Teaching Special Relativity with a Handheld Model

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Abstract

Recent experimental and theoretical work has shown that classical wave processes can produce phenomena previously thought to be beyond the scope of classical physics. We describe how a simple hand-held model can be used to demonstrate the connection between classical waves and modern physics. The model consists of illustrations of two sets of wave crests. A stationary particle is modeled by wave crests propagating in circles. A moving particle is modeled by rotating the orientation of the wave crests so that they would propagate along helical paths rather than circular paths. An internal clock is assumed to tick each time a wave completes a full revolution around the cylinder common to both wave packets. These two model wave packets demonstrate relativistic frequency shift, time dilation, length contraction, and the de Broglie wavelength.

Background

Several generations of physicists have been taught that special relativity and quantum mechanics have no analogues in classical physics. However, experimental and theoretical research in the past few decades has refuted that claim. Bohmian mechanics, or pilot-wave theory, was a initially a theoretical attempt to explain relationships between deterministic wave processes and quantum statistics.[2] However, such pilot waves have since been produced in the laboratory using silicone oil droplets bouncing on a vibrating fluid surface. These pilot waves have produced quantum statistics in myriad experiments, including single-particle diffraction and interference, wave-like probability distributions, tunneling, quantized orbits, and orbital level splitting.[3;4;15;19;20;23;25]

On the theoretical side, the Dirac equation at the heart of quantum mechanics has been used by various researchers to describe classical wave dynamics.[1;11–14;18;28;31;32;39] The description of shear waves in an elastic solid is not only mathematically similar to relativistic quantum mechanics, but also has the same dynamical interpretation of the variables.[12–14] Just as there are two types of momentum - that of the medium (canonical) and that of the wave (dynamical), there are also two types of angular momentum - that of the medium (spin) and that of the wave (orbital). Specifically, spin density is the field whose curl is equal to twice the canonical momentum density of the medium.

A key insight is that the first-order Dirac equation can be understood as a factorization of an ordinary second-order vector wave equation.[11–14] It has also been shown that measurements made entirely with a single type of wave are related by Lorentz transformations.[9] This should not be surprising since the wave equation itself is Lorentz-invariant. The Michelson-Morley experiment, rather than disproving the existence of an aether, is instead a validation of the wave nature of matter.

William Thomson (Lord Kelvin) once remarked that "I am never content until I have constructed a mechanical model of the subject I am studying. If I succeed in making one, I understand. Otherwise, I do not."[38] To help students achieve this level of understanding of modern physics, we describe how to use a simple hand-held model to demonstrate the connection between classical waves, special relativity, and quantum mechanics. The model is designed to be printed on a single sheet of transparency film or paper. The model itself is readily available.[10] This paper describes how the model relates to real physical descriptions of elementary particles.

Circulating Wave Model

The model consists of illustrations of two sets of wave crests as shown in Fig. 1. The wave crests are drawn as black lines of equal length, and are equally spaced along the vertical direction on the page. Each set of wave crests is accompanied by a gray arrow indicating the direction of propagation and the distance traveled in one unit of time. Since the waves are assumed to propagate with equal speed (the speed of light), the gray arrows are of equal length.

When the sheet is rolled into a cylinder as in Fig. 2, the wave packet (a) on the left models a stationary or standing wave (or half of a standing wave) with wave frequency assumed to be $f_0 = m_0c^2/h$ and wavelength

