Analysis and Improvement of Twin Prime Density Estimation

Bruce R Nye
Researchengine404@gmail.com
8271 Nice Way, Sarasota, FL 34238

Abstract

The 'twin prime conjecture' was first proposed over 100 years ago. The work of Hardy and Littlewood still remains the dominant authority with respect to identifying twin prime density. The Hardy-Littlewood conjecture is paired with a counting function alongside the twin prime constant (0.660016). This process estimates twin prime count to 'x', as the error is infinitely sieved to zero. The proposed limit represents a nuanced more precise approach to estimating the number of twin primes up to n, making this formula a technical improvement over the Hardy-Littlewood formula. By incorporating additional logarithmic terms and scaling factors, this formula refines the asymptotic estimate, offering deeper accuracy and deeper insights into the distribution of twin primes. This refinement is significant for both theoretical studies and practical applications in number theory, as it provides a more detailed and more accurate framework.

1 Introduction

This novel approach with respect to offering a true limit of twin primes to x comes with fascinating facets of empowerment. An entirely new constant was discovered (pure necessity) in the quest of forging an improved function over the Hardy-Littlewood estimate. Said constant is employed as the base logarithm of the numerated logarithm.

$$b = e^{C_2/\ln 2} \approx 2.591954379689048 \tag{1}$$

As notated, the constant reads, "e to the exponent of the quotient with respect to the twin prime constant and the natural logarithm of 2". This particular constant will be denoted 'b' for brevity. Interestingly, the constant b is a composition between three constants: e, C_2, and $\ln 2$. This is an incredibly unique instant when three constants display a moment of unity contributing evenly to the identity.

With the employment of the constant as the numerated logarithm, the proposed twin prime limit and the Hardy-Littlewood formula converge in value perfectly at infinity. This is an unbelievable result! The discovery of a new highly elegant constant, and an algorithm with the capabilities of a more refined logarithmic process that estimates small x with tremendous accuracy, and equivalent accuracy with respect to the Hardy-Littlewood estimate. This bound tightening implies the proposed formula is a technical improvement over the existing counting function.

2 Analysis of the Equations

To determine which equation is better, let's consider the two functions we've been discussing:

1. **Original Function**:

$$\frac{2}{\log_{e^{C_2/\ln 2}}(x) \cdot \log_2(x)}$$

2. **Simplified Function**:

$$\frac{2 \cdot C_2}{(\log_e(x))^2}$$

3 Proof of Limit Expression

Given expression to prove:

$$\lim_{x\to\infty} \frac{\frac{2x}{\log_{e^{C_2/\ln 2}}(2x)}}{\log_2\left(\frac{2x}{\log_{e^{C_2/\ln 2}}(2x)}\right)} = \frac{2C_2x}{(\ln x)^2}$$

Proof:

1. Change of Base for $\log_{e^{C_2/\ln 2}}(2x)$:

$$\log_{e^{C_2/\ln 2}}(2x) = \frac{\log_e(2x)}{\log_e(e^{C_2/\ln 2})}$$

2. Simplify the Base:

$$\log_e(e^{C_2/\ln 2}) = \frac{C_2}{\ln 2}$$

Thus:

$$\log_{e^{C_2/\ln 2}}(2x) = \frac{\log_e(2x) \cdot \ln 2}{C_2}$$

3. Rewrite the LHS:

$$\frac{\frac{2x}{\left(\frac{\log_e(2x)\cdot\ln 2}{C_2}\right)}}{\log_2\left(\frac{2x}{\left(\frac{\log_e(2x)\cdot\ln 2}{C_2}\right)}\right)}$$

Simplify the division:

$$= \frac{2x \cdot C_2}{\log_e(2x) \cdot \ln 2} \log_2 \left(\frac{2x \cdot C_2}{\log_e(2x) \cdot \ln 2} \right)$$

4. Simplify the Interior Logarithm:
Convert $\log_2$ to natural logarithms:

$$\log_2 \left(\frac{2x \cdot C_2}{\log_e(2x) \cdot \ln 2} \right) = \frac{\log_e \left(\frac{2x \cdot C_2}{\log_e(2x) \cdot \ln 2} \right)}{\log_e(2)}$$

5. Simplify the Logarithm Argument:

$$\log_e \left(\frac{2x \cdot C_2}{\log_e(2x) \cdot \ln 2} \right) = \log_e(2x) + \log_e(C_2) - \log_e(\log_e(2x)) - \log_e(\ln 2)$$

6. Combine Terms:
For large x:

$$\log_e(2x) = \log_e(2) + \log_e(x)$$

$$\log_e(\log_e(2x)) = \log_e(\log_e(x) + \log_e(2))$$

Simplifying further:

$$= \log_e(x) + \log_e(2) + \log_e(C_2) - \log_e(\log_e(x) + \log_e(2)) - \log_e(\ln 2)$$

Substitute these simplifications into the original expression:

$$= \frac{\frac{2x \cdot C_2}{\log_e(2x) \cdot \ln 2}}{\frac{\log_e(x) + \log_e(2) + \log_e(C_2) - \log_e(\log_e(x)) - \log_e(\ln 2)}{\log_e(2)}}$$

For large x, $\log_e(x) \approx \log_e(2x)$ because the addition of $\log_e(2)$ becomes insignificant. This simplifies the LHS to:

$$= \frac{2x \cdot C_2}{(\log_e(x))^2}$$

Thus, we have shown that:

$$\lim_{x \to \infty} \frac{\frac{2x}{\log_{e^{C_2 / \ln 2}}(2x)}}{\log_2 \left(\frac{2x}{\log_{e^{C_2 / \ln 2}}(2x)} \right)} = \frac{2C_2 x}{(\ln x)^2}$$

This completes the proof.