The breather-soliton model of elementary particles

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Abstract. In this paper we show that a unification of gravity and inertia, as it is achieved in the context of a correct implementation of Mach‘s principle, strongly suggests that elementary particles are oscillatory solitons in the gravitational field (or more general: a unified field including the gravitational one). We show how the properties of elementary particles then give rise to the phenomenology of special relativity as well as quantum mechanics in the usual classical framework and in flat, 3-dimensional Euclidean space. The oscillatory solitons exhibit the same structure as was originally postulated by De Broglie for the quantum wave function in his double solution theory. This structure of the elementary particles naturally gives rise to elementary quantum phenomena, like their wave-particle duality, the uncertainty relation, the De Broglie relations $E = \hbar\omega$ and $p = \hbar k$ and discrete energy levels for bound states. A formula for h can in principle be obtained. This opens up the possibility of explaining the origin of quantum mechanics in a purely classical framework. At the same time, also the special relativistic phenomena like length contraction, time dilation, the relativistic energy-momentum relation, and the apparent constancy of the speed of light can be explained from just the structure of the solitons in flat, 3-dimensional space. The speed of light is just an apparent constant when measured with co-moving rulers and clocks, provided by the elementary particles themselves. It obeys the usual vector addition, just like all other velocities and vectors do, too. Ultimately, mass itself can be explained as entirely of (gravitational) field origin, as the field energy which is concentrated within the soliton. This will also yield an explanation for the energy-mass equivalence. No additional scalar field like the Higgs field is needed. Further, also problems like the twin paradox, the measurement problem, and the infinite self-energies of elementary particles can be resolved in a soliton model.

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Contents

1. Introduction 3
2. Particles as breather-solitons in the gravitational field 7
3. Relativistic phenomena 9
4. Quantum phenomena 16
5. Self-energy, the Coulomb singularity, and renormalization 22
6. N-soliton solution: The Quantum-relativistic N-particle model 23
7. Breather interaction 24
8. Quarks 26
9. Conclusion 27
10. Acknowledgement 28
References 28

1. Introduction

An explanation of inertia as of gravitational origin and inertia in non-relativistic, classical mechanics, as was attempted in [1-7], and successfully in [6,7], has far-reaching consequences for the nature of mass, the structure of elementary particles, and the origin of relativistic and quantum effects. The origin of inertia has long been an unsolved problem in physics. Already very early it was assumed, based on the empirical equality of gravitational and inertial accelerations and masses, that inertia has its origin in gravity [8,9]. Indeed, many attempts were undertaken to explain gravity and inertia from a unified law in classical, non-relativistic physics. They assumed velocity-dependent gravitational potentials, only containing relative quantities (to fulfill Mach's principle), like the Weber potential [2-5]

$$V_{Weber} = -\frac{Gm_1m_2}{r_{12}}(1 - \frac{\dot{r}_{12}^2}{2c^2}), \tag{1.1}$$

or the Riemann-potential [6,7]

$$V_{Riemann} = -\frac{Gm_1m_2}{r_{12}}(1 - \frac{\mathbf{v}_{12}^2}{2c^2}). \tag{1.2}$$

Here, $r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ and $\mathbf{v}_{12} = \mathbf{v}_1 - \mathbf{v}_2$, G is the gravitational constant, and c is the speed of light. It was shown, that the velocity-dependent part of the potential then gives rise to an inertial term, instead of the usual Newtonian kinetic energy. Although the Weber potential (1.1) leads to anisotropic inertial masses and is thus ruled out by experiment, the Riemann potential indeed allows us to explain inertia as entirely of gravitational origin, as was shown in [6,7]. As a consequence of such a theory, inertial mass is no longer an intrinsic, a priori property of the particles, but a derived quantity from the gravitational mass and the gravitational field itself. In concrete terms, in [7], we found that the inertial mass m_k^* of a particle k is given by[1])

$$m_k^* = \frac{2\varphi_k}{c^2} m_k \tag{1.3}$$

Here, m_k is the gravitational mass, c the speed of light and the gravitational potential

$$\varphi_k = \sum_{j \neq k} \frac{m_j}{r_{kj}}$$

As a consequence, inertial mass is no longer an intrinsic property of the particles. Unlike in current theories, where inertial mass is an a priori property of the particles, which they possess independently of the existence of any fields, the situation has

[1] Similar expressions were also obtained from the other theories based on the Weber potential (1.1), however, those expressions were tensorial

now changed completely. It is a derived property from their gravitational mass and other field quantities. Especially, if there were no gravitational field, a particle would possess no inertial mass, as follows directly from (1.3). Only gravitational mass remains as an intrinsic, a priori property of the particles. But this mass only has a meaning in relation to the gravitational field. The only thing it does is quantify the strength of the coupling to the gravitational field. Without a gravitational field, the gravitational mass has no meaning at all. This puts (gravitational) mass and gravitational field on the same fundamental level: There is no gravitational field without mass, but, in turn, the concept of mass is meaningless in the absence of a gravitational field. Thus, it is an obvious conclusion that mass is nothing more than a specific manifestation of the gravitational field itself. Particles are then a kind of localized "lump" of field energy in the gravitational field. The natural candidate for such a "lump" is the *soliton*, since it possesses all the properties that particles have: It is a stable, localized wave packet that propagates shape-persevering and even retains its form after collisions with other solitons.

This also connects to the double solution theory of De Broglie for a deterministic description of quantum mechanics [10]. His initial idea for matter waves [11] was that every particle possesses an internal oscillation, described by some periodic function, which De Broglie wrote as

$$\psi = a_0 \exp\left(\frac{i}{\hbar} E t\right) \tag{1.4}$$

with the oscillation frequency $\omega = E_0/\hbar = m_0 c^2/\hbar$ and a_0 a constant amplitude. For a particle moving with a velocity v this expression then reads

$$\psi = a_0 \exp\left(\frac{i}{\hbar}(Et - \mathbf{p} \cdot \mathbf{x})\right). \tag{1.5}$$

This ψ wave was later further developed by Schrödinger in his wave mechanics. De Broglie soon considered the description with just this ψ wave as incomplete. He wrote that 'the plane monochromatic ψwave [...] did not actually describe reality, but that it could give in a precise way only the *phase* of the wave phenomenon surrounding the particle, since the constant amplitude a could not represent the true amplitude of this phenomenon.' [10, p. 8]. Instead, he argued, that to every such ψ wave should correspond a wave

$$u(\mathbf{x}, t) = f(\mathbf{x}, t)\psi(\mathbf{x}, t), \tag{1.6}$$

with f being an amplitude function, replacing the constant a_0. This function u is then 'the true representation of the physical entity 'particle', which would be an

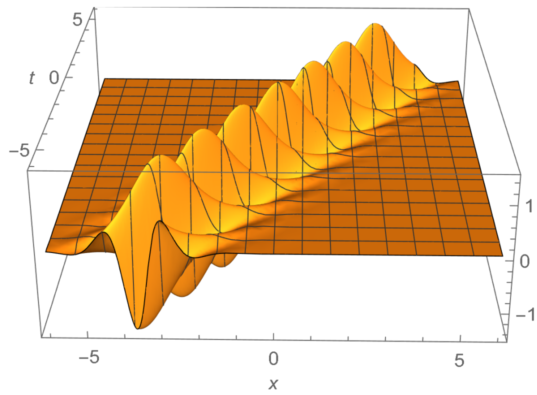


FIGURE 1. The breather solution (1.8) for the parameters $\beta = 0.6$, $q = 1.2$, $d = 0.2$.

extended wave phenomenon centered around a point, which would constitute the particle in the strict sense of the word.'The particle is then represented 'by a very small singular region in space where the function u would take a very large value and obey a non-linear equation, of which the linear equation of wave mechanics would only be an approximate form valid outside the singular region. ' [10, p. 99]. This behavior is guaranteed by the function f, which will have its highest value at the "position" of the particle, and fall off towards zero far away from it.

