Magnetic Monopole as Volumetric Flow Rate: A possible reinterpretation of modern physics

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Abstract

In this paper, we demonstrate that through an elasto-mechanical relationship and the Navier-Stokes equation, a magnetic monopole can be introduced as a volume flux following deformation. This phenomenon is linked to mass generation. By incorporating the magnetic monopole, we derive Maxwell's equations for the magnetic monopole, the electromagnetic waves equation, and we give the Lagrangian formulation. An alternative Lagrangian is also presented, introducing CP violation, the breaking of electromagnetic dual symmetry, and a topological invariant. The monopole results are employed to derive the relativistic mass-energy equation and reinterpret relativistic outcomes. Specifically, the reinterpretation of the event interval enables us to revisit the relativistic metric in elastomechanical terms, related to the longitudinal and transverse speeds of light as defined by Lamé parameters. For an isotropic medium, the Lamé parameters are spatially invariant, corresponding to the validity of the Minkowski metric and constant inertial mass in space. Under anisotropic conditions, the metrics of general relativity are derivable, with inertial mass dependent on spatial coordinates and its gradient described by the convective derivative of longitudinal velocity. Furthermore, we demonstrate that the magnetic continuity equation predicts the fundamental equations of quantum mechanics in the Bohmian interpretation. Finally, evaluating Dirac quantization as predicted by quantum mechanics reveals its incompatibility with the hypothesis of inertial mass. However, is possible to correct this inconsistency by considering the general mass formula obtained from magnetic monopole considerations

Contents

Introduction

Magnetic monopoles are particles that have a single magnetic pole, either north or south. Unlike electric charges that can exist in isolation (positive or negative charges), magnetic poles have only ever been seen to exist in pairs (dipoles). Paul Dirac first proposed the idea in 1931. He suggested that if there were any magnetic monopoles in the universe, they would explain why electric charges come in discrete units (the quantum of electric charge). [1] More recently, in 1954, Chinese physicist Cai Qi and American physicist Robert Mills introduced the theory of Yang-Mills monopoles, which laid the foundation for understanding electromagnetism and weak nuclear interactions from the viewpoint of gauge fields. [2] In the 1970s, the Grand Unified Theories emerged as an attempt to unify electromagnetism, weak and strong nuclear forces, into a unified theoretical framework. These theories naturally predicted that magnetic monopoles would exist in the early universe as topological defects.[3] On the other hand, in 1977, Gerard ‘t Hooft (with the help of Alexander Polyakov) independently demonstrated that the presence of a single monopole would result in a quantization of the electric charge, which was in line with Dirac’s earlier prediction. [4] The existence of a quantization condition motivated further research on the subject. There have been numerous attempts to find magnetic monopoles in experiments, but there is no definitive proof of magnetic monopole existence. The fact that the magnetic monopole has never been detected may be due to a phenomenological interpretation other than its true nature. In this paper, The magnetic monopole is introduced as a volumetric flow generating a mass flow by taking up a mechanical theory of electromagnetism proposed by Zareski. [5] The existence of a mechanical medium in which the Navier-Cauchy equation with stress couple is valid, it is possible to introduce the existence of the magnetic monopole and provide the basis for new physical development from classical considerations.

1 Navier-Cauchy equation: Transverse and Longitudinal Waves

The Navier-Cauchy equation in the presence of a couple stress is given by:

$$\rho\frac{\partial^2\overrightarrow{u}}{\partial t^2} = \eta\nabla^2\overrightarrow{u} + (\eta+\sigma)\overrightarrow{\nabla}(\overrightarrow{\nabla}\cdot\overrightarrow{u}) + \frac{1}{2}\overrightarrow{\nabla}\times\overrightarrow{C} + \overrightarrow{f} \text{ [5]} \tag{1}$$

where $\overrightarrow{u}$ is the displacement vector, ρ is the density of the medium, η and σ are Lamé coefficients, $\overrightarrow{f}$ is the volume force, and $\overrightarrow{C}dV = (\overrightarrow{r}\times\overrightarrow{f})dV$ represents the couple stress generated by the volume force. According to Landau-Lifshitz elastic theory, [6] the Navier-Cauchy equation 1 can be rewritten as :

$$\frac{\partial^2\overrightarrow{u}}{\partial t^2} = c_t^2\nabla^2\overrightarrow{u} + (c_l^2 - c_t^2)\overrightarrow{\nabla}(\overrightarrow{\nabla}\cdot\overrightarrow{u}) + \frac{1}{2\rho}\overrightarrow{\nabla}\times\overrightarrow{C} + \frac{\overrightarrow{f}}{\rho} \tag{2}$$

with $c_t^2 = \frac{\eta}{\rho}$ and $c_l^2 = \frac{\sigma+2\eta}{\rho}$, where c_t is the transverse velocity and c_l is the longitudinal velocity of the medium. We refer from this point onwards as c_l and c_t as the longitudinal and transverse speed of light respectively. The transverse velocity represents the well-known speed of light in vacuum. The displacement vector $\overrightarrow{u}$ can be decomposed into a longitudinal component $\overrightarrow{u}_l$ with zero curl and a transverse component $\overrightarrow{u}_t$ with zero divergence:

$$\overrightarrow{u} = \overrightarrow{u}_t + \overrightarrow{u}_l, \quad \overrightarrow{\nabla}\cdot\overrightarrow{u}_t = 0, \quad \overrightarrow{\nabla}\times\overrightarrow{u}_l = 0 \tag{3}$$

Considering the decomposition in 3 and applying the divergence operator to the Navier-Cauchy equation 2:

$$\frac{\partial^2\overrightarrow{u}_l}{\partial t^2} - c_l^2\nabla^2\overrightarrow{u}_l = \frac{\overrightarrow{f}}{\rho} \implies \overrightarrow{\nabla}\times\overrightarrow{f} = 0 \tag{4}$$

This indicates that, according to Zareski’s hypothesis, the volume force is irrotational. By taking the curl of equation 2:

$$\frac{\partial^2\overrightarrow{u}_t}{\partial t^2} - c_t^2\nabla^2\overrightarrow{u}_t = \frac{1}{2\rho}\overrightarrow{\nabla}\times\overrightarrow{C} \tag{5}$$

Hence, from the Navier-Cauchy equation, it is possible to derive two wave equations: a longitudinal one, where volume changes $\vec{\nabla} \cdot \vec{u}_l \neq 0$ are associated, and a transverse one, with no volume changes $\vec{\nabla} \cdot \vec{u}_t = 0$. From elasticity theory, it is well known that the divergence of longitudinal displacement is related to volume changes:

$$\vec{\nabla} \cdot \vec{u}_l = \frac{d\tau' - d\tau}{d\tau} \text{ [7]} \tag{6}$$

where $d\tau'$ is the elementary volume after compression or dilation, and $d\tau$ is the initial elementary volume.

2 Maxwell Equation and Transverse Equation

Maxwell's equations can be derived by defining the elastomechanical transformation of electromagnetic quantities as presented by Zareski and refer to the transverse component introduced in the previous section:

$$\vec{E} = -\eta \vec{\nabla} \times \vec{u}_t + \frac{\vec{C}}{2} \quad \eta = \frac{1}{\epsilon_0}, \quad \vec{B}_t = -\rho \frac{\partial \vec{u}_t}{\partial t}, \quad \rho = \mu_0, \quad \vec{J}_e = -\frac{1}{2\eta}\frac{\partial \vec{C}}{\partial t}, \quad \rho_e = \frac{1}{2\eta}\vec{\nabla} \cdot \vec{C} \text{ [5]} \tag{7}$$

where $\vec{E}$ is the electric field, $\vec{B}_t$ is the transverse magnetic induction, $\vec{J}_e$ is the electric current density, and ρ_e is the charge density. Here, ϵ_0 is the permittivity of free space and μ_0 is the magnetic permeability of free space. From these definitions of the magnetic and electric fields, it becomes evident that the transverse wave equation 5 corresponds to Maxwell's third equation:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}_t}{\partial t} \tag{8}$$

