Effectiveness of Navier-Stokes equations in modeling turbulence or chaos

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Abstract The motion of fluids which are incompressible could be described by the Navier-Stokes differential equations. Although they are relatively simple-looking, the three-dimensional Navier-Stokes equations misbehave very badly. Even with nice, smooth, reasonably harmless initial conditions, the solutions could wind up being extremely unstable. The field of fluid mechanics would be dramatically altered through a mathematical understanding of the outrageous behavior of these equations. The three-dimensional Navier-Stokes equations are apparently not solvable, i.e., the equations cannot be used to model turbulence or chaos (which is a three-dimensional phenomenon).

Keywords: Navier-Stokes equations; turbulence; forecast; calculus; continuity; discontinuities.
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1 How effective are the Navier-Stokes equations in modeling turbulence or chaos?

The scientist normally utilizes the Navier-Stokes equations as a model to make a forecast of the outcome of a flow. However, for the case of turbulence or chaos making this forecast would be very difficult, if it could be done at all. If turbulence or chaos could be predicted, forecasted or modeled by the Navier-Stokes equations it is by definition not turbulence or chaos, as turbulence or chaos implies lack of predictability, lack of pattern or order and puzzlement. As turbulent flows are three-dimensional, nonlinear and highly unsteady, it would be practically impossible for the Navier-Stokes equations to model them.

Though a lot is known about linear systems of equations, the Navier-Stokes equations contain nonlinear terms which render them intractable. The only practical way of solving the Navier-Stokes equations, which depend on initial conditions, is to do so numerically by utilizing powerful computers.

Differential equations, such as the Navier-Stokes equations, could only make forecasts on phenomena characterized by smooth, regular, continuous flows, which turbulence is definitely not – turbulence, on the other hand, is characterized by great irregularities, discontinuities, disruptions and sharp jumps. With smooth, regular, continuous flows, which are each graphically represented by a smooth, continuous curve with gentle gradients, it would be possible to extrapolate and interpolate, i.e., forecasts are possible. This is not the case with turbulence, which does not display any discernable, set pattern or regularity at all. Hence, the Navier-Stokes equations fail when there is turbulence.

2 Conclusion

Turbulence or chaos, like the lucky numbers in a lottery, is very difficult if not impossible to forecast, particularly by utilizing the Navier-Stokes equations. However, there are some methodologies and ideas for forecasting and modeling turbulence or chaos with mathematics, which are all explicated in the three papers in the references list below. Thus, instead of repeating a long exposition in this article with all the minutiae, the author would let the three papers referenced below do the explaining.

References

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