The form described by De Broglie is exactly the structure that a specific type of soliton possesses, namely the *breather*. Breathers are localized lumps of energy in space and oscillatory in time. This was first noticed by Enz [12] for the one-dimensional Sine-Gordon equation

$$\Box\varphi = \frac{1}{d^2}\sin(\varphi). \tag{1.7}$$

Here, d is some length parameter. This equation has breather solutions given by

$$\varphi(x,t) = 4\arctan\left(\cot(q)\frac{\cos\left(\frac{\gamma\sin(q)}{d}(ct - x\beta)\right)}{\cosh\left(\frac{\gamma\cos(q)}{d}(x - vt)\right)}\right), \tag{1.8}$$

with v the velocity of the breather, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. q is a real parameter and one can see that d is a 1/e radius of the breather. We can see that the solution indeed has the form (1.6) proposed by De Broglie. We can identify the internal oscillation as

$$\psi = \cos\left(\frac{\gamma\sin(q)}{d}(ct - x\beta)\right), \tag{1.9}$$

and the amplitude function f as

$$f = \cot(q)\,\mathrm{sech}(\frac{\gamma\cos(q)}{d}(x - vt)). \tag{1.10}$$

It has its highest value at the particle's position $x = vt$ and falls exponentially far away from it. We can thus write

$$\varphi = 4\arctan(u).$$

The function arctan doesn't change the general structure of the solution since it is strictly monotonic and point symmetric; thus, it especially maps zero to zero. φ and hence also u obeys a non-linear equation, for the former this is eq. (1.7). The structure of the breather solution (1.8) is not specific to the Sine-Gordon equation, but general for breather solitons. They are localized lumps of energy in space and oscillatory in time, thus they always possess the form (1.8) with an oscillatory function ψ and an amplitude function f which falls off to zero far away from the "position" of the lump. Further, solitons are an intrinsically non-linear phenomenon; they owe their stability to a balance between dispersion and non-linear effects. Thus, they always obey a non-linear equation. This is exactly the structure proposed by De Broglie. It thus suggests itself that particles are breather solitons in the gravitational field, or more generally, a unified field that will eventually include gravity and electromagnetism. A famous objection against soliton models of elementary particles, originally put forward by Derrick [13], does not apply to breathers. They are oscillatory and thus are no stationary solutions. This was already pointed out by Derrick himself.

In this paper, we want to show how from such a breather structure of the elementary particles, all special relativistic as well as quantum phenomena can be derived in a purely classical framework, in flat, three-dimensional Euclidean space. They arise solely from the structure of the elementary particles. Their soliton nature naturally gives rise to their wave and particle properties, explaining the wave-particle duality. Further, it is not necessary to introduce any notion of particle or mass a priori. Instead, both can be derived from purely fields and energy. This will allow us to actually explain the origin of the energy-mass equivalence by showing that the expression

$$m = \frac{E}{c^2},$$

takes the role of the mass, with E the field energy of the soliton. The latter will have a well-defined, finite value because solitons are extended particles, removing the problem of infinite self-energies. Finally, the breather-soliton nature of the particles will allow us to explain the origin of the De Broglie relations

$$E = \hbar\omega,$$

$$\mathbf{p} = \hbar \mathbf{k},$$

and the quantum constant $\hbar$. We show, that a formula for $\hbar$ can be obtained from a breather-soliton theory. The form, which the obtained formula possesses agrees with the known 'coincidence'

$$\hbar \sim m_p r_p c, \tag{1.11}$$

with m_p and r_p the mass and radius of the proton.

It must be emphasized that we do not possess the final field equations with the corresponding breather-soliton solutions yet. This paper aims to demonstrate how such a theory would work in principle and what its possibilities are. Finding the correct field equations and breather-soliton solutions will be a task in future research.

2. Particles as breather-solitons in the gravitational field

We now want to demonstrate how the idea of particles being a part of the gravitational field can be expressed mathematically. Therefore, we consider the simplest relativistic field equation, which takes into account a finite propagation speed of light

$$\Box \varphi = 4\pi G \rho.$$

If we assume mass now as a part of the gravitational field, we can write this equation in the form

$$\Box \varphi = -\frac{1}{d^2} V'(\varphi), \tag{2.1}$$

where the field potential V is some function of the gravitational field φ, and d is a length parameter. This equation now for example possesses traveling wave solutions for any function V. They are implicitly given by

$$\frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{V(\varphi)}} \, d\varphi = \frac{1}{d} \gamma (x - vt - x_0),$$

If the solution φ satisfies the condition that it falls off to zero far away from the 'position' $x = vt + x_0$ of the particle, the solution is a solitary wave. One example is the non-linear Klein-Gordon equation, with the potential V given by

$$V(\varphi) = a\varphi^2 - b\varphi^n, \qquad n \in \mathbb{N}$$

with $a, b > 0$ real parameters[2]). The solitary wave solutions to this potential are given by

$$\varphi(x,t) = [\frac{a}{b} \operatorname{Sech}^2 (\sqrt{\frac{a}{2}} (n-2) \frac{\gamma}{d} (v - vt - x_0))]^{\frac{1}{n-2}}. \tag{2.2}$$

[2]For $a < 0$ or $b < 0$ and n even, the solutions are imaginary, and thus unphysical

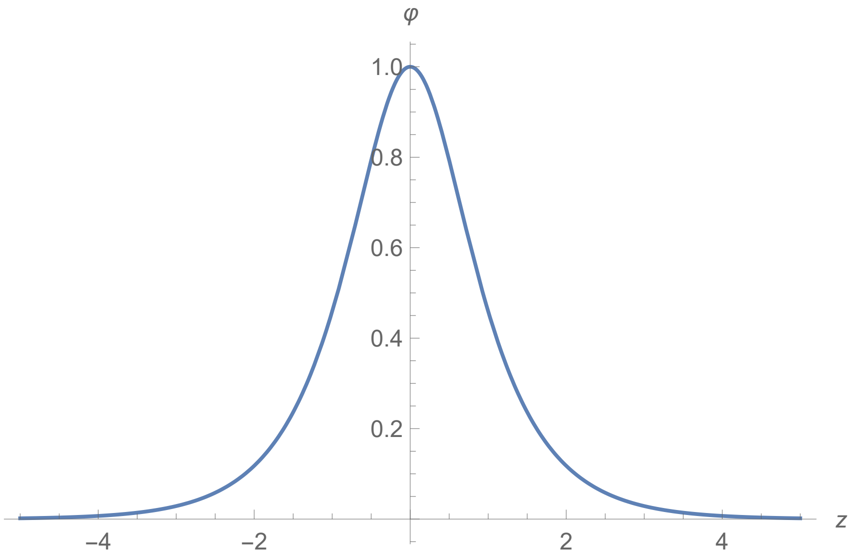


FIGURE 2. The solitary wave solution (2.2) for the values a=b=1 and n=4. It is plotted the field amplitude φ against the distance $z = \frac{\gamma}{d}(x - vt - x_0)$ to the position of the particle.

