

# From the Svedberg ultracentrifuge to the rotating carbon nanotube

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## Abstract

The classical and the quantum motion of a massive body in the rotating carbon nanotube is considered. Photon is included. The spin motion described by the Bargmann-Michel-Telegdi equation is considered in the rotation tube and rotating system. The crucial problem is the Lamm equation in the rotating Carbon nanotube.

**Key words.** Rotating tube. carbon nanotube, Lorentz equation, Bargmann-Michel-Telegdi equation.

## 1 Introduction

The ultracentrifuge was invented by Svedberg. Svedberg worked at the Upsala university with colloids in order to support the theories of Brownian motion by Einstein and Smoluchowski. During his work, he developed the technique of analytical ultracentrifugation, for distinguishing pure proteins one from another.

The process of the ultracentrifugation was demonstrated theoretically by the Lamm equation (Lamm, 1929; Mazumdar, 1999) which describes the sedimentation and diffusion of a solute under the centrifuge forces. Lamm, professor of physical chemistry at the Royal Institute of Technology, derived it under Svedberg direction. The Lamm differential equation involves  $\rho$  as the solute concentration,  $t$  and  $r$  being the time and radius from

the centre of rotation, and the parameters  $D$ ,  $s$ , and  $\omega$  being the solute diffusion constant, sedimentation coefficient and the rotor angular velocity, respectively.

The diffusion constant  $D$  can be estimated from the hydrodynamic radius and shape of the solute, whereas the buoyant mass  $m_b$  can be determined from the ratio of  $s$  and  $D$ , or,  $s/D = m_b/K_B T$ , where  $K_B T$  is the thermal energy, i.e., Boltzmann's constant  $K_B$  multiplied by the temperature  $T$  in kelvins.

We use here the Lamm differential equation as the motivation for the new formulation of the problem with acceleration. The accelerators accelerate only charged particles. It is surprising that neutral particles such as photons, neutrons and so on can be accelerated by rotating tube. We show the mechanics and quantum mechanics of the motion of such particle in the rotating tube. We also include the general relativistic view on the rotating plane in order to generalize Bargmann-Michel-Telegdi equation for rotating systems. In order to see the difference between physics in rotation tube and in accelerating systems, let us first consider the massive body in the noninertial frame.

## 2 Mechanics in the rotating framework

The specific characteristics of the mechanical systems in the rotating framework follow from the differential equations describing the massive body in the noninertial systems (Landau et al., 1965). We will see later that the motion of a body in the rotating tube cannot be described by the formalism for rotating disk. We start by the text of Landau et al. (1965).

Let be the Lagrange function of a point particle in the inertial system as follows:

$$L_0 = \frac{m\mathbf{v}_0^2}{2} - U \quad (1)$$

with the following equation of motion

$$m \frac{d\mathbf{v}_0}{dt} = -\frac{\partial U}{\partial \mathbf{r}}, \quad (2)$$

where the quantities with index 0 correspond to the inertial system.

The Lagrange equations in the noninertial system is of the same form as that in the inertial one, or,

$$\frac{d}{dt} \frac{\partial L}{\partial \mathbf{v}} = \frac{\partial L}{\partial \mathbf{r}}. \quad (3)$$

However, the Lagrange function in the noninertial system is not the same as in eq. (1) because it is transformed.

Let us first consider the system  $K'$  moving relatively to the system  $K$  with the velocity  $\mathbf{V}(t)$ . If we denote the velocity of a particle with regard to system  $K'$  as  $\mathbf{v}'$ , then evidently

$$\mathbf{v}_0 = \mathbf{v}' + \mathbf{V}(t). \quad (4)$$

After insertion of eq. (4) into eq. (1), we get

$$L'_0 = \frac{m\mathbf{v}'^2}{2} + m\mathbf{v}'\mathbf{V} + \frac{m}{2}\mathbf{V}^2 - U. \quad (5)$$