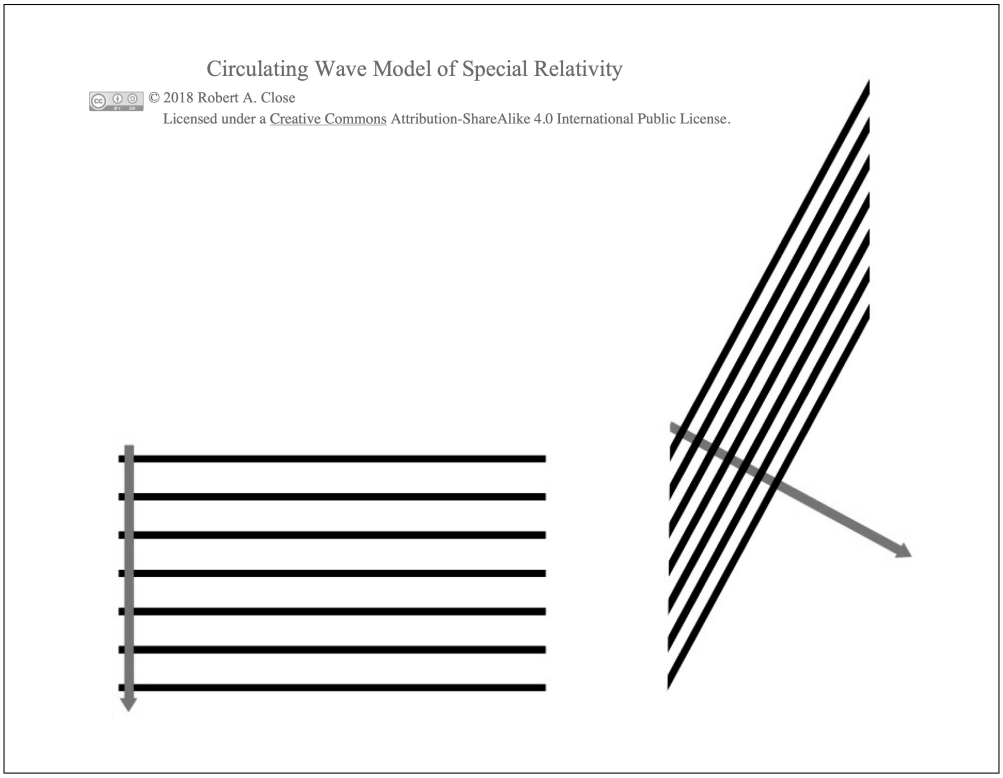

Figure 1: (a) Model of circular wave propagation with the vertical axis representing the azimuthal direction. (b) Model of helical wave propagation with speed $v = 0.866c$ and $\gamma = 2$. These patterns are designed to be enlarged and printed on a transparency sheet, then rolled into a cylindrical tube (if printed on ordinary paper, shine a light into the tube to illuminate the wave pattern).

$\lambda_0 = h/m_0c$. These relationships are based on the two equations for particle energy: $\mathcal{E} = mc^2$ and $\mathcal{E} = hf$, where $m = \gamma m_0$ is the relativistic effective mass, m_0 is rest mass, $\gamma = c/\sqrt{c^2 - v^2}$ is the Lorentz factor, c is the speed of light, v is the particle velocity, and h is Planck's constant. The gray arrow represents the distance light travels in one unit of time, as measured by a stationary observer. The internal clock ticks once each time the wave traverses the circle. If printed such that the length of the arrow is 6 cm, the time for light to make one revolution would be 200 picoseconds.

The theoretical model of particles as circulating waves offers a simple means for understanding special relativity.[6;7;9;24] De Broglie waves satisfying the Klein-Gordon equation in a central potential also propagate in circles,[36] and the mass term in the Dirac equation can be interpreted as representing circular motion.[12;26] This model is a simplification because it restricts the circular motion to a single radius. If the circular motion is assumed to be in another dimension, then this could be a model of string theory.

Figure 2: Circulating wave model of an elementary particle rolled into a cylindrical tube.(a) Stationary particle wave packet. (b) Moving particle wave packet with $v = 0.866c$ and $\gamma = 2$.

Rotating the wave crests as in Fig. 1(b) results in helical wave propagation with average velocity $v =$

$0.866c$ along the axis of the cylinder, and Lorentz factor $\gamma = 2$. The moving wave packet has wavelength λ_0/γ and relativistic frequency γf_0. This is the usual relativistic frequency shift. Applying the relationship $\mathcal{E} = hf$ shows that the moving wave packet has higher energy than the stationary packet. The difference is the relativistic kinetic energy.