For $n > 2$, they fulfill the requirement of falling off to zero far away from the particle's position, thus, they indeed are solitary waves. They describe a localized lump of field energy in the gravitational field, propagating along the trajectory $x = vt + x_0$. Such a lump of field energy then represents an (extended) particle. The solution (2.2) is plotted in figure 2. To describe a universe consisting of N particles, fully integrable equations are needed with N soliton solutions (cf. section 6). In such solutions, the solitons do, unlike it is the case for solitary waves, retain their original shape after collisions with each other, which is an essential property of particles. Such equations of the form (2.1) are for example the Sine-Gordon equation. But also three-dimensional equations with oscillatory soliton solutions, as demanded by De Broglie, have been found for various V [14]. Searching and finding soliton solutions to equations of the form (2.1) is a subject of current, active research. Finding the correct field equations and the corresponding breather-soliton solutions remains a task to be done. For now, we will assume they exist and focus on showing the capabilities of such a breather-soliton theory of elementary particles. As we already elaborated, the solutions have the form (1.6)

$$\varphi(x,t) = \varphi_0(\psi(\frac{ct}{d})f(\frac{r}{d})) \tag{2.3}$$

for a breather resting in the coordinate origin. Here, $r = |\mathbf{r}|$ is the distance and φ_0 is a possible scaling function like the arctan in the Sine-Gordon breather, which we allowed for more generality. The function ψ is periodic in time and the function f decays towards zero as r goes towards infinity. For simplicity, we also assumed that the amplitude function is spherical; its decay depends only on the distance from the 'position' of the particle. For a moving breather, we then have due to the Lorentz-symmetry of the underlying field equation (2.1)

$$\varphi(x,t) = \varphi_0(\psi(\frac{\gamma}{d}(ct - \beta \cdot \mathbf{r}))f(\frac{\sqrt{\gamma^2(\mathbf{r}_{||} - \mathbf{v}t)^2 + \mathbf{r}_{\perp}^2}}{d})). \tag{2.4}$$

Here, $\mathbf{r}_\perp$ and $\mathbf{r}_{||}$ are the position vectors perpendicular and parallel to the direction of motion of the particle. In the following two sections, we're going to show now, how the phenomenology of special relativity and quantum mechanics can be derived from this breather-structure of the particles.

3. Relativistic phenomena

We first want to show that all special relativistic phenomena come out as a consequence of the breather-soliton nature of the elementary particles in flat, three-dimensional Euclidean space. No Minkowski space has to be introduced. Neither time dilation, length contraction, nor the constancy of the speed of light have to be postulated, but can be derived from the breather-soliton nature of the particles. The same also applies to relativistic mechanics. Because the particles are described as a part of the field, it is unnecessary to make mechanics Lorentz-symmetric separately, as was done by introducing special relativity. The Lorentz-symmetry of the field equations directly results in the Lorentz-symmetry of particle (soliton) mechanics. The soliton theory also allows for a resolution of the twin paradox: All relativistic effects occur when a particle moves relative to the rest frame of the field, which serves as an absolute rest frame like Lorentz's aether.

3.1. **Relativistic kinetic energy and momentum.** We start with the expressions for the kinetic energy and momentum. The field energy and momentum derived from equation (2.1) are given by the expressions

$$E = \int_{\mathbb{R}^3} \frac{1}{2}(\frac{1}{c^2}(\frac{\partial \varphi}{\partial t})^2 + (\nabla\varphi)^2) + \frac{1}{d^2}V(\varphi)\, d^3\mathbf{x}, \tag{3.1}$$

$$\mathbf{p} = \frac{1}{c}\int_{\mathbb{R}^3} (\frac{\partial \varphi}{\partial t}\nabla\varphi)\, d^3\mathbf{x}. \tag{3.2}$$

Those expressions form a 4-vector [15], that is they transform under Lorentz transformations into a frame moving with a velocity -$\mathbf{v}$ as

$$E \to \gamma(E + v\mathbf{p}), \tag{3.3}$$

$$\mathbf{p} \to \gamma(\mathbf{p} + \frac{\mathbf{v}}{c^2}E). \tag{3.4}$$

Therefore, we have for a resting breather

$$E = E_0, \tag{3.5}$$

$$\mathbf{p} = 0, \tag{3.6}$$

with E_0 its rest energy. For a moving one, we have

$$E = \gamma E_0, \tag{3.7}$$

$$\mathbf{p} = \frac{E_0}{c^2}\gamma \mathbf{v}. \tag{3.8}$$

Those are the well-known relations for the relativistic energy and momentum of a moving particle, with an important difference. The particles are part of the fields and are not postulated to exist a priori. The energy E_0 is part of the field energy and is calculated via equation (3.1). Further, we have not introduced any notion of mass a priori. From equation (3.8) we can see that the role of the rest mass is played by

$$m_0 = \frac{E_0}{c^2} \tag{3.9}$$

Taking (3.7,3.8) and (3.9) together, we obtain the relativistic energy-momentum relation

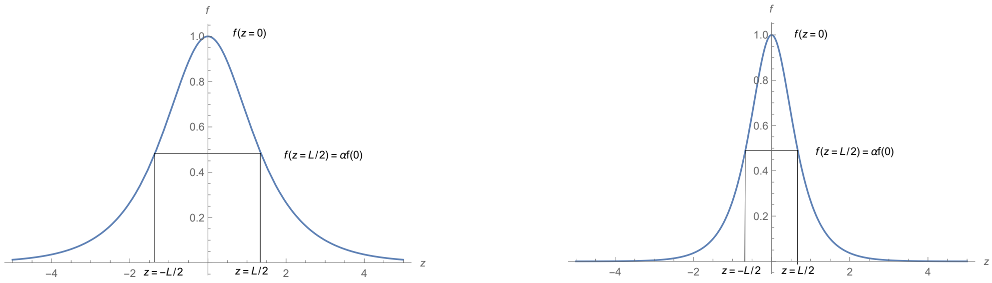
$$E = \sqrt{m_0^2 c^4 + c^2 \mathbf{p}^2} \tag{3.10}$$

Thus, particle mechanics inherit their Lorentz-covariance from the Lorentz-symmetry of the underlying field equation. This doesn't need to be done 'by hand', like was done with the introduction of special relativity. When at the end of the 19th century, physicists saw that Maxwell's equations were Lorentz-invariant, but mechanics were not, mechanics was made Lorentz-covariant by introducing special relativity. If particles are solitons in the (gravitational) field, then the Lorentz-covariance of mechanics directly follows from the Lorentz-invariance of the field equations.

3.2. **Length contraction.** We now show how the phenomenon of length contraction arises from the soliton nature of the particles. For simplicity, we first show this in the example of the one-dimensional Sine-Gordon breather [3]), and later for the general breather-type soliton (2.4).

The elementary particles themselves provide elementary rulers. Their "size" L is characterized by a certain α (e.g. 1/e) decay of the amplitude function f (and thus also the whole function φ) of the breather soliton, as is shown in fig. 3 below.

[3]This was done in a different way also by Günther [16,17] in the context of the Sine-Gordon model



(A) For $\beta = 0$ (B) For $\beta = \frac{\sqrt{3}}{2}$

FIGURE 3. The amplitude function (1.10) is plotted against the distance $z = x - vt - x_0$ to its "position" $x = vt + x_0$; exemplarily for the values $q = \pi/4$ and $d = 1/\sqrt{2}$. The particle's "size" is defined as the length L at which the value of the field amplitude has dropped to a fraction $\alpha < 1$ of its highest value at $x = vt + x_0$.

