1648-16, Kiryu 376-0041, Japan
e-mail: saburou.saitoh@gmail.com
***Japanese Society for the History of Mathematics
e-mail: oznjun@gmail.com

Abstract. We show that the result in [1] holds in the limiting cases using 1/0.

Keywords. $\dfrac{1}{0} = 0$, $\tan \dfrac{\pi}{2} = \cot 0 = 0$.

Mathematics Subject Classification (2010). 01A27, 51M04

1. Introduction

We consider oriented circles and oriented lines. Two figures are said to touch if the orientations at the point of tangency are the same. Circles with counterclockwise orientation have radius with plus sign, otherwise minus. A line segment AB has also length with signs, and the line segment with initial point B and end point A is denoted by $-AB$. Hence we have $AB + (-AB) = 0$, which is denoted by $AB - AB = 0$. The line having the opposite orientation to t is denoted by $-t$. In [1], we obtain the following result (see Figure 1).

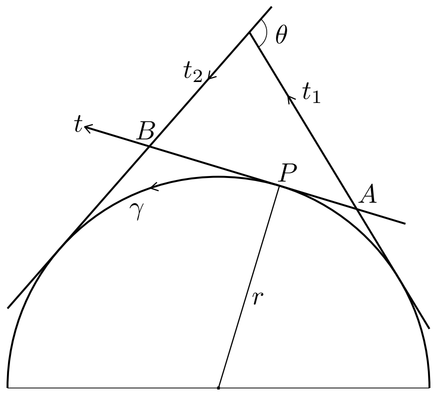

Figure 1.

Theorem 1. *Two lines t_1 and t_2 touch a circle γ of radius r. Another line t touches γ at a point P and meets t_1 and t_2 in points A and B, respectively. If θ is the angle between t_1 and t_2, then we have*

$$\cot \frac{\theta}{2} = \frac{r}{AP + PB} - \frac{r^{-1}}{AP^{-1} + PB^{-1}}. \tag{1}$$

The angle between t_1 and $-t_2$ is considered in [1], which is denoted by 2θ in the paper. If we denote the same angle by $2\theta'$ in this paper, then we have $\theta + 2\theta' = \pi$, i.e., $\cot \theta/2 = \tan \theta'$.

2. Main result

In this section we give the next theorem using the fact $1/0 = 0$ ([2]).

Theorem 2. *Equation* (1) *in Theorem 1 holds if* $\theta = 0$ *or* $\theta = \pi$.

Proof. Assume $\theta = 0$ (see Figure 2). Then the lines t_1 and t_2 overlap. Hence we have $|AP| = |PB|$, i.e., $AP + PB = 0$. This also implies $AP^{-1} + PB^{-1} = 0$. Therefore the right side of (1) equals 0. Assume $\theta = \pi$ (see Figure 3). Then the lines t_1 and t_2 are parallel. We consider using Cartesian coordinate with origin at the center of the circle γ so that the line t_1 has equation $x = r > 0$. Let $(r\cos\rho, r\sin\rho)$ be the coordinates of the point P. We may assume $0 < \rho < \pi$. The line t has equation $x\cos\rho + y\sin\rho = r$. Hence the points A and B have coordinates $(r, r(1-\cos\rho)/\sin\rho)$ and $(-r, r(1+\cos\rho)/\sin\rho)$, respectively. Therefore we get

$$|AP| = r\tan\frac{\rho}{2} \quad \textit{and} \quad |PB| = r\cot\frac{\rho}{2}.$$

Since the line segments AP and PB overlap, they have the same sign, which is plus in this case. Hence we have

$$\frac{r}{AP+PB} = \frac{\sin\rho}{2} \quad \textit{and} \quad \frac{r^{-1}}{AP^{-1}+PB^{-1}} = \frac{\sin\rho}{2}.$$

Therefore (1) holds. □

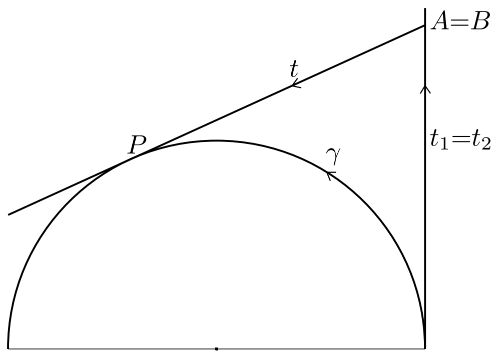

Figure 2: $\theta = 0$.

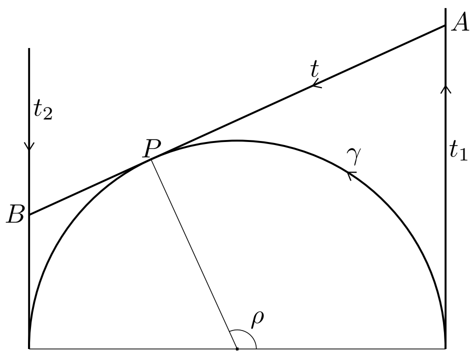

Figure 3: $\theta = \pi$.

References

[1] H. Okumura, Yoshimasa Michiwaki, Sabrou Saitoh, Jun Ozone, On a problem of "Seiyō Sampō", Sūgakushi Kenkyū, (107) (1985) 44–47.
[2] S. Saitoh, Introduction to the Division by Zero Calculus, 2021, Scientific Research Publ., Inc..