Applying the divergence operator to the magnetic field yields Maxwell's second equation, because of $\vec{\nabla} \cdot \vec{u}_t = 0$:

$$\vec{\nabla} \cdot \vec{B}_t = 0; \tag{9}$$

This implies that in a transverse magnetic field, a magnetic monopole does not exist. By applying the divergence operator to the electric field and using the definition of charge density, we obtain Maxwell's first equation:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\epsilon_0} \tag{10}$$

Applying the curl operator to the magnetic field and using $\vec{\nabla} \times \vec{u}_t = \frac{\vec{C}}{2\eta} - \frac{\vec{E}}{\eta}$, derived from the definition of the electric field and current density, we obtain Maxwell's fourth equation:

$$\vec{\nabla} \times \vec{B}_t = \mu_0 \vec{J}_e + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \quad c_t^2 = \frac{1}{\epsilon_0 \mu_0} \equiv \frac{\eta}{\rho} \tag{11}$$

Finally, the electric continuity equation is obtained by taking the divergence of $\vec{J}_e$:

$$\vec{\nabla} \cdot \vec{J}_e + \frac{\partial \rho_e}{\partial t} = 0 \tag{12}$$

2.1 Electric Charge and Dimensional Analysis

A consequence of the elastomechanical interpretation of electromagnetism is the conceptualization of electric charge. From the definition of electric charge q_e:

$$q_e = \iiint_\tau \rho_e d\tau \tag{13}$$

where τ is the volume of integration. Utilizing the elastomechanical transformation of the electric charge density in 7:

$$q_e = \frac{1}{2\eta} \iiint_\tau \vec{\nabla} \cdot \vec{C} d\tau = \frac{1}{2\eta} \oiint_{\partial\tau} \vec{C} \cdot \hat{n} dS \tag{14}$$

where $\partial\tau$ is the boundary of volume τ, $\hat{n}$ is the unit normal vector to surface dS, and the divergence theorem is applied. This indicates that the electric charge represents the flow of stress couples divided by the shear modulus η. This elastomechanical perspective imposes a dimensional constraint on the electric charge, as the flux of stress couples divided by the Lamé coefficient dimensionally corresponds to a squared length:

$$[C] = [L^2] \tag{15}$$

Here, the following conventions are used: $[M]$ = Kilogram, $[T]$ = Seconds, $[L]$ = Meter, $[N]$ = Newton, $[C]$ = Coulomb. By imposing this dimensional constraint, it becomes possible to describe all electromagnetic quantities in mechanical terms:

- Current Density:
  $[\overrightarrow{J}_e] = [CT^{-1}L^{-2}] \implies [\overrightarrow{J}_e] = [T^{-1}]$
- Magnetic Transverse Induction:
  $[\overrightarrow{B}_t] = [NTC^{-1}L^{-1}] \implies [\overrightarrow{B}_t] = [MT^{-1}L^{-2}]$
- Electric Field
  $[\overrightarrow{E}] = [NC^{-1}] \implies [\overrightarrow{E}] = [NL^{-2}]$
- Magnetic Permeability Constant in Vacuum:
  $[\mu_0] = [NC^{-2}T^2] \implies [\rho] = [ML^{-3}]$
- Dielectric Constant in Vacuum:
  $[\frac{1}{\epsilon_0}] = [C^{-2}NL^2] \to [\eta] = [NL^{-2}]$
- Charge Density:
  $[\rho_e] = [CL^{-3}] \implies [\rho_e] = [L^{-1}]$;

Therefore, this dimensional constraint transforms all electromagnetic quantities into elastomechanical terms, suggesting that a mechanical interpretation of electromagnetism is possible. Moreover, experimentally, all known elementary particles that can be isolated as individual particles have an electric charge equal in magnitude to the electron's charge or zero. [8] Hence, the flux of stress couples must be quantized. For an elementary particle:

$$\frac{1}{2\eta}\oiint_{\partial\tau} \overrightarrow{C} \cdot \hat{n} dS = \pm 1.6 \times 10^{-19} m^2 \tag{16}$$

$$\oiint_{\partial\tau} \overrightarrow{C} \cdot \hat{n} dS \simeq \pm 2.5 \times 10^{-13} N \tag{17}$$

In continuum mechanics, stress couples represent moments or rotational effects within a material. They describe the tendency of a material to rotate under the application of forces. The stress couple $\overrightarrow{C}$ is a vector that characterizes this rotational tendency.The surface integral in formula 17 is evaluating the flux of this stress couple through the surface. This can be interpreted as the amount of rotational effect passing through the surface per unit area. Dividing this flux by the Lamé parameter η introduces a scaling factor. The Lamé parameter is a measure of the resistance of a material to deformation under stress. So, dividing by η essentially scales the rotational effect by the material's resistance to deformation. In simpler terms, the formula quantifies how much rotational tendency is passing through a surface per unit area, and it adjusts this quantity by considering how resistant the material is to deformation under stress. The medium considered illustrates that this nature is quantised according to the charge of the electron

3 Maxwell Monopole equations

3.1 Magnetic Monopole

In the preceding sections, we have demonstrated how the transverse wave and elastomechanical transformation explain classical elctromagnetism in elastomechanical terms. We extend this analysis to the longitudinal wave equation. To do so, we introduce the concept of the magnetic monopole and its elastomechanical interpretation. We define the longitudinal magnetic field $\overrightarrow{B}_l$ analogously to the transverse magnetic field $\overrightarrow{B}_t$ as per equation 7:

$$\overrightarrow{B}_l = -\rho \frac{\partial \overrightarrow{u}_l}{\partial t} \tag{18}$$

The implication of this equation is the theoretical existence of the magnetic monopole, as the divergence of the longitudinal magnetic field is nonzero. Analogous to the electric field, the magnetic charge density and Maxwell's fifth equation can thus be introduced:

$$\overrightarrow{\nabla} \cdot \overrightarrow{B}_l = \rho \rho_m, \quad \rho_m = -\frac{\partial(\overrightarrow{\nabla} \cdot \overrightarrow{u}_l)}{\partial t}, \quad [\rho_m] = T^{-1} \tag{19}$$

Here, $\rho_m = \frac{dg}{d\tau}$ is the magnetic charge density and g represents the magnetic monopole. Utilizing equation 6, the magnetic charge density becomes:

$$\rho_m = -\frac{\partial}{\partial t}\left(\frac{d\tau' - d\tau}{d\tau}\right) \tag{20}$$

Thus, the magnetic monopole is associated with volume variations and their time derivative. From the definition of magnetic charge density:

$$\rho \iiint_\tau \rho_m d\tau = \iiint_\tau \overrightarrow{\nabla} \cdot \overrightarrow{B}_l d\tau = \oiint_{\partial\tau} \overrightarrow{B}_l \cdot \hat{n} dS := -\dot{m} \tag{21}$$

where $\dot{m} = \frac{dm}{dt}$ represents mass flow, and the divergence theorem is applied. From this result, it is derived that the magnetic charge is given by the ratio of the mass flux to the density:

$$\iiint_\tau \rho_m d\tau = -\frac{\dot{m}}{\rho} \implies g = -\frac{\dot{m}}{\mu_0} \tag{22}$$

Continuing this calculation, we can determine how volume changes in the mechanical medium are a fundamental mechanism for particle generation. By substituting the definition of ρ_m from 20 into 21 and defining $\Upsilon = \tau' - \tau$:

$$\rho\frac{d\Upsilon}{dt} = \dot{m} \implies \rho d\Upsilon = dm \quad \text{and} \quad g = \frac{d\Upsilon}{dt} \tag{23}$$

A similar analysis can be performed by considering that relation 20 is equal to:

$$\rho_m = -\frac{\partial}{\partial t}\left(\frac{d\tau'}{d\tau}\right) \tag{24}$$

Repeating the calculation with this relation yields:

$$\rho d\tau' = dm \quad \text{and} \quad g = \frac{d\tau'}{dt} \tag{25}$$

Comparing equations 23 and 25, which are mathematically consistent, it follows that:

$$\frac{d\tau}{dt} = 0 \tag{26}$$

This indicates that the magnetic monopole represents the rate of change of the deformed volume over time, with the magnetic elementary charge given by the same relation. Moreover, the density of the mechanical medium $\rho = \mu_0$ remains constant over time. Consequently, an elementary mass in the initial state, dm_0, can be introduced:

$$dm_0 = \rho d\tau \tag{27}$$

Combining relations 20, 25, and 27 yields:

$$\overrightarrow{\nabla} \cdot \overrightarrow{u}_l = \frac{d\tau' - d\tau}{d\tau} = \frac{dm - dm_0}{dm_0} \tag{28}$$