The function  $\mathbf{V}^2$  is the function of time only and it can be expressed as the total derivation of time of some new function. It means that the term with the total derivation in the Lagrange function can be removed from the Lagrangian. We also have:

$$m\mathbf{v}'\mathbf{V}(t) = m\mathbf{V}\frac{d\mathbf{r}'}{dt} = \frac{d}{dt}(m\mathbf{r}'\mathbf{V}(t)) - m\mathbf{r}'\frac{d\mathbf{V}}{dt}. \quad (6)$$

After inserting the last formula into the Lagrange function and after removing the total time derivation we get

$$L' = \frac{mv'^2}{2} - m\mathbf{W}(t)\mathbf{r}' - U, \quad (7)$$

where  $\mathbf{W} = d\mathbf{V}/dt$  is the acceleration of the system  $K'$ .

The Lagrange equations following from the Lagrangian (7) are as follows:

$$m\frac{d\mathbf{v}'}{dt} = -\frac{\partial U}{\partial \mathbf{r}'} - m\mathbf{W}(t). \quad (8)$$

We see that after acceleration of the system  $K'$  the new force  $m\mathbf{W}(t)$  appears. This force is fictitious one because it is not generated by the internal properties of some body.

In case that the system  $K'$  rotates with the angle velocity  $\boldsymbol{\Omega}$  with regard to the system  $K$ , vectors  $\mathbf{v}$  and  $\mathbf{v}'$  are related as (Landau et al., 1965)

$$\mathbf{v}' = \mathbf{v} + \boldsymbol{\Omega} \times \mathbf{r}. \quad (9)$$

The Lagrange function for this situation is (Landau et al., 1965 )

$$L = \frac{mv^2}{2} - m\mathbf{W}(t)\mathbf{r} - U + m\mathbf{v} \cdot (\boldsymbol{\Omega} \times \mathbf{r}) + \frac{m}{2}(\boldsymbol{\Omega} \times \mathbf{r})^2. \quad (10)$$

The corresponding Lagrange equations for the last Lagrange function are as follows (Landau et al., 1965 ):

$$m\frac{d\mathbf{v}}{dt} = -\frac{\partial U}{\partial \mathbf{r}} - m\mathbf{W} + m\mathbf{r} \times \dot{\boldsymbol{\Omega}} + 2m\mathbf{v} \times \boldsymbol{\Omega} + m\boldsymbol{\Omega} \times (\mathbf{r} \times \boldsymbol{\Omega}). \quad (11)$$

We observe in eq. (11) three so called inertial forces. The force  $m\mathbf{r} \times \dot{\boldsymbol{\Omega}}$  is connected with the nonuniform rotation of the system  $K'$  and the forces  $2m\mathbf{v} \times \boldsymbol{\Omega}$  and  $m\boldsymbol{\Omega} \times \mathbf{r} \times \boldsymbol{\Omega}$  correspond to the uniform rotation. The force  $2m\mathbf{v} \times \boldsymbol{\Omega}$  is so called the Coriolis force and it depends on the velocity of a particle. The force  $m\boldsymbol{\Omega} \times \mathbf{r} \times \boldsymbol{\Omega}$  is called the centrifugal force. It is perpendicular to the rotation axes and the magnitude of it is  $m\varrho\omega^2$ , where  $\varrho$  is the distance of a particle from the rotation axis.

### 3 The massive point moving in the rotating tube

Let us consider a force  $\mathbf{F}$  acting at a massive body with mass  $m$ , where the mathematical form of this force is as follows:

$$F_i = m\varepsilon_{ijk}x_j\Omega_k, \quad (12)$$

where the dimensionality of this quantity is  $\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$  if  $\Omega$  has the dimensionality of frequency. We can easily reduce the formula (12) to the vectorial form

$$\mathbf{F} = m\Omega(\mathbf{r} \times \boldsymbol{\Omega}), \quad (13)$$

and  $\boldsymbol{\Omega}$  is supposed to be angular velocity and  $\mathbf{r}$  is the radius vector of the position of the body with mass  $m$ . The force defined in such a way is perpendicular to the angular velocity  $\boldsymbol{\Omega}$  and to the radius vector  $\mathbf{r}$  and it can be physically interpreted as force acting by rotating tube on the massive body which motion is restricted to the motion inside of the tube AB, where the tube AB rotates in the x-y plane in such a way that point A is  $A \equiv 0(0, 0, 0)$ . The force (13) is formally similar to the Lorentz force acting on a charged particle moving in the constant magnetic field, but the physical meaning is diametrically different from the Lorentz force.