The width of the moving wave packet is reduced by a factor of $1/\gamma$. This length contraction was proposed by Fitzgerald and made quantitative by Lorentz in order to explain the null result of the Michelson-Morley experiment.[22;30;34]

Propagation speed in the azimuthal direction, which measures time, is also reduced by a factor of $1/\gamma$. The arrow showing propagation of the moving packet in one unit of time (as measured by the stationary particle) only travels halfway around the cylinder. This corresponds to half of a tick on the clock, thereby demonstrating relativistic time dilation: a moving clock ticks slower than a stationary clock.

In addition to demonstrating Lorentz transformations, this model also demonstrates an important aspect of quantum mechanics. The distance along the propagation axis between wave crests of the moving wavepacket is $(\lambda_0/\gamma)(c/v) = h/(\gamma m_0 v) = h/p$. This is the de Broglie wavelength of a moving "particle". More generally, the de Broglie wavelength results from a Lorentz boost of a stationary oscillation. The de Broglie wavelength is observable in interference and diffraction just like wavelengths of ordinary plane waves. For example, electrons passing through a crystal exhibit the same diffraction as x-rays whose wavelength matches the electron's de Broglie wavelength.[17;37]

Consider the velocity triangle in Fig. 3 with hypotenuse c, one side representing average motion v, and a third side $\sqrt{c^2 - v^2}$ representing circulating motion perpendicular to the average motion. The Pythagorean theorem states that:

$$c^2 = v^2 + \left(\sqrt{c^2 - v^2}\right)^2 . \tag{1}$$

This relationship is valid, averaging over the cyclical motion, even if the average motion is in the plane of circulation.[9]

Simply multiply each side by $\gamma m_0 c$ to obtain the energy-momentum-mass triangle. The Pythagorean theorem now yields:

$$(\gamma m_0 c^2)^2 = (\gamma m_0 c v)^2 + (m_0 c^2)^2 , \tag{2}$$

which is equivalent to:

$$E^2 = (pc)^2 + (m_0 c^2)^2 . \tag{3}$$

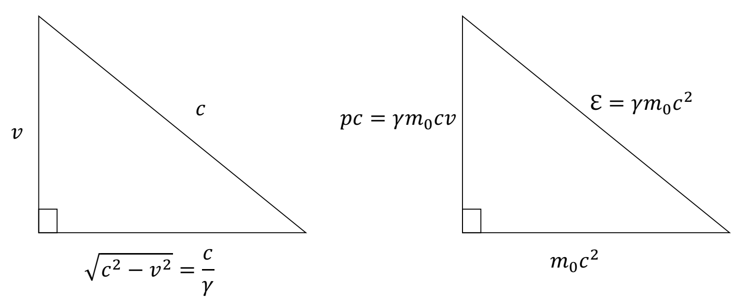

Figure 3: Left: Velocity triangle with the lower side representing azimuthal propagation. Right: Energy-momentum triangle obtained from multiplication by $\gamma m_0 c$.

If we use the relativistic mass $m = \gamma m_0$, then the right-hand side of Eq. 2 or Eq. 3 becomes simply $(mc^2)^2$. Hence we see the remarkable fact that Einstein's famous equation $\mathcal{E} = mc^2$ is simply a restatement of the Pythagorean theorem!

Since the wave equation is Lorentz-invariant and also arises for many different types of waves, special relativity may be understood as a general property of waves rather than a property of spacetime.[9;24] A unifying principle, applicable to all waves, is this:

> Measurements made by differently moving inertial observers, using a particular type of wave, are related by Lorentz transformations based on the characteristic wave speed.

An explanation of this principle using animations is available online.[8]

Although the model does not incorporate measurement tools, its properties can also be used to explain why it is impossible to determine absolute motion. Suppose that there is an absolute reference frame (e.g. a solid medium), and that our stationary wave packet is at rest with respect to that frame (except for the circular propagation). Also suppose that observer A is in the rest frame and observer B is in the reference frame of the moving wave packet. Both observers believe that their frame is the rest frame. Primed variables will indicate measurements in the actual rest frame.