This leads to the following conditions:

$d\tau' < d\tau$; $dm < dm_0$; $g = -\frac{\dot{m}}{\mu_0} > 0$ (Volumetric Compression/Mass Rarefaction/North Monopole)

$d\tau' > d\tau$; $dm > dm_0$; $g = -\frac{\dot{m}}{\mu_0} < 0$ (Volumetric Dilatation/Mass Condensation/South Monopole)

Thus, volumetric compressions of the longitudinal wave are responsible for particle generation mechanisms. In these mechanisms, density remains constant. During compressions, the final volume $d\tau'$ is smaller than the initial volume $d\tau$, leading to mass rarefaction for constant density, and vice versa for volumetric expansion.
Even under the assumption that the magnetic permeability is material dependent, the same relations can be obtained by considering that the magnetic monopole relation is given:

$$\iiint_{\partial\tau} \rho_{m,r} d\tau = -\frac{\dot{m}}{\rho}, \quad \rho_{m,r} = \mu_r \rho_m, \quad \rho_{m,r} d\tau' = dm, \quad \rho_{m,r} d\tau = dm_0, \quad \overrightarrow{\nabla} \cdot \overrightarrow{B}_l = \rho\rho_{m,r} \tag{29}$$

Here μ_r is the relative magnetic permeability. For simplicity's sake, the results of the following sections will be obtained under the assumption of $\overrightarrow{\nabla} \cdot \overrightarrow{B}_l = \rho\rho_m$

3.2 Complete Maxwell Equation

This subsection extends the elastic description of longitudinal wave mechanics. Specifically, just as the equation of the transverse wave corresponds to Maxwell's third equation, the equation of the longitudinal wave analogously corresponds to Maxwell's sixth equation through the appropriate definition of relationships between elastic and electromagnetic quantities. We hereby define the magnetic current density $\overrightarrow{J}_m$:

$$\overrightarrow{J}_m = c_l^2 \nabla^2 \overrightarrow{u}_l + \frac{\overrightarrow{f}}{\rho} \qquad [\overrightarrow{J}_m] = LT^{-2} \qquad \overrightarrow{\nabla} \times \overrightarrow{J}_m = 0 \tag{30}$$

The magnetic current density represents a volumetric acceleration term. By combining the newly defined quantities in 18 and 30 with the longitudinal wave equation 4, we obtain the Maxwell's sixth equation:

$$-\frac{\partial \overrightarrow{B}_l}{\partial t} = \mu_0 \overrightarrow{J}_m \tag{31}$$

The complete Maxwell equations with a magnetic monopole are:

$$\begin{cases} \overrightarrow{\nabla} \cdot \overrightarrow{E} = \frac{\rho_e}{\epsilon_0} \quad \textit{I Maxwell Equation} \\ \overrightarrow{\nabla} \cdot \overrightarrow{B}_t = 0 \quad \textit{II Maxwell equation} \\ \overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}_t}{\partial t} \quad \textit{III Maxwell Equation/Transverse Wave Equation} \\ \overrightarrow{\nabla} \times \overrightarrow{B}_t = \mu_0 \overrightarrow{J}_e + \epsilon_0\mu_0 \frac{\partial \overrightarrow{E}}{\partial t} \quad \textit{IV Maxwell Equation} \\ \overrightarrow{\nabla} \cdot \overrightarrow{B}_l = \mu_0 \rho_m \quad \textit{V Maxwell Equation} \\ -\frac{\partial \overrightarrow{B}_l}{\partial t} = \mu_0 \overrightarrow{J}_m \quad \textit{VI Maxwell Equation/Longitudinal Wave Equation} \end{cases} \tag{32}$$

Maxwell's equations can be further consolidated by introducing the total magnetic field.

$$\overrightarrow{B} = \overrightarrow{B}_l + \overrightarrow{B}_t \qquad \begin{cases} \overrightarrow{\nabla} \cdot \overrightarrow{E} = \mu_0 c_t^2 \rho_e \\ \overrightarrow{\nabla} \cdot \overrightarrow{B} = \mu_0 \rho_m \\ \overrightarrow{\nabla} \times \overrightarrow{E} + \frac{\partial \overrightarrow{B}}{\partial t} = -\mu_0 \overrightarrow{J}_m \text{ Navier-Stokes Equation with Stress Couple} \\ \frac{\partial \overrightarrow{E}}{\partial t} - c_t^2 \overrightarrow{\nabla} \times \overrightarrow{B} = -\mu_0 c_t^2 \overrightarrow{J}_e \end{cases} \tag{33}$$

Additionally, We derive the continuity equation for the magnetic component, analogous to the electrical continuity equation, by applying the divergence to the third Maxwell equation from 33:

$$\frac{\partial \rho_m}{\partial t} + \overrightarrow{\nabla} \cdot \overrightarrow{J}_m = 0 \tag{34}$$

The magnetic continuity equation implies that magnetic charge is conserved. Hence, from equation 21:

$$\overrightarrow{B}_l = -\frac{\dot{m}}{4\pi r^2}\hat{r}, \quad \overrightarrow{\nabla}\cdot\overrightarrow{B}_l = -\frac{\dot{m}}{\rho}\delta(\overrightarrow{r}) \tag{35}$$

Where $\delta(\overrightarrow{r})$ is the Dirac delta. From this result, it is also possible to rewrite the Lorentz force as:

$$\overrightarrow{F}_L = e(\overrightarrow{E} + \overrightarrow{v}_e \times \overrightarrow{B}_t) + \frac{\dot{m}}{\mu_0}(\overrightarrow{B}_l + \overrightarrow{v}_m \times \frac{\overrightarrow{E}}{c_t^2}) \tag{36}$$

Where $\overrightarrow{v}_e$ and $\overrightarrow{v}_m$ are the velocity of electric and magnetic charge respectivel. It is possible to further compact Maxwell's equations and the electrical and magnetic continuity equations by defining the following quantities:

$$\begin{gathered}\overrightarrow{J}_m = \rho_m\overrightarrow{v}_m, \quad \overrightarrow{J}_e = \rho_e\overrightarrow{v}_e, \quad \rho_{e+m} = c_t\rho_e + \rho_m, \quad \overrightarrow{v}_{e+m} = \frac{c_t\rho_e\overrightarrow{v}_e + \rho_m\overrightarrow{v}_m}{c_t\rho_e + \rho_m}, \quad \overrightarrow{J} = c_t\overrightarrow{J}_e + \overrightarrow{J}_m, \\ \overrightarrow{B}_f = \tfrac{\overrightarrow{E}}{c_t} + \overrightarrow{B} \text{ Final Magnetic Field} \qquad \overrightarrow{E}_f = \overrightarrow{E} - c_t\overrightarrow{B} \text{ Final Electric Field}\end{gathered} \tag{37}$$

The Maxwell Equation of 33 and continuity equation becomes:

$$\begin{cases}\overrightarrow{\nabla}\cdot\overrightarrow{B}_f = \mu_0\rho_{e+m} \\ \overrightarrow{\nabla}\times\overrightarrow{E}_f + \frac{\partial\overrightarrow{B}_f}{\partial t} = -\mu_0\overrightarrow{J} \\ \frac{\partial\rho_{e+m}}{\partial t} + \overrightarrow{\nabla}\cdot\overrightarrow{J} = 0\end{cases} \tag{38}$$

Thanks to this result, Maxwell's equations with a magnetic monopole can be rewritten in covariant form is the continuity equation:

$$\partial_\mu = (\frac{1}{c_t}\frac{\partial}{\partial t}; \overrightarrow{\nabla}) \qquad J^\mu = \begin{pmatrix} c_t\rho_{e+m} \\ \overrightarrow{J}\end{pmatrix} \tag{39}$$

$$G^{\mu\nu} = \begin{pmatrix} 0 & B_{fx}c_t & B_{fy}c_t & B_{fz}c_t \\ -B_{fx}c_t & 0 & -E_{fz} & E_{fy} \\ -B_{fy}c_t & E_{fz} & 0 & -E_{fx} \\ -B_{fz}c_t & -E_{fy} & E_{fx} & 0\end{pmatrix} \tag{40}$$