The equation of motion under the force (13) is evidently as follows:

$$m\ddot{\mathbf{r}} = m\Omega(\mathbf{r} \times \boldsymbol{\Omega}) = m\Omega \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ 0 & 0 & \Omega \end{vmatrix}. \quad (14)$$

If we write for the circular motion in the x-y plane

$$\mathbf{r} = r(\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t + \mathbf{k}z), \quad (15)$$

we get the equations of motion in the form

$$\ddot{x} = -r\Omega^2 \cos \Omega t, \quad \ddot{y} = r\Omega^2 \sin \Omega t, \quad (16)$$

from which follows the differential equations for  $r$ :

$$\ddot{r} = \Omega^2 r \quad (17)$$

with the solution

$$r = c_1 e^{\Omega t} + c_2 e^{-\Omega t}. \quad (18)$$

Supposing  $r(0) = r_0, \dot{r}(0) = 0$ , we get the solution in the form

$$r(t) = r_0 \cosh \Omega t. \quad (19)$$

In case that  $r(0) = r_0, \dot{r}(0) = v$ , we get the solution in the form

$$r(t) = r_0 \cosh \Omega t + \frac{v}{\Omega} \sinh \Omega t. \quad (20)$$

Equations (19), or (20) can be immediately applied to the situation, where the tube is joined with the Earth (North Pole), where the frequency of rotation is  $\Omega = 1/\text{day}$ . If we put  $r_0 = 1\text{m}, t = \text{day}$ , then with regard to the formula  $\cosh x = 1 + x^2/2! + x^4/4! + \dots$ , we get  $r \approx 1,5\text{m}$ . So, we see that the rotation of Earth can be confirmed by the experiment with the rotating tube. The experiment, if performed, is the physical proof of the Earth rotation. This experiment was not considered in the textbooks on mechanics including the Euler famous opus "Teoria motus corporum solidorum seu rigidorum". (Euler, 1790). Only Foucault pendulum is discussed (Parady, 2007).

Let us remark that we can identify the equation  $\ddot{r} = \Omega^2 r$  with the equation for harmonic oscillator if we put  $\Omega \rightarrow i\Omega$ . At the same case we can say that this equation follows immediately from the incorrect physical assumption that the motion of a point particle,

in a rotating tube is caused by the centrifugal force  $F = m\Omega^2 r$  to which corresponds the "potential energy"

$$W = \int_{r_0}^r F dr = \frac{1}{2} m\Omega^2 (r^2 - r_0^2). \quad (21)$$

The kinetic energy at point  $r$ , at the direction of a tube, is (as follows from eq. (19))

$$E_{kin} = \frac{1}{2} mv^2 = \frac{1}{2} m\Omega^2 (r^2 - r_0^2). \quad (22)$$

So,  $W = E_{kin}$ . However, It is evident that there is no real centrifugal force inside of the rotating tube.

The rotating tube in the form of the carbon nanotube can be applied to intercalate different atoms and molecules. The carbon nanotube with such intercalated atoms and molecules has new nonexpected physical properties including superconductivity behavior. So, the substantial ingredient of every science, surprise, is established.

## 4 The motion of a photon in a rotating tube

The mass of moving photon is not zero but is given by the Einstein relation  $m_\gamma = (\hbar\omega)/c^2$ , where  $\omega$  is the frequency of the photon and  $c$  is the velocity of photon in vacuum. We consider the tube rotating in vacuum and the initial of photon velocity is  $c$ . With regard to the fact that the photon velocity is constant in the rotating tube, the result of the rotation is the change of frequency of photon. Or, the final frequency is

$$\hbar\omega' = \hbar\omega + \Delta E_\gamma, \quad (23)$$

where  $E_\gamma$  is the additional energy of photon which is obtained by photon under the process of acceleration along the trajectory of photon in the tube AB. Using equation (21), the equation for the shift of photon frequency (23) can be expressed as

$$\hbar\omega' = \hbar\omega + \Delta E_\gamma = \hbar\omega + \frac{1}{2} \left( \frac{\hbar\omega}{c^2} \right) \Omega^2 (r^2 - r_0^2), \quad (24)$$

Although such derivation of the change of the photon frequency is heuristical, it is necessary because there is still no theory of photons in the rotating tube.