We have at a distance $z = x - vt - x_0 = \pm L/2$ away from the 'position' of the breather for the amplitude function (1.10)

$$f(z = \pm\frac{L}{2}) = \cot(q)\,\mathrm{Sech}(\frac{\gamma\cos(q)}{d}\frac{L}{2}) = \alpha f(0) = \alpha\cot(q).$$

Solving for L gets us

$$L = \frac{2d}{\gamma\cos(q)}\,\mathrm{sech}^{-1}(\alpha)$$

For a resting breather, we then have

$$L = \frac{2d}{\cos(q)}\,\mathrm{sech}^{-1}(\alpha) =: L_0, \tag{3.11}$$

since $\gamma = 1$. For a moving one, we get

$$L = \gamma^{-1}L_0 = \sqrt{1-\beta^2}L_0. \tag{3.12}$$

Thus, every moving particle is contracted by the known Lorentz factor. For a general, three-dimensional breather, we can define its length in the same as above. However, we have to distinguish between its length $L_{||}$ in the direction of the breather's motion, and $L_\perp$ the one perpendicular to it. For the first, we have at a distance $L/2$ away from the position of the breather in the direction of motion $\mathbf{r}_\perp = 0$ and $\mathbf{r}_{||} - \mathbf{v}t = \pm L/2$ and thus

$$f(\frac{\gamma L_{||}}{2d}) = \alpha f(0) \leftrightarrow L_{||} = \gamma^{-1}2df^{-1}(\alpha f(0))$$

For a resting breather, we therefore have

$$L_{||} = 2df^{-1}(\alpha f(0)) =: L_0. \tag{3.13}$$

For a moving one, we get

$$L_{||} = \gamma^{-1}L_0 = \sqrt{1-\beta^2}L_0. \tag{3.14}$$

On the other hand, for the second case, we have at a distance $L/2$ away from the position of the breather perpendicular to the direction of motion $\mathbf{r}_\perp = \pm L/2$ and $\mathbf{r}_{||} - \mathbf{v}t = 0$, thus

$$f(\frac{L_\perp}{2d}) = \alpha f(0) \leftrightarrow L_\perp = 2df^{-1}(\alpha f(0)) = L_0. \tag{3.15}$$

The three-dimensional breather is Lorentz-contracted only in the direction of motion by the known Lorentz factor but remains unchanged perpendicular to it.

Since all macroscopic rulers and also all other material objects are composed of elementary particles, they inherit this behavior. It must be noted, that length contraction here is a real physical effect. It is not space contracting, but matter itself contracts when it moves. Further, like in special relativity, the effect is unobservable for the observer when he uses comoving rulers to measure objects at rest in his frame: Since the rulers themselves change their length by the same factor as the objects, he always measures the same size for any object as he would measure if his frame, together with the object, were at rest.

3.3. **Time dilation.** We now turn to the phenomenon of time dilation. For simplicity, we again first show this for the Sine-Gordon breather, and a general breather-type soliton afterwards. As we already established, every particle (breather), possesses an internal oscillation described by the oscillatory function ψ. These internal oscillations of the particles provide elementary clocks. For the Sine-Gordon breather, this function was given by (1.9)

$$\psi = \cos\left(\frac{\gamma \sin(q)}{d}(ct - x\beta)\right).$$

If the breather rests, we have

$$\psi = \cos\left(\frac{\sin(q)}{d}ct\right),$$

from which we can read of the oscillation frequency

$$\omega = \frac{\sin(q)c}{d} =: \omega_0. \tag{3.16}$$

If the same breather moves with a velocity v, we have at any fixed position on the moving breather $\mathbf{r} = \mathbf{v}t + \mathbf{a}_0$, and thus

$$\psi = \cos\left(\frac{c\sin(q)}{\gamma d}t - \frac{\gamma \sin(q)}{d}\beta \cdot \mathbf{a}_0\right) \qquad (3.17).$$

From this, we can read off the frequency of the moving breather

$$\omega = \frac{c\sin(q)}{\gamma d} = \omega_0\sqrt{1-\beta^2}. \tag{3.18}$$

Thus, a moving elementary clock runs slower by the inverse Lorentz factor. The second term is a trivial phase, which just depends on which co-moving position on the moving particle we evaluate the frequency. As one can see, the frequency itself is independent of this position: The whole moving particle oscillates with the same reduced frequency (3.18). For a general breather-soliton, we can do the same as above. For a breather at rest, we have for the oscillatory function in (2.3)

$$\psi(\frac{c}{d}t).$$

This function oscillates with a frequency of[4])

$$\omega = \frac{sc}{d} =: \omega_0, \tag{3.19}$$

with s a dimensionless parameter. An elementary clock at rest therefore ticks with a frequency ω_0 . We now consider the same breather moving again and evaluate the periodic function at some fixed position $\mathbf{r} = \mathbf{v}t + \mathbf{a}_0$ of it. We get

$$\psi(\frac{\gamma}{d}(ct - \beta \cdot \mathbf{r})) = \psi(\frac{c}{d\gamma}t - \frac{\gamma}{d}\beta \cdot \mathbf{a}_0). \tag{3.20}$$

Thus, the function ψ oscillates with a reduced frequency

$$\omega = \frac{sc}{d\gamma} = \omega_0\sqrt{1-\beta^2}. \tag{3.21}$$

Since all matter is composed of elementary particles, all macroscopic clocks and macroscopic processes inherit this behavior. As for the length contraction, this is a real physical effect. It is not time running slower, but just all processes including clocks. Again, this effect is unobservable for the observer when he measures processes of objects at rest inside his moving frame, when he uses comoving clocks. All comoving clocks run slower by the same factor as all processes, thus he always measures the same time elapsing for any processes as he would measure if both were at rest.

3.4. **Lorentz transformation and Einstein velocity addition.** From the length contraction and time dilation effects derived in the two previous sections, the Lorentz transformations and the relativistic velocity addition formula can be derived. This was done in great detail by Günther[17] for the Sine Gordon model. Since the derivations only rely on the elementary rulers and clocks exhibiting contraction and dilation as was derived in the previous two sections, the derivations carry over to the general case considered here.

An important thing to point out though is how the Lorentz-transformations and velocity addition formula come about. As we already pointed out, it is not space

[4]For the Sine-Gordon model, we have $s = \sin(q)$, and the period of the oscillating function $P = 2\pi$.

and time changing in a moving frame, but the elementary rulers and clocks contracting, respectively dilating. This allows for a very clear and simple explanation of the origin of the Lorentz transformation, and especially where the difference to the Galilean transformation arises from. We demonstrate this in the example of the Lorentz transformation of the x-coordinate; the time-coordinate and the velocity addition formula follow the same underlying idea (cf. Günther[17]).

Take two reference frames S and S', with S' moving with a velocity v relative to S. Then the Galilean transformation between the two reads

$$x' = x - vt$$

Now, since the observer moving with S' has his rulers contracted according to (3.12), he will measure for the same distance

$$x' = \gamma(x - vt)$$

which is the Lorentz transformation of x. It naturally arises due to the change of the (elementary) measuring instruments in the moving frame and the resulting changes in the measured quantities. Apart from that, the normal Euclidean vector addition holds. The difference between the Galilean transformation and the Lorentz transformation is a pure *measurement-effect* coming from the used rulers and clocks changing. The same also applies to the relativistic velocity addition: The usual vector addition of velocities remains valid, the difference is again a pure measurement effect, coming from the use of comoving rulers and clocks.

3.5. **Apparent constancy of the speed of light.** From the derived phenomena of time dilation and length contraction, we can show now that the speed of light appears constant in any moving reference frame. Unlike in special relativity, this does not have to be postulated either, it comes out as a natural consequence of the properties of the solitons providing elementary clocks and rulers. The speed of light is only apparently constant when measured with comoving rulers and clocks, but otherwise, again, obeys the normal velocity addition. This was shown similarly also by Günther [16,17] in the context of the Sine-Gordon model.