Where ∂_μ is the four-gradient, J^μ is the four density current representing four acceleration in elastic terms and G is the electromagnetic tensor with magnetic monopole. Hence, the compact equations

$$\begin{cases}\partial_\nu G^{\mu\nu} = \mu_0 J^\mu \text{ Maxwell Equations} \\ \partial_\mu J^\mu = 0 \text{ Continuity Equation}\end{cases} \tag{41}$$

G corresponds to an electric field, which, in the framework of elasticity, represents a surface force, thus the divergence of $[\partial_\nu G^{\mu\nu}] = NL^{-3}$. This tensor is the 4×4 analogue of the Cauchy stress tensor. Analogously to the Cauchy tensor, $G^{\mu\nu}$ represents the net internal force per unit volume due to the stress distribution within the material. The external force, is given by the vacuum permeability constant, that denotes a mass density, multiplied by a four-acceleration J^μ. Moreover, the metric tensor G depends of A electromagnetic four potential because is given by the sum of the electromagnetic tensor F and is its dual $\widetilde{F}$:

$$A^\mu = (\frac{\varphi_E}{c_t}; \overrightarrow{A}), \qquad \overrightarrow{B}_t = \overrightarrow{\nabla}\times\overrightarrow{A}, \qquad [A] = [ML^{-1}T^{-1}] \tag{42}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \qquad F^{\mu\nu} = \begin{pmatrix} 0 & \frac{E_x}{c_t} & \frac{E_y}{c_t} & \frac{E_z}{c_t} \\ -\frac{E_x}{c_t} & 0 & B_z & -B_y \\ -\frac{E_y}{c_t} & -B_z & 0 & B_x \\ -\frac{E_z}{c_t} & B_y & -B_x & 0\end{pmatrix} \tag{43}$$

$$\widetilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\lambda\omega}F_{\lambda\omega} \qquad \widetilde{F}^{\mu\nu} = \begin{pmatrix} 0 & B_{t,x} & B_{t,y} & B_{t,z} \\ -B_{t,x} & 0 & -\frac{E_z}{c_t} & \frac{E_y}{c_t} \\ -B_{t,y} & \frac{E_z}{c_t} & 0 & -\frac{E_x}{c_t} \\ -B_{t,z} & -\frac{E_y}{c_t} & \frac{E_x}{c_t} & 0 \end{pmatrix} \tag{44}$$

$$G^{\mu\nu} = c_t(F^{\mu\nu} + \widetilde{F}^{\mu\nu}) = c_t[\partial^\mu A^\nu - \partial^\nu A^\mu + \frac{1}{2}\varepsilon^{\mu\nu\lambda\omega}(\partial_\lambda A_\omega - \partial_\omega A_\lambda)] \tag{45}$$

With this result, it is possible to describe how the volume force is correlated by the electromagnetic four-potential. By combining 41 and 45:

$$J^\mu = \frac{c_t}{\mu_0}[\partial^\mu(\partial_\nu A^\nu) - \Box A^\mu + \frac{1}{2}\partial_\nu \varepsilon^{\mu\nu\lambda\omega}(\partial_\lambda A_\omega - \partial_\omega A_\lambda)] \tag{46}$$

Finally, we note that the form of the Maxwell equations with a magnetic monopole is mathematically analogous to the non-homogeneous Maxwell equations without the magnetic monopole $\partial_\nu F^{\mu\nu} = \mu_0 J^\mu_e$ which is well known to be Lorentz invariant. Using the same algebraic procedure, one can prove that Maxwell's equations with magnetic monopole are Lorentz invariant in accordance with the First Principle of Relativity.

3.3 Wave Equations

In this short subsection, we will give the complete equations for the electric and magnetic field in the presence of magnetic monopoles. Applying the rotor to the third and fourth equations of 32 obtains:

$$\begin{cases} \Box\overrightarrow{E} = -\mu_0 c_t^2 \overrightarrow{\nabla}\rho_e - \mu_0 \frac{\partial \overrightarrow{J}_e}{\partial t} \\ \\ \Box\overrightarrow{B} = \mu_0 \overrightarrow{\nabla}\rho_m + \frac{\mu_0}{c_t^2}\frac{\partial \overrightarrow{J}_m}{\partial t} + \mu_0 \overrightarrow{\nabla} \times \overrightarrow{J}_e \end{cases} \tag{47}$$

Where $\Box = \partial_\mu \partial^\mu$ is the D'Alambertian operator. It can be seen that apparently the wave equations do not respect the dual symmetry typical of electromagnetism with a magnetic monopole. However, this result is due to the fact that the rotor of $\overrightarrow{J}_m$ is zero and therefore does not appear in the electrical wave equation. In fact, a term $-\mu_0 \overrightarrow{\nabla} \times \overrightarrow{J}_m$ should appear.

3.4 Lagrangian Formulation

The given Lagrangian density involving the magnetic monopole:

$$\mathcal{L} = -\frac{1}{4\mu_0}[F_{\mu\nu}F^{\mu\nu} + \widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}] + \frac{1}{c_t}J^\mu A_\mu \tag{48}$$

From the action:

$$\mathcal{S} = \int \mathcal{L} d^4x \qquad \delta S = 0 \tag{49}$$

We get the Maxwell Equation with magnetic monopole:

$$\partial_\nu G^{\mu\nu} = \mu_0 J^\mu \qquad J^\mu \equiv \text{ Noether Current} \tag{50}$$

The Noether current implies a conserved quantity:

$$Q = c_t e + \frac{\dot{m}}{\mu_0} \tag{51}$$

The quantity Q conserved is a volumetric flow rate. The constant volumetric flow rate is a direct consequence of the conservation of both electric and magnetic charge. The Lagrangian description provides us with a complete painting of the Maxwell equations representing the divergence of a 4x4 stress tensor. In the presence of an internal equilibrium, the forces are zero and this implies that there are no sources or sinks. Thus, the volumetric flow rate Q is zero. Conversely, in the presence of external forces $\mu_0 J$ A constant volumetric flow rate defined by the relationship 51.

3.5 CP Violation and Topological invariance

We note that in the lagrangian density does not appear the term $G_{\mu\nu}G^{\mu\nu}$. We consider what happens if this term appears in the Lagrangian density:

$$\frac{1}{c_t^2}G_{\mu\nu}G^{\mu\nu} = F_{\mu\nu}F^{\mu\nu} + \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + 2F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{52}$$

Because $F_{\mu\nu}\tilde{F}^{\mu\nu} = \tilde{F}_{\mu\nu}F^{\mu\nu}$. So, We can define a new Lagrangian density:

$$\mathcal{L} = -\frac{1}{4\mu_0}[F_{\mu\nu}F^{\mu\nu} + \widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu} + \frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}] - \frac{1}{c_t}J^{\mu}A_{\mu} \tag{53}$$

The Euler-Lagrange equations for the given Lagrangian density is:

$$c_t\partial_\nu(F^{\mu\nu} - \frac{1}{2}\tilde{F}^{\mu\nu}) = \mu_0 J^{\mu} \qquad J^{\mu} \equiv \text{Noether Current} \tag{54}$$

The Lagrangian density given modifies the Maxwell equations, introducing terms $F_{\mu\nu}\tilde{F}^{\mu\nu}$ that break the duality symmetry and imply CP violation. This term is invariant under charge conjugation but changes sign under parity, thus violating CP symmetry. The Noether current remains the same by not changing the value of Q. This means that the symmetry breaking only affects the internal structure of the fields and not the external force applied to it. Furthermore, to the breaking of dual symmetry, there is a topological invariant, the Pontryagin's number:

$$P = \int F_{\mu\nu}\tilde{F}^{\mu\nu}d^4x \tag{55}$$

This number is a topological invariant because, under continuous, smooth gauge transformations (which do not change the topology of the gauge field), the value of P remains unchanged. It takes integer values and can be interpreted as a measure of the 'number of twists' or 'number of windings' of the gauge field in space-time. In summary, the Lagrangian introduced in this section causes the dual symmetry of the electromagnetic field to break down and introduces the topological invariant.