In case that we consider the situation, where photon is moving from B to A, then we get red shift of the frequency, which of course cannot be considered as the analogy of the red shift of the rotating meta-galaxy.

The second possibility of derivation of the change of photon frequency in the rotating tube is to consider the tube as wave guide and then to calculate the electromagnetic field in the rotating wave guide.

## 5 Motion of the spin-vector in a rotating tube

We suppose here that it is possible to use the problem of motion of the spin in a rotating tube as an analogy with the problem of the spin-vector motion in classical relativistic mechanics presented by Bargmann, Michel and Telegdi (Berestetskii et al., 1989). They derived so called BMT equation for motion of spin in the electromagnetic field, in the form

$$\frac{da_\mu}{ds} = \alpha F_{\mu\nu} a^\nu + \beta v_\mu F^{\nu\lambda} v_\nu a_\lambda, \quad (25)$$

where  $a_\mu$  is so called axial vector describing the classical spin,  $v_\mu$  is velocity and constants  $\alpha$  and  $\beta$  were determined after the comparison of the postulated equations with the non-relativistic quantum mechanical limit. The result of such comparison is the final form of so called BMT equations:

$$\frac{da_\mu}{ds} = 2\mu F_{\mu\nu} a^\nu - 2\mu' v_\mu F^{\nu\lambda} v_\nu a_\lambda, \quad (26)$$

where  $\mu$  is magnetic moment of electron following directly from the Dirac equation and  $\mu'$  is anomalous magnetic moment of electron which can be calculated as the radiative correction to the interaction of electron with electromagnetic field and follows from quantum electrodynamics.

The BMT equation has more earlier origin. The first attempt to describe the spin motion in electromagnetic field using the special theory of relativity was performed by Thomas (1926). However, the basic ideas on the spin motion was established by Frenkel (1926, 1958). After appearing the Frenkel basic article, many authors published the articles concerning the spin motion (Ternov et al., 1980; Tomonaga, 1997). At present time, spin of electron is its physical attribute which follows only from the Dirac equation.

It was shown by Rafanelli and Schiller (1964), (Parady, 1973) that the BMT equation can be derived from the classical limit, i.e. from the WKB solution of the Dirac equation with the anomalous magnetic moment.

If we introduce the average value of the vector of spin in the rest system by the quantity  $\zeta$ , then the 4-pseudovector  $a^\mu$  is of the form  $a^\mu = (0, \zeta)$  (Berestetskii et al., 1989; Parady, 2009). The momentum four-vector of a particle is  $p^\mu = (m, 0)$  in the rest system of a particle. Then the equation  $a^\mu p_\mu = 0$  is valid not only in the rest system of a particle but in the arbitrary system as a consequence of the relativistic invariance. The following general formula is also valid in the arbitrary system  $a^\mu a_\mu = -\zeta^2$ .

The components of the axial 4-vector  $a^\mu$  in the reference system, where particle is moving with the velocity  $\mathbf{v} = \mathbf{p}/\varepsilon$  can be obtained by application of the Lorentz transformation to the rest system and they are as follows (Berestetskii et al., 1989):

$$a^0 = \frac{|\mathbf{p}|}{m} \zeta_{\parallel}, \quad \mathbf{a}_{\perp} = \zeta_{\perp}, \quad a_{\parallel} = \frac{\varepsilon}{m} \zeta_{\parallel}, \quad (27)$$

where suffices  $\parallel, \perp$  denote the components of  $\mathbf{a}$ ,  $\zeta$  parallel and perpendicular to the direction  $\mathbf{p}$ . The formulas for the spin components can be also rewritten in the more compact form as follows (Berestetskii et al., 1989):

$$\mathbf{a} = \zeta + \frac{\mathbf{p}(\zeta\mathbf{p})}{m(\varepsilon + m)}, \quad a^0 = \frac{\mathbf{a}\mathbf{p}}{\varepsilon} = \frac{\zeta\mathbf{p}}{m}, \quad \mathbf{a}^2 = \zeta^2 + \frac{(\mathbf{p}\zeta)^2}{m^2}. \quad (28)$$