Before we start showing this, we first have to remark, that it is not possible to measure the *one-way speed of light*, but only the *two-way speed of light* can be measured directly. Any measurement of the one-way speed of light depends on a convention as to how to synchronize clocks at the source and the detector. Thus, the only thing required to show is that the measured two-way speed of light equals c in any frame of reference.[5])

To show this, consider a test section of rest length Δx_0 . A light signal is sent back and forth and the time it takes to get back to the start is measured. The measured speed of light is always

$$c = \frac{2L}{T}, \tag{3.22}$$

with L the measured length and T the measured time. If the section is at rest, the time needed for a light signal to travel back and forth within the test section is

$$\Delta t = \frac{2\Delta x_0}{c_0}.$$

The measured time is just $T = \Delta t$, and the measured length of the section $L = \Delta x_0$. The measured speed of light then is thus

$$c = \frac{2\Delta x_0}{\Delta t} = c_0.$$

Consider now the test section moving with a velocity v, and a co-moving experimenter. In this case, the time the light signal needs to travel back and forth is

$$\Delta t_{\rightarrow} = \frac{2\Delta x}{c_0 - v}, \tag{3.23}$$

$$\Delta t_{\leftarrow} = \frac{2\Delta x}{c_0 + v}, \tag{3.24}$$

$$\Delta t = \Delta t_{\leftarrow} + \Delta t_{\rightarrow} = \frac{2\Delta x}{c_0}\frac{1}{1-\beta^2} = \frac{2\Delta x_0}{c_0}\gamma.$$

In the last step, we have made use of the fact that the test section is contracted by $\Delta x = \sqrt{1-\beta^2}\Delta x_0$ according to (3.12). The time, measured again with a co-moving clock, is reduced by $T = \sqrt{1-\beta^2}\Delta t$, according to (3.21). Thus, we have for the measured time

$$T = \sqrt{1-\beta^2}\Delta t = \frac{2\Delta x_0}{c_0}$$

The measured length for the test section stays $L = \Delta x_0$, since the rulers are contracted by the same factor as the section. Thus, the experimenter will always measure the same length for it, as was already pointed out in the section about length contraction. This yields for the measured speed of light

$$c = \frac{2L}{T} = c_0$$

Thus, we measure the same value for the speed of light in any frame. Notice, that we assumed standard velocity addition to be valid for the speed of light, as is expressed

[5]It was shown by Günther that, like in special relativity, it is possible to choose the synchronization convention in such a way, that also the one-way speed of light is the same in any moving frame [17]

by the equations (3.23-3.24). No assumptions about its constancy were postulated. The fact that it *appears* constant in a moving frame is a natural consequence of the rulers and clocks, provided by the elementary particles, changing in this frame, according to what was demonstrated in section 3.2 & 3.3.

3.6. **The twin paradox.** The soliton model also allows for a satisfactory solution to the twin paradox without relying on iffy assumptions about a very extraordinary behavior of time dilation during accelerated motions. In a soliton model, all relativistic effects, like time dilation, length contraction, mass increase, etc., always occur when a particle moves relative to the rest frame provided by the field that generates it. Therefore, the field plays the same role as an absolute rest frame, like Lorentz's aether. This immediately removes the twin paradox in the same way Lorentz's aether model does. Only the twin that moves relative to the absolute rest frame of the field experiences time dilation, the other one doesn't.

This concludes our section about the special relativistic effects. As we have seen, all the phenomenology of special relativity comes out as a consequence of the soliton properties. We emphasize again, that no Minkowski space is needed. No change in time or space occurs in a moving frame of reference. It is just the elementary particles that change their properties in a moving frame and thus give rise to all the relativistic phenomena in flat, 3-dimensional Euclidean space. Further, the Lorentz-symmetry of particle mechanics follows directly from the Lorentz-symmetry of the field equations.

4. Quantum phenomena

Next, we will turn our attention to quantum phenomena. Like for the special relativistic ones, they will come out entirely of the properties of the solitons in a purely classical framework. It is then unnecessary to postulate the validity of a special 'quantum mechanics' at a microscopic scale. It will emerge by itself from the soliton nature of the elementary particles. The soliton model explains the wave-particle duality in a natural way since solitons are localized waves exhibiting particle properties. Therefore, particles naturally possess wave and particle properties when described as solitons. Further, due to particles not being points, but spread out distributions in space, one naturally obtains the uncertainty principle via the *bandwidth theorem*. However, unlike in quantum mechanics, the waves are not statistical in nature, but purely classical wavepackets of field energy. Further, discrete energy levels can arise for bound states in the same way they occur for standing waves in classical mechanics. Of course, it has to be shown, that the soliton model leads to the same quantitative predictions (especially for the energy levels) as quantum mechanics, once the correct soliton equations are found. Also, it is still unclear what spin, and therefore entanglement looks like in the soliton model.

4.1. **The Uncertainty relation.** The soliton model naturally gives rise to the quantum phenomenon of uncertainty. The wave nature of the particles gives rise to the uncertainty principle due to the bandwidth theorem for signals. Since a wave is a spread-out phenomenon, one cannot simply ascribe a position to it, as one would do for a point particle. Instead, one has to define its position as a suitable average with the amplitude of the wave. Suppose our wave is given again by the function $\varphi(x,t)$. We can define the weighted average for some quantity g analogous to the expectation value in quantum mechanics by

$$\langle g \rangle = \frac{\int_{\mathbb{R}^3} \varphi(\mathbf{x},t)^2 g(\mathbf{x})\, d^3\mathbf{x}}{\int_{\mathbb{R}^3} \varphi(\mathbf{x},t)^2\, d^3\mathbf{x}}, \tag{4.1}$$

and the mean square deviation for position and wavenumber variables in the well-known way as

$$\Delta x^2 = \langle (\mathbf{x} - \langle \mathbf{x} \rangle)^2 \rangle, \tag{4.2}$$

$$\Delta k^2 = \langle (\mathbf{k} - \langle \mathbf{k} \rangle)^2 \rangle. \tag{4.3}$$

k here is defined as the operator

$$\mathbf{k} = -i\nabla,$$

just like in quantum mechanics. Then, the bandwidth theorem states the inequality

$$\Delta x^2 \Delta k^2 \geq \frac{1}{4}. \tag{4.4}$$

This is exactly the Heisenberg uncertainty principle, apart from the De Broglie relation $p = \hbar k$. It comes out naturally just from the soliton nature of the particles. For the De Broglie relations themselves, we will show in the next section, how they emerge as a necessary consequence of breather solutions to relativistic field equations. If one defines the mean square deviations for angular frequency and time in the same way as done above for k and x as, with the frequency operator

$$\omega = i\frac{\partial}{\partial t},$$

then one also gets the second uncertainty relation

$$\Delta \omega^2 \Delta t^2 \geq \frac{1}{4}. \tag{4.5}$$

This is again equivalent to the Heisenberg uncertainty relation for energy and time, apart from the relation $E = \hbar\omega$, which we will also derive in the next paragraph.

It must be emphasized again, that the average value (4.1) has no statistical character, and neither has the function $\varphi(x, t)$. It is just the weighted average with the amplitude of the wave φ which is a classical 'lump' of field energy, thus a real wave in the classical sense.