4 Special relativity

Special relativity, formulated by Albert Einstein in 1905 [10], is based on two fundamental postulates and Lorentz transformation:

- *Principle of Relativity:* The laws of physics are the same in all inertial reference frames (i.e., reference frames that are not subject to acceleration). This means that there is no absolute reference frame, and absolute motion cannot be determined.
- *Constancy of the Speed of Light:* The speed of light in a vacuum c_t is the same for all observers, regardless of the motion of the light source or the observer.

  Lorentz transformations (Boost along x-axis):

$$\begin{cases} X = \gamma(x - vt) \\ Y = y \\ Z = z \\ T = \gamma(t - \frac{\beta x}{c_t}) \end{cases} \tag{56}$$

  Where $\gamma = \sqrt{1-\beta^2}$ is the Lorentz parameter and $\beta = \frac{v}{c_t}$ where v is the velocity of moving frame. The Lorentz transformations and the principle of relativity are interconnected. The Lorentz transformations are necessary to maintain the invariance of the laws of physics (including the constancy of the speed of light) among inertial reference frames, in accordance with the principle of relativity.

A fundamental consequence of the theory of relativity is the mass-energy relationship, which represents a cornerstone of modern physics. In the following paragraphs, We will demonstrate how it is possible to derive this relationship using classical considerations based on results obtained with the magnetic monopole. In particular, We will see that the mass-energy relation arises in the specific case of inertial mass. This assumption, coupled with the hypothesis of isotropy, namely that the Lamé parameters do not depend on spatial coordinates, allows us to justify why the event interval must be invariant for inertial reference systems, thus validating the Lorentz transformations.

4.1 Inertial Mass and Relativistic Mass-Energy

In this section, we will derive the mass-energy relationship by utilizing the results pertaining to the magnetic monopole. We begin by describing the Lorentz factor. According to the elastic theory proposed by Landau and Lifshitz, [6] in an isotropic medium, the longitudinal velocity is related to the transverse one according to the following relation:

$$c_l^2 - c_t^2 = \frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)} - \frac{E}{2\rho(1+\nu)} = \frac{\sigma+\eta}{\rho} \tag{57}$$

Where E is Young module, ν is the Poisson Ratio. We note that if we impose that $c_l \leq c_t$:

$$\begin{cases} (c_l^2 - c_t^2) \leq 0 \implies \sigma+\eta \leq 0 \implies \sigma \leq -\eta = -\frac{1}{\varepsilon_0} \\ c_l^2 \geq 0 \implies \sigma + 2\eta \geq 0 \implies \sigma \geq -2\eta = -\frac{2}{\varepsilon_0} \end{cases} \implies \sigma\epsilon[-\frac{2}{\varepsilon_0}, -\frac{1}{\varepsilon_0}] \tag{58}$$

By defining $\beta = \frac{c_l}{c_t}$ and $\gamma = (1-\beta^2)^{-1/2}$ positive parameter we obtain from 57 the following relation:

$$\gamma = \sqrt{2\nu - 1} \quad \text{Where } \nu\epsilon[\frac{1}{2}; +\infty) \text{ If } c_l\epsilon[0; c_t] \tag{59}$$

The Lorentz factor γ represents an effective contraction due to the Poisson ratio. The relationship 59 allows to rewrite the divergence term of Navier-Cauchy equation. By taking in account that:

$$(c_l^2 - c_t^2)\vec{\nabla}\vec{u}_l = -\frac{c_t^2}{\gamma^2}\vec{\nabla}\vec{u}_l \tag{60}$$

It can be written, using the relation 6 and 59 as:

$$-\frac{c_t^2}{\gamma^2}\vec{\nabla}\vec{u}_l = \frac{c_t^2}{1-2\nu}\frac{d\tau' - d\tau}{d\tau} \tag{61}$$

From elastic theory, it is well known that last term of 61 can be written, for small displacement hypothesis, as:

$$\frac{d\tau' - d\tau}{d\tau} = (1-2\nu)\frac{dx' - dx}{dx} \tag{62}$$

Unifying the relations 60, 61 and 62 we obtain that the term related to volume changes can be written as:

$$-\frac{1}{\gamma^2}\vec{\nabla}\vec{u}_l = \frac{dx' - dx}{dx} \tag{63}$$

Using the relation 28 with the the last relation:

$$\frac{dm_0 - dm}{dm_0} = \gamma^2\frac{dx' - dx}{dx} \tag{64}$$

From this relationship, we obtain that the mass generated by the magnetic monopole and the initial mass is:

$$dm = (\gamma^2\frac{dx'}{dx} - \gamma^2 + 1)dm_0 \tag{65}$$

This formula generally applies since no assumptions were made about longitudinal deformation. To obtain the inertial mass assumed by special relativity, we assume that for relativistic velocity:

$$\frac{dx'}{dx} = 1 + \frac{1}{\gamma} - \frac{1}{\gamma^2} \implies dm = \gamma dm_0 \quad d\tau' = \gamma d\tau \tag{66}$$

Where $\frac{dx'}{dx}\epsilon[2;1[$ for $c_l\epsilon[0;c_t[$. As long as there is inertial mass generated by the motion of the longitudinal wave we introduce longitudinal momentum:

$$\overrightarrow{p}_l = m\overrightarrow{c}_l = m_0\gamma\overrightarrow{c}_l \tag{67}$$

This term represent the energy due to the inertia of the medium. Similarly, the transverse momentum can be introduced. This momentum does not exhibit volumetric deformation terms since $\overrightarrow{\nabla}\cdot\overrightarrow{u}_t$ is zero. This result implies that the mass of the momentum remains as the initial mass, meaning the transverse momentum is given:

$$\overrightarrow{p}_t = m_0\overrightarrow{c}_t \tag{68}$$

By calculating the force classically, the relativistic force can be obtained. It is observed that the derivative of the transverse momentum is zero because both m_0 and c_t are constants. Therefore, the force $\overrightarrow{F}$ is given by:

$$\overrightarrow{F} = \frac{d(\overrightarrow{p}_l + \overrightarrow{p}_t)}{dt} = \frac{d\overrightarrow{p}_l}{dt} \tag{69}$$

Using the formula 67:

$$\overrightarrow{F} = m_0\gamma^3\frac{d\overrightarrow{c}_l}{dt} \tag{70}$$

Calculating the kinetic energy of the force:

$$T_l = \int_0^{c_l}\overrightarrow{F}\cdot d\overrightarrow{x} = (\gamma-1)m_0c_t^2 \implies m_0\gamma c_t^2 = T_l + m_0c_t^2 \tag{71}$$

Where was used in the calculation $\overrightarrow{c}_l = \frac{d\overrightarrow{x}}{dt}$. Another method to calculate relativistic energy is by considering the quadrature sum of the momenta. Defining $\overrightarrow{p}$ the total momentum:

$$p^2 = p_l^2 + p_t^2 = m_0^2c_t^2(\gamma^2\beta^2+1) = m_0^2\gamma^2c_t^2 \tag{72}$$

We note that the direction of total momentum is exclusively determined by the transverse speed of light. Therefore, the total energy is given:

$$\overrightarrow{p} = m_0\gamma\overrightarrow{c}_t \implies E = \overrightarrow{p}\cdot(\overrightarrow{c}_t + \overrightarrow{c}_l) = m_0\gamma c_t^2 \tag{73}$$