The equation for the change of polarization can be obtained immediately from the BMT equation in the following form (Berestetskii et al., 1989):

$$\begin{aligned} \frac{d\mathbf{a}}{dt} = & \frac{2\mu m}{\varepsilon} \mathbf{a} \times \mathbf{H} + \frac{2\mu m}{\varepsilon} (\mathbf{a}\mathbf{v})\mathbf{E} - \frac{2\mu'\varepsilon}{m} \mathbf{v}(\mathbf{a}\mathbf{E}) + \\ & + \frac{2\mu'\varepsilon}{m} \mathbf{v}(\mathbf{v}(\mathbf{a} \times \mathbf{H})) + \frac{2\mu'\varepsilon}{m} \mathbf{v}(\mathbf{a}\mathbf{v})(\mathbf{v}\mathbf{E}), \end{aligned} \quad (29)$$

where we used the relativistic relations  $c = 1$ ,  $ds = dt\sqrt{1 - v^2}$ ,  $\varepsilon = m\sqrt{1 - v^2}$  and the following components of the electromagnetic field (Landau et al., 1988):

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{pmatrix} \stackrel{d}{=} (\mathbf{E}, \mathbf{H}); \quad F_{\mu\nu} = (-\mathbf{E}, \mathbf{H}). \quad (30)$$

Inserting equation **a** from eq. (28) into eq. (29) and using equations

$$\mathbf{p} = \varepsilon\mathbf{v}, \quad \varepsilon^2 = \mathbf{p}^2 + m^2, \quad \frac{d\mathbf{p}}{dt} = e\mathbf{E} + e(\mathbf{v} \times \mathbf{H}), \quad \frac{d\varepsilon}{dt} = e(\mathbf{v}\mathbf{E}), \quad (31)$$

we get after long but simple mathematical operations the following equation for the polarization  $\zeta$

$$\begin{aligned} \frac{d\zeta}{dt} &= \frac{2\mu m + 2\mu'(\varepsilon - m)}{\varepsilon} \zeta \times \mathbf{H} + \\ &\frac{2\mu'\varepsilon}{\varepsilon + m} (\mathbf{v}\mathbf{H})(\mathbf{v} \times \zeta) + \frac{2\mu m + 2\mu'\varepsilon}{\varepsilon + m} \zeta \times (\mathbf{E} \times \mathbf{v}). \end{aligned} \quad (32)$$

The equation of motion of spin in electric field as far as first order terms in velocity  $v$  is obtained from eq. (32) in the form

$$\frac{d\zeta}{dt} = (\mu + \mu')\zeta \times (\mathbf{E} \times \mathbf{v}) = \left( \frac{e}{2m} + 2\mu' \right) \zeta \times (\mathbf{E} \times \mathbf{v}). \quad (33)$$

It follows from equation (31) that  $e\mathbf{E}$  is the electric force interacting with spin of an electron. If we want to express eq. (14) as the equation of spin motion in the rotating tube, then it is easy to show that  $\mathbf{E}$  must be identified by  $\mathbf{F}/e$ . Or,

$$\frac{d\zeta}{dt} = \frac{1}{e}(\mu m + \mu')\zeta \times (\mathbf{F} \times \mathbf{v}) = \frac{1}{e} \left( \frac{e}{2m} + 2\mu' \right) \zeta \times (\mathbf{F} \times \mathbf{v}). \quad (34)$$

The equation was never derived in the framework of the general theory of relativity and gravitation. The force which causes motion of spin is in case of the rotation tube the electric force and not the gravitational force. There is not principle equivalence between electric field and gravity.