4.2. **The De Broglie relations.** We are now going to show how the De Broglie relations can be derived from the breather-soliton theory. For the Sine-Gordon model, this was already shown by Enz [12]. We will show later, that this is not restricted to the Sine-Gordon breather, but a consequence of the Lorentz-symmetry of the underlying field equation when applied to a breather solution. Therefore, any breather solution to a relativistic field equation provides the De Broglie relations. We recall that for the Sine Gordon field, we had for the periodic function (1.9)

$$\psi(x,t) = \cos\left(\frac{\gamma \sin(q)}{d}(ct - \beta x)\right),$$

which described the internal oscillation of the particle. From this, we can read off the frequency and wavenumber as

$$\omega = \frac{\sin(q)c}{d}\gamma, \tag{4.6}$$

$$k = \frac{\sin(q)}{cd}v\gamma. \tag{4.7}$$

On the other hand, the energy and momentum of the breather can be calculated using equations (3.1-3.2), which yields

$$E = 16d\cos(q)\gamma, \tag{4.8}$$

$$p = \frac{E_0}{c^2}v\gamma. \tag{4.9}$$

From equations (4.6-4.7) and (4.8-4.9), we find the De Broglie relations

$$E = \hbar\omega, \tag{4.10}$$

$$p = \hbar k, \tag{4.11}$$

with Planck‘s constant h given by

$$\hbar = \frac{E_0 d}{\sin(q)c}. \tag{4.12}$$

The De Broglie relations are, however, not just coincidentally included in the Sine-Gordon breather. All breather solutions to relativistic field equations exhibit them. They are a result of the Lorentz-Symmetry of the underlying field equations. A resting breather has the general form (2.3)

$$\varphi(x,t) = \varphi(\psi(\frac{ct}{d})f(\frac{r}{d})),$$

where ψ is an oscillatory function periodic in time in the sense of De Broglie and f is the corresponding function of spatially variable amplitude. Now, ψ has an oscillation frequency ω=sc/d with s some dimensionless parameter. If we consider the same breather moving, due to the Lorentz-symmetry of the underlying field equation, we have for the periodic function

$$\psi(\frac{\gamma}{d}(ct - \beta \cdot \mathbf{r})). \tag{4.13}$$

From this, we can read off

$$\omega = \frac{sc}{d}\gamma \qquad (4.14),$$

$$\mathbf{k} = \frac{s}{cd}\mathbf{v}\gamma. \tag{4.15}$$

Recalling $E = E_0\gamma$ and $\mathbf{p} = m_0\gamma\mathbf{v}$, which are also valid again according to (3.7-3.8), we obtain the relations (4.10-4.11) with h given by (4.12) (the parameters s and d are, of course, different in general here). It is very interesting to compare the expression (4.12) with the „coincidence“ (1.11)

$$\hbar \sim m_p r_p c = \frac{E_p r_p}{c}$$

Since $s \sim 1$, one can see that this expression has the same structure as (4.12). In (4.12), E_0 is the energy of the fundamental particle (the breather) and d its 1/e length, a fundamental length scale, entering via the field equation (1.7). The 'coincidence' (1.11) has the same structure, with the fundamental particle being the proton (or the neutron). This also suggests that protons and neutrons are elementary particles in the soliton picture. It is remarkable and non-trivial, that in all relativistic breather models, the expression for h depends on the rest-energy and radius of a fundamental particle (the breather solution) in the exact same way that is found in the „coincidence“ (1.11).

It is also interesting to notice, that the reason why breather solutions to relativistic field equations exhibit the De Broglie relations, is the structure of the Lorentz-Transformations, under which these equations are invariant. More precisely, the transformation of time

$$t \to \gamma(t - \frac{\mathbf{v}\cdot\mathbf{r}}{c^2})$$

If the field equations would only exhibit Galilean symmetry, time would transform as $t \to t$ and therefore (4.13) would yield neither of the De Broglie relations (4.10-4.11). This is even more remarkable since the De Broglie relations are already fundamental in non-relativistic quantum mechanics. It shows again, how quantum mechanics and relativistic phenomena in a soliton theory have a common origin in the breather structure of the elementary particles.

4.3. **Discrete energy levels.** The soliton model can also give rise to discrete energy levels for bound states. This discrete nature of the energy levels of bound states can arise from the modes of the trapped breathers in a potential, similar to the modes of a standing wave in classical physics, with a wave and a reflected wave superimposed. As an example of how this works, we consider a Sine-Gordon breather trapped in an infinite potential well of width L. The breather is reflected at the walls of the well, resulting in a second breather, traveling in the opposite direction with the same velocity and phase shifted by π. Thus, the entire solution is a two-breather solution to the Sine-Gordon equation, which can be found analytically. This solution has to fulfill the boundary conditions

$$\varphi(0,t) = \varphi(L,t) = 0$$

for all times t, as well as satisfy the Sine-Gordon equation (1.7). The explicit calculations are carried out in a separate paper [18], as they are a bit more technical. Demanding that the solution fulfills the boundary conditions, one obtains

$$kL = n\pi, n \in \mathbb{N}, \tag{4.16}$$

for the wavenumber k of the breather, as defined by equation (4.7). The solution is then a breather oscillating back and forth between both ends of the potential well. The condition (4.16) exactly agrees with the quantization condition derived from quantum mechanics. If we combine this with the De Broglie relation (4.11) and plug both in the relativistic energy-momentum relation (3.10), we obtain

$$E_n = \sqrt{(m_0c^2)^2 + c^2\hbar^2(\frac{n\pi}{L})^2}. \tag{4.17}$$

Those are the well-known energy levels for the infinite potential well obtained from the Klein-Gordon equation of relativistic quantum mechanics. To show that also for more sophisticated potentials, in general, the energy levels agree with those of the Klein-Gordon equation, remains a task to be done. Of course, this is to be expected from the full theory.

4.4. **The measurement problem.** The soliton model can also resolve the measurement problem. Or more precisely, it doesn't arise in the first place. Since the particles in the soliton model are real waves instead of probability waves, no 'collapse' occurs when a measurement is performed. The wavefunction does not collapse to a point particle with a determined, discrete position or momentum, but instead stays a spread out distribution in space, still fulfilling the uncertainty relation (4.4) at every time.

Yet still, unlike in 'hidden variable theories', there is no deeper reality behind the wave. No discrete particle with a well-defined trajectory 'sits' behind the wave. The wave *is* the deepest reality, similar to the Copenhagen interpretation of Quantum mechanics. However, unlike in Quantum mechanics, the wave is not statistical, but a true, classical wave.

4.5. **Bell's theorem.** This also brings us to Bell's theorem, one of the major bottlenecks for classical, deterministic theories of quantum phenomena. A final discussion of this topic has to wait until it is clear what the mechanism for spin, and thus entanglement in the framework of the soliton theory is. Nevertheless, it is worth looking at what we already know about the soliton theory to shed light on what a possible answer to the EPR experiment looks like. Therefore, we first recall what the violation of Bell's inequality implies that a quantum theory cannot at the same time be *deterministic* and *local*. Here, it is especially important to recall what *deterministic* means in this context: That behind the, according to quantum mechanics, statistically distributed quantities (like position, momentum, spin-direction, etc.) exist 'hidden variables', possessing a well-defined, specific value, like it is the case for a classical point particle moving on a well-defined trajectory. That behind the quantum mechanical wavefunction ψ hides a deeper reality, which cannot be known, due to, for example, unknown initial conditions, like it is the case for the De Broglie-Bohm theory. As we already saw, this is not the case for solitons. There are no hidden variables behind them. No deeper reality hides behind the wave. Instead, the wave itself *is* the true reality, similar to the Copenhagen interpretation of Quantum mechanics. Thus, the soliton model is non-deterministic according to the definition given above, despite being a purely classical theory. At every time, the particles obey the uncertainty relations (4.4-4.5). Therefore, it is likely that the soliton theory will indeed violate Bell's inequalities in the same way as Quantum mechanics does.

