Bridging the Gap between Higgs Mechanism and Complex Dynamics

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Abstract

The Standard Model of particle physics postulates that the (mass) ^ 2 term of the Higgs potential is negative. This choice is considered unnatural and leads to the *tachyonic mass problem*. It is known that the formulation of the Higgs mechanism relies on the standard Ginzburg-Landau equation describing equilibrium phase transitions. It is also known that the Complex Ginzburg-Landau equation (CGLE) is a universal model of complex dynamics outside equilibrium. This brief note suggests that the tachyonic mas problem goes away upon switching from the standard Ginzburg-Landau equation to the CGLE.

Key words: Higgs mechanism, tachyonic mass problem, complex Ginzburg-Landau equation, nonequilibrium dynamics.

The standard Ginzburg-Landau potential underlying the Higgs mechanism of spontaneous symmetry breaking is given by,

$$V_{GLE}(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \tag{1}$$

where a positive self-interaction coupling ($\lambda > 0$) forces (1) to be bounded below as the field goes to infinity, $(\varphi^\dagger \varphi)^{1/2} \to \infty$. The potential (1) has a minimum at

$$(\varphi^\dagger \varphi)_{GLE} = \frac{-\mu^2}{2\lambda} = v^2 \tag{2}$$

in which the mass parameter $\mu^2 < 0$ and v represents the vacuum expectation value of the Higgs boson. It is known that the CGLE defines the generic dynamics of a complex order parameter z and assumes the form [1]

$$\partial_t z = az + (1 + ic_1)\Delta z - (1 - ic_3) z |z|^{2\sigma} \tag{3}$$

where a and σ are positive and c_1, c_3 are real. In the absence of spatial dependence ($\nabla z = \Delta z = 0$) and upon taking $c_3 = 0, \sigma = 1, z = \text{real}$, (3) can be rescaled to [1-3]

$$\partial_t z = -\frac{\partial V(z)}{\partial z} = \alpha z - \beta z^3 \tag{4}$$

in which $\alpha > 0, \beta > 0$. Side by side evaluation of (1) and (4) yields

$$V(z) = -\frac{\alpha}{2} z^2 + \frac{\beta}{4} z^4 \tag{5}$$

$$z = (\varphi^\dagger \varphi)^{1/2} \tag{6}$$

$$-\frac{\alpha}{2} = -\mu^2 \tag{7}$$

$$\frac{\beta}{4} = \lambda \tag{8}$$

By (5)-(8), the CGLE potential of the Higgs boson gets changed from (1) to,

$$V_{CGLE}(\varphi) = -\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \tag{9}$$

with a positive mass square $\mu^2 = \alpha/2 > 0$ and a minimum at

$$(\varphi^\dagger \varphi)_{CGLE} = \frac{\mu^2}{2\lambda} = v^2 \tag{10}$$

In closing, we note that the tachyonic mass problem can also be naturally alleviated by identifying the Higgs scalar with the *nontrivial fixed point* of the classical Landau-Ginzburg theory [4].

References

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