## 6 Quantum mechanics of a particle in a rotating tube

The rigorous formulation of the problem of quantum mechanical motion of a charged particle in a rotating tube is to consider the situation of a charged particle where the motion is restricted by the moving boundary conditions. Such problem was still not defined in quantum mechanical monographs or solved, or published. So we here use the heuristic approach which represents the most simple approach to the problem.

The elementary solution is to consider quantity  $V(r) = \frac{1}{2}\Omega^2 r^2$  as the potential energy of a body in the tube and in the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{d^2 \psi}{dr^2} + V(r)\psi(r) \quad (35)$$

However, potential  $V(r)$  in the rotating tube is the potential of the harmonic oscillator with frequency  $i\Omega$ . All formulas of harmonic oscillator are here mathematically valid only the problem is that they involve imaginary frequency which cannot be physically correctly interpreted in classical physics. In other words it is necessary to consider  $\psi$  as a wave in the rotating wave guide. It means to consider the Schrödinger equation in the rotating tube. It is equivalent to consider the Schrödinger equation with the potential of the harmonic oscillator with the imaginary frequency  $\Omega$ . The equation for the stationary states is then as it follows:

$$\frac{d^2\psi}{dr^2} + \frac{2m}{\hbar^2} \left( E + \frac{m\Omega^2 r^2}{2} \right) \psi = 0 \quad (36)$$

The corresponding energies of the "stationary" states are:

$$E_n = -i\hbar\Omega\left(n + \frac{1}{2}\right) \quad (37)$$

So, we see that the eigenvalue-problem leads to the states which are decaying. The classical limit of the solution is evidently the motion of a charged particle accelerating by potential  $V(r) = \frac{1}{2}\Omega^2 r^2$ .

## 7 Discussion

We have presented the Lagrange theory of the non-inertial classical systems, classical particle motion and spin motion in the rotating tube and quantum motion in the rotating tube. There are other effects which is possible to consider. For instance, Mössbauer effect in the rotating tube, Pound-Rebka effect in the rotating tube, the Čerenkov effect in the rotating dielectric tube, conductivity and superconductivity in the rotating tube and so on.

The solved problems were not involved in the framework of the GRG. On the other hand GRG is able to define geometry on the rotating disk which cannot be composed from the rotating ribbons or nanoribbons. Let us discuss geometry of the rotating disk and show some physical consequences which differs from the physics in the rotating tube.

In the present time, the rotating tube can be realized by the carbon nanotube, which plays the fundamental interest in all areas of science. Carbon nanotubes, the walls of which are made up of a hexagonal lattice of carbon atoms analogous to that of graphite, are cylinder-shaped macromolecules where radius of a cylinder is a few nanometers and length up to 20 cm. In the most general case, a Carbon nanotube is composed of a concentric arrangement of many cylinders. Such multi-walled nanotubes can reach diameters of up to 100 nm and arbitrary length. The formation of a Carbon nanotube can be visualized through the rolling of a graphene sheet.

If we use the the Minkowski element

$$ds^2 = -c^2 dt'^2 + dx'^2 + dy'^2 + dz'^2 \quad (38)$$

and the nonrelativistic transformation to the rotation system (Matsuo, 2011)

$$d\mathbf{r}' = d\mathbf{r} + (\boldsymbol{\Omega} \times \mathbf{r})dt \quad (39)$$

then we get that space-time element can be expressed in the vectorial form:



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = [-c^2 + (\boldsymbol{\Omega} \times \mathbf{r})^2] dt^2 + (d\mathbf{r})^2 + 2(\boldsymbol{\Omega} \times \mathbf{r}) dt d\mathbf{r}. \quad (40)$$

Thus the metric in the rotating frame can be written by the matrix:

$$g_{\mu\nu} = \begin{pmatrix} -1 + \mathbf{u}(x)^2 & u_x & u_y & u_z \\ u_x & 1 & 0 & 0 \\ u_y & 0 & 1 & 0 \\ u_z & 0 & 0 & 1 \end{pmatrix}, \quad (41)$$

where

$$\mathbf{u}(x) = \boldsymbol{\Omega}(t) \times \mathbf{r}/c. \quad (42)$$

Matsuo et al. (2011) applied the derived metric in order to derive the Dirac equation in the rotating system in order to solve the quantum mechanical problems of the spin-dependent inertial force and spin current in accelerating system. Nevertheless, the knowledge of space-time metric of the rotation system leads to the results which cannot be involved in the physics of the rotated tube because of different formalism, as can be easily seen.