This concludes our section about quantum phenomena. We saw that basic quantum phenomena like the wave-particle duality and uncertainty relation arise naturally from a soliton model of particles in a purely classical framework. Nevertheless, it is non-deterministic according to the definition used in the context of hidden-variable theories. The solitons (the waves) are the most fundamental nature; no deeper reality 'hides' behind them, like, for example, hidden trajectories of point particles. It was shown, how in principle discrete energy levels arise for bound states, in the same way they do for standing waves. Finally, the breather-soliton nature of elementary particles enables one to explain the origin of the quantum constant h and another 'coincidence' (1.11). This highly suggests that particles indeed exhibit such a breather-soliton nature and that this is at the heart of quantum mechanics.

5. Self-energy, the Coulomb singularity, and renormalization

Due to the particles not being points, but spread out distributions in space, the soliton model immediately removes the problem of infinite self energies, as well as the unphysical 1/r singularity in the potentials, be it gravitational or electromagnetic. As is well known, both problems can be tracked down to particles being described as points. We will restrict ourselves here to the gravitational potential, but all arguments apply to the electromagnetic as well. The energy of a mass distribution in a stationary gravitational field is given by

$$E = \frac{1}{8\pi G} \int_{\mathbb{R}^3} (\nabla\varphi)^2 \, d^3\mathbf{r}. \tag{5.1}$$

For a point particle, which we without loss of generality assume to be at the center, the gravitational potential φ reads

$$\varphi = \frac{Gm}{r},$$

which, if plugged into (5.1), yields an infinite energy. One can see, that this infinite self-energy is a consequence of the likewise unphysical singularity of the 1/r field, which in turn is a result of point particles being the source. This, one can easily see by plugging the Dirac distribution into the equation for the potential

$$\varphi = -G \int_{\mathbb{R}^3} \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \, d^3\mathbf{r}'. \tag{5.2}$$

On the other hand, one can easily see that the potential (5.2), as well as its derivative, remains finite for $\mathbf{r} \to 0$ for any density which remains finite in this limit, too. Now, a particle described by a soliton, obviously fulfills this requirement. Therefore, the singularity in the potential as well as the resulting infinite self energy are removed. Indeed, as mentioned at the beginning, the solitons are part of the field. Thus, the energy calculated by equation (3.1) contains both the combined energy of the particle (the core field) and its (far-) field. As we saw, it yields a well-defined, finite value for its rest mass. No renormalization is required.

It must be emphasized, that from a physical perspective, there is no reason to demand that elementary particles are point-like, as is claimed in [19]. As is the case in the soliton model, a particle can be at the same time elementary in the sense that all other matter is built out of it, and still be extended and possess an internal structure. At the same time, it is not 'built up' of any more fundamental building blocks, like for example point particles. As we already saw, the wave (the soliton) is the most fundamental object; nothing else 'sits behind it'. The assumption, that elementary particles are such whose '[...] mechanical state is fully described by three space coordinates and three components of velocity' [19, p. 53], is invalid for solitons. Consequently, also the conclusion that an elementary particle, if extended, would need to be undeformable (because otherwise it could not be described by a single set of three space coordinates), is not correct. Indeed, solitons deform under external perturbations and mutual collisions and retain their original shape afterward. Thus, no instantaneous action at a distance needs to take place between different 'parts' of an extended particle, like this were the case if it indeed consisted of rigidly moving sub-parts. Further, as was shown in section 3.1, the solitons exhibit the known relativistic energy and momentum relations (3.7-3.8) as a consequence of the underlying field equations. Thus, it is impossible to accelerate them to the speed of light, or even beyond it, just as in special relativity.

6. N-soliton solution: The Quantum-relativistic N-particle model

In non-relativistic, classical mechanics, the universe is described by an N-particle model. Many soliton equations possess *analytic* N-soliton solutions, as well as N-breather solutions [20,21]. They are then the natural quantum-relativistic generalization of the N particle model in classical mechanics. Those solutions describe particles (the soliton core fields), their (far-) fields as well as their mutual interactions (and the resulting motion) in a *single* solution to the underlying field equations. As an example, we give the N-breather solution of the 1-dimensional sine-Gordon equation (1.7)

$$\Box\varphi + \frac{1}{d^2}\sin(\varphi) = 0.$$

Its N-soliton solution is given by [21]

$$\varphi = \arctan\left(\frac{f_i}{f_r}\right), \tag{6.1}$$

$$f = f_r + if_i = W(\psi_1, ..., \psi_N). \tag{6.2}$$

Here, f_r, f_i denote the real and imaginary part of f and W is the Wronskian with the entry vector $\psi = (\psi_1, ..., \psi_N)^T$

$$\begin{aligned} W(\psi_1, ..., \psi_N) &= |\psi^{(0)}, \psi^{(1)}, ..., \psi^{(N-1)}| \\ &= \begin{vmatrix} \psi_1^{(0)} & \psi_1^{(1)} & ... & \psi_1^{(N-1)} \\ \psi_2^{(0)} & \psi_2^{(1)} & ... & \psi_2^{(N-1)} \\ ... & ... & ... & ... \\ \psi_N^{(0)} & \psi_N^{(1)} & ... & \psi_N^{(N-1)} \end{vmatrix}, \end{aligned} \tag{6.3}$$

with $\psi_k^{(j)} = \partial^j \psi_k / \partial X^j$. The number N is the number of solitons in the solution. The functions ψ_k are given by

$$\psi_k = a_k \exp\left(\frac{\zeta_k}{2}\right) + ib_k \exp\left(-\frac{\zeta_k}{2}\right), \tag{6.4}$$

$$\zeta_k = \alpha_k X + \frac{1}{\alpha_k} T + \zeta_k^{(0)}$$

$X = (x + ct)/2d$ and $T = (x - ct)/2d$ are the light cone coordinates. α, a and b are complex parameters, $\zeta^{(0)}$ a complex phase.

The N-breather solution is obtained from the 2N-soliton solution by setting

$$\psi = (\psi_{11}, \psi_{12}, \psi_{21}, \psi_{22} ..., \psi_{N1}, \psi_{N2})^T, \tag{6.5}$$

with

$$\psi_{k1} = a_k \exp\left(\frac{\zeta_k}{2}\right) + b_k \exp\left(-\frac{\zeta_k}{2}\right), \qquad \psi_{k2} = a_k^* \exp\left(\frac{\zeta_k^*}{2}\right) + ib_k^* \exp\left(-\frac{\zeta_k^*}{2}\right). \tag{6.6}$$

N is the number of breathers, respectively particles.

It is expected that the correct gravitational field equations possess such N-breather solutions, too. They are then the relativistic generalization of the N-particle model in classical mechanics.

7. Breather interaction

Of course, one does not just want to recover the phenomenology of special relativity and Quantum mechanics, but also the known behavior of particles in classical physics. As stated at the beginning, it is well known that solitons exhibit particle behavior in the sense that they propagate and collide shape-preserving. In [22], it was shown that in the Sine-Gordon model, two colliding breathers behave like classical particles of equal masses under collision. To show that the breather-solitons in the full theory behave the same way under collisions, but for in general different masses, remains to be shown once the correct field equations are found. However, an important difference between the collisions of solitons and classical particles is already apparent in the Sine-Gordon model: The momentum transfer is not discrete in time, but continuously mediated via the solitons fields. For example, for two colliding Sine-Gordon breathers one obtains a trajectory in the centre of mass frame given by

$$x_{\pm}(t) = \pm\frac{1}{a}\cosh^{-1}(g\cosh(bt) + k), \tag{7.1}$$

for both particles "+" and "-". Here, a, b, g and k are constants depending on the initial velocities v_1, v_2 of the breathers as well as the parameter q. The trajectory (7.1) belongs to an accelerated motion of both breathers, during which momentum is transferred from one breather to the other. This acceleration is caused by the breathers' fields. It shows again, how particles and their fields are indeed described in a single framework. One can also see, that the motion due to these fields is indeed included in the multi-soliton solutions to the underlying field equations.