We note that relationship 73 is nothing but the energy-momentum relationship. Indeed if we multiply for c_t^2 the relation 72, because c_l contribution disappears in the scalar product:

$$E^2 = (p_lc_t)^2 + (m_0c_t^2)^2 \tag{74}$$

4.1.1 Consequences Inertial Mass Hypothesis

The implications of the inertial mass hypothesis can be evaluated by reassessing the term $(c_l^2 - c_t^2)\overrightarrow{\nabla}(\overrightarrow{\nabla}\overrightarrow{u})$ as it appears in the Navier-Cauchy equation 2. It is observed that within the inertial mass hypothesis, $\overrightarrow{\nabla}(\overrightarrow{\nabla}\overrightarrow{u}) = \overrightarrow{\nabla}(\overrightarrow{\nabla}\overrightarrow{u}_l) = \overrightarrow{\nabla}(\gamma-1) = \overrightarrow{\nabla}\gamma$. Under isotropic conditions, the Lamé parameters defining the velocities c_l and c_t do not vary with spatial coordinates, leading to the Lorentz factor being independent of spatial coordinates as well. Consequently, the gradient term vanishes in the Navier-Cauchy equation, implying that the longitudinal wave equation 4 becomes, in the context of the inertial mass hypothesis, the second principle of dynamics express in volume force:

$$\overrightarrow{f} = \rho\frac{\partial^2\overrightarrow{u}_l}{\partial t^2} \qquad \overrightarrow{J}_m = \frac{\overrightarrow{f}}{\rho} \quad \rho_m = -\frac{d(\gamma-1)}{dt} \tag{75}$$

In this scenario, the magnetic current density represents volume acceleration, and the relationships 31 and 34 remain valid. To calculate the magnetic monopole, we consider that the magnetic density does not depend on spatial coordinates

$$g = \rho_m\iiint_\tau d\tau = \tau\frac{d(\gamma-1)}{dt} \tag{76}$$

The same result is obtained if we assume that the inertial mass hypothesis:

$$g = -\frac{\dot{m}}{\rho} = -\frac{m_0}{\rho}\frac{d\gamma}{dt} = -\tau\frac{d(\gamma-1)}{dt} \tag{77}$$

Explicitly, the magnetic monopole takes a form of the type

$$g = -\beta\gamma^3\frac{d\beta}{dt}\tau \qquad Q = c_t e - \frac{m_0}{\mu_0}\beta\gamma^3\frac{d\beta}{dt} \tag{78}$$

These results implies that $\frac{d\gamma}{dt} =$ Constant, in order to theory to be consistent. Nevertheless, consideration can also be given to situations where anisotropies are present and Lamé coefficients vary spatially. It is worth noting that the anisotropy hypothesis contradicts the second principle of relativity, as the transverse speed of light must remain independent of the reference system in accordance with the Michelson-Morley experiment. Moreover, anisotropy implies that magnetic monopole is not conserved because it would depend on the spatial coordinates through γ. However, for the sake of mathematical analysis, calculations are carried out, and the physical outcomes of the Navier-Cauchy equation are assessed.

$$(c_l^2 - c_t^2)\overrightarrow{\nabla}(\overrightarrow{\nabla}\overrightarrow{u}) = (c_l^2 - c_t^2)\frac{\gamma^3}{c_t^2}(\overrightarrow{c}_l \cdot \overrightarrow{\nabla})\overrightarrow{c}_l \tag{79}$$

Considering that $\frac{(c_l^2-c_t^2)}{c_t^2}\gamma^3 = -\frac{\gamma^3}{\gamma^2}$ we obtain:

$$(c_l^2 - c_t^2)\overrightarrow{\nabla}(\overrightarrow{\nabla}\overrightarrow{u}) = -\gamma(\overrightarrow{c}_l \cdot \overrightarrow{\nabla})\overrightarrow{c}_l \tag{80}$$

The Navier-Cauchy equation 2 under the assumption of inertial mass becomes and anisotropy hypothesis:

$$\Box\overrightarrow{u} = +\frac{1}{2\rho}\overrightarrow{\nabla}\times\overrightarrow{C} + \frac{\overrightarrow{f}}{\rho} - \gamma(\overrightarrow{c}_l \cdot \overrightarrow{\nabla})\overrightarrow{c}_l \tag{81}$$

The terms $(\overrightarrow{c}_l \cdot \overrightarrow{\nabla})\overrightarrow{c}_l$ represents the advective term and describes how the velocity of a medium particle changes due to its motion through a spatially varying velocity field. This is critical for understanding complex medium flows where the velocity field varies in space. The inertial mass hypothesis entail the emergence of an advective term in the Navier-Cauchy equation under the assumption of anisotropy. However, the experimental result of Michelson-Morley and the second principle of relativity dictate in our model that velocities do not vary with spatial coordinates, as otherwise, the speed of light c_t would be contingent upon the chosen reference frame. It is noted that the advective term vanishes since the longitudinal velocity c_l, akin to its transverse counterpart, remains independent of spatial coordinates under the isotropy hypothesis, although it may be time-dependent. Therefore, the theory of relativity holds true under the assumption of inertial mass and medium isotropy. Therefore, for relativity to be valid, the Navier-Cauchy equation 2 and waves equation 4 and 5 becomes:

$$\Box\overrightarrow{u} = +\frac{1}{2\rho}\overrightarrow{\nabla}\times\overrightarrow{C} + \frac{\overrightarrow{f}}{\rho} \tag{82}$$

$$\overrightarrow{f} = \rho\frac{\partial^2\overrightarrow{u}_l}{\partial t^2} \tag{83}$$

$$\Box u_t = \frac{1}{2\rho}\overrightarrow{\nabla}\times\overrightarrow{C} \tag{84}$$

These new results do not alter the form of Maxwell's equations with magnetic monopole nor the continuity equation derived. The only difference is that in special relativity, the magnetic density current represents a volume acceleration and the longitudinal wave equation degenerates into the second principle of dynamics expressed in terms of volume force. The same set of equation is obtained in the non-relativistic limit $c_l \to 0$.

4.2 New interpetration Interval of events

In special relativity, an event is defined as a point in space-time, characterized by four coordinates: three spatial (x, y, z) and one temporal (t). The interval between two events can be calculated using these coordinates.For two events with coordinates (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2), the space-time interval is given by:

$$\Delta S^2 = c_t^2(t_2 - t_1)^2 - (x_2 - x_1)^2 + (y_2 - y_1)^2 - (z_2 - z_1)^2 \tag{85}$$

The interval can be classified into three categories:

- Time-Like: If $\Delta S^2 < 0$, the events can be connected by a particle traveling at a speed less than that of light. There exists a reference frame in which the events occur at the same location (in space).
- Light-Like: If $\Delta S^2 = 0$, the events are connected by a light beam. The distance between these events is traversed exactly at the speed of light.
- Space-Like: If $\Delta S^2 > 0$, no observer can perceive these events as happening at the same time. They are separated by a spatial distance such that no signal can travel from one to the other without exceeding the speed of light.

A mathematical tool for calculating the interval between two events is the Minkowski metric

$$\zeta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \Delta S^2 = \zeta_{\mu\nu}\Delta x^\mu \Delta x^\nu \tag{86}$$

It is well known that the interval between events is invariant under transformations belonging to the Lorentz group, and therefore also under Lorentz transformations. These transformations are crucial for maintaining the invariance of physical laws across different inertial frames. For this reason, the event interval plays a critical role in special relativity. In this model, the invariance of ΔS^2 is not introduced through spacetime considerations, but rather by considering ΔS^2 as the space generated by the transverse and longitudinal speed of light. In differential terms:

$$dS^2 = (c_t^2 - c_l^2)dt^2 \qquad (\frac{dS}{dt})^2 = -\frac{\eta + \sigma}{\rho} = \frac{|\eta + \sigma|}{\rho} \tag{87}$$

This term is equivalent to the event range because $\overrightarrow{c}_l = \frac{d\overrightarrow{x}}{dt}$. The relations of relativistic dynamics have been derived under the assumption of inertial mass and the spatial coordinate independence of the Lamé coefficients. This consideration leads us to conclude that the speeds of light, both transverse and longitudinal, are equal in every chosen reference frame. A consequence of this consideration is that:

$$dS^2 = dS'^2 \tag{88}$$

Consequently, the time intervals can be reinterpreted as:

- Subluminar motion: If $\Delta S^2 < 0$, the time intervals can be reinterpreted as follows: the longitudinal speed is less than the transverse speed. Thus, an inertial motion is established, which becomes more significant as the longitudinal speed approaches the transverse speed.
- Luminar: If $\Delta S^2 = 0$, the longitudinal speed reaches the transverse speed by overcoming the *"light barrier"*, analogous to what occurs when an airplane reaches the speed of sound.
- Superluminar motion: If $\Delta S^2 > 0$, in this interval, the Lorentz factor is complex and corresponds to Poisson ratios $\nu < 1/2$. We can define a new Lorentz factor in analogy with aerodynamics. In various supersonic flow equations, such as those describing shock waves and expansion waves, the term $\sqrt{Ma^2 - 1}$, where Ma is the Mach number. This term in supersonic flow is a critical term that arises in various aerodynamic analyses, reflecting the deviation from sonic speed and playing a key role in the mathematical formulation of shock waves, expansion waves, and other supersonic phenomena. So, in the super luminary regime, one could introduce the term $\gamma = (\beta^2 - 1)^{-\frac{1}{2}}$

to describe the departure from luminary behaviour. However, as a body travelling at a speed greater than the speed of light has never been experimentally detected, these considerations remain purely theoretical speculation

With this new interpretation of the event interval as the space traversed by the difference between the two squares of the transverse and longitudinal speed of light, which also appears in the original Navier-Cauchy equation, it is possible to introduce spacetime. In our model, spacetime represents more of a useful mathematical tool rather than a physical reality in itself, as the spatial components are introduced by replacing in dS^2:

$$(d\overrightarrow{x})^2 = c_l^2 dt^2 \tag{89}$$

This implies that in our model, there exists physical space traversed by the two components of the speed of light, while spacetime is a mathematical entity obtained by explicitly expressing the longitudinal speed of light in spatial terms. In fact, the time interval describes the speed with which the gradient of volume deformations moves (see equation 1). In the case of special relativity, valid in the isotropic medium in this model, the gradient of deformations is zero because it's velocity $c_t^2 - c_l^2$ is constant throughout space regardless of the reference system. This result tells us that an observer solidal to the frame in motion sees a zero longitudinal velocity, but that in reality it cannot be since $c_l \neq 0$ and is constant in all space. To correct this 'apparent' discrepancy we reintroduce the relativistic proper time $d\tau_P = \frac{dt}{\gamma}$. However, the proper time in our model is not a real existing time, but rather a necessary correction to be introduced for an observer in motion to reintroduce the effects of longitudinal velocity in his reference system. It is assessed that this model has no mathematical discrepancy with relativity, however, it possesses a very different epistemological interpetration. In the elastic model proposed here, time and space are distinct because dS^2 is generated by the difference of the velocities of light and that only through the transformation $\overrightarrow{c}_l = \frac{d\overrightarrow{x}}{dt}$ is space-time obtained. We conclude this section by noting that the magnetic monopole is preserved in special relativity because the medium is isotropic and the mass distribution is homogeneous throughout space. This consideration decays in the realm of general relativity.

4.3 Anisotropy Hypothesis and General Relativity

As previously observed, the hypothesis of anisotropy conflicts with the second principle of relativity. However, in general relativity, The metric $g_{\mu\nu}$ is typically determined by the Einstein field equations, which describe the relationship between the distribution of matter and the evolution of spacetime. In a curved spacetime, the concept of the *"speed of light"* becomes more complex. The speed of light is constant in a local inertial frame, but due to spacetime curvature, the path of light can be bent, and the time it takes to travel between two points can be affected by gravitational fields. For example, Clocks in stronger gravitational fields (closer to a massive object) run slower compared to clocks in weaker fields. Thus, the measured speed of light might appear different when comparing signals between regions with different gravitational potentials. For this reason, the aim of this section is to describe how solutions of Einstein's field equation can be obtained by means of the anisotropy hypothesis where light velocity depends on spatial co-ordinates. For this reason, we decompose the light velocities as:

$$\begin{cases} \overrightarrow{c}_t(x,y,z) = \overrightarrow{a}(x,y,z) \cdot \overrightarrow{c}_t \ \hat{c}_t \\ \overrightarrow{c}_l(x,y,z,t) = \overrightarrow{b}(x,y,z) \cdot \overrightarrow{c}_l(t) \ \hat{c}_l \end{cases} \qquad [\overrightarrow{a}] = [\overrightarrow{b}] = [\text{Adimensional}] \tag{90}$$

Where in this notation $\overrightarrow{c}_t$ and $\overrightarrow{c}_l(t)$ represent the light speeds of special relativity, while $\overrightarrow{c}_t(x,y,z)$ and $\overrightarrow{c}_l(x,y,z,t)$ representing those of general relativity. Interval of events becomes

$$dS^2 = a^2 c_t^2 dt^2 - b^2 c_l^2 dt^2 \tag{91}$$

In spherical coordinate:

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = arccos(\frac{z}{r}) \\ \varphi = arcsin(\frac{y}{x}) \end{cases} \tag{92}$$

The space dS^2 becomes:

$$dS^2 = a^2(r,\theta,\varphi,t) c_t^2 dt^2 - b^2(r,\theta,\varphi,t) c_l^2 dt^2 \tag{93}$$

Where:

$$b^2(r,\theta,\varphi,t)c_l^2dt^2 = [\overrightarrow{b}(r,\theta,\varphi,t)\cdot d\overrightarrow{x}]^2 = b_r^2dr^2 + b_\theta^2r^2d\theta^2 + b_\varphi^2r^2sin^2\theta d\varphi^2 \tag{94}$$

Hence, we get:

$$dS^2 = a^2(r,\theta,\varphi,t)c_t^2dt^2 - b_r^2dr^2 - b_\theta^2r^2d\theta^2 - b_\varphi^2r^2sin^2\theta d\varphi^2 \tag{95}$$

The interval between events can be used to determine the distance between two events in spacetime and plays a fundamental role in formulating the laws of physics in General Relativity. In the case of special relativity, the invariance of the interval between events in this model is a consequence of the fact that the Lamé coefficients do not exhibit spatial dependence. This implies that invariance under Lorentz transformations holds. In the event intervals of general relativity, with the elastic interpretation, the Lamé coefficients exhibit anisotropy and therefore depend on spatial coordinates. The consequence of this consideration is the loss of Lorentz invariance. Hence, to achieve physical covariance, it is necessary to impose it as a principle of the theory. Moreover, by including the inertial mass hypothesis:

$$\frac{m}{m_0} = \gamma(x,y,z,t), \quad g = g(x,y,z,t) \tag{96}$$

The interval can be rewritten in its general form as:

$$dS^2 = a^2(\frac{m_0}{m})^2c_t^2dt^2 \tag{97}$$

The same result can be obtained in the case of the Minkoswki metric with the difference that the ratio of masses does not depend on spatial coordinates and the transverse light velocity is the same for each reference system ($a^2 = 1$). In fact, in the case of general relativity, the gradient of the Lorentz factor is related to the convective derivative, i.e.

$$\overrightarrow{\nabla}\gamma = \frac{\gamma^3}{c_t^2}(\overrightarrow{c}_l\overrightarrow{\nabla})\overrightarrow{c}_l \tag{98}$$

$$\overrightarrow{\nabla}m = m_0\frac{\gamma^3}{\beta^2}(\overrightarrow{b}\cdot\overrightarrow{\nabla})\overrightarrow{b} \tag{99}$$

This relation indicates that the gradient of the mass is determined by the advective term. Through this relation, we can see how the metric interval of time depends on the advective term, which is non-zero in the case of anisotropy. This result shows that the spatial distribution of the longitudinal velocity determines the mass gradient, thanks to which we can define a new metric with which the various solutions of the field equations can be explained with our model, such as Schwartzchild, Reissner-Nordström, Kerr, Kerr-Newman, Friedmann-Lemaître-Robertson-Walker (FLRW), de Sitter and Anti-de Sitter Spacetime.

5 Bohm's interpretation of Quantum Mechanics

A brief explanation of Bohm's interpetration [12] [13] of quantum mechanics to understand how the magnetic monopole predicts quantum mechanics must be made. Bohmian Mechanics, also known as the de Broglie-Bohm theory or pilot-wave theory, is an interpretation of quantum mechanics that posits an objective reality where particles have precise positions and velocities at all times. It contrasts with the standard Copenhagen interpretation by providing a clear ontology and avoiding the measurement problem. There are three key concepts concern with the theory:

1. **Pilot Wave**: Particles are guided by a wave function ψ which evolves according to the Schrödinger equation.

2. **Particle Trajectories**: Particles have definite trajectories determined by a guiding equation.