Observer B sends a wave signal with locally measured frequency f_B toward Observer A. Since B's clock ticks slowly, the actual frequency at B is $f'_B = f_B/\gamma = f_B\sqrt{(1 - v^2/c^2}$. The propagating wave detected by Observer A is Doppler shifted by the moving source:

$$f'_A = \frac{f'_B}{1 + v/c} = f_B\sqrt{\frac{1 - v/c}{1 + v/c}}\ . \tag{4}$$

This is the relativistic Doppler shift. Observer A recognizes that the wave is Doppler shifted and that B's clock is slow, so A correctly deduces the frequency f_B measured by Observer B.

Now suppose that Observer A sends an echo of the wave to Observer B. The wavelength observed by A is $\lambda'_A = c/f'_A$. Observer B will receive the wave with relative velocity $(c - v)$, so Observer A sees the wave crests impinge on observer B with frequency $(c - v)/\lambda'_A = f'_A(1 - v/c)$. Since the clock on observer B ticks slowly, observer B measures a higher wave frequency of $f_B = f'_A\gamma(1 - v/c)$. However, Observer B thinks that the wave from observer A is Doppler shifted by a moving source, so that the actual frequency emitted by Observer A would be $f_B(1 + v/c)$. Observer B also thinks that A's clock ticks slowly, so Observer B predicts that the frequency measured by A is $f_B\gamma(1 + v/c)$. We can confirm that this is consistent with the measurements of observer A:

$$f_B\gamma(1 + v/c) = (f'_A\gamma(1 - v/c))\,\gamma\,(1 + v/c) = f'_A\gamma^2(1 - v^2/c^2) = f'_A\,.$$

Although observer B was wrong about the Doppler shift and the clocks, the prediction of observer A's measured frequency was still correct. It is impossible to determine which observer is moving based on wave measurements.

Discussion

A classical model of wave propagation in a Galilean physical spacetime (with wave measurements comprising Minkowski spacetime) is entirely consistent with the laws of special relativity. The reader may recall that Maxwell also derived the equations of electromagnetism with the assumption of Galilean spacetime.[33] Curiously, the success of Maxwell's model is sometimes regarded as evidence that his assumptions were wrong! For example, one textbook states that "Maxwell's equations are not compatible with the Galilean description of spacetime."[16] In the words of the Nobel laureate Robert Laughlin,"Relativity actually says nothing about the existence or nonexistence of matter pervading the universe, only that any such matter must have relativistic symmetry. It turns out that such matter exists."[29]

Einstein's postulate of the constancy of the speed of light may be understood as a recognition that all of our measurements are made using waves (including particle-like or standing waves) whose characteristic propagation speed is the speed of light. The current definition of the "meter" guarantees a constant measured speed of light (even though we know that the actual speed of light varies in a gravitational field).[21]

Many researchers have proposed that stationary elementary particles consist of standing waves or "solitons" rather than point-like singularities.[5;27] The model described here reduces such standing waves to waves propagating in circles at a single radius.

Interpretation of special relativity as a property of matter rather than spacetime clarifies the analysis of relative motion. Although it is impossible to measure absolute velocity, it is possible to measure absolute acceleration. If an inertial observer detects relativistic changes of accelerated clocks and rulers, it is certain that those changes are real, and they are consistent with the wave nature of matter. Acceleration changes matter, not the spacetime in which the matter moves. Likewise, an accelerated observer should realize that apparent changes seen in external inertial clocks and rulers are not real, but are due to changes in the co-accelerated clocks and rulers used for comparison. Poincaré's statement that "we have no means of knowing whether it is the magnitude or the instrument that has changed"[35] does not apply to accelerated reference frames.

Conclusions

The hand-held model of circulating waves provides a visual demonstration of Lorentz transformations and the de Broglie wavelength. It can give students an intuitive understanding of special relativity and quantum mechanics that is lacking in conventional presentations of those topics.

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