If one calculates the potential for this interaction between two Sine-Gordon breathers and evaluates it in the classical, non-relativistic limit, one obtains an interaction potential of the form

$$V = f(r_{12})(1 + \alpha\beta_{12}^2), \tag{7.2}$$

with α a dimensionless parameter of order of unity. This is the same form as exhibited by the potentials (1.1-1.2), on which the Machian theories are based, which initially led us to the conclusion that particles are breather-solitons in the gravitational field. In the full theory, it is expected that the potential (7.2) for the interaction between two particles in the classical, non-relativistic limit will agree with the gravitational potential (1.2).

Generally, the solitons should posses a far field and a core field. The far field should coincide with the gravitational field caused by the particle and thus fall off as 1/r or less[6]). The core field, which is in the conventional sense the actual particle itself, will have a much faster decay than the far field, most likely exponential, like it is the case in the Sine-Gordon model. Should it be true, that the field turns out to be a unified field, the electrical and gravitational fields would be two different components of this far field. This would then also yield a model

[6] In the light of the dark matter problem, it could well be, that the fall off is slower than 1/r.

for electric charge and enable one to also find an expression for the unexplained electric field constant ϵ_0 . Consequently, this would enable us to explain the force ratio between the electric and gravic force[7]).

8. Quarks

In the Sine-Gordon model, the most fundamental soliton solution is not the breather, but the kink and the anti-kink. One can calculate the single soliton solution from (6.1-6.4) by setting N=1 and $a_1, b_1, \alpha_1 \in \mathbb{R}$. This yields the well-known kink/anti-kink solution

$$\varphi_p^{\pm}(z) = 4 \arctan(\exp(\pm z)). \tag{8.1}$$

$$z = \frac{\gamma}{d}(x - vt)$$

Here, '+' corresponds to the kink, and '-' to the anti-kink. A breather is now composed of two such kinks, as one can see from (6.5): the parameters of each of the two single soliton solutions ψ_{k1} and ψ_{k2} are chosen to be complex conjugates to each other, that is $a_{k2} = a_{k1}^*$, $b_{k2} = b_{k1}^*$, $\alpha_{k2} = \alpha_{k1}^*$. The single breather solution (1.8) is then obtained by setting N=1 with (6.5). Now, this solution can also be written in terms of the kink/anti-kink solution as [23]

$$\varphi(x,t) = \varphi_p^+(\Gamma(x - r/2)) + \varphi_p^-(\Gamma(x + r/2)) - 2\pi, \tag{8.2}$$

with

$$r(t) = \frac{2}{\Gamma} \sinh^{-1}(\cot(q)\cos(\omega t)) \tag{8.3}$$

and $\Gamma = \cos(q)/d$, $\omega = \sin(q)c/d$. For simplicity, we restricted ourselves to the case of a stationary breather here, but the decomposition remains valid for a moving breather; it can be obtained by a Lorentz transformation of the equations (8.2-8.3). From (8.2-8.3), we can see that the breather is nothing but a kink and an anti-kink oscillating around their common center of mass. It is a bound state between a kink and an anti-kink. Like the elementary particles, it is composed of two (also extended) particles. It suggests itself that kink and anti-kink play the role of quark and anti-quarks in the Sine-Gordon model. The breather would then correspond to a simple Pion in this model. In the full theory, there should exist more than one type of kink solution, with each corresponding to one of the quarks, and the anti-kink solutions to the anti-quarks. The different types of particles are then the bound (breather) states between those different types of kinks and anti-kinks, like the breather solution in the Sine-Gordon model.

[7]The gravitational constant, the second unknown in this ratio, can already be calculated in the Machian unifed theories of gravity and inertia

9. Conclusion

We have shown that it is suggested by a unification of gravity and inertia that elementary particles are oscillatory solitons in the gravitational field, or more generally, a „unified“ field. We saw, that such breather solitons have exactly the structure which had been proposed by De Broglie for the quantum mechanical wave function in his double solution theory. We have shown, that from such a theory of elementary particles, all the basic phenomena of special relativity and quantum mechanics can be derived in a classical framework, in flat 3-dimensional Euclidean space. However, the theory yields actual physical explanations for the relativistic effects like length contraction, time dilation, and the energy-mass equivalence as well as for the mentioned quantum phenomena. Unlike in current theories, they are not ad hoc postulates, but natural consequences of the soliton nature of the particles. Further, we were able to explain the apparent constancy of the speed of light, instead of having to postulate it. It is not constant but only appears to be constant due to the changes of the elementary rulers and clocks in moving frames, provided by the elementary particles. Also, problems like the twin paradox, the measurement problem, and the need for renormalization don't occur in the soliton model. It is unnecessary to introduce any notion of mass or particle a priori, but instead, both come out of the theory. The particle mechanics inherit the Lorentz symmetry from the field equations automatically,, it also doesn't have to be put in by hand. This also leads us directly to the origin of mass, which could be explained as entirely of field origin, as the energy of the soliton divided by c^2. Ultimately, explaining the origin of quantum mechanics from the breather-soliton nature of the particles enabled us to derive an expression for the unexplained quantum constant h and the De Broglie relations.

The above said leads us to the conviction, that a soliton theory of elementary particles, as was presented here, could be the gateway to a unified field theory from which both relativistic phenomena and Quantum mechanics emerge. It would therefore be worthwhile to direct research in this direction and search for suitable candidates for non-linear field equations with soliton solutions. This especially also applies to *mathematical research* for the soliton solutions, which is an active field of research, but should get more attention in the light of a possible physical application. From our elaborations, we can already list a couple of properties those solutions must have:

1) They must be solutions to a 3-dimensional, Lorentz-invariant non-linear field equation, like (2.1) (but also more general equations are possible)
2) They must be localized in all three space dimensions and oscillatory in time, that is, of breather type.
3) The breather-type solutions (particles) must be composed of multiple single-soliton solutions (quarks).
4) There must exist (analytic) N-soliton solutions, and thus, N-breather solutions to the underlying field equations.
5) The solitons must possess a near and a far field. The near field constitutes the actual particle, the far field what is in current terminology the field generated by the particles. The near field is probably exponential, the far field 1/r.

Since Special relativity is based on space and time changing, while in the soliton theory, it is the elementary particles themselves that are changing in flat, Euclidean space, it is necessary and possible to part ways with this theory when pursuing the soliton approach further. The same, consequently, also applies to General relativity and Quantum mechanics in its current formulation. The soliton nature of the elementary particles then describes the effects currently described by Special relativity and Quantum mechanics. To show, that the soliton theory can reproduce all the successful quantitative predictions of Quantum mechanics, remains a task to be done, once the full, three-dimensional field equations and soliton solutions are found. The same also applies to those experiments, which are currently *correctly* described by General relativity without additional ad hoc assumptions like dark matter or dark energy. For those effects, it can be expected that the soliton theory yields a physical explanation, should it indeed turn out to be the unified field theory physics is looking for.

10. Acknowledgement

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