3. **Determinism**: The theory is fully deterministic; the future states of the particles are entirely determined by their current states and the wave function.

The fundamental equation are:

$$\begin{cases} \frac{\partial \rho_q}{\partial t} + \overrightarrow{\nabla}(\rho_q \overrightarrow{v}) = 0 \\ \overrightarrow{v} = \frac{1}{m}\overrightarrow{\nabla} S \\ -\frac{\partial S}{\partial t} = \frac{|\overrightarrow{\nabla} S|^2}{2m} + V + Q \quad \text{Hamilton-Jacobi Equation for action S} \\ Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho_q}}{\sqrt{\rho_q}} \end{cases} \tag{100}$$

Where $\rho_q = |\psi|^2$ is the is density probability of quantum mechanics, $\overrightarrow{v}$ is Velocity field of Guidance equation, Q is the Quantum Potential and Hamilton-Jacobi Equation for action S is the Schrödinger Equation Hamilton-Jacobi Formalism. The Schrödinger is given:

$$\hat{H}\psi = [\frac{\hbar^2}{2m}\nabla^2 + V(\overrightarrow{r})]\psi = i\hbar\frac{\partial \psi}{\partial t} \tag{101}$$

There are several implication of this theory. Emphasising the main consequences

- **Deterministic Trajectories**: Unlike the probabilistic nature of the Copenhagen interpretation, Bohmian mechanics asserts that particles have well-defined paths.
- **Quantum Equilibrium Hypothesis**: The distribution of particles matches the probability density ρ_q, ensuring consistency with the predictions of standard quantum mechanics.
- **Measurement**: Measurement outcomes are determined by the positions of particles and the configuration of the measuring device, avoiding wave function collapse.

6 Quantum Mechanics

In our model, the magnetic continuity equation can be linked to the quantum continuity equation under the assumption that the magnetic density and magnetic current density are described by:

$$\rho_m = \frac{\dot{m}}{\mu_0}\rho_q \qquad \overrightarrow{J}_m = \frac{i\dot{m}\hbar}{2\mu_0 m}(\psi^*\overrightarrow{\nabla}\psi - \psi\overrightarrow{\nabla}\psi^*) \quad \overrightarrow{J}_m = \frac{\dot{m}}{\mu_0}\overrightarrow{J}_q \tag{102}$$

Indeed, simply multiplying the quantum continuity equation by the magnetic monopole yields the magnetic continuity equation. This physically implies that mass fluxes associated with the magnetic monopole are responsible for quantum phenomena. By combining 100 and 102 we get:

$$\overrightarrow{v}_m \equiv \overrightarrow{v} \qquad \overrightarrow{J}_m = \frac{\rho_m}{m}\overrightarrow{\nabla} S \tag{103}$$

Given the irrotational nature of the magnetic current, it is demonstrated how the action of quantum mechanics is determined by the potential of the magnetic current ϕ_j:

$$S = -\frac{m}{\rho_m}\phi_j \tag{104}$$

A consequence of this relationship is that the action S of the Hamilton-Jacobi equation is described by the magnetic density and the potential of the magnetic current. This implies that the action is entirely determined by the magnetic monopole. Similarly, the quantum potential Q can be rewritten in terms of the magnetic density:

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho_m}}{\sqrt{\rho_m}} \tag{105}$$

These findings demonstrate that all fundamental equations of the pilot wave theory can be reformulated in magnetic terms. This implies that the outcomes of quantum mechanics can be regarded as manifestations of the magnetic monopole.

6.1 Dirac Quantization

Quantisation of the magnetic charge starting from the central field obtained 35 implies:

$$e\dot{m} = 2\pi n\hbar \qquad n\epsilon\mathcal{N} \tag{106}$$

for $n = 1$ the condition is obtained:

$$h = e\dot{m} = \frac{\rho}{2\eta}(\oiint_{\partial\tau} \overrightarrow{C}\hat{n}dS)\frac{d}{dt}(\oiint_{\partial\tau} \overrightarrow{u}_l\hat{n}dS) \quad \text{Dirac's Quantization} \tag{107}$$

The Dirac Quantization implies:

$$\dot{m} \simeq 4.1 \times 10^{-15}\frac{\text{kg}}{\text{s}} \qquad dm = \frac{h}{e}dt \qquad g \simeq \pm 3.3 \times 10^{-9}\frac{m^3}{s} \tag{108}$$

The conserved volume flow rate 51 can also be written in terms of the fine structure constant:

$$\alpha = \frac{e^2}{2\varepsilon_0 h c_t} \qquad Q = \frac{\dot{m}}{\mu_0}(1 + 2\alpha) \tag{109}$$

This means that the mass exchanged depends on the time variation δt between the beginning of the mass flow and its end. We note that this result can be applied only in the realm of quantum mechanics in order to explain the nature of Planck constant. Indeed, the magnetic monopole in the relativistic limit is a particular case under the assumption of inertial mass and possesses an incongruences with Dirac quantization:The magnetic monopole is null $g = 0$ if $c_l = 0$. This implies that for non-relativistic velocities there are no magnetic monopoles and the equation of quantum mechanics would all be trivially null for inertial mass hypothesis. This inconsistency is resolved simply by considering that the inertial mass hypothesis only applies to relativistic regimes. For a non-relativistic regime, the general formula 65 can be used, which unity with Dirac's quantisation gives the result:

$$\delta t = \frac{m_0 e}{h}(\gamma^2\frac{dx'}{dx} - \gamma^2 + 1) \tag{110}$$

This result represents the temporal variation in which the mass flux is responsible for the mechanism of particle generation. We note that the electric charge determines the sign of the mass flux, but in calculating the time variation its value in modulus must be taken. In this relationship the relativistic limit problem does for magnetic monopole is resolved because for $c_l \to 0$:

$$\delta t = \frac{m_0 e}{h}\frac{dx'}{dx} \tag{111}$$

And from relation 61 the relativistic limit implies:

$$\frac{dx'}{dx} = -\frac{d\tau'}{d\tau} + 2 \tag{112}$$

We note that Dirac's condition in the nonrelativistic limit imposes the condition that $\frac{dx'}{dx} > 0$. This, in terms of the magnetic charge results in:

- *South pole*: In this case $d\tau' < d\tau$ and this implies that $\frac{dx'}{dx} > 0$
- *North pole*: In this case $d\tau' > d\tau$ and this impone the condition that $d\tau' < 2d\tau$ in order to get $\frac{dx'}{dx} > 0$

7 Conclusion

In this paper, we demonstrate how, through an elasto-mechanical relationship and the Navier-Stokes equation, it is possible to introduce the magnetic monopole as a volume flux after it has undergone deformation. Furthermore, we show that this phenomenon is associated with the generation of mass because the density of the medium remains constant, and thus, a change in volume represents a change in mass. By incorporating the magnetic monopole, we derive Maxwell's equations for the magnetic monopole, the electromagnetic wave equation, and the Lagrangian formulation. An alternative version of the Lagrangian is also provided, which introduces CP violation, the breaking of electromagnetic dual symmetry, and a topological invariant. The results on the monopole are used to derive the relativistic mass-energy equation and reinterpret relativistic results. In particular, the reinterpretation of the event interval allows us to revisit the relativistic metric in elastomechanical terms related to the longitudinal and transverse speeds of light, as defined by Lamé parameters. In the case of an isotropic medium, the Lamé parameters do not depend on spatial coordinates, which is when the Minkowski metric is valid, and inertial mass is constant in terms of spatial coordinates. Conversely, under the assumption of anisotropy, it is shown how the metrics of general relativity can be derived. In this case, the inertial mass depends on spatial coordinates, and its gradient is described by the convective derivative of the longitudinal velocity. Subsequently, it is demonstrated how the magnetic continuity equation predicts the base set of equations that describe quantum mechanics in the Bohmian interpretation. Finally, Dirac quantization predicted by quantum mechanics is evaluated, and it is found to be incompatible with the hypothesis of inertial mass. Thus, the inertial mass hypothesis was considered to be a special case of mass for relativistic velocities, implying that relativity is a special case of this theory. This inconsistency can be easily corrected by considering the general mass formula obtained from the magnetic monopole considerations.

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