The transformation between inertial and rotation system is necessary because it enables to describe the motion of the particle and spin in the LHC by the general relativistic methods. The basic idea is the generalization of the so called Lorentz equation for the charged particle in the electromagnetic field  $F^{\mu\nu}$  (Landau et al., 1988):

$$mc \frac{dv^\mu}{ds} = \frac{e}{c} F^{\mu\nu} v_\nu. \quad (43)$$

In other words, the normal derivative must be replaced by the covariant one and we get the general relativistic equation for the motion of a charged particle in the electromagnetic field and gravity (Landau et al., 1988):

$$mc \left( \frac{dv^\mu}{ds} + \Gamma_{\alpha\beta}^\mu v^\alpha v^\beta \right) = \frac{e}{c} F^{\mu\nu} v_\nu, \quad (44)$$

where

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g^{\mu\lambda} \left( \frac{\partial g_{\lambda\alpha}}{\partial x^\beta} + \frac{\partial g_{\lambda\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\lambda} \right) \quad (45)$$

are the Christoffel symbols derived in the Riemann geometry theory (Landau et al., 1988).

In case that we consider motion in the rotating system, then it is necessary to insert the metrical tensor  $g_{\mu\nu}$ , following from the Minkowski element for the rotation system. The construction of LHC with orbiting protons must be in harmony with equation (44) because orbital protons respect the Coriolis force caused by the rotation of the Earth.

The analogical situation occurs for the motion of the spin. While the original Bargmann-Michel-Telegdi equation for the spin motion is as follows (Berestetzki et al., 1988)

$$\frac{da^\mu}{ds} = 2\mu F^{\mu\nu} a_\nu - 2\mu' v^\mu F^{\alpha\beta} v_\alpha a_\beta, \quad (46)$$

where  $\mu' = \mu - e/2m$  and  $a_\mu$  is the axial vector, which follows also from the classical limit of the Dirac equation with  $\psi i\gamma_5 \gamma_\mu \psi \rightarrow a_\mu$  (Rafanelli et al., 1964; Pardy, 1973), the general

relativistic generalization of the Bargmann-Michel-Telegdi equation can be obtained by the analogical procedure which was performed with the Lorentz equation. Or,

$$\left(\frac{da^\mu}{ds} + \Gamma_{\alpha\beta}^\mu v^\alpha a^\beta\right) = 2\mu F^{\mu\nu} a_\nu - 2\mu' v^\mu F^{\alpha\beta} v_\alpha a_\beta, \quad (47)$$

where in case of the rotating system the metrical tensor  $g_{\mu\nu}$  must be replaced by the metrical tensor of the rotating system. Then, the last equation will describe the motion of the spin in the rotating system.

The motion of the polarized proton in LHC will be described by the last equation because our Earth rotates. During the derivation we wrote  $\Gamma_{\alpha\beta}^\mu v^\alpha a^\beta$  and not  $\Gamma_{\alpha\beta}^\mu v^\alpha v^\beta$ , because every term must be the axial vector. In other words, the last equation for the motion of the spin in the rotating system was not strictly derived but created with regard to the philosophy of author that physics is based on creativity, phantasy and logic.

On the other hand, the equation (47) must evidently follow from the Dirac equation in the rotating system, by the same WKB methods which were used by Rafanelli, Schiller and Pardy (Rafanelli and Schiller, 1964; Pardy, 1973). The derived BMT equation in the metric of the rotation of the Earth are fundamental for the proper work of LHC because every orbital proton of LHC respects the rotation of the Earth and every orbital proton spin respects the Earth rotation too.

We hope that the named problems are interesting and their solution will be integral part of the theoretical physics.

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